

AI1103-Assignment 2

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Download all python codes from

https://github.com/DineshAvulaMohanaDurga/AI1103/blob/main/assignment_2/codes/ai1103_assignment2.py

and latex codes from

https://github.com/DineshAvulaMohanaDurga/AI1103/blob/main/assignment_2/main.tex

1 QUESTION

(GATE-1999 problem-1.31) The joint probability density function of the random variables X, Y and Z is

$$\begin{cases} f(x, y, z) = 8xyz, 0 < x, y, z < 1 \\ = 0 \text{ otherwise} \end{cases}$$

Then $P(X < Y < Z)$ is

- 1) $\frac{1}{8}$
- 2) $\frac{1}{3}$
- 3) $\frac{1}{6}$
- 4) $\frac{1}{2}$

2 ANSWER

Given joint probability density function j.d.f

$$\begin{cases} f(x, y, z) = 8xyz, 0 < x, y, z < 1 \\ = 0 \text{ otherwise} \end{cases}$$

we know that if probability distribution function (p.d.f) = $f(X)$ then

$$\Pr(X \leq x) = \int_{-\infty}^x f(x) dx \quad (2.0.1)$$

from the given probability function we can say that $0 < x, y, z < 1$ as the probability is zero otherwise, So

$$\Pr(x < y < z) = \int_{z=0}^1 \int_{y=0}^z \int_{x=0}^y f(x, y, z) dx dy dz \quad (2.0.2)$$

$$= \int_{z=0}^1 \int_{y=0}^z 8yz \left(\int_{x=0}^y x dx \right) dy dz$$

$$= \int_{z=0}^1 4z \left(\int_{y=0}^z y^3 dy \right) dz$$

$$= \int_{z=0}^1 z^5 dz$$

$$= \frac{1}{6} \quad (2.0.3)$$

\therefore The value of $\Pr(X < Y < Z)$ is $\frac{1}{6}$

\therefore option C is correct

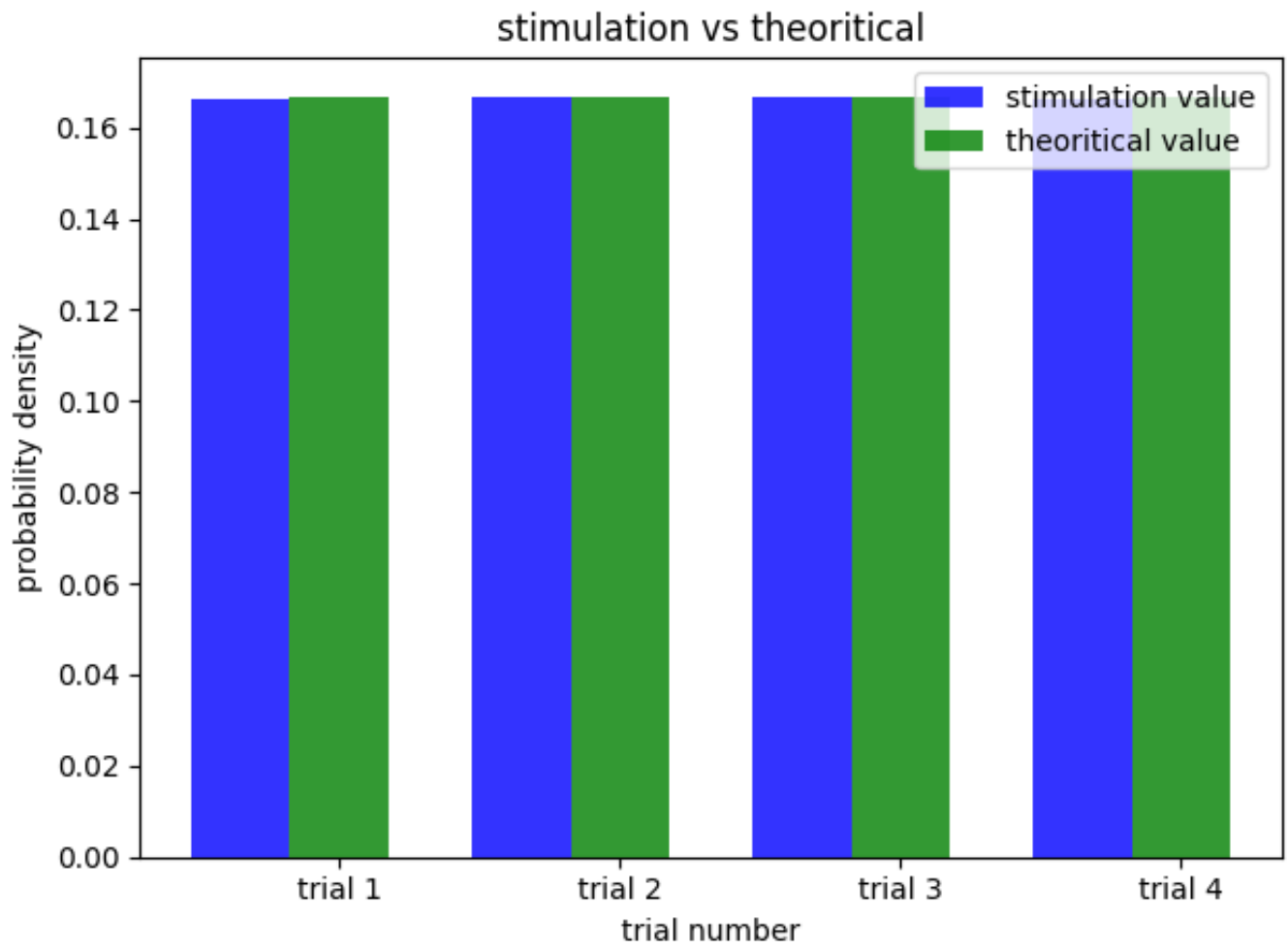


Fig. 4: Simulation vs Theoretical