[This question paper contains 8 printed pages.]

Your Roll No. 22312915020

Sr. No. of Question Paper : 1907 **G**

Unique Paper Code : 3122612301

Name of the Paper : Modeling Continuous Changes Through Ordinary Differential Equations

Name of the Course : **B.Tech. (Information Technology & Mathematical Innovations**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt any **SIX** questions.

3. **All** questions carry equal marks.

P.T.O.

1. (a) Suppose that the population P(t) of a country satisfies the differential equation

$$\frac{dP}{dt} = kP(200 - P)$$

with k constant. Its population in 1940 was 100 million and was then growing at the rate of 1 million per year. Predict this country's population for the year 2000. (5)

(b) Construct a slope field for the differential equation $\frac{dy}{dx} = x^2 + y^2 - 1$ and use it to sketch an approximate solution curve that passes through the point origin. (5)

(c) Explain radius of convergence of a power series. Find the radius of convergence of the following series. (5)

$$\sum_{n=0}^{\infty} \frac{(-1)^n . 2^n}{n!} x^n$$

2. (a) Find a solution to the differential equation (8)

$$\frac{d^2x}{dt^2} - 3\frac{dx}{dt} - 4x = \sin 4t + 2e^{4t} + e^{5t} - t.$$

(b) Determine the eigenvalues and eigenfunctions of the boundary value problem

$$\frac{d^2y}{dx^2} + \mu y = 0$$

(7)

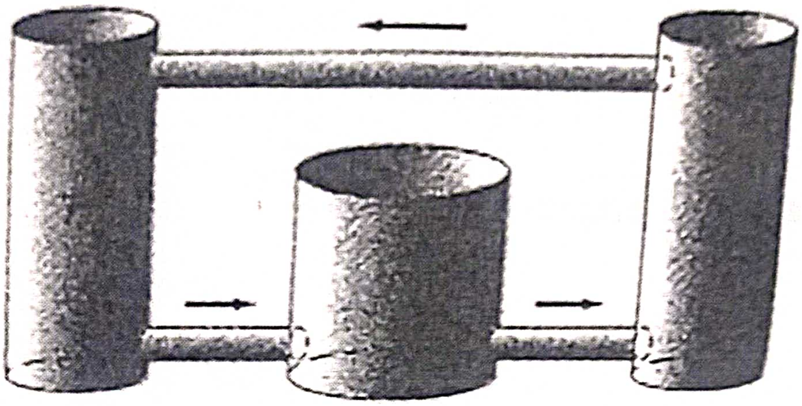
$$\frac{dy}{dx}(0) = 0, y(L) = 0.$$

3. (a) The roots of the characteristic equation of a certain differential equation are 0, 2, 3, −5, 0, 0, 0,1, $2 \pm 3i$ and $2 \pm 3i$. Write a general solution of this homogeneous differential equation. (5)

(b) Solve the following initial value problem using Laplace transformation method. (10)

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0, y(0) = 2, \frac{dy}{dx}(0) = -1.$$

4. Three 100-gal fermentation vats are connected as shown in the following figure and the mixtures in each tank are kept uniform by stirring. Suppose that the mixture circulates between the tanks at the rate of 10 gal/min.



Let $X_k(t)$ is the amount (in pounds) of alcohol in tank T_k at time t (k=1, 2, 3). Derive the mathematical equations and find the solution. (15)

5. Consider two species (of animals, plants, or bacteria, for instance) with population x(t) and y(t) at time t and which compete with each other for the food available in their common environment. We assume

that competition has the effect of a rate of decline in each population that is proportional to their product xy. The populations x(t) and y(t) satisfy the following differential equations

$$\frac{dx}{dt} = 60x - 4x^2 - 3xy$$
$$\frac{dy}{dt} = 42y - 2y^2 - 3xy$$

Find all critical points of the above competition system. Also, discuss the stability of the system. (15)

6. (a) Find an integrating factor for the equation

$$(3xy + y^2) + (xy + x^2)\frac{dy}{dx} = 0$$

and then solve the equation. (7)

(b) Find the general solution in powers of x of the differential equation (8)

$$(x^2 - 4)\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = 0.$$

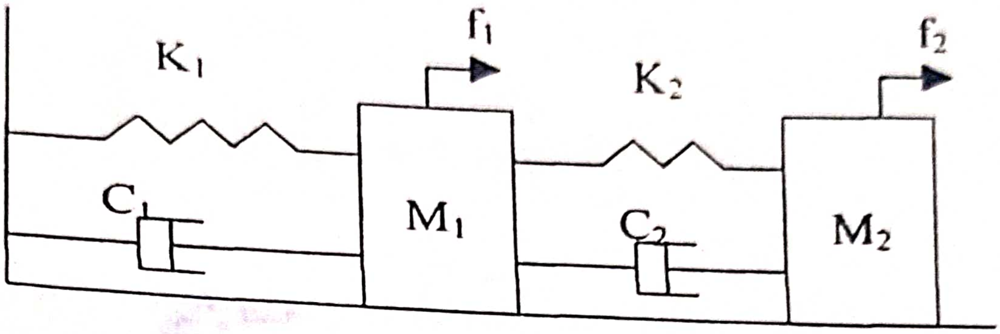
Also, find the particular solution with

$$y(0) = 4, \frac{dy}{dx}(0) = 1.$$

7. Suppose that the mass in a mass-spring-dashpot system with m=1, c=2, and k=100 is set in motion with $x(0) = 1, \frac{dx}{dt}(0) = -5$. Find the position function, frequency, period of motion and time lag. (15)

OR

Consider the following system with $C_1 = C_2 = 0.001$, $K_1 = K_2 = 1$, $M_1 = 1$ and $M_2 = 2$.



Write the ordinary differential equation of the motion and find the transfer function.