

AI1103-Assignment 4

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Download all python codes from

https://github.com/DineshAvulaMohanaDurga/AI1103/blob/main/assignment_4_new/codes/ai1103_assignment4_new.py

and latex codes from

https://github.com/DineshAvulaMohanaDurga/AI1103/blob/main/assignment_4_new/main.tex

∴ The total probability is 1

$$\int_0^{\infty} f(X) dX = 1 \quad (2.0.5)$$

$$\int_0^{\infty} \frac{1}{\theta} e^{-x/\theta} dX = 1$$

$$\frac{1}{\theta} = 1$$

$$\Rightarrow \theta = 1 \quad (2.0.6)$$

$$\text{So } f(X) = e^{-X} \quad (2.0.7)$$

Hence lemma 2.2 is proved. \square

Lemma 2.3. pdf of Y is

$$p(Y = k) = e^{-k} (1 - e^{-1}) \quad (2.0.8)$$

and is in the form of geometric distribution.

Proof. Also given that $Y=k$ if $k \leq X < k+1$
 $k=0,1,2,\dots$

$$p(Y = k) = \int_k^{k+1} p(X = x) dx \quad (2.0.9)$$

$$= \int_k^{k+1} e^{-x} dx \quad (2.0.10)$$

$$= e^{-k} (1 - e^{-1})$$

Hence using lemma 2.1 and (2.0.10) lemma 2.3 is proved. \square

1 QUESTION

(CSIR-UGC-NET June 2015 Q 110) Suppose X has density $f\left(\frac{x}{\theta}\right) = \frac{1}{\theta} e^{-x/\theta}, x > 0$ where $\theta > 0$ is unknown. Define Y as follows:

$Y=k$ if $k \leq X < k+1, k=0,1,2,\dots$

Then the distribution of Y is :

- 1) Normal
- 2) Binomial
- 3) Poisson
- 4) Geometric

2 ANSWER

Lemma 2.1. If the pdf of X is in the form of

$$p(X = x) = (1 - p)^{x-1} p \quad (2.0.1)$$

Then the distribution is said to be in the form of geometric distribution.

Lemma 2.2. pdf of X is

$$P(X = x) = e^{-x} \quad (2.0.2)$$

Proof. Given pdf of X is

$$f\left(\frac{x}{\theta}\right) = \frac{1}{\theta} e^{-x/\theta}, x > 0, \text{ where } \theta > 0 \text{ is unknown} \quad (2.0.3)$$

$$f(X) = \frac{1}{\theta} e^{-X/\theta}, X > 0 \text{ as } x > 0, \theta > 0 \quad (2.0.4)$$

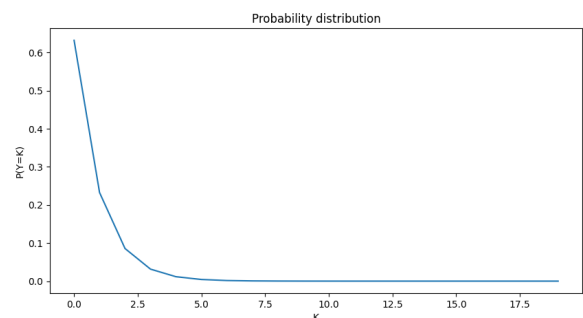


Fig. 4: Probability distribution of Y