Appendix A
Online Learning Paradigm

To solve the learning problems in the bidding machine model, several algorithms have been proposed, such as SGD [9], FOBOS [2] and RDA [12]. In [5], the authors derived the relationship between FTRL-Proximal algorithm and other mirror descent algorithms and showed the better sparsity of it. For sequentially update the embedded model of the bidding machine, we derive FTRL-Proximal algorithm [6] to dynamically control the learning process while maintaining satisfying sparsity.

We first denote $\boldsymbol{g}_t$ as the gradient of the t^{th} instance. To update the CTR estimation model and winning probability model, the corresponding equation are

$$\boldsymbol{g}_t^{\boldsymbol{\theta}} = \frac{\partial U_t}{\partial \boldsymbol{\theta}}, \ \ \boldsymbol{g}_t^{\boldsymbol{\phi}} = \frac{\partial U_t}{\partial \boldsymbol{\phi}}. \tag{1}$$

and $\boldsymbol{g}_{1:t} = \sum_{s=1}^{t} \boldsymbol{g}_s$.

In online gradient descent stype, the model parameters $\boldsymbol{\theta}$ and $\boldsymbol{\phi}$ will be respectively updated as

$$\begin{aligned} \boldsymbol{\theta}_{t+1} &= \boldsymbol{\theta}_t - \eta_t^{\boldsymbol{\theta}} \boldsymbol{g}_t^{\boldsymbol{\theta}}, \\ \boldsymbol{\phi}_{t+1} &= \boldsymbol{\phi}_t - \eta_t^{\boldsymbol{\phi}} \boldsymbol{g}_t^{\boldsymbol{\phi}}, \end{aligned} \tag{2}$$

where η_t is a non-increasing learning rate schedule. Instead, we use FTRL-Proximal paradigm for parameter updating:

$$\begin{aligned} \boldsymbol{\theta}_{t+1} &= \arg\min_{\boldsymbol{\theta}} (\boldsymbol{g}_{1:t}^{\boldsymbol{\theta}} \cdot \boldsymbol{\theta}_t + \frac{1}{2} \sum_{s=1}^{t} \delta_s \|\boldsymbol{\theta} - \boldsymbol{\theta}_s\|_2^2 + \lambda_1 \|\boldsymbol{\theta}\|_1), \\ \boldsymbol{\phi}_{t+1} &= \arg\min_{\boldsymbol{\phi}} (\boldsymbol{g}_{1:t}^{\boldsymbol{\phi}} \cdot \boldsymbol{\phi}_t + \frac{1}{2} \sum_{s=1}^{t} \delta_s \|\boldsymbol{\phi} - \boldsymbol{\phi}_s\|_2^2 + \lambda_1 \|\boldsymbol{\phi}\|_1), \end{aligned} \tag{3}$$

where we define $\delta_{1:t} = \frac{1}{\eta_t}$ as in terms of the learning rate schedule.

As is shown in [6], only one number per coefficient needs to be stored and we can rewrite the argmin equation as a quadratic function of the two parameter

$$\begin{aligned} &\frac{1}{\eta_t} \|\boldsymbol{\theta}\|_2^2 + (\boldsymbol{g}_{1:t}^{\boldsymbol{\theta}} - \sum_{s=1}^{t} \delta_s \boldsymbol{\theta}_s) \cdot \boldsymbol{\theta} + \lambda_1 \|\boldsymbol{\theta}\| + (\text{const}), \\ &\frac{1}{\eta_t} \|\boldsymbol{\phi}\|_2^2 + (\boldsymbol{g}_{1:t}^{\boldsymbol{\phi}} - \sum_{s=1}^{t} \delta_s \boldsymbol{\phi}_s) \cdot \boldsymbol{\phi} + \lambda_1 \|\boldsymbol{\phi}\| + (\text{const}). \end{aligned} \tag{4}$$

