

In this lecture, we discuss the representation of discrete time signals through a decomposition as a linear combination of complex exponentials. The motivation for representing discrete-time signals is identical in both continuous time and discrete time. The effect of an LTI system on each basic signal is simply a complex amplitude change. For periodic signals this representation becomes the discrete time Fourier series, and for aperiodic signals it becomes the discrete Fourier transform.

Suppose that the Fourier series coefficients of a discrete time periodic signal are unique only as the frequency variable spans a range of  $2\pi$ . In many ways, this simplifies the analysis of the discrete time Fourier transform because there are only  $N$  distinct complex exponentials of this type available to use. The resulting analysis equation is a summation very similar in form to the synthesis equation and suggests a strong duality between the analysis and the discrete-time Fourier transforms.

The Fourier series coefficients can be represented as a periodic sequence with period  $N$ , that is the same period as the time sequence  $x_n$ . This periodicity is illustrated in several examples. Partly in anticipation of the fact that we will want to follow an approach similar to that used in the continuous time case for a Fourier decomposition of aperiodic signals, it is useful to represent the Fourier coefficients as samples of an envelope.

As the period of the sequence increases, the Fourier series coefficients are samples of the same envelope function with increasingly finer spacing along the frequency axis specifically, a spacing of  $2\pi/N$  where  $N$  is the period. As the period approaches infinity, this envelope function corresponds to a Fourier representation of the aperiodic signal corresponding to one period.

The discrete-time Fourier transform is a decomposition of an aperiodic signal as a linear combination of a continuum of complex exponentials. The synthesis equation is then the limiting form of the Fourier series sum, specifically an integral.

The Fourier series coefficients are obtained using the analysis equation that we used previously in the discrete-time Fourier transform. The analysis equation is the same one we used to obtain the envelope of the series coefficients. While there was a duality between the discrete and continuous Fourier transforms, the duality is lost when the synthesis equation is an integral and the analysis equation is a summation. This represents one difference between the continuous and discrete Fourier transformations. Another important difference is that the discrete time Fourier transformation is always a periodic function of frequency.

The Fourier series coefficients for a discrete-time periodic signal are compared to the continuous-time Fourier transform which extends over an infinite frequency range. The Fourier Series is completely defined by its behavior over a frequency range of  $2\pi$  in contrast to its continuous time Fourier Transform. Consequently, it is completely characterized by the behavior of the Fourier transforms over a certain frequency range and is completely described by the frequency of the signal. In this lecture, we discuss the response of the discrete time LTI system to complex exponentials.

Suppose that the discrete-time Fourier series coefficients are given as samples of an envelope. We demonstrate that as the period increases, the samples of the discrete time series coefficients become more closely spaced and the samples become more evenly spaced. We show that this is true for the case when the period is increased.

The Fourier representation of an aperiodic signal is discussed in the context of the Fourier theory of signals. The Fourier representations are used to derive the frequency of the signal in a periodic signal. This is a review of the approach to developing a Fourier Representation of a periodic signal. It is shown that the Fourier representation can be used to determine the frequency and frequency of a given signal.

Aperiodic signals can be represented as the limiting form of a periodic signal with the period increasing. Fourier series can be used to represent discrete-time aperiodic signal. The Fourier representation of discrete time signals can also be obtained by using the Fourier transform of the signal. In order to represent a discrete time signal, use a series to represent the periodic signal. A periodic signal can then be represented by a series.

The Fourier series coefficients for a periodic signal approach the continuous envelope function as the period increases. Transparencies are used to illustrate how the Fourier Series coefficients for periodic signals approach the envelope function for the periodic signal. The series coefficients can be used to determine the frequency of the signal and the period of the noise.

The discrete-time Fourier transform for a rectangular pulse can be described by the summation equation. The upper limit in the summation in the second equation should be  $n - 1$ . The discrete time Fourier series can be used to determine the magnitude and phase of the discrete time series for an exponential sequence.

The Fourier series coefficients of a periodic signal and the Fourier transform of one period are discussed. The relationship between the series coefficients and Fourier transforms of one periodic signal is discussed. Note that  $a$  is real. The Fourier Series coefficients are defined as times samples of Fourier transformations of one period.

The Fourier series coefficients and the Fourier transform associated with periodic sequences are discussed in the lecture. The Fourier transforms can be used to determine the frequency of a given periodic sequence. The relationship between the frequency and frequency of the series coefficients of a periodic sequence is illustrated in the lectures.