CBSE QUESTIONS

1 Calculus

1. Find the value of $\tan^{-1}\sqrt{3} - \cot^{-1}\left(-\sqrt{3}\right)$
2. Prove that:
$$3\sin^{-1}(x) = \sin^{-1}\left(3x^2 - 4x^3\right), x \in \left[-\frac{1}{2}, \quad \frac{1}{2}\right]$$
3. Differntiate $\tan^{-1}\left(\frac{1+\cos(x)}{\sin(x)}\right)$ with respect to x.
4. The total cost $C(x)$ associated with the production of x units of an item is given by $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$. Find the marginal cost when 3 units are produced, where by marginal cost we mean the instanteaneous rate of change of total cost at any level of output.
5. Evaluate:
$$\int \frac{cos2x + 2\sin^2 x}{\cos^2 x} dx$$
6. Find the differntial equation representing the family of curves $y = ae^{bx+c}$, where a and b are arbitrary constants.
7. If $\left(x^2 + y^2\right)^2 = xy$, find $\frac{dy}{dx}$.
8. If $x = a(2\theta - \sin(2\theta))$ and $y = a(1 - \cos(2\theta))$, find $\frac{dy}{dx}$ when $\theta = \frac{\pi}{3}$.
9. If $y = \sin(\sin(x))$, prove that $\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y\cos^2 x = 0$.
10. Find the equations of the tangent and the normal, to the curve $16x^2 + 9y^2 = 145$ at the point (x_1, y_1), where $x_1 = 2$ and $y_1 > 0$.
11. Find the intervals in which the function $f(x) = \frac{x^4}{4} - x^3 - 5x^2 + 24x + 12$ is (a) strictly increasing, (b) strictly decreasing.

12. An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of material will be least when depth of the tank is half of its width. If the cost is to be borne by nearby settled lower income families, for whom water will be provided, what kind of value is hidden in this question ?

13. Find:
$$\int \frac{2\cos x}{(1-\sin x)(1-\cos x)}dx$$

14. Find the particular solution of the differential equation $e^x \tan y dx + (2-e^x)\, sec^2 y dy = 0$, given that $y = \frac{\pi}{4}$ when $x = 0$.

15. Find the particular solution of the differential equation $\frac{dy}{dx} + 2y\tan x = \sin x$, given that $y = 0$ when $x = \frac{\pi}{3}$

16. Using integration, find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

17. Evaluate:
$$\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{16 + 9\sin 2x}dx$$

18. Evaluate:
$$\int_1^3 \left(x^2 + 3x + e^x\right) dx,$$
as the limit of the sum.