If we store $\boldsymbol{z}_{t-1}^{\boldsymbol{\theta}} = \boldsymbol{g}_{1:t-1} - \sum_{s=1}^{t-1} \delta_s \boldsymbol{\theta}_s$ and the same for $\boldsymbol{\phi}$, and we update at t^{th} instance as

$$\begin{aligned} \boldsymbol{z}_t^{\boldsymbol{\theta}} &= \boldsymbol{z}_{t-1}^{\boldsymbol{\theta}} + \boldsymbol{g}_t^{\boldsymbol{\theta}} + (\frac{1}{\eta_t} - \frac{1}{\eta_{t-1}}) \boldsymbol{\theta}_t, \\ \boldsymbol{z}_t^{\boldsymbol{\phi}} &= \boldsymbol{z}_{t-1}^{\boldsymbol{\phi}} + \boldsymbol{g}_t^{\boldsymbol{\phi}} + (\frac{1}{\eta_t} - \frac{1}{\eta_{t-1}}) \boldsymbol{\phi}_t. \end{aligned} \tag{5}$$

So that we can update per-coordinate of the parameter as

$$\begin{aligned} \boldsymbol{\theta}_{t+1,i} &= \begin{cases} 0 & \text{if} |z_{t,i}^{\boldsymbol{\theta}}| \le \lambda_1, \\ -\eta_t^{\boldsymbol{\theta}} (z_{t,i}^{\boldsymbol{\theta}} - sgn(z_{t,i}^{\boldsymbol{\theta}}) \lambda_1) & \text{otherwise}, \end{cases} \\ \boldsymbol{\phi}_{t+1,i} &= \begin{cases} 0 & \text{if} |z_{t,i}^{\boldsymbol{\phi}}| \le \lambda_1, \\ -\eta_t^{\boldsymbol{\phi}} (z_{t,i}^{\boldsymbol{\phi}} - sgn(z_{t,i}^{\boldsymbol{\phi}}) \lambda_1) & \text{otherwise}, \end{cases} \end{aligned} \tag{6}$$

where the second subscript i is the coordinate index of the parameter.

For learning rates η_t, we implement as per-coordinate rate updating as

$$\eta_{t,i} = \frac{\alpha}{\beta + \sqrt{\sum_{s=1}^{t} \boldsymbol{g}_{s,i}^2}}, \tag{7}$$

where i is the index of the coordinate.

Appendix B
Optimality under Second-price Auction

In this section, we give the proof of the optimal bidding function under the second-price auction.

Theorem 1. *The optimal bidding function under the second-price auction is linear to the estimated utility.*

Proof. Recall that, under the second price auction, the winning probability $w(b)$ w.r.t. the bid price b is integral over $[0, b]$ for the market price distribution $p_z(z)$ as

$$w(b) = \int_0^b p_z(z) dz. \tag{8}$$

And $c(b)$ is the expected cost of bidding with price b.

$$c(b) = \frac{\int_0^b z p_z(z) dz}{\int_0^b p_z(z) dz} \ . \tag{9}$$

We use r to represent the predicted user response of the given bid request, while $b(r)$ is the bidding function w.r.t. r and $u(r)$ is the utility function set by the advertiser.

Our optimization problem is to maximize the *proft* of the advertiser with the budget constraint B under the second-price auction, which is formulated as

$$\begin{aligned} \max_{b()} \quad & T \int_r [u(r) - c(b(r))] w(b(r)) p_r(r) dr \ , \\ s.t. \quad & T \int_r c(b(r)) w(b(r)) p_r(r) dr = B \ , \end{aligned} \tag{10}$$

where T is the total number of the bid requests.

The Lagrangian of the optimization problem Eq. (10) is

$$\begin{aligned} \mathcal{L}(b(r), \lambda) = &\int_r [u(r) - c(b(r))] w(b(r)) p_r(r) dr \\ &- \lambda \cdot \int_r c(b(r)) w(b(r)) p_r(r) dr + \frac{\lambda B}{T} \ , \end{aligned} \tag{11}$$

where λ is the Lagrangian multiplier.

