MATH7501 Exercise sheet 2 — to be done by Friday 26th January

THIS SET OF EXERCISES IS THE FIRST ASSESSMENT

To assist the marking of this assessment, please write your name CLEARLY at the top of each sheet of paper, and indicate clearly (e.g. by underlining) the final answer to each part of a question.

All solutions must be your own work.

1. For each case below, state whether or not the binomial distribution is suitable. If it is, give the values of n and p. If it is not, explain why not.

   (a) The number of sixes obtained in three successive throws of a fair die.

   (b) The number of aces in a hand of four cards dealt from a standard pack.

   (c) The number of students in a class of 40 whose birthday falls on a Sunday this year.

   (d) The number of throws of a fair coin until the first head is obtained. **4 marks**

2. An exam paper consists of ten multiple choice questions, each offering four suggested answers, only one of which is correct. If a candidate chooses answers completely at random, what is the probability that

   (a) at least 8 correct answers are chosen?

   (b) the last of the ten answers given is the eighth one that is correct? **4 marks**

   Note: for full marks, here and elsewhere you are required to explain your reasoning clearly.

3. It costs £150 to repair a washing machine when it breaks down. Suppose that the number of washing machine breakdowns in a year has a geometric distribution with parameter 0.8.

   (a) Find the expected value and the variance of the total cost of repairs in a year.

   (b) The machine owner has the option of taking out a breakdown protection insurance policy. Under the terms of the policy, the insurance company will pay £100 towards the cost of repair whenever the machine breaks down. The policy itself costs £60 per year. Find the expected value and variance of the total cost of repairs (including the cost of taking out the policy) in this case. **5 marks**

4. Suppose that X has a binomial distribution with parameters n and p. Find the probability generating function of X, and use it to show that $E(X) = np$ and $Var(X) = np(1-p)$. **7 marks**

   (**Hint**: you will need to use the binomial expansion $(a+b)^m = \sum_{k=0}^{m} \binom{m}{k} a^k b^{m-k}$ for appropriate choices of a, b and m).