Assignment-3

1) A meteorologist is modelling the weather forecast for a city. The weather can be in one of two states: Sunny (state 1), Rainy (state 2), ...

Ans→ Transition Matrix →

$$P = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix}$$

Initial state vector → $v_0 = [1\ 0]$

Probability that the weather will be Rainy two days from now $= v_2 = v_0 \cdot P^2$

$$P^2 = P \cdot P = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix} \cdot \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix} = \begin{bmatrix} 0.7\cdot0.7 + 0.3\cdot0.4 & 0.7\cdot0.3 + 0.3\cdot0.6 \\ 0.4\cdot0.7 + 0.6\cdot0.4 & 0.4\cdot0.3 + 0.6\cdot0.6 \end{bmatrix}$$

$$= \begin{bmatrix} 0.49 + 0.12 & 0.21 + 0.18 \\ 0.28 + 0.24 & 0.12 + 0.36 \end{bmatrix} = \begin{bmatrix} 0.61 & 0.39 \\ 0.52 & 0.48 \end{bmatrix}$$

Now, $v_2 = v_0 \cdot P^2 = [1\ 0] \begin{bmatrix} 0.61 & 0.39 \\ 0.52 & 0.48 \end{bmatrix} = [0.61\ \ 0.39]$

∴ Probability that the weather will be Rainy two days ~~after~~ from now is ; 0.39.

2) A company produces widgets using a machine that can either be in one of two states : Operational (state 1) and Broken (state 2)...

Ans→ Transition Matrix, P → $\begin{bmatrix} 0.95 & 0.05 \\ 0.40 & 0.60 \end{bmatrix}$

Initial state, $v_0 = [1\ 0]$

Now, ~~Let~~ $P^3 = \begin{bmatrix} 0.907375 & 0.092625 \\ 0.741 & 0.259 \end{bmatrix}$

So, probability that the machine is Operational after 3 days, $v_3 = v_0 \cdot P^3$

$$= [1\ 0] \begin{bmatrix} 0.907375 & 0.092625 \\ 0.741 & 0.259 \end{bmatrix}$$

$$= [0.907375\ \ 0.092625]$$

$$= 0.907375 \quad \text{(Ans)}$$

3) Consider an MDP with 3 states $S_0 = \{S_1, S_2, S_3\}$ and 2 possible actions $A = \{A_1, A_2\}$.

Ans→ Given, ~~So~~ States, $S = \{S_1, S_2, S_3\}$, Actions, $A = \{A_1, A_2\}$, Transition Probabilities $P(s'|s,a)$ Rewards $R(s,a)$, Discount factor $\gamma = 0.9$, Initial value of states $= V_0(S1) = V_0(S2) = V_0(S3) = 0$

Value Iteration Formula → $V_{K+1}(s) = \max_{a \in A} \left[R(s,a) + \gamma \sum_{s'} P(s'|s,a) \cdot V_k(s') \right]$

i) State S1:

- Action A1:
  - $P(S2|S1,A1) = 1,\ R(S1,A1) = 5$
  - $V_1(S1,A1) = 5 + 0.9 \cdot V_0(S2) = 5 + 0.9 \cdot 0 = 5$
- Action A2:
  - $P(S1|S1,A2) = 1,\ R(S1,A2) = 0$
  - $V_1(S1,A2) = 0 + 0.9 \cdot V_0(S1) = 0 + 0.9 \cdot 0 = 0$

$$V_1(S1) = \max(5,0) = 5$$

ii) State S2:

- Action A1:
  - $P(S3|S2,A1) = 1,\ R(S2,A1) = 10$
  - $V_1(S2,A1) = 10 + 0.9 \cdot V_0(S3) = 10 + 0.9 \cdot 0 = 10$
- Action A2:
  - $P(S1|S2,A2) = 1,\ R(S2,A2) = 2$
  - $V_1(S2,A2) = 2 + 0.9 \cdot V_0(S1) = 2 + 0.9 \cdot 0 = 2$

$$V_1(S2) = \max(10,2) = 10$$

iii) State S3:

- Action A1:
  - $P(S1|S3,A1) = 1,\ R(S3,A1) = 2$
  - $V_1(S3,A1) = 2 + 0.9 \cdot V_0(S1) = 2 + 0.9 \cdot 0 = 2$
- Action A2:
  - $P(S3|S3,A2) = 1,\ R(S3,A2) = 0$
  - $V_1(S3,A2) = 0 + 0.9 \cdot V_0(S3) = 0 + 0.9 \cdot 0 = 0$

$$V_1(S3) = \max(2,0) = 2$$

∴ After iteration, $V_1(S1) = 5$, $V_1(S2) = 10$, $V_1(S3) = 2$

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4) From S1:

- A1:

  $R(S1,A1)=5$

  $P(S1|S1,A1)=0.7,\ P(S2|S1,A1)=0.3$

- A2:

  $R(S1,A2)=1$

  $P(S1|S1,A2)=0.4,\ P(S2|S1,A2)=0.6$

From S2:

- A1:

  $R(S2,A1)=2$

  $P(S1|S2,A1)=0.6,\ P(S2|S2,A1)=0.4$

- A2:

  $R(S2,A2)=4$

  $P(S1|S2,A2)=0.2,\ P(S2|S2,A2)=0.8$

Iteration 1: Compute $V_1(S1)$, $V_1(S2)$

For S1 (using A1):

$$V_1(S1) = R(S1,A1) + \gamma \cdot [0.7 \cdot V_0(S1) + 0.3 \cdot V_0(S2)] = 5 + 0.9(0+0) = 5$$

For S2 (using A2):

$$V_1(S2) = R(S2,A2) + \gamma [0.2 \cdot V_0(S1) + 0.8 \cdot V_0(S2)] = 4 + 0.9(0+0) = 4$$

Iteration 2: Compute $V_2(S1)$, $V_2(S2)$

For S1 (using A1):

$$V_2(S1) = 5 + 0.9(0.7 \cdot 5 + 0.3 \cdot 4] = 5 + 0.9(3.5 + 1.2) = 5 + 0.9 \cdot 4.7 = 5 + 4.23 = \boxed{9.23}$$

For S2 (using A2):

$$V_2(S2) = 4 + 0.9(0.2 \cdot 5 + 0.8 \cdot 4) = 4 + 0.9(1 + 3.2) = 4 + 0.9(4.2) = 4 + 3.78 = \boxed{7.78}$$

5) Write down the definition of an irreducible Markov Chain.

Ans→ A Markov Chain with state space S is irreducible if for every pair of states $i, j \in S$, there exists an integer $n \geq 0$ such that

$$P^n(i,j) > 0,$$

where $P^n(i,j)$ is the probability of transitioning from state i to state j in n steps.

6) Write down the definition of transient states and recurrent states with examples.

Ans→ Recurrent State→ A state i is recurrent if the probability of returning to i at some future time, given that the process starts in i, is 1. That is:

$$P(X_n = i \text{ for some } n \geq 1 \mid X_0 = i) = 1$$

Example → A simple random walk on a line with just two states → $P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

This alternates between state 0 & state 1. Both state 0 & state 1 are recurrent, since the process returns to each state infinetly often with probability 1.

Transient State→ A state i is transient if the probability of returning to i at some future time, given that the process starts in i, is less than 1. That is:

$$P(X_n = i \text{ for some } n \geq 1 \mid X_0 = i) < 1$$

Example → Consider the following Markov chain with 3 states : $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

Here → • State 1 moves to state 2
• State 2 moves to state 3
• State 3 remains in state 3 forever

In this case, states 1 & 2 are transient, because once the process leaves them, it will never return. State 3 is recurrent.

7) Express the statement "Some student in this class has visited Mexico" and "Every student in this class has visited either Canada or Mexico" using predicates & quantifiers.

Ans→ Domain→ All students in this class

Predicates → • $M(x)$ = "x has visited Mexico"
• $C(x)$ = "x has visited Canada"

i) Some student in this class has visited Mexico. This is an existential statement: $\exists x\, M(x)$

ii) Every student in this class has either Canada or Mexico. This is an universal statement:

$$\forall x\, (C(x) \vee M(x))$$

8) Express the statement using predicates & quantifiers where domain for quantification consists of all students at your school: "Every student at your school either can speak Russian or knows C++"

Ans→ Domain → All students in this class

Predicates → • $R(x)$: "x can speak Russian"
• $C(x)$: "x knows C++"

Expressions → $\forall x\, (R(x) \vee C(x))$

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9) Translate each of these statements into logical expressions: "Not everyone is perfect.", "No one is perfect".