Solving $b()$**.** Based on calculus of variations, the Euler-Lagrangian condition of $b(r)$ is

$$\frac{\partial \mathcal{L}(b(r), \lambda)}{\partial b(r)} = 0 \ , \tag{12}$$

which can be derived as

$$\begin{aligned} & 0 = \frac{\partial \mathcal{L}(b(r), \lambda)}{\partial b(r)} \\ \Rightarrow \quad & 0 = u(r) p_r(r) \frac{\partial w(b(r))}{\partial b(r)} - (\lambda + 1) p_r(r) \\ & \quad \cdot \left[\frac{\partial c(b(r))}{\partial b(r)} w(b(r)) + c(b(r)) \frac{\partial w(b(r))}{\partial b(r)} \right] \\ \Rightarrow \quad & (\lambda + 1) p_r(r) \frac{\partial c(b(r))}{\partial b(r)} w(b(r)) \\ & \quad = [u(r) - (\lambda + 1) c(b(r))] \, p_r(r) \frac{\partial w(b(r))}{\partial b(r)} \ . \end{aligned} \tag{13}$$

Since

$$\frac{\partial c(b(r))}{\partial b(r)} = \frac{p_z(b(r))\left[b(r)\int_0^b p_z(z)dz - \int_0^b zp_z(z)dz\right]}{(w(b(r)))^2}\ , \tag{14}$$

and

$$\frac{\partial w(b(r))}{\partial b(r)} = p_z(b(r))\ , \tag{15}$$

taking Eq. (14) and Eq. (15) into Eq. (13), we can then derive the Euler-Lagrangian condition as

$$\begin{aligned} (\lambda+1)b(r) &= u(r) \\ \Rightarrow \qquad b(r) &= \frac{u(r)}{\lambda+1}\ . \end{aligned} \tag{16}$$

■

Appendix C
Game Theoretic Analysis

In this section, we conduct a theoretic analysis of the optimal bidding strategy under the symmetric game of repeated auctions with budget constraints following [13], [14]. First, we will derive the optimal bidding function in the equilibrium of the second price auction. Second, based on the derived bidding function, we discuss that a *tragedy of the commons* situation exists among multiple advertisers with the same optimal bidding strategy in RTB display advertising. Note that the analysis may not be first proposed in this work and we present it here to make this paper self-contained.

C.1 Problem Settings

At first we present some preliminaries and describe the problem settings. We add subscripts b, z, r to the c.d.f. and p.d.f. functions to make differences among these variables.

Monotonicity of the Bidding Function. In a clean game theoretic analysis setting [7], there are n ($n \geq 2$) advertisers with the same bidding strategy $b(r)$ which takes the estimated CTR r and outputs the bid price b. It is reasonable that $b(r)$ is monotonically increasing w.r.t. CTR r, i.e.

$$b(r_1) > b(r_2) \Leftrightarrow r_1 > r_2\ . \tag{17}$$

Later we will prove this monotonicity. Each time when an impression is auctioned, for each advertiser the CTR r follows the same p.d.f. $p_r(r)$ independently (i.i.d.) and the corresponding c.d.f. is $F_r(r)$ as

$$F_r(r) = \int_0^r p_r(t)dt, \quad \frac{\partial F_r(r)}{\partial r} = p_r(r)\ . \tag{18}$$

We also define $F_b(b)$ as the c.d.f. of the bid price b, i.e. the probability of performing a bid less than b:

$$F_b(b) = \int_0^b p_b(a)da\ . \tag{19}$$

Note that

$$F_b(b(r)) = P(b(r) > b(r_2) = P(r > r_2) = F_r(r)\ , \tag{20}$$

since $b(r)$ monotonously increases w.r.t. r. Thus, for the market price variable z, which is defined as the highest bid price across $(n-1)$ competitors, its c.d.f. $F_z(z)$ is

$$F_z(z) = F_b(z)^{n-1}\ , \tag{21}$$

and the corresponding p.d.f. $p_z(z)$ is

$$p_z(z) = \frac{\partial F_z(z)}{\partial z} = (n-1)F_b(z)^{n-2}p_b(z)\ . \tag{22}$$

