[This question paper contains 8 printed pages.]

Your Roll No. 22312915020

Sr. No. of Question Paper : 1337   **D**

Unique Paper Code   : 3122611102

Name of the Paper   : Discrete Mathematics and its Applications

Name of the Course   : **B.Tech. (Information Technology & Mathematical Innovation)**

Semester   : I

Duration : 3 Hours   Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. This question paper contains **seven** questions, out of which any **five** are to be attempted.

3. Each question carries equal marks. Marks for each question are indicated.

1. (a) A detective has interviewed four witnesses to a crime. From the stories of the witnesses the

P.T.O.

detective has concluded that if the butler is telling the truth then so is the cook; the cook and the gardener cannot both be telling the truth; the gardener and the handyman are not both lying; and if the handyman is telling the truth then the cook is lying. For each of the four witnesses, can the detective determine whether that person is telling the truth or lying? Explain your reasoning. (9)

(b) A deposit of $ 100,000 is made to an investment fund at the beginning of a year. On the last day of each year two dividends are awarded. The first dividend is 20% of the amount in the account during that year. The second dividend is 45% of the amount in the account in the previous year. Find a recurrence relation for $\{P_n\}$, where P_n is the amount in the account at the end of n years if no money is ever withdrawn. How much is in the account after n years if no money has been withdrawn? (9)

2. (a) Suppose that a computer science laboratory has 15 workstations and 10 servers. A cable can be used to directly connect a workstation to a server. For each server, only one direct connection to that server can be active at any time. We want to

guarantee that at any time any set of 10 or fewer workstations can simultaneously access different servers via direct connections. Although we could do this by connecting every workstation directly to every server (using 150 connections), what is the minimum number of direct connections needed to achieve this goal? (9)

(b) The Head of the Department of Mathematical Sciences at a certain university has 12 mathematicians, seven computer scientists, and three statisticians in his employ. He wishes to appoint some committees from among these 22 people. How many five- member committees can he appoint? How many five-member committees, each containing at least one statistician, can he appoint? A certain professor of mathematics, Dr. G, and a certain colleague, Dr. P, refuse to serve together on the same committee. How many five-member committees can be formed so as not to contain both Dr. G and Dr. P? How many five-member committees can be formed so that the number of mathematicians is greater than the number of computer scientists and the number of computer scientists is greater than the number of statisticians? (9)

3. (a) You and a friend meet three other couples at a party and several handshakes take place. Nobody shakes hands with himself or herself, there are no handshakes within couples, and no one shakes hands with the same person more than once. The numbers of hands shaken by the other seven people (excluding you) are all different. How many hands did you shake? How many hands did your partner shake? Use a graph to aid your solution.

(9)

(b) The weighted graphs in the figure given below; show some major roads of Haryana and the vertices represent the cities connected by that road. If the weights on each arc represent the time, in hours, required to travel along each road, find the shortest distance between two cities A and E.

(9)
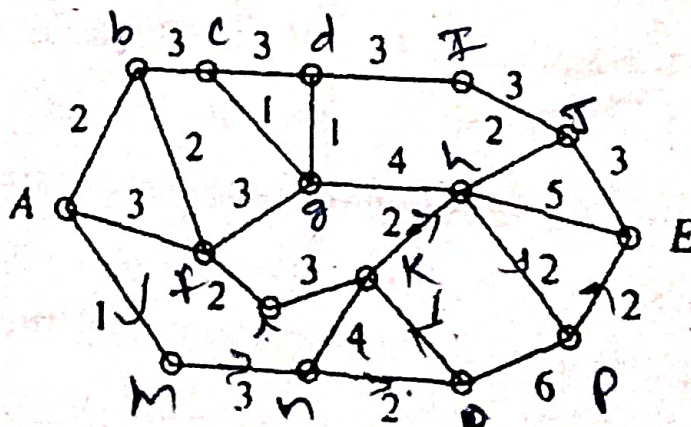
4. (a) Solve the Chinese Postman Problem for the given graph. (9)
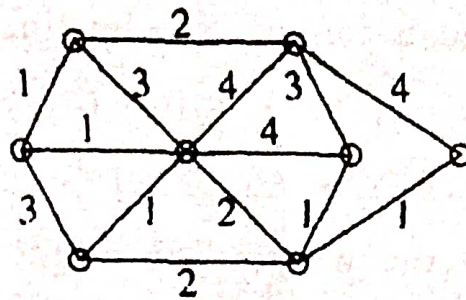
(b) The following chart lists a number of tasks that must be completed in order for a crew of workers to construct a glynskz.

