Pumping Lemma for Regular Languages

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Non-regular Languages

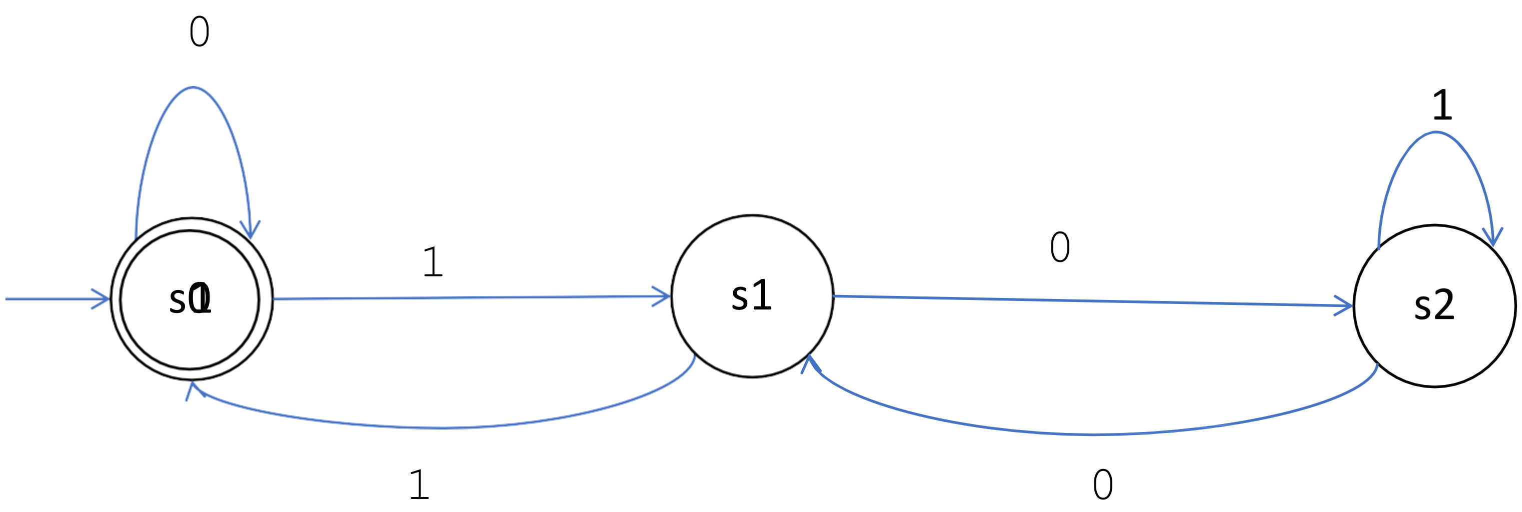
- All regular languages have a special property.
- We use **pumping lemma** to prove that a language is not regular.
- Non-regular languages can not be recognized by any finite automaton (DFA, NFA, or Regex)

What is Pumping Lemma?

- The **pumping lemma** describes a property of all regular languages.
- If a language doesn’t have that property, then it is not regular.
- The **pumping lemma** is used to prove that a language is not regular.
- We can’t use any (DFA, NFA, Regex) to prove a language isn’t regular.