The Winning Probability in a Symmetric Game. In such a setting, the winning probability $w_r(r)$ of Advertiser 1, without loss of generality, w.r.t. the given CTR r is the largest one among the n advertisers that

$$w_r(r) = P(r > r_2, r > r_3, \ldots, r > r_n) = F_r(r)^{n-1}\ . \tag{23}$$

Note that, according to Eqs. (20) and (21), we can also derive the winning probability function $w_b(b(r))$, which is equivalent to $w(b(r))$ in the main part of our paper, w.r.t. the bid price is that

$$\begin{aligned} w_b(b(r)) &= P(b(r) > b_2, b(r) > b_3, \ldots, b(r) > b_n) \\ &= F_b(b(r))^{n-1} = F_r(r)^{n-1} \\ &= P(r > r_2, r > r_3, \ldots, r > r_n) \\ &= w_r(r)\ . \end{aligned} \tag{24}$$

The Expected Utility and the Expected Cost. We follow our previously conducted results in Eq. (9) and derive the expected utility of profit $R(r, b)$ as that

$$R(r,b) = u(r) - c(b) = u(r) - \frac{\int_0^b zp_z(z)dz}{\int_0^b p_z(z)dz}\ . \tag{25}$$

where b is the output variable of the bidding function $b(r)$.

C.2 Optimal Bidding Function under Symmetric Game

Now that we have defined the problem settings with the utility and the cost function, we will derive the optimal bidding function under the symmetric game scenario, where each advertiser participating in the game adopts the same bidding function.

Theorem 2. *The optimal bidding function under a symmetric game of repeated auctions with budget constraints is linear to the estimated utility, the bid price is monotonously increasing w.r.t. the number of the participating advertiser bidders.*

Proof. Our optimization problem is to maximize the *profit* of each participating advertiser with the budget constraint B under the second price auction, which is formulated as

$$\begin{aligned} \max_{b()} \quad & T\int_r [u(r) - c(b(\tau))]w_b(b(\tau))p_r(r)dr\ , \\ s.t. \quad & T\int_r c(b(\tau))w_b(b(\tau))p_r(r)dr = B\ , \end{aligned} \tag{26}$$

here we assume that the bidding is based on a *signal* τ related with the CTR r.

The Lagrangian function $\mathcal{L}(\tau, \lambda)$ is

$$\begin{aligned} \mathcal{L}(\tau,\lambda) =& \frac{\lambda B}{T} + \int_r u(r)w_b(b(\tau))p_r(r)dr \\ & - (\lambda+1)\int_r c(b(\tau))w_b(b(\tau))p_r(r)dr\ , \end{aligned} \tag{27}$$

where λ is the Lagrangian multiplier. Note that the utility function $u(r)$ is only influenced by the true CTR r and the cost is dependent on the bid price which is based on the known CTR signal τ.

Taking Eqs. (24) and (25) into consideration, the Lagrangian function can be derived as

$$\begin{aligned}\mathcal{L}(\tau,\lambda) =& \frac{\lambda B}{T} + \int_r \Big[u(r) F_r(\tau)^{n-1} \\ & -(\lambda+1)\int_0^{b(\tau)} z p_z(z) dz \Big] p_r(r) dr \ ,\end{aligned} \tag{28}$$

Solving $b()$**.** We can calculate the gradient w.r.t. τ as that

$$\begin{aligned}\frac{\partial \mathcal{L}(\tau,\lambda)}{\partial \tau} =& \int_r \Big[u(r)(n-1)F_r(\tau)^{n-2} p_r(\tau) \\ & -(\lambda+1) b(\tau) p_z(b(\tau))\ \frac{\partial b(\tau)}{\partial \tau} \Big] p_r(r) dr \ .\end{aligned} \tag{29}$$

In a symmetric equilibrium, the objective is maximized when using the true signal, i.e., at $\tau = r$ [7]. Therefore, we have

$$\frac{\partial \mathcal{L}(\tau,\lambda)}{\partial \tau} = 0 \Big|_{\tau=r} \Rightarrow \tag{30}$$
$$u(r)(n-1)F_r(r)^{n-2} p_r(r) = (\lambda+1) b(r) p_z(b(r)) \frac{\partial b(r)}{\partial r} \ .$$

