MATH 7501: Probability and Statistics

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Exercise sheet 3 (completion)

Exercise 3, part (b)

Towards the end of the relevant problem class, we completed part (a) of the relevant exercise, by noting that the computation that was provided suggests that the customer is not justified in her conclusion, before noting that the Poisson process is probably not appropriate in the kind of situation described in this exercise.

A Poisson process involves assuming independence between non-overlapping time intervals; here, its use involves assuming that knowledge of the number of past battery replacements, e.g. in the last few hours, will not affect the probabilities of future battery replacements, e.g. in the next hour.

In practice, this assumption of independence probably does not hold in a situation such as the one described in this exercise. In particular, it is less likely that a new battery (i.e. a battery that has just been replaced) will require immediate replacement, when compared, for example, to a battery that has been in the camera for several hours.

As a result, it seems that the assumption of independence probably does not hold; the Poisson process is probably not appropriate in this kind of situation.

Exercise 4

Let us refer to the moment generating function of the $U(a,b)$ distribution as M.
We may then describe M by using:

$$M(t) \;=\; \int_{\mathbb{R}} e^{tx} f(x)\,dx$$

where f denotes the probability density function of the $U(a,b)$ distribution; this is given by:

$$f(x) \;=\; \frac{1}{b-a} \;\text{ for } a \leq x \leq b \quad , \quad f(x) \;=\; 0 \;\text{ otherwise}$$

By substituting this into the the integral above, we may compute $M(t)$, as required:

$$\begin{aligned}
M(t) \;&=\; \int_{\mathbb{R}} e^{tx} f(x)\,dx \\
&=\; \int_a^b \left(e^{tx} \times \frac{1}{b-a} \right) dx \\
&=\; \frac{1}{b-a} \int_a^b e^{tx}\,dx \\
&=\; \frac{1}{b-a} \left[\frac{e^{tx}}{t} \right]_a^b \qquad \text{for } t \neq 0 \\
&=\; \frac{1}{b-a} \left(\frac{e^{tb} - e^{ta}}{t} \right) \\
&=\; \frac{e^{tb} - e^{ta}}{t(b-a)}
\end{aligned}$$

Hence, we deduce that the moment generating function of the $U(a,b)$ distribution is given by

$$M(t) \;=\; \frac{e^{tb} - e^{ta}}{t(b-a)} \qquad \text{for } t \in \mathbb{R}\backslash\{0\}$$

We note that, when evaluating the required moment generating function at $t = 0$, we obtain:

$$M(0) \;=\; \int_{\mathbb{R}} f(x)\,dx \;=\; \int_a^b \frac{1}{b-a}\,dx \;=\; 1$$