AI1103-Assignment 2

Name: Avula Mohana Durga Dinesh Reddy , Roll Number: CS20BTECH11005

Download all python codes from

https://github.com/DineshAvulaMohanaDurga/AI1103/blob/main/assignment_2/codes/ai1103_assignment1.py

and latex codes from

https://github.com/DineshAvulaMohanaDurga/AI1103/blob/main/assignment_2/main.tex

1 Question

(GATE-1999 problem-1.31) The joint probability density function of the random variables X, Y and Z is

$$\begin{cases} f(x,y,z) &= 8xyz, 0 < x,y,z < 1 \\ &= 0 \text{ otherwise} \end{cases}$$

Then P($X < Y < Z$) is
(A) $\frac{1}{8}$ (B) $\frac{1}{3}$ (C) $\frac{1}{6}$ (D) $\frac{3}{8}$

2 Answer

Given joint probability density function j.d.f

$$\begin{cases} f(x,y,z) &= 8xyz, 0 < x,y,z < 1 \\ &= 0 \text{ otherwise} \end{cases}$$

we know that if probability distribution function (p.d.f) = f(X) then

$$\Pr(X \le x) = \int_{-\infty}^{x} \text{f(x) dx} \tag{2.0.1}$$

from the given probability function we can say that $0 < x,y,z < 1$ as the probability is zero otherwise,So

$$\Pr(x < y < z) = \int_0^1 \int_0^z \int_0^y f(x,y,z) \text{ dx dy dz} \tag{2.0.2}$$

$$\begin{aligned} &= \int_0^1 \int_0^z 8yz \left(\int_0^y x \text{ dx} \right) \text{ dy dz} \\ &= \int_0^1 4z \left(\int_0^z y^3 \text{ dy} \right) \text{ dz} \\ &= \int_0^1 z^5 dz \\ &= \frac{1}{6} \end{aligned} \tag{2.0.3}$$

$\therefore$ The value of $\Pr(X < Y < Z)$ is $\frac{1}{6}$
$\therefore$ option C is correct

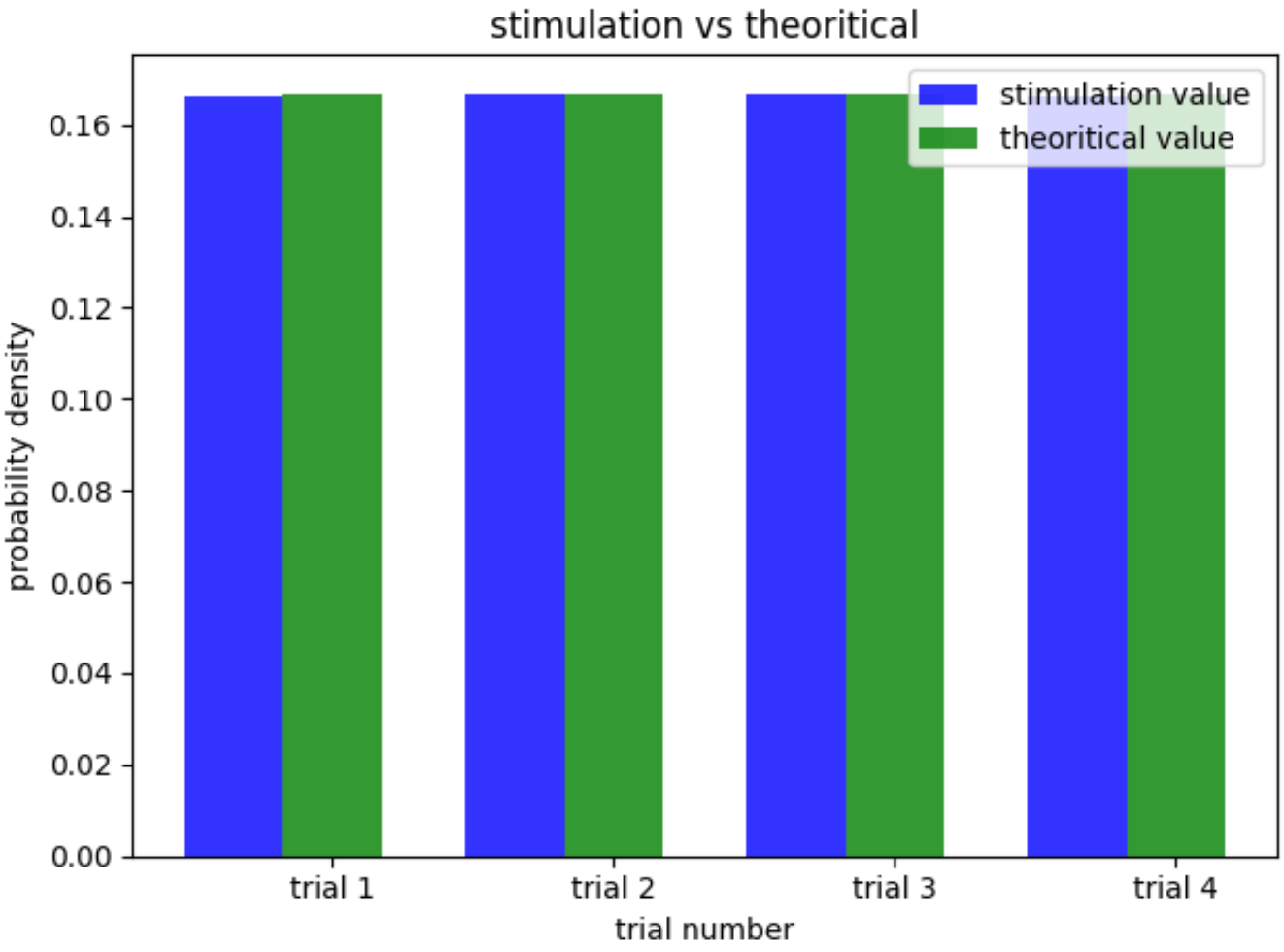

Fig. 4: Simulation vs Theoritical