As $b(r)$ is monotonously increasing w.r.t. r, then their p.d.f.s $p_r(r)$ and $p_b(b)$ have the following relationship

$$p_r(r) = p_b(b(r)) \frac{\partial b(r)}{\partial r} \ . \tag{31}$$

Taking Eqs. (22) and (31) into Eq. (30), we can have that

$$\begin{aligned} & u(r)(n-1)F_r(r)^{n-2} p_r(r) \\ &= (\lambda+1) b(r) (n-1) F_b(b(r))^{n-2} p_b(b(r)) \frac{\partial b(r)}{\partial r} \\ \Rightarrow \quad & u(r)(n-1)F_r(r)^{n-2} p_r(r) \\ &= (\lambda+1) b(r)(n-1)F_b(b(r))^{n-2} p_r(r) \\ \Rightarrow \quad & b(r) = \frac{u(r)}{\lambda+1} \ .\end{aligned} \tag{32}$$

We can easily find that the bidding function is linear w.r.t. the utility $u(r)$. Specifically, in Sec. 3.1 of our paper, we adopt a utility function as in Eq. (4) that

$$u(r) = vr \ , \tag{33}$$

where v is the click value of the advertiser. Therefore,

$$b(r) = \frac{vr}{\lambda+1} \ . \tag{34}$$

Till now, we have derived the optimal bidding function under a symmetric game of repeated auctions with multiple advertisers adopting the same bidding strategy. Comparing the derived bidding function with that in the single bidder situation, the only difference is the value of λ. Next, we will illustrate that λ has a strong relationship with the number of the participating bidders.

Solving λ**.** Take gradient of the Lagrangian function w.r.t. λ, we can get the budget constraint equation as

$$\begin{aligned} & \frac{\partial \mathcal{L}(\tau,\lambda)}{\lambda} = 0 \\ & \frac{B}{T} = \int_r \int_0^{\frac{vr}{\lambda+1}} z p_z(z) dz \ p_r(r) dr \\ & \frac{B}{T} = \int_r \int_0^{\frac{vr}{\lambda+1}} z(n-1)F_b(z)^{n-2} p_b(z) dz \ p_r(r) dr \\ & \frac{B}{T} = \int_r \int_0^{\frac{vr}{\lambda+1}} z(n-1)F_r\left(\frac{(\lambda+1)z}{v}\right)^{n-2} p_r\left(\frac{(\lambda+1)z}{v}\right) \\ & \qquad \cdot \frac{(\lambda+1)}{v} dz \ p_r(r) dr \\ & \frac{B}{T} = \int_r \int_0^r \frac{vt}{\lambda+1}(n-1)F_r\left(t\right)^{n-2} p_r\left(t\right) dt \ p_r(r) dr \\ & \frac{v}{\lambda+1} = \frac{B}{T \int_r \int_0^r t(n-1)F_r\left(t\right)^{n-2} p_r\left(t\right) dt \ p_r(r) dr} \ .\end{aligned} \tag{35}$$

The third equation is derived with Eq. (22) and the forth equation considers Eq. (20). Thus we can get the optimal bidding function as

$$b(r) = \frac{vr}{\lambda+1} = \frac{Br}{T \int_r \int_0^r t(n-1)F_r\left(t\right)^{n-2} p_r\left(t\right) dt \ p_r(r) dr} . \tag{36}$$

From Eq. (36), we can easily find that the denominator is positive and its gradient w.r.t. $n \geq 2$ is negative, which means that the bid price $b(r)$ is monotonously increasing when the bidder number n increases. ■

Analytic Solution with a Special Case. Here we propose an analytic solution for $b(r)$ with a special case of $p_r(r)$. Assume that CTR value r is uniformly distributed, which means that

$$p_r(r) = 1 \ , F_r(r) = r \ . \tag{37}$$

Thus the closed form of the optimal bidding function is

$$\begin{aligned} b(r) &= \frac{vr}{\lambda+1} = \frac{Br}{T \int_{r'} \int_0^{r'} t(n-1)t^{n-2}\left(t\right) dt \ dr'} \\ &= \frac{Br}{T \int_{r'} \frac{n-1}{n} r'^n \ dr'} = \frac{Br \cdot n(n+1)}{T(n-1)} \ .\end{aligned} \tag{38}$$

In this case, we find that the optimal bidding function is linear w.r.t. the average budget per auction $\frac{B}{T}$ and the number n of the participating bidders in the market. When there are more than two advertisers (i.e., $n \geq 2$), the optimal bid price is monotonously increasing when n increases.

