AI1103-Assignment 4

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Download all python codes from

https://github.com/DineshAvulaMohanaDurga/AI1103/blob/main/assignment_4_new/codes/ai1103_assignment4 new.py

and latex codes from

https://github.com/DineshAvulaMohanaDurga/AI1103/blob/main/assignment_4_new/main.tex

1 Question

(CSIR-UGC-NET June 2015 Q 110)Suppose X has density $f\left(\frac{x}{\theta}\right) = \frac{1}{\theta}e^{-x/\theta}, x > 0$ where $\theta > 0$ is unknown.Define Y as follows:
Y=k if $k \leq X < k+1$, k=0,1,2 $\cdots$
Then the distribution of Y is :

1) Normal
2) Binomial
3) Poisson
4) Geometric

2 Answer

Lemma 2.1. *If the pdf of X is in the form of*

$$p(X = x) = (1-p)^x p, \ \textit{where} \ (x = 0, 1, 2, \cdots) \tag{2.0.1}$$

Then the distribution is said to be in the form of geometric distribution.

Lemma 2.2. *pdf of X is*

$$P(X = x) = e^{-x} \tag{2.0.2}$$

Proof. Given pdf of X is

$$f\left(\frac{x}{\theta}\right) = \frac{1}{\theta}e^{-x/\theta}, x > 0, \text{where } \theta > 0 \text{ is unknown} \tag{2.0.3}$$

$$f(X) = \frac{1}{\theta}e^{-X}, X > 0 \text{ as } x > 0, \theta > 0 \tag{2.0.4}$$

$\because$ The total probability is 1

$$\int_0^\infty f(X)\,dX = 1 \tag{2.0.5}$$

$$\int_0^\infty \frac{1}{\theta}e^{-X}dX = 1$$

$$\frac{1}{\theta} = 1$$

$$\implies \theta = 1 \tag{2.0.6}$$

$$\text{So } f(X) = e^{-X} \tag{2.0.7}$$

Hence lemma 2.2 is proved. □

Lemma 2.3. *pdf of Y is*

$$p(Y = k) = e^{-k}\left(1 - e^{-1}\right) \tag{2.0.8}$$

and is in the form of geometric distribution.

Proof. Also given that Y=k if $k \leq X < k+1$ k=0,1,2,$\cdots$

$$p(Y = k) = \int_k^{k+1} p(X = x)\,dx \tag{2.0.9}$$

$$= \int_k^{k+1} e^{-x}dx$$

$$= e^{-k}\left(1 - e^{-1}\right) \tag{2.0.10}$$

Hence using lemma 2.1 and (2.0.10) lemma 2.3 is proved. □

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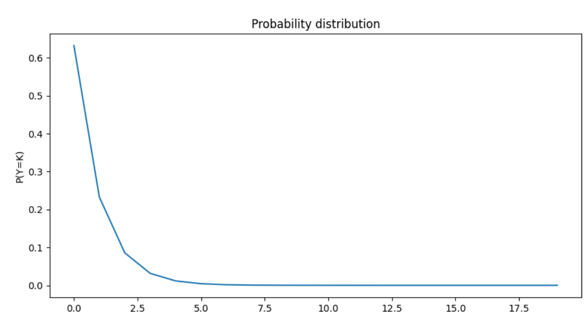

Fig. 4: Probability distribution of Y