Q>1) The output Y of a 2-bit comparator is logic 1 whenever the 2-bit input A is greater than the 2-bit input B. The number of combinations for which the output is logic 1, is ______.

Ans → The only possible combinations are

A = 01 & B = 00

A = 10 & B = 00, 01

A = 11 & B = 00, 01, 10

So, there are 6 combinations

Q>2) Find all prime implicants of the following function and then find all minimum solutions using petrick's method →

$F(A,B,C;D) = \Sigma m\,(9,12,13,15) + \Sigma d\,(1,4,5,7,8,11,14)$

Ans →

Column 1		Column 2		Column 3		
1	0001	1,5	0_01	1,5,9,13	_ _01	C'D
4	0100	1,9	_001	1,9,5,13	_ _01	C'D
8	1000	4,5	010_	4,5,12,13	_10_	BC'
5	0101	4,12	_100	4,12,5,13	_10_	BC'
9	1001	8,9	100_	5,7,13,15	_1_1	BD
12	1100	8,12	1_00	5,13,7,15	_1_1	BD
7	0111	5,7	01_1	8,9,12,13	1_0_	AC'
11	1011	5,13	_101	8,12,9,13	1_0_	AC'
13	1101	9,11	10_1	9,11,13,15	1_ _1	AD
14	1110	9,13	1_01	9,13,11,15	1_ _1	AD
15	1111	12,13	110_	12,13,14,15	11_ _	AB
		12,14	11_0	12,14,13,15	11_ _	AB
		7,15	_111			
		11,15	1_11			
		13,15	11_1			
		14,15	111_			

Prime implicants → C'D, BC', BD, AC', AD, AB

Petrick's table →

		9	12	13	15
P_1 (1,5,9,13)	C'D	X		X	
P_2 (4,5,12,13)	BC'		X	X	
P_3 (5,7,13,15)	BD			X	X
P_4 (8,9,12,13)	AC'	X	X	X	
P_5 (9,11,13,15)	AD	X		X	X
P_6 (12,13,14,15)	AB		X	X	X

By using Petrick's method,

$$P = (P_1+P_3+P_5)(P_2+P_3+P_6)(P_1+P_2+P_3+P_4+P_5+P_6)(P_4+P_5+P_6)$$

$$= (P_3+(P_1+P_5)(P_2+P_6))(P_4+P_5+P_6(P_1+P_2+P_3+P_6))$$

$$= (P_3+P_1P_2+P_1P_6+P_5P_2+P_5P_6)(P_4+P_5+P_1P_6+P_2P_6+P_3P_6+P_6)$$

$$= (P_3+P_1P_2+P_1P_6+P_5P_2+P_5P_6)(P_4+P_5+P_6(P_1+P_2+P_3+1))$$

$$= (P_3+P_1P_2+P_1P_6+P_2P_5+P_5P_6)(P_4+P_5+P_6)$$

$$= \underline{P_3P_4} + P_1P_2P_4 + P_1P_6P_4 + P_2P_5P_4 + P_4P_5P_6 + \underline{P_3P_5} + P_1P_2P_5 + P_1P_6P_5 + \underline{P_2P_5} + \underline{P_5P_6}$$
$$+ \underline{P_3P_6} + P_1P_2P_6 + P_2P_5P_6 + P_5P_6 + \underline{P_1P_6}$$

Simplified SOP → $F_1 = P_3P_4 = AC' + BD$

$F_2 = P_3P_5 = AC' + AD$

$F_3 = P_2P_5 = BC' + AD$

$F_4 = P_5P_6 = AD + AB$

$F_5 = P_3P_6 = AC' + AB$

$F_6 = P_1P_6 = C'D + AD$

4) Determine a minimum sum of products expression for the function $F = \Sigma m(0,1,2,5,6,7)$ using Quine-McCluskey method and prime implicant chart.

Ans → $F = \Sigma m(0,1,2,5,6,7)$

Column 1	Column 2	Prime implicants
0 → 000	0,1 → 00_	a'b'
1 → 001	0,2 → 0_0	a'c'
2 → 010	1,5 → _01	b'c
5 → 101	2,6 → _10	bc'
6 → 110	5,7 → 1_1	ac
7 → 111	6,7 → 11_	ab

Prime implicant chart →

	0	1	2	5	6	7
(0,1) a'b'	*	*				
(0,2) a'c'	*		*			
(1,5) b'c		*		*		
(2,6) bc'			*		*	
(5,7) ac				*		*
(6,7) ab					*	*

For the 1st solution,
$F = a'b' + bc' + ac$

For the 2nd solution,
$F = a'c' + b'c + ab$

Q3) Determine a minimum sum of products expression for the function $F = \Sigma m(0,1,2,5,6,7)$ using Petrick's method.