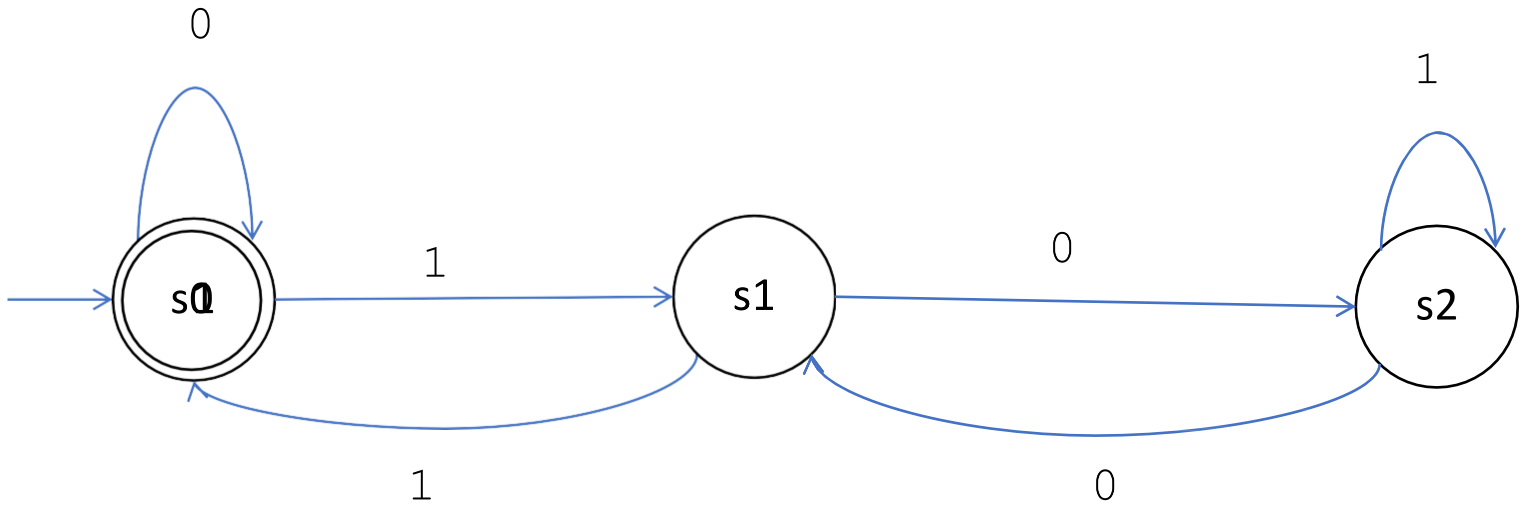
Basic idea of pumping lemma

- Example: language L = { w over {0, 1} | w is the binary representation of an integer that is divisible by 3}
  - Accepted strings: ε, 0, 11, 110, 1001, 1100



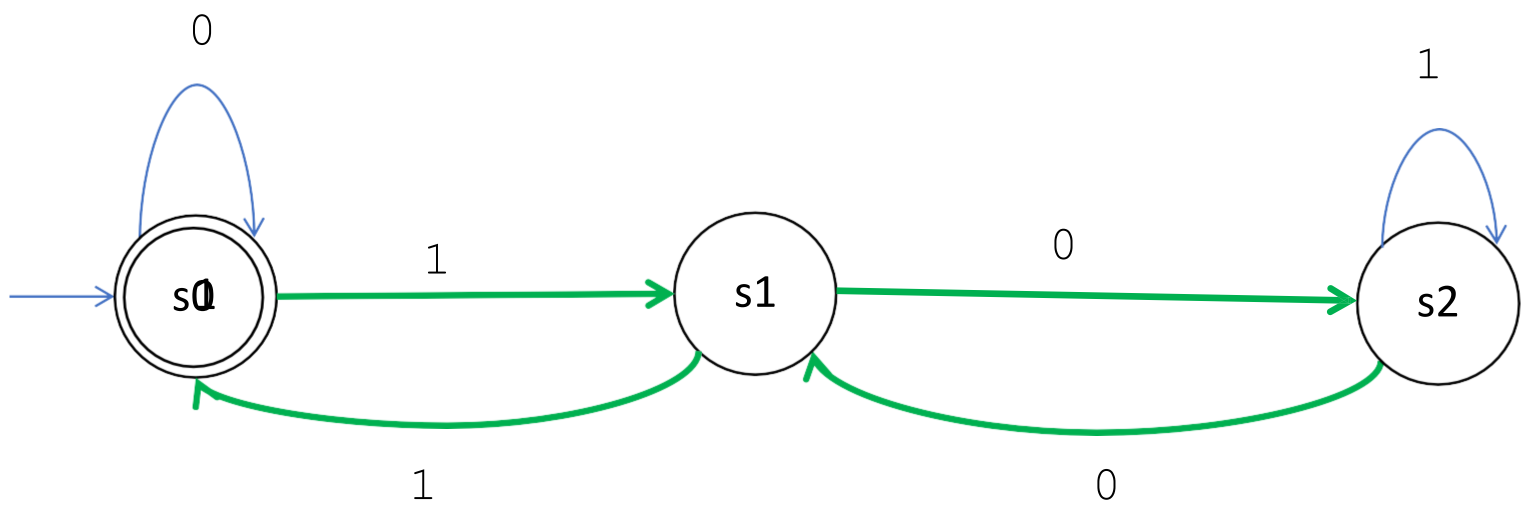
Basic idea of pumping lemma

- Example: language L = { w over {0, 1} | w is the binary representation of an integer that is divisible by 3}
  - Let's take this string: 1001, which is 9 in decimal



