Summary of Continuous Distributions for a random Variable X

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1 Uniform $X \sim U(a, b)$

1.1 Probability Density Function $f(x)$

$$f(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{Otherwise} \end{cases}$$

1.2 Cumulative Distribution Function $F(x)$

$$F(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ 1 & x > b \end{cases}$$

1.3 Moment Generating Function $M(t) = E(e^{tX})$

$$M(t) = \frac{e^{tb} - e^{ta}}{t(b-a)}$$

1.4 $E(X)$

$$E(X) = \frac{a+b}{2}$$

1.5 $Var(X)$

$$Var(X) = \frac{(b-a)^2}{12}$$

2 Exponential $X \sim Exp(\lambda)$

2.1 Probability Density Function $f(x)$

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0 \\ 0 & \text{Otherwise} \end{cases}$$

2.2 Cumulative Distribution Function $F(x)$

$$F(x) = 1 - e^{-\lambda x} \text{ where } x > 0$$

2.3 Moment Generating Function $M(t) = E(e^{tX})$

$$M(t) = \frac{\lambda}{\lambda - t}$$

2.4 $E(X)$

$$E(X) = \frac{1}{\lambda}$$

2.5 $Var(X)$

$$Var(X) = \frac{1}{\lambda^2}$$

3 Gamma $X \sim \Gamma(\alpha, \lambda)$

3.1 Probability Density Function $f(x)$

$$f(x) = \begin{cases} \frac{\lambda^\alpha x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)} & x \geq 0 \\ 0 & \text{Otherwise} \end{cases}$$

Where we have

$$\begin{aligned} \Gamma(\alpha) &= \int_0^\infty x^{\alpha-1} e^(-x) dx \\ &= (\alpha - 1)! \end{aligned}$$

3.2 Moment Generating Function $M(t) = E(e^{tX})$

$$M(t) = \frac{\lambda^\alpha}{(\lambda - t)^\alpha}$$

3.3 $E(X)$

$$E(X) = \frac{\alpha}{\lambda}$$

3.4 $Var(X)$

$$Var(X) = \frac{\alpha}{\lambda^2}$$

4 Beta $X \sim B(\alpha, \beta)$

4.1 Probability Density Function $f(x)$

$$f(x) = \begin{cases} \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)} & x \in (0,1) \\ 0 & \text{Otherwise} \end{cases}$$

Where we have

$$B(\alpha, \beta) = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1} dx$$

4.2 $E(X)$

$$E(X) = \frac{\alpha}{\alpha + \beta}$$

4.3 $Var(X)$

$$Var(X) = \frac{\alpha\beta}{(\alpha\beta)^2(\alpha + \beta + 1)}$$

5 Normal $X \sim N(\mu, \sigma^2)$

5.1 Probability Density Function $f(x)$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

Where we have $x \in \mathbb{R}$

5.2 Moment Generating Function $M(t)$

$$M(t) = exp\left[\mu t - \frac{\sigma^2 t^2}{2}\right]$$

5.3 $\boldsymbol{E(X)}$

$$E(X) = \mu$$

5.4 $\boldsymbol{Var(X)}$

$$Var(X) = \sigma^2$$

Also for normal distributions we can have that X can become the standard normal distribution $Z \sim N(0, 1)$ by the calculation

$$\frac{X - \mu}{\sigma^2}$$