

- Assignment - I -

1) What are the 4 aspects of AI?

Ans → The four aspects of Artificial Intelligence (AI) are:

- i) Perception → AI's ability to collect data from the environment through various sensors, like cameras, microphones, or other detection devices and interpret that data.
- ii) Reasoning and Problem-Solving → AI's ability to process information and make decisions.
- iii) Learning → AI's capability to improve its performance based on past experiences, typically through machine learning algorithms.
- iv) Natural Language Processing (NLP): AI's ability to understand, interpret & generate human language.

2) Discuss perceptions and understandings of AI and their applications.

Ans → Perceptions and understanding of AI vary widely, ranging from fears of job displacement and autonomous weapons to hopes for improved health care & transportation. Some common applications include:

- i) Virtual assistance
- ii) Image and speech recognition
- iii) Healthcare
- iv) Customer support

3) Discuss the role of AI in natural language processing and social media.

Ans → AI plays a crucial role in natural language processing (NLP) and social media, with applications including:

- i) Language translation: AI is used to translate text and speech in real-time.
- ii) Sentiment analysis: AI powered sentiment analysis systems are used to analyze social media posts and determine the sentiment of users.
- iii) Content generation: AI powered content generation systems are used to generate social media posts and chatbot responses, and other content.

4) What is the difference between real data & simulated data?

Ans → Real data refers to the data is collected from real world sources , such as sensors, surveys and transactions . Simulated data, on the other hand is artificially generated data that mimics real data. The key differences between real and simulated data are :

i) Source : Real data comes from real -world sources while simulated data is generated artificially.

ii) Accuracy : Real data may contain errors and inconsistencies , while simulated data can be generated to be error free.

iii) Purpose : Real data is often used for analysis and decision -making, while simulated data is often used for testing and training AI systems.

5) An AI system is trained to predict the response time of a chatbot for a given set of inputs.

$$[120, 150, 130, 140, 150, 135, 125, 150, 140, 160]$$

Ans → a) Mean = $\frac{120+150+130+140+150+135+125+150+140+160}{10}$

$$= 141 \text{ ms}$$

b) $[120, 125, 130, 135, 140, 140, 150, 150, 150, 160]$

$$\text{Median} = \frac{140+140}{2} = 140 \text{ ms}$$

c) Mode = 150 ms

d) First we calculate deviations from the mean :

$$[-21, 9, -11, -1, 9, -6, -16, 9, -1, 19]$$

Square of each deviation :

$$[441, 81, 121, 1, 81, 36, 256, 81, 1, 361]$$

$$\therefore \text{Variance} = \frac{(441+81+121+1+81+36+256+81+1+361)}{9}$$

$$= 174.33 \text{ ms.}$$

6) An AI-based recommendation system is designed to suggest products based on a user's browsing time and their final purchase amount.

Browsing Time (in mins.)	10	15	8	20	12	18	25	7	14	22
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Purchase Amount (in dollars.)	50	65	40	90	85	80	120	30	60	95
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Ans \rightarrow

$$r_c = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2} \sqrt{\sum (y_i - \bar{y})^2}}$$

x	y	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})^2$	$(y_i - \bar{y})^2$
10	50	-5.1	-18.5	26.01	342.25
15	65	-0.1	-3.5	0.01	12.25
8	40	-7.1	-28.5	50.41	812.25
20	90	4.9	21.5	24.01	462.25
12	85	-3.1	-13.5	9.61	182.25
18	80	2.9	11.5	8.41	132.25
25	120	9.9	81.5	98.01	2642.25
7	30	-8.1	-38.5	65.61	1482.25
14	60	-1.1	-8.5	1.21	72.25
22	95	6.9	26.5	47.61	702.25
				$\Sigma = 330.9$	$\Sigma = 7345$

$$\bar{x} = 15.1 \text{ minutes}$$

$$\bar{y} = 68.5 \text{ dollars}$$

$$\therefore r_c = \frac{1484.3}{\sqrt{330.9} * \sqrt{7345}} = \underline{\underline{0.95}}$$