Ans→ $P(x)$: "x is perfect"

Domain: All people

i) Not everyone is perfect. → $\neg \forall x P(x)$ or $\exists x \neg P(x)$

ii) No one is perfect → $\forall x \neg P(x)$

10) Let $P(x)$ be the statement "x spends more than 7 hours every weekday in office", when the domain for x consists of all employees. Express each of these in English:

a) $\forall x \neg P(x)$

b) $\exists x \neg P(x)$

Ans→ $P(x)$: "x spends more than 7 hours every weekday in office"

Domain: All employees

a) $\forall x \neg P(x)$ → No employees spends more than 7 hours every weekday in office"

b) $\exists x \neg P(x)$ → There is at least one employee who does not spend more than 7 hours every weekday in office.

11) What is fuzzy logic? Give some real life examples of fuzzy logic.

Ans→ Fuzzy logic is a form of logic that deals with approximate reasoning rather than fixed, exact values. Unlike traditional binary logic, fuzzy logic allows for degrees of truth - values between 0 & 1.

Real life Examples → Washing Machine, Air Conditioners, Cameras, Automatic Transmission in Cars, etc.

12) Write the difference between fuzzy logic & probability.

Ans→

Aspect	Fuzzy Logic	Probability
Purpose	Models degree of truth or vagueness.	Models uncertainity due to randomness or lack of knowledge.
Value Range	Value range from 0 to 1, representing truth dance.	Values range from 0 to 1, representing likelihood.
Interpretation	How true a statement is.	How likely an event is to happen
Nature of information	Deals with imprecise, vague or linguistic terms	Deals with randomness, chance & statistical data

13) Discretize and solve the boundary value problem $y'' = \frac{3}{2}y^2$, with $y(0) = 4$, $y(1) = 1$ by taking $h = \frac{1}{3}$.

Ans→ ~~Discrete~~ Divide the interval $[0, 1]$ into 3 subintervals for given $h = \frac{1}{3}$ →

$x_0 = 0$, $x_1 = \frac{1}{3}$, $x_2 = \frac{2}{3}$, $x_3 = 1$

Let $y_i \approx y(x_i)$, for $i = 0, 1, 2, 3$. We are given: $y_0 = 4$, $y_3 = 1$ Unknowns $= y_1, y_2$

Now, $y''(x_i) \approx \dfrac{y_{i-1} - 2y_i + y_{i+1}}{h^2}$, $y'' = \frac{3}{2}y^2 \Rightarrow \dfrac{y_{i-1} - 2y_i + y_{i+1}}{h^2} = \frac{3}{2}y_i^2$

At x_1 → $\dfrac{y_0 - 2y_1 + y_2}{h^2} = \frac{3}{2}y_1^2 \Rightarrow \dfrac{4 - 2y_1 + y_2}{(1/3)^2} = \frac{3}{2}y_1^2 \Rightarrow 9(4 - 2y_1 + y_2) = \frac{3}{2}y_1^2 \Rightarrow 36 - 18y_1 + 9y_2 = \frac{3}{2}y_1^2$ (eq (i))

At x_2 → $\dfrac{y_1 - 2y_2 + y_3}{h^2} = \frac{3}{2}y_2^2 \Rightarrow \dfrac{y_1 - 2y_2 + 1}{(1/3)^2} = \frac{3}{2}y_2^2 \Rightarrow 9(y_1 - 2y_2 + 1) = \frac{3}{2}y_2^2 \Rightarrow 9y_1 - 18y_2 + 9 = \frac{3}{2}y_2^2$ (eq (ii))

14) Discretize and solve the boundary value problem $y'' + y + 1 = 0$, with $y(\pm 1) = 0$ by taking $h = 0.5$.

Ans→ The interval is $[-1, 1]$ & step size is $h = 0.5$.

