AI1103-Assignment 4

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Download all Latex codes from

https://github.com/DineshAvulaMohanaDurga/AI1103/blob/main/assignment_4/main.tex

1 Question

(CSIR-UGC-NET June 2015 Q 104)Let X and Y be random variables with joint cumulative distribution function F(x, y). Then which of the following cnditions are sufficient for $(x, y) \in R^2$ to be a point of continuity of F?

1) P$(X = x, Y = y)$=0
2) Either P$(X = x)$=0 or P$(Y = y)$=0.
3) P$(X = x)$=0 and P$(Y = y)$=0.
4) P$(X = x, Y \leq y)$=0 and P$(X \leq x, Y = y)$=0.

2 Answer

Let F(x, y) be joint cumulative distribution function and P(x, y) be joint probability distribution function. From the definitoion of cummulative distribution function

$$
\begin{aligned}
pdf &= \frac{d}{dx} cdf \\
P(X) &= \frac{d}{dx} F(X) \\
&or \\
F(x) &= \int_{X=-\infty}^{x} P(X)\, dX && (2.0.1) \\
\Rightarrow F(x, y) &= \int_{Y=-\infty}^{y} \int_{X=-\infty}^{x} P(X, Y)\, dX dY && (2.0.2)
\end{aligned}
$$

For F to be continuous at (x, y)

$$\lim_{k\to 0}\lim_{h\to 0} \left(F(x+h, y+k) - F(x, y)\right) = 0 \quad (2.0.3)$$

$$
\begin{aligned}
\lim_{k\to 0}\lim_{h\to 0} &\int_{Y=-\infty}^{y+k} \int_{X=-\infty}^{x+h} P(X, Y)\, dX dY \\
&- \int_{Y=-\infty}^{y} \int_{X=-\infty}^{x} P(X, Y)\, dX dY = 0 \quad (2.0.4)
\end{aligned}
$$

On expanding the limits of integrals we get

$$
\begin{aligned}
\lim_{k\to 0}\lim_{h\to 0} &\int_{Y=y}^{y+k} \int_{X=-\infty}^{x} P(X, Y)\, dX dY \\
&+ \int_{Y=-\infty}^{y} \int_{X=x}^{x+h} P(X, Y)\, dX dY \\
&+ \int_{Y=y}^{y+k} \int_{X=x}^{x+h} P(X, Y)\, dX dY = 0 \quad (2.0.5)
\end{aligned}
$$

Which is zero if P(X, Y) is defined over the integrating region i.e P(X, Y) should be defined over the intergrating region of

1) $$\lim_{k\to 0}\lim_{h\to 0} \int_{Y=y}^{y+k} \int_{X=-\infty}^{x} P(X, Y)\, dX dY$$
2) $$\lim_{k\to 0}\lim_{h\to 0} \int_{Y=-\infty}^{y} \int_{X=x}^{x+h} P(X, Y)\, dX dY$$
3) $$\lim_{k\to 0}\lim_{h\to 0} \int_{Y=y}^{y+k} \int_{X=x}^{x+h} P(X, Y)\, dX dY$$

The integrating regions tend to be equal to

1) $-\infty < X < x, Y = y$
2) X=x,$-\infty < Y < y$
3) point (x, y)

Let these be regions 1,2,3 respectively

$\Rightarrow$ P(X, Y) should be defined over regions 1,2,3. i.e

1) P$(-\infty < X < x, Y = y)$
2) P$(X = x, -\infty < Y < y)$
3) P(x, y) should be defined

Which is satisfied by options 3,4.

Let's consider an example similar to dirac-delta function.

$$
\begin{aligned}
G(X, Y) &= \infty \qquad (X, Y) = (x, y_1) \text{ where } y_1 > y \\
&= 0 \qquad \text{otherwise} \qquad (2.0.6)
\end{aligned}
$$

example satisfies options 1,2 but F(x, y) fails to be continous at (x, y)