Ans → For Petrick's method we can directly use Q.4's prime implicant chart

	0	1	2	5	6	7
P_1 - (0,1) a'b'	×	×				
P_2 - (0,2) a'c'	×		×			
P_3 - (1,5) b'c		×		×		
P_4 - (2,6) bc'			×		×	
P_5 - (5,7) ac				×		×
P_6 - (6,7) ab					×	×

$$P = (P_1+P_2)(P_1+P_3)(P_2+P_4)(P_3+P_5)(P_4+P_6)(P_5+P_6)$$
$$= (P_1 + P_2P_3)(P_4 + P_2P_6)(P_5 + P_3P_6)$$
$$= (P_1P_4 + P_2P_3P_4 + P_1P_2P_6 + P_2P_3P_6)(P_5 + P_3P_6)$$
$$= \underline{P_1P_4P_5} + P_2P_3P_4P_5 + P_1P_2P_5P_6 + P_2P_3P_5P_6 + P_1P_3P_4P_6 + P_2P_3P_4P_6 + P_1P_2P_3P_6 + \underline{P_2P_3P_6}$$

So, minimum SOP expression of the function,

for $P_1P_4P_5$ = $F = a'b' + bc' + ac$

for $P_2P_3P_6$ = $F = a'c' + b'c + ab$

Q)5) Determine the a minimum sum of products expression for the function $F=\sum m(1,3,4,5,6,7,10,12,13)+\sum d(2,9,15)$ using Quine-McCluskey method and prime implicant chart.

Ans→ $F=\sum_m(1,3,4,5,6,7,10,12,13)+\sum_d(2,9,15)$

Column-1	Column 2	Column-3	Prime Implicants
1→ 0001	1,3→ 00_1	1,3,5,7→ 0_ _1	A'D
2→ 0010	1,5→ 0_01	~~1,5,3,7→ 0_ _1~~	
4→ 0100	1,9→ _001	1,5,9,13→ _ _01	C'D
3→ 0011	2,3→ 001_	~~1,9,5,13→ _ _01~~	
5→ 0101	2,6→ 0_10	2,3,6,7→ 0_1_	A'C
6→ 0110	2,10→ _010	~~2,6,3,7→ 0_1_~~	
9→ 1001	4,5→ 010_	4,5,6,7→ 01_ _	A'B
10→ 1010	4,6→ 01_0	~~4,6,5,7→ 01_ _~~	
12→ 1100	4,12→ _100	4,12,5,13→ _10_	BC'
7→ 0111	3,7→ 0_11		
13→ 1101	5,7→ 01_1	5,13,7,15→ _1_1	BD
15→ 1111	6,7→ 011_	~~5,7,13,15→ _1_1~~	
	5,13→ _101	2,10→ _010	B'CD'
	9,13→ 1_01	~~4,5,12,13→ _10_~~	
	12,13→ 110_		
	7,15→ _111		
	13,15→ 11_1		

Prime implicant chart →

	1	3	4	5	6	7	10	12	13
(1,3,5,7) A'D	X	X		X		X			
(1,5,9,13) C'D	X			X					X
(2,3,6,7) A'C		X			X	X			
(4,5,6,7) A'B			X	X	X	X			
(4,12,5,13) BC'			X	X				⊗	X
(5,13,7,15) BD				X		X			
(2,10) B'CD'							⊗		

2 possible sol^n are → $F = BC' + B'CD' + C'D + A'C$

$F = BC' + B'CD' + A'B + A'D$

Q6) Using the method of map-entered variables, use four-variable maps to find a minimum sum-of-products expression for $F(A,B,C,D,E) = \Sigma_m(0,4,6,13,14) + \Sigma_d(2,9) + E(m_1 + m_{12})$.

Ans→ $F(A,B,C,D,E) = \Sigma_m(0,4,6,13,14) + \Sigma_d(2,9) + E(m_1 + m_{12})$

AB \ CD	00	01	11	10
00	1			X
01	1			1
11		1		1
10		X		

For $E=0$

$MS_0 = A'D + BCD' + AC'D$

AB \ CD	00	01	11	10
00	X	1		X
01	X			X
11	1	X		X
10		X		

For $E=1$

$MS_1 = A'B'C' + BD'$

So, minimum SOP expression of $F = MS_0 + E(MS_1)$

$= A'D + BCD' + AC'D + E(A'BC' + BD')$

Q7) Using the method of map-entered variables, use four-variable maps to find a minimum sum-of-products expression for $F(A,B,C,D,E,F,G) = \Sigma m(2,5,6,9) + \Sigma d(1,3,4,13,14) + E(m_{11} + m_{12}) + F(m_{10}) + G(m_0)$.