Task	A B C D E F G H I J
Time (in days)	2 2 3 1 1 2 3 4 3 3

Task A must be carried out before any other tasks can commence. Task B must precede tasks E and F, and both E and F must be completed before H can begin. Tasks C and D must precede task G, which in turn must precede I. Task J must be carried out last. It is assumed that there are enough workers to carry out any number of tasks simultaneously. What is the fewest number of days needed to construct this glynskz? Describe the critical path and illustrate with a directed network, showing all labels, also calculate the slack for tasks, not on the critical path. (9)

5. (a) A company plans to build a communications network connecting its five computer centers. Any pair of these centers can be linked with a leased telephone line. The following network shows 6 computer centers, edges represent possible leased lines, and the weights on edges are the monthly lease rates, in thousand rupees, of the lines. Which links should be made to ensure that there is a path between any two computer centers so that the total cost of the network is minimized? (9)
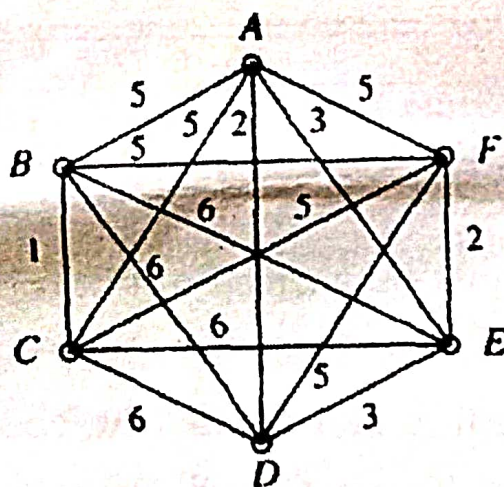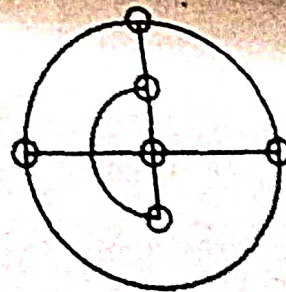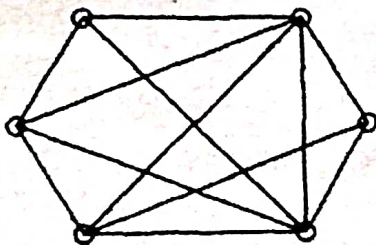

(b) Given that the digraph is acyclic and in canonical labelling. Find the lengths of shortest paths from the root v_0 to all other vertices, specifying the predecessor vertex (as defined in Bellman's algorithm). (9)
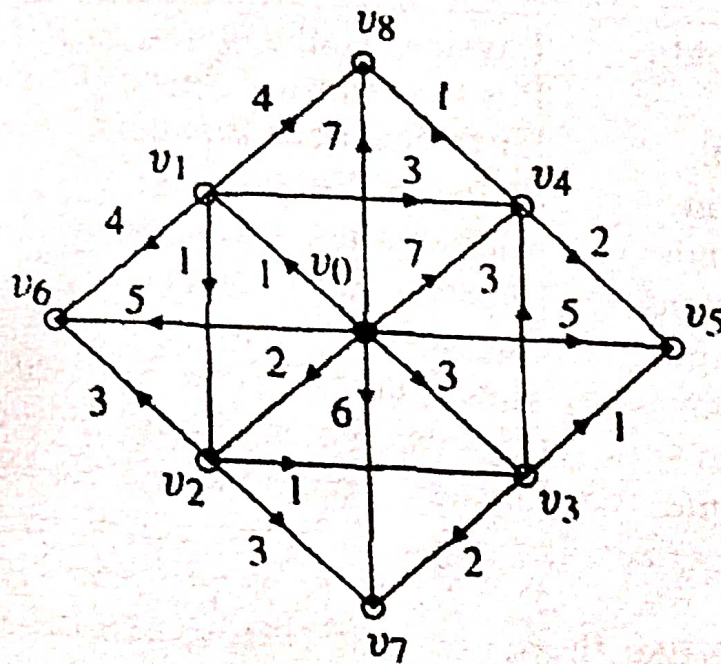
(a) Use Kuratowski theorem to determine whether the given graphs are planar or not. (9)

(b) There are ten students who, in the coming semester, will be taking the courses shown in the following table. How many time periods must be allowed in order for these students to take the courses they want without conflicts? (9)

Arnold	Physics, Mathematics, English
Bill	Physics, Earth Science, Economics
Carol	Earth Science, Business
Calvin	Statistics, Economics
Eleanor	Mathematics, Business
Frederick	Physics, Earth Science
George	Business, Statistics
Huber	Mathematics, Earth Science
Ingrid	Physics, Water Skiing, Statistics
Jacquie	Physics, Economics, Water Skiing

7. Suppose that there are four employees in the computer support group of the School of Engineering of a large university. Each employee will be assigned to support one of four different areas: hardware, software, networking, and wireless. Suppose that Ping is qualified to support hardware, networking, and wireless; Quiggley is qualified to support software and networking; Ruiz is qualified to support networking and wireless, and Sitea is qualified to support hardware and software.

(a) Use a bipartite graph to model the four employees and their qualifications.

(6)

(b) Use Hall's theorem to determine whether there is an assignment of employees to support areas so that each employee is assigned one area to support.

(6)

(c) If an assignment of employees to support areas so that each employee is assigned to one support area exists, find one.

(6)