Basic idea of pumping lemma

- string: 1001
- The path of states is: s0, s1, s2, s1, s0
- The path goes over s1 twice.
- So, there’s a loop in the path that starts at s1 and ends at s1



- Now, we divide the string (1001) into three parts: **xyz**

x	y		z
1	0	0	1

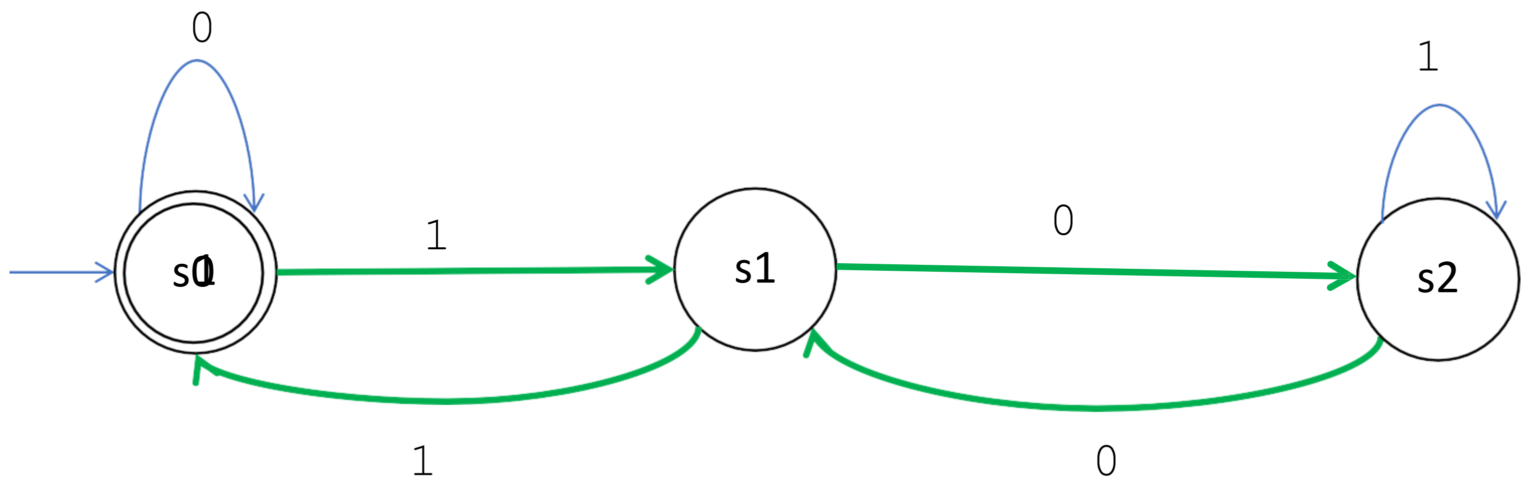
Path = s0 s1 s2 s1 s0

loop

x	z
1	1

Path = s0 s1 s0

- Take another string: **xz,** which is string 11
- It is accepted by the DFA. We get it when deleting the loop.

