

Find the reduced row echelon form and treating the input matrix as an augmented matrix, solve the corresponding system of equations.

In the first sample input, the system of equations corresponding to the matrix is:

$$x_1 + 2x_2 + 3x_3 = 1$$

$$4x_1 + 5x_2 + 6x_3 = 0$$

$$6x_1 + 8x_2 + 9x_3 = -1$$

Sample inputs:

1.

Input:

1,2,3,1

4,5,6,0

6,8,9,-1

Output:

The RREF is:

1,0,0,0

0,1,0,-2

0,0,1,5/3

The matrix rank is 3.

The system has a unique solution: (0,-2,5/3).

[1.67 instead of 5/3 is acceptable]

2.

Input:

0,2,3,1

0,1,1

4,0,5

-2,3,1

1,2,1

Output:

The RREF is:

1,0,0

0,1,0

0,0,1

0,0,0

0,0,0

The system has no solution.

3.

Input:

1,1,1

2,2,2

The RREF is:

1,1,1

0,0,0

The matrix rank is 1.

The system has infinitely many solutions, described by an affine subspace of dimension 2.

4.

Input:

0,2,3,0,-1,1

4,1,2,3,0,0

2,-2,0,0,1,-1

Output:

The RREF is:

1,0,0,9/11,3/22,-3/22

0,1,0,9/11,-4/11,4/11

0,0,1,-6/11,-1/11,1/11

The matrix rank is 3.

The system has infinitely many solutions, described by an affine subspace of dimension 2.

5.

Input:

0,2,3,0,0,1

4,1,-1,0,2,0

0,1,1,0,0,0

1,3,1,7,-2,4

Output:

The RREF is:

1,0,0,0,1/2,1/2
0,1,0,0,0,-1
0,0,1,0,0,1
0,0,0,1,-5/14,11/14

The matrix rank is 4.

The system has infinitely many solutions, described by an affine subspace of dimension 1