

MATH7501 Exercise sheet 0: revision of basic topics

1. Explain the difference between the sample space and the event space. Write down the elements of the two spaces for the experiment where a coin is tossed once (use the notation H, T to denote heads and tails). **3 marks**
2. Consider an experiment with a countable (but possibly infinite) sample space Ω in which the individual outcomes are denoted by $\omega_1, \omega_2, \dots$. Let p_1, p_2, \dots be a corresponding set of non-negative 'weights' with $\sum_i p_i = 1$. For any event $E \subseteq \Omega$, define $P(E) = \sum_{\{\omega_i \in E\}} p_i$. Show that the function $P(\cdot)$ is a probability function (i.e. that it satisfies Kolmogorov's three axioms). **4 marks**
3. Use the axioms of probability to prove that $P(E^c) = 1 - P(E)$ (**Hint:** note that $\Omega = E \cup E^c$). **3 marks**
4. Use the axioms of probability to prove that for any two events E and F , $P(E \cup F) = P(E) + P(F) - P(E \cap F)$. **4 marks**
5. Is it possible for two events to be mutually exclusive and also independent? **3 marks**
6. Suppose that E and F are two events with $P(F) > 0$. Show that if $P(E|F) > P(E)$ then $P(F|E) > P(F)$. Interpret this result in words. **4 marks**
7. A proportion of 1% of a population are known to have a particular disease. A screening test has a 95% chance of detecting the disease (*i.e.*, of giving a positive result) in an individual who has the disease. There is also a 6% chance that a disease-free individual will give a positive result and be thought to have the disease. What is the probability that a positive result is obtained on an individual chosen at random from the population? If a positive result is obtained, what is the probability that the individual does have the disease? **2 marks**
8. An urn contains five red balls and one white ball. A ball is drawn and then it and another ball of the same colour are placed back in the urn. Finally a second ball is drawn.
 - (a) What is the probability that the second ball is white?
 - (b) If the second ball is white, what is the probability that the first was red?

2 marks