C.3 Discussion about Tragedy of the Commons

In this part, we discuss about the derived results above. We define the performance comparison scheme. First the advertiser will compare the achieved utility $u(r)$, e.g., $u(r) = \sum vy$ and y is the click indicator of each ad impression. At this point, the higher utility, i.e., more gained clicks, the better; Second, if the utility values are the same, the lower cost for achieving this utility is better.

From Eqs. (36) and (38), the optimal bid price is monotonously increasing when the number of the competitor gets larger, which means that each bidder tries to maximize the objective utility with the cost lower than the

TABLE 1: Regression performances over campaigns. AUC: the higher, the better. RMSE: the smaller, the better.

	AUC				RMSE ($\times 10^{-2}$)			
iPinYou	SE	CE	EU	RR	SE	CE	EU	RR
1458	.948	.987	.987	.977	3.01	1.94	2.42	2.32
2259	.542	.692	.674	.691	2.01	1.77	1.76	1.79
2261	.490	.569	.622	.619	1.84	1.68	1.71	1.68
2821	.511	.620	.608	.639	2.56	2.43	2.39	2.46
2997	.543	.610	.606	.608	5.98	5.82	5.84	5.82
3358	.863	.974	.970	.980	3.07	2.47	3.32	2.67
3386	.593	.768	.761	.778	2.95	2.84	3.32	2.85
3427	.634	.976	.976	.960	2.78	2.20	2.61	2.34
3476	.575	.957	.954	.950	2.50	2.32	2.39	2.33
Average	.633	.794	.795	**.800**	2.97	**2.61**	2.86	2.69
YOYI	.882	.891	**.912**	**.912**	11.9	11.7	11.8	**11.6**

budget. When the number of the participating bidders with the same optimal bidding function is n, each bidder will win the auction with $\frac{1}{n}$ probability. Such that each bidder will try to spend all the budget to maximize the objective utility. However, such an equilibrium is not efficient and it will result in a situation with very low social welfare since all the advertiser will exhaust all the budget while winning the same utility, i.e., $\frac{1}{n}$ impressions and clicks.

A better situation is that each advertiser spends B/n budget and still gets the same utility (the same number of impressions and clicks as in the previous case). Extremely when $n \to \infty$, each advertiser bids 0 so that the winner will be selected randomly across all the bidders pays 0 for each auction. However, this situation is never realistic since each bidder will compete with each other rather than cooperation. In such an unstable case, every advertiser will propose higher bid price to win the auction to maximize the expected utility given the current market situation, i.e., $F_z(z)$. Finally, the whole system will get into the equilibrium of Eqs. (36) and (38) where every advertiser spend out all the budgets.

This is a *tragedy of the commons* [3] reflecting a *prisoner's dilemma* [8] in the RTB market competition with budget constraints. Note that such a tragedy of the commons result is not just for the proposed bidding strategies in this work, it applies to any bidding fuctions that is monotonously increasing w.r.t. the expected utility. More discussions are provided in [13].

Appendix D
Accuracy Comparison of CTR Estimation

In this section, we compare the accuracy of the CTR estimation models, measured by AUC and RMSE. As our utility estimation models are designed to optimize campaign profit rather than user response prediction accuracy, the evaluation here is to see whether our proposed solutions would still be able to achieve comparable performance against the conventional estimators that directly optimize the prediction accuracy. Table 1 shows the AUC and RMSE for each model over all campaigns. First, the baseline CE achieves better performance than the baseline SE on all campaigns, confirming the previous study that cross entropy as an objective naturally works better on the binary classification problem with probabilistic predictions. Second, both our EU and RR models achieve similar or higher AUC values over the strong baseline CE model, while maintaining comparable RMSE performances. From our derivation in the main theory section, we know that a key advantage of our EU model over the baseline SE model is that it considers the market price in the gradient updating. Here, we find that our EU model not only compensates the relatively weakness of the SE model, but also gains better in some campaigns, e.g., iPinYou campaign 2261 and YOYI. Moreover, the EU model achieves similar (sometimes better) performances compared with the CE model. Finally, we also observe that our RR model performs more stably in most campaigns and achieves higher AUC than other three models in most campaigns, e.g., iPinYou campaigns 2821, 3358, 3386 and YOYI, suggesting that combining the cross entropy loss with the market price density is the best option.