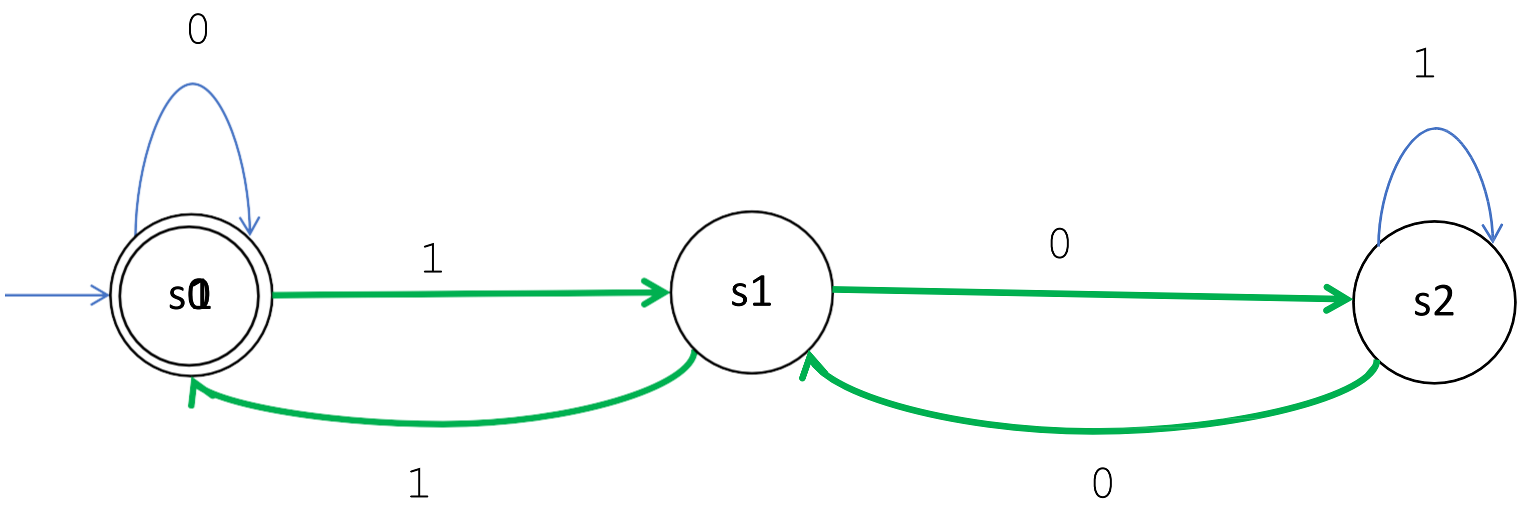


- Also, take a third string: **xyyz**

x	y		y		z
1	0	0	0	0	1

Path = s0 s1 s2 s1 s2 s1 s0

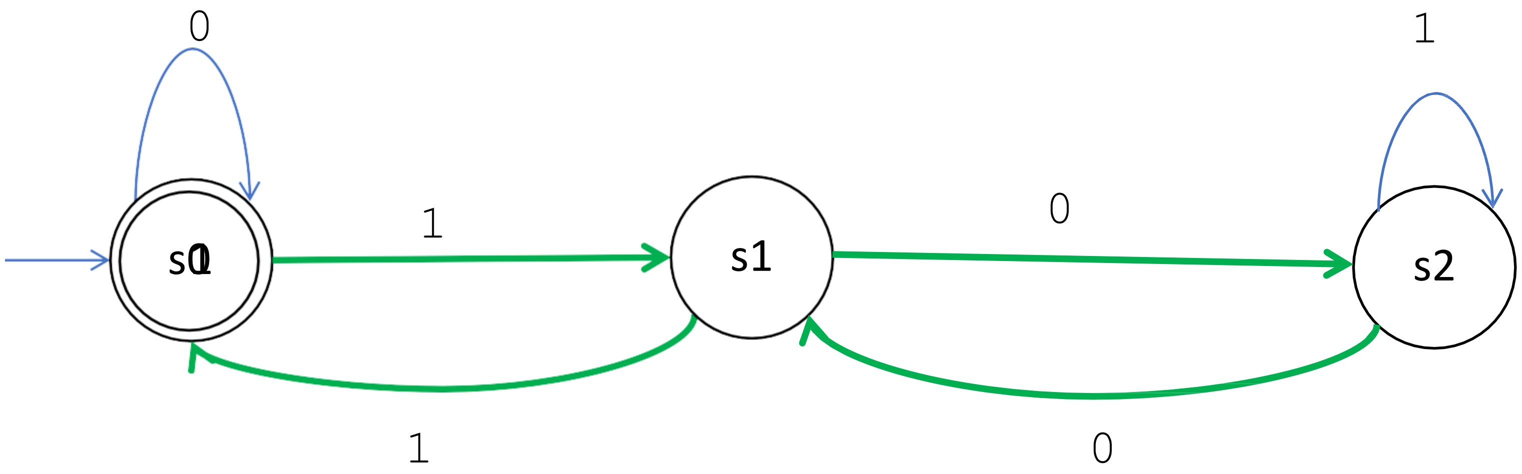
- It is accepted by the DFA. We get it when adding another loop.
- String 100001 in binary, equals to 33 in decimal



