AI1103-Assignment 2

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Download all python codes from

```
https://github.com/DineshAvulaMohanaDurga/
    AI1103/blob/main/assignment_3/codes/
    ai1103_assignment3.py
```

and latex codes from

```
https://github.com/DineshAvulaMohanaDurga/
    AI1103/blob/main/assignment_3/main.tex
```

1 Question

$(GATE - 2021\,(ME, set - 1)\,problem - 5)$
Consider a binomial random variable X. If $X_1, X_2, ..., X_n$ are independent and identically distributed samples from the distribution of $\mathbf{X}$ with sum Y=$\Sigma_{i=1}^{n} X_i$, then the distribution of $\mathbf{Y}$ as $n \to \infty$ can be approximated as

1) Exponential
2) Bernoulli
3) Binomial
4) Normal

2 Answer

Given a binomial random variable $\mathbf{X}$

$$\Rightarrow \Pr(X = x) = \binom{n}{x} p^x q^{n-x} \tag{2.0.1}$$

also given that $X_1, X_2, ..., X_n$ are independent and identically distributed samples

$$\Rightarrow \Pr(X_i = x) = \binom{n}{x} p_i^x q_i^{n-x} \tag{2.0.2}$$

also given that

$$Y = \Sigma_{i=1}^{n} X_i \tag{2.0.3}$$

$$\Rightarrow \Pr(Y = nx) = \Sigma_{i=1}^{n} \binom{n}{x} p_i^x q_i^{n-x} \tag{2.0.4}$$

Mean of Y:-

$$\text{Mean of Y}\,(\mu_Y, E(Y)) = \frac{\Sigma_{i=1}^{n} \text{Mean of } X_i\,(\mu_{X_i})}{n} \tag{2.0.5}$$

$$= \frac{\Sigma_{i=1}^{n} E(X_i)}{n}$$

$$= \frac{\Sigma_{i=1}^{n} n p_i}{n}$$

$$= \Sigma_{i=1}^{n} p_i \tag{2.0.6}$$

Variance of Y:-

$$\text{Variance of Y}\,(\sigma_Y) = (E(Y))^2 - E\left(Y^2\right) \tag{2.0.7}$$

$$= \left(\Sigma_{i=1}^{n} p_i\right)^2 - \frac{\Sigma_{i=1}^{n} E\left(X_i^2\right)}{n}$$

$$= \left(\Sigma_{i=1}^{n} p_i\right)^2 - \frac{\Sigma_{i=1}^{n} n p_i q_i}{n}$$

$$= \left(\Sigma_{i=1}^{n} p_i\right)^2 - \Sigma_{i=1}^{n} p_i q_i \tag{2.0.8}$$

Central Limit Theorem:-
It states that "In probability theory, the central limit theorem (CLT) establishes that, in many situations, when independent random variables are added, their properly normalized sum tends toward a normal distribution (or Gaussian distribution informally a bell curve) even if the original variables themselves are not normally distributed".

So here as X_i are independent variables we can apply CLT to all X_i's and the outcome is distribution of Y which is according to CLT a **normal distribution**.

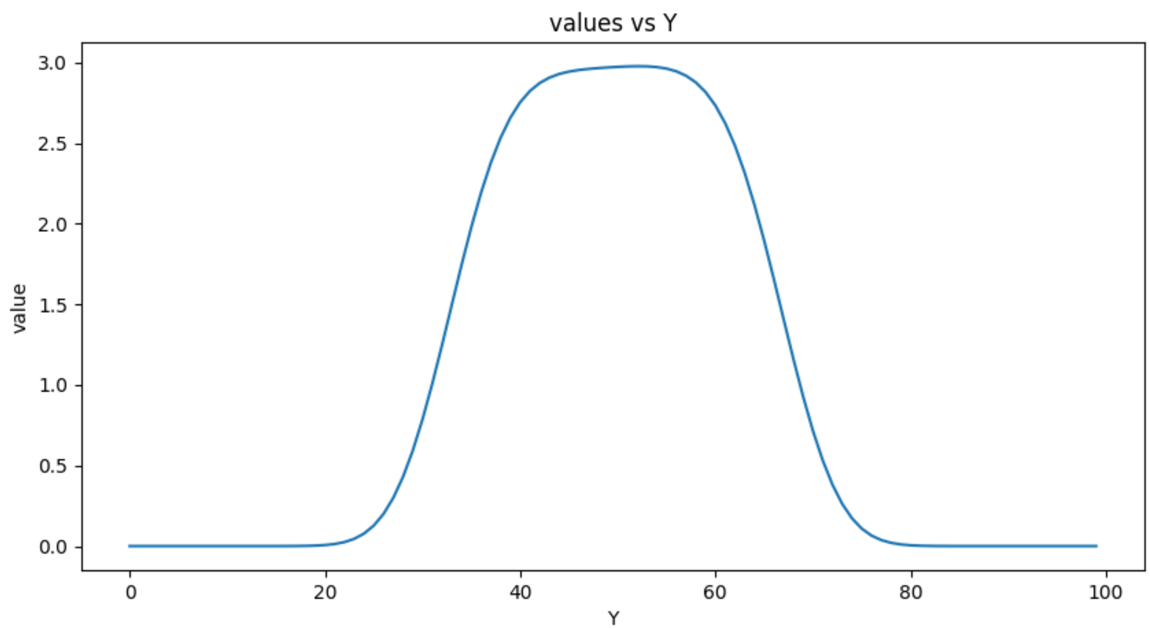

Fig. 4: distribution of Y