MATH7501 Exercise sheet 5 — to be done by Friday 23rd February, 2018

THIS SET OF EXERCISES IS THE THIRD ASSESSMENT

To assist the marking of this assessment, please write your **student number** CLEARLY at the top of each sheet of paper, and indicate clearly (e.g. by underlining) the final answer to each part of a question.

All solutions must be your own work.

1. A random variable Y, taking values in $\mathbb{R}^+$, is said to have an *inverse gamma distribution with parameters α and λ* if $1/Y \sim \Gamma(\alpha, \lambda)$. Find the probability density function of Y.

   6 marks

2. Suppose that X_1 and X_2 are independent discrete random variables with $P(X_1 = x_1, X_2 = x_2) = p(x_1, x_2)$. Consider the definition $E\left[\phi\left(X_1, X_2\right)\right] = \sum_{x_1} \sum_{x_2} \phi(x_1, x_2) p(x_1, x_2)$ (this is the definition given in Section 4.1.1 of the lecture notes, with $n = 2$). Show that if $\phi(x_1, x_2) = x_1$, this reduces to the usual definition of $E(X_1)$ from Chapter 2.

   4 marks

3. Let X denote the number of heads obtained when a fair coin is tossed twice. If $X = k$, another fair coin is tossed k times. Let Y denote the number of heads obtained on the second coin.

   (a) Give a table showing the joint probability mass function of X and Y.

   (b) Find the marginal probability mass functions of X and Y.

   (c) Are X and Y independent? Justify your answer clearly.

   10 marks