- So, any integer ($i \geq 0$) greater than or equal to zero:
- **xy^iz** is accepted by the machine

x	y		z
1	0	0	1

- For our string (1001):
- **xy^0z** = xz = 11, *equals to 3 in decimal*
- **xy^1z** = xyz = 1001, *equals to 9 in decimal*
- **xy^2z** = xyyz = 100001, *equals to 33 in decimal*
- **xy^3z** = xyyyz = 10000001, *equals to 129 in decimal*
- *The DFA accepts all of them.*



What’s the special property?

- The property:

All strings in the language can be “pumped” if they are at least as long as a certain special value, called the *pumping length*.

- It means that each string has a section that can be repeated any number of times with the resulting string remaining in the language.

- All regular languages have this special property.

Theorem: Pumping Lemma

- If **A** is a regular language, then there is a number *p* (called the pumping length)
- If *s* is any string in language A ($s \in A$) with $|s| \geq p$, then:
- String *s* can be divided into three pieces $s = xyz$, satisfying the following conditions:

1. for each $i \geq 0$, $xy^iz \in A$
2. $|y| > 0$ *(cannot be empty)*
3. $|xy| \leq p$

Example:

- Let's go back to our language L = { w over {0, 1} | w is the binary representation of an integer that is divisible by 3}
- We pick, for example, pumping length $p = 3$, string $s = 1001 \in L$, $|s| = 4 \geq p$
- We divide string **s = xyz**:
  - x = 1
  - y = 00
  - z = 1
- Condition 1 of Pumping Lemma **(for each $i \geq 0$, $xy^iz \in L$)**
  - *$i = 0$, $xy^0z = xz = 11 \in L$, equal to 3 in decimal*
  - *$i = 2$, $xy^2z = 100001 \in L$, equal to 33 in decimal*
- Condition 2 of Pumping Lemma (**$|y| > 0$**)
  - *$y = 00$, $|y| = 2 > 0$*
- Condition 3 of Pumping Lemma (**$|xy| \leq p$**)
  - *$xy = 100$, $|xy| = 3 \leq p$*

Another Solution:

- Let's go back to our language L = { w over {0, 1} | w is the binary representation of an integer that is divisible by 3}
- We pick, for example, pumping length p = 3, string s = 110 ∈ L, |s| = 3 ≥ p
- We divide string **s = xyz**:
  - $x = \varepsilon$
  - $y = 110$
  - $z = \varepsilon$
- Condition 1 of Pumping Lemma **(for each *i* ≥ *0*, $xy^iz \in L$)**
  - $i = 0$, $xy^0z = xz = \varepsilon \in L$
  - $i = 2$, $xy^2z = 110110 \in L$, *equal to 54 in decimal*
- Condition 2 of Pumping Lemma (**|y| > 0**)
  - $y = 110$, $|y| = 3 > 0$
- Condition 3 of Pumping Lemma (**|xy| ≤ p**)
  - $xy = 110$, $|xy| = 3 \leq p$

How to prove a language isn't regular?

- Prove that a language A is not regular
  1. For the purpose of contradiction, assume that A is regular
  2. Let p be the pumping length
  3. Pick a string $s \in A$ with $|s| \geq p$
  4. Identify **all** possible decompositions of s into xyz, with $|xy| \leq p$ and $|y| > 0$
  5. Show that for each decomposition, there exists an $i \geq 0$ such that $xy^iz \notin A$
  6. Conclude that the assumption is wrong. Hence A isn't regular

Links for Pumping Lemma and Examples

1-Pumping Lemma
https://www.youtube.com/watch?v=dikEDuepOtI
2- Example 1:
https://www.youtube.com/watch?v=Ty9tpikilAo
3- Example 2:
https://www.youtube.com/watch?v=kZzH8E-s-9o
4- Example 3: prove that the primes language={0^n: n is a prime} is not regular Language.
https://www.youtube.com/watch?v=I3FuVKVgLCA

END