This gives us nodes: $x_0 = -1$, $x_1 = -0.5$, $x_2 = 0$, $x_3 = 0.5$, $x_4 = 1$

So, we have: $y_0 = y(-1) = 0$

$y_4 = y(1) = 0$

Unknowns $= y_1 = y(-0.5)$, $y_2 = y(0)$, $y_3 = y(0.5)$

Now, $y''(x_i) \approx \dfrac{y_{i+1} - 2y_i + y_{i-1}}{h^2} + y_i + 1 = 0$

$y_{i+1} - 2y_i + y_{i-1} + 0.25y_i + 0.25 = 0$

We apply the discretized form to $i = 1, 2, 3$:

For $i=1$: $y_0 + y_2 - 1.75y_1 = -0.25$, but $y_0 = 0 \Rightarrow y_2 - 1.75y_1 = -0.25$

For $i=2$: $y_1 + y_3 - 1.75y_2 = -0.25$

For $i=3$: $y_2 + y_4 - 1.75y_3 = -0.25$, but $y_4 = 0 \Rightarrow y_2 - 1.75y_3 = -0.25$

We solve the 3 equations in 3 unknowns: $y_2 = 1.75y_1 - 0.25$

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Now, $y_1 + y_3 - 1.75(1.75y_1 - 0.25) = -0.25 \Rightarrow y_1 + y_3 - 3.0625y_1 + 0.4375 = -0.25 \Rightarrow -2.0625(y_1) + y_3 = -0.6875$

Then, $1.75y_1 - 0.25 - 1.75y_3 = -0.25 \Rightarrow 1.75y_1 - 1.75y_3 = 0 \Rightarrow y_1 = y_3$

Now, $-2.0625y_1 + y_1 = -0.6875 \Rightarrow -1.0625y_1 = -0.6875 \Rightarrow y_1 = \frac{0.6875}{1.0625} = \frac{11}{17} \Rightarrow y_1 = y_3 = \frac{11}{17} \approx 0.647$

$y_2 = 1.75 \cdot 11/17 - 0.25 = \frac{19.25}{17} - 0.25 = \frac{19.25 - 4.25}{17} = \frac{15}{17} \approx 0.882$

$\therefore y(-1) = 0,\ y(-0.5) = 11/17,\ y(0) = 15/17,\ y(0.5) = 11/17,\ y(1) = 0$

15) Discretize the Heat conduction equation $\frac{dT}{dt} = c\frac{d^2T}{dx^2}$.

Ans → Given, $\frac{dT}{dt} = c\frac{d^2T}{dx^2}$

Forward Difference → $\frac{dT}{dt} \approx \frac{T_i^{n+1} - T_i^n}{\Delta t}$

Central Difference → $\frac{d^2T}{dx^2} \approx \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\Delta x^2}$

Substituting in the heat equation →

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} = c \cdot \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\Delta x^2}$$

$$T_i^{n+1} = T_i^n + \frac{c\Delta t}{\Delta x^2}(T_{i+1}^n - 2T_i^n + T_{i-1}^n)$$

$$T_i^{n+1} = T_i^n + \lambda(T_{i+1}^n - 2T_i^n + T_{i-1}^n)$$

16) Discretize the wave equation of a vibrating membrane $\frac{\partial^2 u}{\partial t} = c\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right]$

Ans → $\frac{\partial^2 u}{\partial t^2} = c\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$

Let;

$x_i = i\Delta x,\ y_j = j\Delta y,\ t^n = n\Delta t$

$u(i,j,n) = u_{ij}^n \approx u(x_i, y_j, t^n)$

$$\frac{\partial^2 u}{\partial t^2} \approx \frac{u_{ij}^{n+1} - 2u_{ij}^n + u_{i,j}^{n-1}}{(\Delta t)^2}$$

$$\frac{\partial^2 u}{\partial x^2} \approx \frac{u_{i+1,j}^n - 2u_{ij}^n + u_{i-1,j}^n}{(\Delta x)^2}$$

$$\frac{\partial^2 u}{\partial y^2} \approx \frac{u_{i,j+1}^n - 2u_{i,j}^n + u_{i,j-1}^n}{(\Delta y)^2}$$

