

AI1103-Assignment 4

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Download all python codes from

https://github.com/DineshAvulaMohanaDurga/AI1103/blob/main/assignment_4_new/codes/ai1103_assignment4_new.py

and latex codes from

https://github.com/DineshAvulaMohanaDurga/AI1103/blob/main/assignment_4_new/main.tex

Also given that $Y=k$ if $k \leq X < k+1$ $k=0,1,2,\dots$

$$p(Y=k) = \int_k^{k+1} e^{-x} dx \quad (2.0.6)$$

$$= e^{-k} (1 - e^{-1}) \quad (2.0.7)$$

Which is of the form of geometric distribution $p(X=x) = (1-p)^{x-1} p$ where $p = e^{-1}$ and $\therefore k \in (0, 1, \dots, \infty)$ the forward shift by one unit doesn't matter.

Hence the distribution is geometric distribution.

1 QUESTION

(CSIR-UGC-NET June 2015 Q 110) Suppose X has density $f\left(\frac{x}{\theta}\right) = \frac{1}{\theta} e^{-x/\theta}, x > 0$ where $\theta > 0$ is unknown. Define Y as follows:

$Y=k$ if $k \leq X < k+1$, $k=0,1,2,\dots$

Then the distribution of Y is :

- 1) Normal
- 2) Binomial
- 3) Poisson
- 4) Geometric

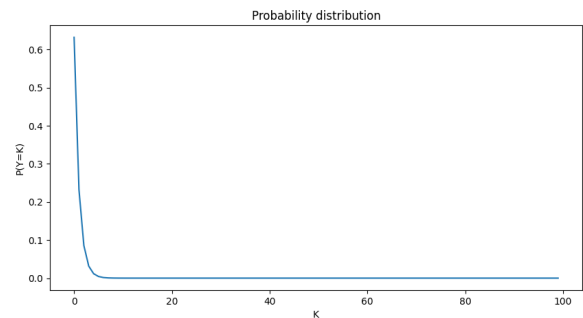


Fig. 4: Probability distribution of Y

2 ANSWER

Given pdf of X is

$$f\left(\frac{x}{\theta}\right) = \frac{1}{\theta} e^{-x/\theta}, x > 0, \text{ where } \theta > 0 \text{ is unknown} \quad (2.0.1)$$

$$f(X) = \frac{1}{\theta} e^{-X}, X > 0 \text{ as } x > 0, \theta > 0 \quad (2.0.2)$$

\therefore The total probability is 1

$$\int_0^{\infty} f(X) dX = 1 \quad (2.0.3)$$

$$\int_0^{\infty} \frac{1}{\theta} e^{-X} dX = 1$$

$$\frac{1}{\theta} = 1$$

$$\Rightarrow \theta = 1 \quad (2.0.4)$$

$$\text{So } f(X) = e^{-X} \quad (2.0.5)$$