TABLE 2: Experimental results of bid landscape forecasting models over iPinYou dataset. ANLP: The lower, the better.

Model	1458	2259	2261	2821	2997	3358	3386	3427	3476
NM	5.36	6.76	5.53	6.55	5.36	5.83	5.27	4.88	5.28
MM	5.78	7.32	7.02	7.26	6.70	7.17	6.14	6.18	6.02
Linear	7.54	7.38	7.28	7.27	7.28	7.54	7.50	7.66	8.01
Quadratic	6.56	9.18	9.15	11.4	6.48	8.50	6.84	6.95	6.64

Appendix E
Results of Market Competition Modeling

In this section, we present the experimental results of our market competition model, which is learned landscape p.d.f. $p_z(z, \boldsymbol{x}; \boldsymbol{\phi})$.

The metric is the Averaged Negative Log Probability (ANLP) [10], which is to measure the averaged log-likelihood of fitting the observed market price in the test data:

$$P_{NL} = -\frac{1}{n} \sum_{i=1}^{n} \log p_z(z_i, \boldsymbol{x}_i; \boldsymbol{\phi}) \ , \tag{39}$$

where i is the index of the sample in the test dataset and $p_z(z_i, \boldsymbol{x}_i; \boldsymbol{\phi})$ is the corresponding probabilistic density calculated by the bid landscape model. The better fitting performance is, the lower ANLP value it achieves.

We report the ANLP performance over the market modeling model in our paper and the state-of-the-art model: the Mix Model (MM) [11] using linear regression and censored regression altogether, and the normal model (NM) as in [10] using only observed data without lost censored data. Our model includes the performance of linear form and quadratic form of market modeling function, as described in our paper.

As is illustrated in Table 2, we find that the NM model achieves the best ANLP results and our models are in the same level as the MM model. The results are reasonable since NM fits the raw market price distribution better while MM pays more attention on the lost auctions which are censored in the true market. Our model not only optimizes the direct profits but also learns the bid landscape information very well.

Appendix F
Significance Test

In this section, we present the results of the significance test in our experiments.

TABLE 3: p-values under the AUC evaluation (MannWhitney U test, one-tailed).

Models	EU	RR	BM(MKT)
SE	$< 10^{-6}$	$< 10^{-6}$	$< 10^{-6}$
CE	0.442676	0.192533	0.09581

TABLE 4: p-values under the RMSE evaluation.

Models	EU	RR	BM(MKT)
SE	$< 10^{-6}$	$< 10^{-6}$	$< 10^{-6}$
CE	$< 10^{-6}$	$< 10^{-6}$	$< 10^{-6}$

For the AUC metric, we conducted a MannWhitney U test [4] and list the results in Table 3. We find that our proposed models has significantly beaten the linear regression model with squared loss (SE). However, the linear regression with cross entropy loss (CE) has similar AUC performance with our models. It is reasonable since our models aim at optimizing the overall revenue of the advertiser, rather than the classification accuracy.

For the RMSE metric, we tested p-values [1] of the predicted CTR values across all the models, which is illustrated in Table 4. From the results we can find that the improvement of EU and RR model over linear regression models are significant, either for the BM(MKT) model against all the baselines.

We also report the results of p-values under ANLP metric, which is shown in Table 5. The results have shown the significant results for our bid landscape modeling against other baselines.

TABLE 5: p-values under the ANLP Metric.

Models	Linear	Quadratic
NM	$< 10^{-6}$	$< 10^{-6}$
MM	$< 10^{-6}$	$< 10^{-6}$

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