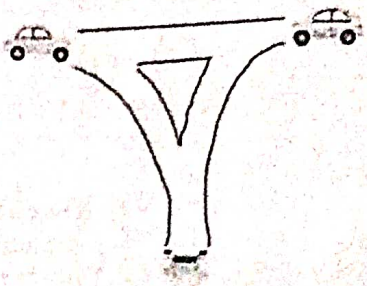
Cluster Innovation Centre, University of Delhi, Delhi-110007

End Semester Examination – May 2024

Name of the Course	: B.Tech (Information Technology and Mathematical Innovations)
Semester	: IV
Paper Title	: Applied Probability & Statistics
Paper Code	: MA 312261240 1
Maximum Marks	: 90
Instructions	Duration: 3 Hours All questions are compulsory. Each question carries 18 marks. Use of scientific calculator is allowed but not on mobile.

Q1) Attempt any two parts

a. Three cars arrive at a tri-junction. There is an equal chance of them taking the two routes available to them. Draw a tree diagram to represent the probability of each possible outcome. Under what outcomes can you expect them to crash? (These are just single lane paths). **(9 marks)**



A random variable X has the following distribution function

x	-2	-1	0	1	2
$f_X(x)$	1/8	2/8	2/8	2/8	c

i. For the above to be a probability distribution function, what should be the value of c?
ii. Find the cumulative distribution function of the random variable X.
iii. Find the following probability

(i) $P(X \le 2)$ (ii) $P(0 \le X \le 1)$ (iii) $P(-0.5 \le X \le 1.5)$

(3 + 3 + 3 marks)

c. A random variable X has the moment generating function

$$m_x(t) = e^t\left(\frac{1}{2}\right) + e^{2t}\left(\frac{1}{3}\right) + e^{3t}\left(\frac{1}{6}\right)$$

a. Find the mean and the variance of the random variable
b. Define the probability distribution function and hence find the cumulative distribution function of X.

(4 + 5 marks)

Q2) Attempt any one part

a. A particularly long traffic light on your morning commute is green 20% of the time that you approach it. Assume that each morning represents an independent trial.
   i. Over 20 mornings, what is the probability that the light is green on more than four days?
   ii. What is the probability that you will find the first green light on your 5th day?
   iii. What is the probability that your 5th green light will be on the 20th day?

(6 + 6 + 6 marks)

b. a and b are two positive integers. A random variable takes all integer values from a to b. If it follows a discrete uniform distribution, find its mean, variance and moment generating function. **(6 + 6 + 6 marks)**

Q3) Attempt any two parts

a. The serum cholesterol level (X) of males aged between 25–34 follows a distribution with pdf

$$\frac{(x-80)}{2500} e^{-(x-80)^2/50}, \quad 80 < x < \infty.$$

i. What are the mean and the variance of this distribution?
ii. Find M so that $P(X < M) = 0.8$

iii. What percentage of males has a serum cholesterol level less than 200?

(5 + 2 + 2 marks)

b. The line width of for semiconductor manufacturing is assumed to be normally distributed with a mean of 0.5 micrometer and a standard deviation of 0.05 micrometer.

i. What is the probability that a line width is greater than 0.62 micrometer?

ii. What is the probability that a line width is between 0.47 and 0.63 micrometer?

iii. The line width of 90% of samples is below what value? **(3 + 3 + 3 marks)**

c. Some college professors and students examined 137 Canadian geese for patent schistosome in the year they hatched. Of these 137 birds, 54 were infected. The professors and students were interested in estimating p, the proportion of infected birds of this type.

i. Find a 95% confidence interval for p

ii. For future studies, determine the sample size n so that the estimate of p is within $\varepsilon =$ 0.04 of the unknown p with 90% confidence. **(4 + 5 marks)**

Q4) Attempt any one part

a. A manufacturing company employs two inspecting devices to sample a fraction of their output for quality control purposes. The first inspection monitor is able to accurately detect 99.3% of the defective items it receives, whereas the second is able to do so in 99.7% of the cases. Assume that four defective items are produced and sent out for inspection. Let X and Y denote the number of items that will be identified as defective by inspecting devices 1 and 2, respectively. Assume the devices are independent.

i. Determine the joint distribution function of X and Y

ii. Find the distribution function of Y given X =2.

iii. Find E[Y|X = 2] and Var[Y|X = 2] **(6 + 6 + 6 marks)**

b. Consider the function $f(x, y) = ce^{-0.001(x+2y)}$ for $x \le y$ and $y < \infty$.

i. Determine the value of c such that the function $f(x, y) = ce^{-0.001(x+2y)}$ for $x \le y$ and $y < \infty$ satisfies the properties of a joint probability density function.

ii. Determine the marginal probability distribution of the random variable X and Y

iii. Find the joint mgf of X and Y and the mgf of X and Y respectively. Using the result state whether X and Y are independent or not. **(5 + 6 + 7 marks)**

Q5) Attempt any one part

a. A manufacturer is interested in the output voltage of a power supply used in a PC. Output voltage is assumed to be normally distributed, with standard deviation 0.25 Volts, and the manufacturer wishes to test $H_0 : \mu = 5$ Volts by using a sample of 8.

i. If the average output voltage of these 8 units is measured to be 4.9 Volts, is there evidence to support the claim that the average output voltage is less than 5 Volts at 5% level of confidence?

ii. If the acceptable region is (4.85, 5.15) what should be the level of confidence? Compute the power of the test if the true mean battery life is 4.5 hours.

iii. Find the power of the test for detecting a true mean output voltage of 5.1 Volts. **(6 + 6 + 6 marks)**

b. An annual report of CIC claimed that nearly one-third of all its B.Tech students continue acaemic studies beyond the B.Tech degree, ultimately receiving either an M.S. or a Ph.D. degree. The alumni data available on the website indicate that in the last five years 51 of 211 went in for higher studies.

i. Is the claim made in the annual report of *CIC* consistent with the alumni data available on the website at 5% level of confidence?

ii. Find the P-value for the test.

iii. If the actual proportion of the students going for the higher studies is 0.3, what is the power of the test? **(6 + 6 + 6 marks)**

Discrete Distributions

Binomial Distribution $f_x(x) = {}^nC_x p^x (1-p)^{n-x}, \quad x = 0,1,2,\ldots n$

Poisson Distribution $f_x(x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0,1,2,\ldots$

Geometric Distribution $f_x(x) = p(1-p)^x, \quad x = 0,1,2,\ldots$

Hypergeometric Distribution $f_x(x) = \frac{{}^KC_x \, {}^{N-K}C_{n-x}}{{}^NC_n}, \quad K,n = 0,1,2,\ldots N$

Negative Binomial Distribution $f_x(x) = {}^{x+n-1}C_x p^n (1-p)^x, \quad x = 0,1,2,\ldots$

S. ıdard Normal Cumulative Probability Table

Cumulative probabilities for NEGATIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641