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$$\frac{u_{i,j}^{n+1} - 2u_{i,j}^{n} + u_{i,j}^{n-1}}{(\Delta t)^2} = c\left[\frac{u_{i+1,j}^{n} - 2u_{i,j}^{n} + u_{i-1,j}^{n}}{(\Delta x)^2} + \frac{u_{i,j+1}^{n} - 2u_{i,j}^{n} + u_{i,j-1}^{n}}{(\Delta y)^2}\right]$$

17) Discretize the given PDE using central difference method $\frac{d^2f}{dx^2} + 4f = 0,\ 0 \le x \le 1$ subject to the boundary conditions $f(0)=1, f(1)=1$ and assume $h=0.5$.

Ans→ $\frac{d^2f}{dx^2} + 4f = 0,\ 0 \le x \le 1$ with boundary $f(0)=1, f(1)=1, h=0.5$

- $x_0 = 0,\ x_1 = 0.5,\ x_2 = 1$
- $f_0 = f(0) = 1,\ f_2 = f(1) = 1$
- Unknown $f_1 \approx f(0.5)$

Central difference for second derivative → $\frac{d^2f}{dx^2} \approx \frac{f_{i+1} - 2f_i + f_{i-1}}{h^2}$

For $i=1$: $\frac{f_2 - 2f_1 + f_0}{(0.5)^2} + 4f_1 = 0$

Now, $\frac{1 - 2f_1 + 1}{0.25} + 4f_1 = 0 \Rightarrow \frac{2-2f_1}{0.25} + 4f_1 = 0 \Rightarrow 8(1-f_1) + 4f_1 = 0 \Rightarrow 8 - 8f_1 + 4f_1 = 0 \Rightarrow 8 - 4f_1 = 0$
$\Rightarrow f_1 = 2$

$\therefore f(0)=1,\ f(0.5)=2,\ f(1)=1$

18) What is Good AI? Give example.

Ans→ Good AI refers to artificial intelligence systems designed and user responsibly, in ways that benefit society, respect ethical principles, and avoid harm.

Example → AI-powered accessibility tools: Like voice-to-text or image descriptions for the visually impaired.
AI for climate modelling → Predicting weather patterns or natural disasters.

19) Discuss about AI policies and its type.

Ans→ AI policies are rules, guidelines and strategies created by governments, institutions and organizations to regulate the development and use of artificial intelligence. Their main goals are to ensure safety, fairness, accountability and ethical use of AI technologies.

Types of AI policies →
i) Ethical Policies → Focus on moral principle in AI use.
ii) Regulatory Policies → Legal frameworks to govern AI behaviour and its impact.
iii) Research and innovation Policies → Promote AI R&D through funding, collaboration & infrastructure.

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20) Write down the characteristics of good AI.

Ans→ i) Accuracy → • Produces correct and precise results.
• Learns and improves with more data over time.

ii) Fairness → • Avoid bias and discrimination.
• Treats all users equally, regardless of gender, race, or background.

iii) Transparency → • The decision-making process is explainable and understandable.
• Users can trace how conclusions are reached.

iv) Accountability → • Clearly defines who is responsible for AI outcomes.
• Allows for audits and error tracing.

v) Privacy & security → • Protects user's personal data.
• Prevents unauthorised access and data leaks.

vi) Robustness → • Performs reliably even in unexpected or complex conditions.
• Resistant to adversarial attacks and failures.

21) Distinguish between the terms Bias and Discrimination in AI ethics.

Ans→ Bias in AI →

• Definition → Bias refers to systematic errors or prejudices in AI data, design or algorithms that lead to inaccurate or unfair outcomes.

• Cause → Can be introduced through training data, human assumptions, or flawed algorithms.

• Example → An AI model trained mostly on images of light-skinned faces may perform poorly on darker-skinned faces.

Discrimination in AI →

• Definition → Discrimination is the unfair or prejudiced treatment of individuals or groups based on characteristics like race, gender, age, etc., often as a result of bias.

• Effect → It leads to unequal access, oppurtunities, or outcomes.

• Example → A hiring algorithm that favours male applicants over equally quallified female applicants due to biased training data.