Ans→ $F(A,B,C,D,E,F,G) = \Sigma m(2,5,6,9) + \Sigma d(1,3,4,13,14) + E(m_{11} + m_{12}) + F(m_{10}) + G(m_0)$

For $E=G=F=0$,

AB \ CD	00	01	11	10
00		X	X	1
01	X	1		1
11		X		X
10		1		

$MS_0 = C'D + A'CD'$

For $E=1, F=0, G=0$

AB \ CD	00	01	11	10
00		X	X	X
01	X	X		X
11	1	X		X
10		X	1	

$MS_1 = BC' + B'D$

For, $E=G=0, F=1$

AB \ CD	00	01	11	10
00		X	X	X
01	X	X		X
11		X		X
10	0	X		1

$MS_2 = CD'$

For $E=F=0, G=1$

AB \ CD	00	01	11	10
00	1	X	X	X
01	X	X		X
11		X		X
10		X		

$MS_3 = A'B'$

So, minimum SOP of $F = MS_0 + E(MS_1) + F(MS_2) + G(MS_3)$

$= C'D + A'CD' + E(BC' + B'D) + F(CD')$

$+ G(A'B')$

Q)8) Design a combinational circuit with 3 inputs X, Y & Z and one output F. The output is 1 when the binary value of the inputs is less than or equal to 2. The output is 0 otherwise.

Ans → Truth table →

Inputs			Output
X	Y	Z	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

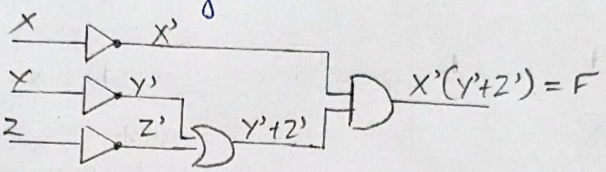
K-map →

X \ YZ	00	01	11	10
0	1	1		1
1				

$F = X'Z' + X'Y'$

$= X'(Y'+Z')$

Circuit Diagram →



Q)9) Design a combinational circuit with four inputs A, B, C & D and one output F. The output F value is 0 if three or four of the inputs are 1; otherwise the value of F is 1.

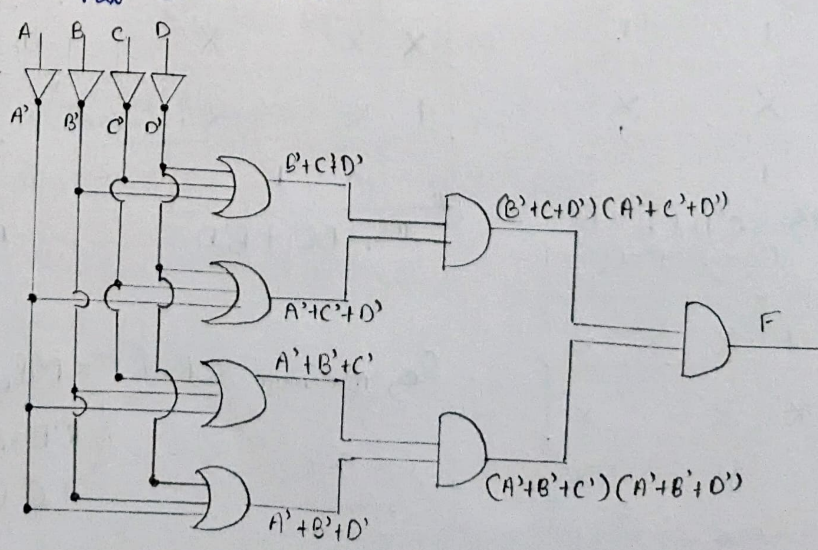
Ans → Truth Table →

Inputs				Outputs
A	B	C	D	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

K-map for F_{max} →

AB \ CD	00	01	11	10
00				
01			0	
11		0	0	0
10			0	

$F_{max} = (B'+C'+D')(A'+C'+D')(A'+B'+C')(A'+B'+D')$



Q) 10) Design a 4-bit BCD to Excess-3 code converter.

Ans → Truth table →

Inputs				Outputs			
A	B	C	D	W	X	Y	Z
0	0	0	0	0	0	1	1
0	0	0	1	0	1	0	0
0	0	1	0	0	1	0	1
0	0	1	1	0	1	1	0
0	1	0	0	0	1	1	1
0	1	0	1	1	0	0	0
0	1	1	0	1	0	0	1
0	1	1	1	1	0	1	0
1	0	0	0	1	0	1	1
1	0	0	1	1	1	0	0

K-map →

For W →

AB \ CD	00	01	11	10
00				
01		1	1	1
11	X	X	X	X
10	1	1	X	X

$W = A + BD + BC = A + B(C+D)$

For X →

AB \ CD	00	01	11	10
00		1	1	1
01	1			
11	X	X	X	X
10		1	X	X

$X = B'D + B'C + BC'D'$

$= B'(C+D) + BC'D'$

For Y →

AB \ CD	00	01	11	10
00	1		1	
01	1		1	
11	X	X	X	X
10	1		X	X

$Y = C'D' + CD$

$= C \odot D$

For Z →

AB \ CD	00	01	11	10
00	1			1
01	1			1
11	X	X	X	X
10	1		X	X

$Z = D'$

Circuit Diagram →