MATH7501 Exercise sheet 3 — to be done by Friday 2nd February

1. (a) A continuous random variable X has distribution function $F_X(\cdot)$. Let $Y = aX+b$, where a and b are constants. If $a > 0$, find an expression for $P(Y \leq y)$, giving your answer in terms of a, b and $F_X(\cdot)$. How, if at all, would this expression change if $a < 0$?

   (b) Suppose $X \sim U(0,1)$ and that $Y = aX + b$ with $a > 0$. Use the results from part (a), together with the expression given in the lecture notes for the distribution function of a uniform distribution, to find the distribution function of Y. Name the corresponding distribution and give the values of its parameters.

6 marks

2. For some value $\alpha > 1$, a continuous random variable X has probability density function

$$f(x) = \begin{cases} K/x^\alpha & x > 1 \\ 0 & \text{otherwise,} \end{cases}$$

where K is an appropriately chosen constant. Find K in terms of α. Without carrying out any detailed calculations, state the values of r for which the rth moment $E(X^r)$ exists. Explain your answer.

4 marks

3. In a digital camera, battery replacements are assumed to occur in a Poisson process of rate λ per hour of use. For long-life batteries, $\lambda = 0.1$ whereas for normal batteries, $\lambda = 0.3$. A customer buys a new camera along with a large pack of long-life batteries, and is surprised when she has to replace the batteries 3 times in the first 10 hours of use. Upon making some enquiries, she discovers that there are some counterfeit long-life batteries in circulation, which are normal batteries that have been repackaged. It is believed that 5% of long-life battery packets are counterfeit. The customer concludes that her batteries are counterfeits.

   (a) Use an appropriate probability calculation to determine whether or not the customer is justified in her conclusion.

   (b) Do you think the Poisson process model is appropriate in this kind of situation? Justify your answer.

7 marks

4. Find the moment generating function of the $U(a,b)$ distribution.

3 marks