AI1103-Assignment 4

Name: Avula Mohana Durga Dinesh Reddy , Roll Number: CS20BTECH11005

Download all Latex codes from

https://github.com/DineshAvulaMohanaDurga/AI1103/blob/main/assignment_4/main.tex

1 Question

(CSIR-UGC-NET June 2015 Q 104)Let X and Y be random variables with joint cumulative distribution function F(x, y). Then which of the following cnditions are sufficient for $(x, y) \in R^2$ to be a point of continuity of F?

1) P$(X = x, Y = y)$=0
2) Either P$(X = x)$=0 or P$(Y = y)$=0.
3) P$(X = x)$=0 and P$(Y = y)$=0.
4) P$(X = x, Y \leq y)$=0 and P$(X \leq x, Y = y)$=0.

2 Answer

Let F(x, y) be joint cumulative distribution function and P(x, y) be joint probability distribution function. From the definitoion of cummulative distribution function

$$pdf = \frac{d}{dx} cdf$$

$$P(X) = \frac{d}{dx} F(X)$$

$$or$$

$$F(x) = \int_{X=-\infty}^{x} P(X)\, dX \tag{2.0.1}$$

$$\Rightarrow F(x, y) = \int_{Y=-\infty}^{y} \int_{X=-\infty}^{x} P(X, Y)\, dXdY \tag{2.0.2}$$

For F to be continuous at (x, y)

$$\lim_{k\to 0} \lim_{h\to 0} (F(x + h, y + k) - F(x, y)) = 0 \tag{2.0.3}$$

$$\lim_{k\to 0} \lim_{h\to 0} \int_{Y=-\infty}^{y+k} \int_{X=-\infty}^{x+h} P(X, Y)\, dXdY - \int_{Y=-\infty}^{y} \int_{X=-\infty}^{x} P(X, Y)\, dXdY = 0 \tag{2.0.4}$$

On expanding the limits of integrals we get

$$\lim_{k\to 0} \lim_{h\to 0} \int_{Y=y}^{y+k} \int_{X=-\infty}^{x} P(X, Y)\, dXdY + \int_{Y=-\infty}^{y} \int_{X=x}^{x+k} P(X, Y)\, dXdY + \int_{Y=y}^{y+k} \int_{X=x}^{x+h} P(X, Y)\, dXdY = 0 \tag{2.0.5}$$

Which is zero if P(X, Y) is defined over the integrating region i.e P(X, Y) should be defined over the intergrating region of

1) $$\lim_{k\to 0} \lim_{h\to 0} \int_{Y=y}^{y+k} \int_{X=-\infty}^{x} P(X, Y)\, dXdY$$
2) $$\lim_{k\to 0} \lim_{h\to 0} \int_{Y=-\infty}^{y} \int_{X=x}^{x+k} P(X, Y)\, dXdY$$
3) $$\lim_{k\to 0} \lim_{h\to 0} \int_{Y=y}^{y+k} \int_{X=x}^{x+h} P(X, Y)\, dXdY$$

The integrating regions tend to be equal to

1) $-\infty < X < x, Y = y$
2) X=x,$-\infty < Y < y$
3) point (x, y)

Let these be regions 1,2,3 respectively
$\Rightarrow$ P(X, Y) should be defined over regions 1,2,3. i.e

1) P$(-\infty < X < x, Y = y)$
2) P$(X = x, -\infty < Y < y)$
3) P(x, y) should be defined

Which is satisfied by options 3,4.