CSIR-UGC NET-June 2015-Question-110

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Question

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Suppose X has density $f(x|\theta) = \frac{1}{\theta}e^{-x/\theta}, x > 0$ where $\theta > 0$ is unknown.Define Y as follows:

$$Y = k \text{ if } k \leq X < k+1, k = 0, 1, 2 \cdots \tag{1}$$

Then the distribution of Y is :

1. Normal
2. Binomial
3. Poisson
4. Geometric

Definitions

Definition of Normal Distribution

The distributions which have probability distribution function in the form of

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \tag{2}$$

are known as normal distribution.
Where μ is mean(and also median and mode) and σ is standard distribution of *PDF*.

Other Properties of Normal Distribution

1. Normal distribution is also known as Gauss or Gaussian or Laplace-Gauss distribution.
2. It is a symmetric distribution function about its mean.

Definitions Contd.

Binomial Distribution

Binomial distribution is a common probability distribution that models the probability of obtaining one of two outcomes under a given number of parameters.It's probability distribution function is in the form of

$$f(k, n, p) = \Pr(k, n, p) = \Pr(X = k) = \binom{n}{k} p^k (1-p)^{n-k} \tag{3}$$

Where

- k is the number of occurences.
- p is the probability of an outcome being true
- n is total number of trials

Definitions Contd.

Poisson distribution

A random variable X is said to have poisson distribution with parameter $\lambda > 0$,if it has probability mass function in the form of

$$f(k,\lambda) = \Pr(k,\lambda) = \frac{\lambda^k e^{-\lambda}}{k!} \tag{4}$$

Where

- k is the number of occurences($k = 0, 1, 2..$).
- e is the eulers number.($e = 2.71828$)

Definitions Contd.

Geometric distribiution

A distribution is said to be a geometric distribution if it is one of the two following distributions

1. The probability distribution of X number of bernoulli trials needed to get one success, supported on the set $(1, 2, 3..)$

$$\Pr(X = k) = (1-p)^{k-1} p, (k = 1, 2, 3, \cdots) \tag{5}$$

2. The probability distribution of number Y=X-1 of failures before the first success, supported on the set $(0, 1, 2, 3..)$

$$\Pr(Y = k) = \Pr(X = k+1) = (1-p)^{k} p, (k = 0, 1, 2, \cdots) \tag{6}$$

Solution

Lemma

PDF of X is

$$\Pr(X) = \frac{1}{\theta} e^{-\frac{x}{\theta}} \tag{7}$$

Proof.

Given *PDF* of X is

$$f(x|\theta) = \frac{1}{\theta} e^{-x/\theta}, x > 0, \text{where } \theta > 0 \text{ is unknown} \tag{8}$$

$$f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}} \text{since x and } \theta \text{ are independent} \tag{9}$$

Hence lemma 2.1 is proved. □

Solution Contd.

Lemma

PDF of Y is in the form of geometric distribution and is

$$p(Y=k)=e^{-\frac{k}{\theta}}\left(1-e^{-\frac{k}{\theta}}\right) \tag{10}$$

Proof.

Also given that Y=k if $k \leq X < k+1$ k=0,1,2,$\cdots$

$$p(Y=k)=\int_{k}^{k+1} p(X=x)\,dx \tag{11}$$

$$=\int_{k}^{k+1} \frac{1}{\theta} e^{-\frac{x}{\theta}}\,dx$$

$$=e^{-\frac{k}{\theta}}\left(1-e^{-\frac{k}{\theta}}\right) \tag{12}$$

Hence using (6) and (12) lemma 2.2 is proved. □

Example figure

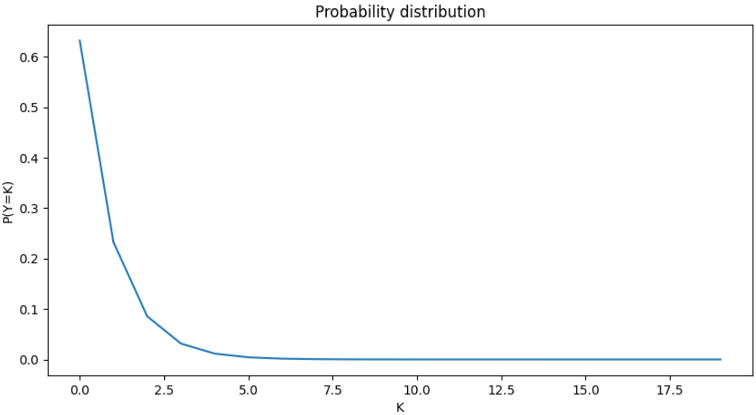

Figure: Probability distribution of Y