\Rightarrow we know,

$$f(x) = \frac{1}{B-A} = \frac{1}{100-100} = 0.01$$

$$\text{So, } F(x) = \frac{x-a}{b-a} = \frac{x-100}{100}$$

$$a) P(X < 150) = F(150) = 0.5$$

$$b) P(120 < X < 180) = F(180) - F(120)$$
$$= 0.8 - 0.2$$
$$= 0.6$$

$$c) \text{Variance, } \text{Var}(x) = \frac{(b-a)^2}{12} = \left(\frac{200-100}{12}\right)^2$$

$$\text{Mean, } E(x) = a + \frac{b}{2} = 100 + \frac{200}{2} = 150 \text{ ms}$$

$$8) Z = \frac{x - \mu}{\sigma} = \frac{480 - 150}{20} = -1$$

$$\therefore P(X \leq 480) = P(Z \leq -1) = 0.1587$$

$$9) n=15, p=0.9, q=0.1$$

$$a) P(X=13) = \binom{15}{13} (0.9)^{13} (0.1)^2$$
$$= 0.2503$$

$$b) P(X=15) = \binom{15}{15} (0.9)^{15} (0.1)^0$$
$$= 0.205891$$

$$c) P(X \geq 12) = 1 - P(X \leq 11)$$
$$= 1 - 0.0618$$
$$= 0.9382$$

$$10) \lambda = 4$$

$$a) P(X=3) = \frac{e^{-4} \cdot 4^3}{3!} = 0.1954$$

$$b) P(X \geq 5) = 1 - P(X \leq 4)$$
$$= 1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)]$$
$$= 1 - [0.0183 + 0.0733 + 0.1468 + 0.1954 + 0.1954]$$
$$= 0.3711$$

11) Consider the density function. $f(x) = \begin{cases} k\sqrt{x}, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$

a) Evaluate k .

b) Find $F(x)$ and use it to evaluate $P(0.3 < X < 0.6)$

$$\text{Ans} \rightarrow \text{a) } \int_{-\infty}^{\infty} f(x) dx = 1$$

$$\Rightarrow \int_0^1 k\sqrt{x} dx = 1$$

$$\Rightarrow k \left[\frac{2}{3} x^{3/2} \right]_0^1 = 1$$

$$\Rightarrow k \left[\frac{2}{3} \right] = 1$$

$$\Rightarrow k = \frac{3}{2}$$

$$\text{b) } P(0.3 < X < 0.6) = F(0.6) - F(0.3)$$

$$= 0.6^{3/2} - 0.3^{3/2}$$

$$= 0.46476 - 0.16432 = 0.3004$$

12) $f(x,y) = \begin{cases} 10xy^2, & 0 < x < y \\ 0, & \text{elsewhere} \end{cases}$

$$\text{a) } g(x) = \int_{-\infty}^{\infty} f(x,y) dy = \int_x^y 10xy^2 dy$$

$$= \left[\frac{10xy^3}{3} \right]_x^y = \frac{10}{3}x(y^3 - x^3), \quad 0 < x < 1$$

$$h(y) = \int_{-\infty}^{\infty} f(x,y) dx = \int_0^y 10xy^2 dx =$$

$$= \left[\frac{10x^2y^2}{2} \right]_0^y = 5y^2y^2 = 5y^4, \quad 0 < y < 1$$

$$\text{Now, } f(y|x) = \frac{f(x,y)}{g(x)} = \frac{10xy^2}{\frac{10}{3}x(1-x^3)} = \frac{3y^2}{1-x^2}, \quad 0 < x < y < 1$$

$$\text{b) } P(Y|X=0.25) = \frac{3y^2}{1-(0.25)^3} = \frac{3y^2}{0.984375}$$

$$\text{So, } P(Y>0.5 | X=0.25) = \frac{1}{0.984375} (1 - 0.5^2) \\ = 0.888$$

13) The cumulative distribution function dist. function of X is

$$F(x) = \begin{cases} 0, & x < 1 \\ 0.4, & 1 \leq x < 3 \\ 0.6, & 3 \leq x < 5 \\ 0.8, & 5 \leq x < 7 \\ 1.0, & x \geq 7 \end{cases}$$

- a) What is the probability mass function of X ?
 b) Compute $P(4 < X \leq 7)$

Ans → a) For $1 \leq x < 3$,

$$\begin{aligned} P(X=1) &= F(3) - F(1) \\ &= 0.4 - 0 \\ &= 0.4 \end{aligned}$$

$3 \leq x < 5$

$$\begin{aligned} P(X=3) &= F(5) - F(3) \\ &= 0.6 - 0.4 \\ &= 0.2 \end{aligned}$$

$5 \leq x < 7$

$$\begin{aligned} P(X=5) &= F(7) - F(5) \\ &= 0.8 - 0.6 \\ &= 0.2 \end{aligned}$$

$$\begin{aligned} x \geq 7 \\ P(X=7) &= 1.0 - 0.8 \\ &= 0.2 \end{aligned}$$

b) $P(4 < X \leq 7)$

$$\begin{aligned} &= P(X=5) + P(X=7) \\ &= 0.2 + 0.2 \\ &= 0.4 \end{aligned}$$

14)

		x		$\phi(y)$
		2	4	
y	1	0.1	0.15	0.25
	3	0.2	0.3	0.5
	5	0.1	0.15	0.25
		$g(x)$	0.4	0.6
				1