MATH7501: Exercise 9 Solutions

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1 Question 1 (12 marks)

1.1 Test hypothesis that $\sigma_A = \sigma_B$ against the alternative that $\sigma_A \neq \sigma_B$ at 5% hyothesis level

To test $H_0 : \sigma_A = \sigma_B$ against $\sigma_A \neq \sigma_B$ the test statistic is $F = \frac{s_A^2}{s_B^2}$ as $\frac{s_A^2/\sigma_A^2}{s_B^2/\sigma_B^2}$ has a F distribution and in this case $\sigma_A = \sigma_B$

So under H_0 we have $F \sim F_{n_A^{-1}, n_B^{-1}}$ i.e. $F \sim F_{8,10}$ and we have that $\frac{1}{F} \sim F_{10,8}$ which is $\frac{s_B^2}{s_A^2}$

So using the tables we will reject H_0 if

$$\frac{s_A^2}{s_B^2} > 3.855$$

or if

$$\frac{s_B^2}{s_A^2} > 4.295 \text{ i.e.} \frac{s_A^2}{s_B^2} < 0.233 \text{ (3dp)}$$

To write it out clearer reject H_0 if

$$\left(\frac{s_A^2}{s_B^2} > 3.855\right) \vee \left(\frac{s_A^2}{s_B^2} < 0.233\right)$$

The observed value of F is $\frac{5100^2}{5900^2} = 0.747$ to 3 decimal places. This value is in the acceptance region for the test so we **DO NOT REJECT** H_0 and conclude that there is **NO EVIDENCE** that the standard deviation differs

1.2 Next hypothesis test on the mean lifetimes

Now assuming that $\sigma_A = \sigma_B$ we test that $H_0 : \mu_A = \mu_B$ against $H_1 : \mu_A \neq \mu_B$ The relevant test statistic is

$$T = \frac{\bar{X_A} - \bar{X_B}}{s_p\sqrt{\frac{1}{n_A} + \frac{1}{n_B}}}$$

where

$$s_p = \frac{(n_A - 1)s_A^2 + (n_B - 1)s_B^2}{n_A + n_B - 2}$$

Under $H_0 : T \sim t_{n_A+n_B-2}$ i.e. here $T \sim t_{18}$ (under H_0) So using the relevant table we will reject H_0 if

$$|T| > 2.101 \text{ (The upper 2.5\% of } t_{18})$$

i.e. if $T < -2.101$ or $T > 2.101$

The observed values of s_p and T are given by

$$s_p^2 = \frac{8 \times (5100^2) + 10 \times (5900^2)}{18}$$

$$= 3089889 \text{ approximately}$$

$$T = \frac{37900 - 39800}{s_p\sqrt{\frac{1}{9} + \frac{1}{11}}}$$

$$= -0.76 \text{ to 2 dp}$$

This lies within the acceptance region hence there is NO EVIDENCE for a difference between the lifetimes of the two tyre types

1.3 The 95% confidence interval

A 95% confidence interval for $\mu_A - \mu_B$ is given by

$$(\bar{x_A} - \bar{x_B}) \pm (t_{18})_{2.5\%} s_p \sqrt{\frac{1}{n_A} + \frac{1}{n_B}} \tag{1.3.1}$$

i.e.

$$(37900 - 39800) \pm (2.101) s_p \sqrt{\frac{1}{9} + \frac{1}{11}} \tag{1.3.2}$$

This leads to the interval $\boxed{(-7149.4, 3349.5)}$ km (approximately) for $\mu_A - \mu_B$

This interval includes zero agreeing with the hypthesis test result (the test for $\mu_A = \mu_B$ i.e. $\mu_A - \mu_B = 0$)

The interval 1.3.1 arises from

$$-2.101 < \frac{\bar{x}_A - \bar{x}_B - (\mu_A - \mu_B)}{s_p \sqrt{\frac{1}{n_A} + \frac{1}{n_B}}} < 2.101$$

$$(\bar{x}_A - \bar{x}_B) - 2.101 \sqrt{\frac{1}{n_A} + \frac{1}{n_B}} < \mu_A - \mu_B < (\bar{x}_A - \bar{x}_B) + 2.101 \sqrt{\frac{1}{n_A} + \frac{1}{n_B}}$$

2 Question 2 (8 marks)

Since the test scores between the first and second test are not independent we use a paired t-test. Let d be the improvement score for the i^{th} child and let

$$E(d_i) = \mu_D$$
$$Var(d_i) = \sigma_D^2$$

To test $H_0 : \mu_0$ against $H_1 : \mu_0 \neq 0$ we use the test statistic

$$T = \frac{\bar{d}}{s_D / \sqrt{n}}$$

which under H_0 is distributed by t_7. For a 5% test we reject H_0 if

$$|T| > 2.365$$

otherwise we do not reject.

Here $\bar{d} = 2$ and $s_D^2 = 7.142$ (3 dp). This leads to the observed of T being 2.117 (3 dp) we do not reject H_0 at the 5% significance level, there is no evidence to show that the new teaching method makes a difference

We can comment on the fact that it is the same group of children being tested. A better test could have been done if we had two samples of children one that used that the new methods and one did not.