AI1103-Assignment 3

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Download all python codes from

```
https://github.com/DineshAvulaMohanaDurga/
    AI1103/blob/main/assignment_3/codes/
    ai1103_assignment3.py
```

and latex codes from

```
https://github.com/DineshAvulaMohanaDurga/
    AI1103/blob/main/assignment_3/main.tex
```

1 Question

$(GATE - 2021\,(ME, set-1)\,problem-5)$
Consider a binomial random variable X. If $X_1, X_2, ..., X_n$ are independent and identically distributed samples from the distribution of **X** with sum Y=$\Sigma_{i=1}^{n} X_i$, then the distribution of **Y** as $n \to \infty$ can be approximated as

1) Exponential
2) Bernoulli
3) Binomial
4) Normal

2 Answer

Given a binomial random variable **X**

$$\Rightarrow X \sim B(r, p) \tag{2.0.1}$$

also given that $X_1, X_2, ..., X_n$ are independent and identically distributed samples

$$\Rightarrow X_1 = X_2 = ... = X_n = X \sim B(r, p) \tag{2.0.2}$$

also given that

$$Y = \sum_{i=1}^{n} X_i \tag{2.0.3}$$

We know that the characteristic equation of binomial trials with n elements is

$$\phi_X(t) = \left(1 - p + pe^{it}\right)^n \tag{2.0.4}$$

consider two sets of Bernoulli trials containing $r_1 \& r_2$ elements respectively where both trials have the same probability 'p'$(X \sim B(r_1, p), Y \sim (r_2, p))$. Now considering both as a whole set

$$B(r_1, p) + B(r_2, p) = \Phi_{X+Y}(t) \tag{2.0.5}$$

$$= \Phi_X(t) \times \Phi_Y(t)$$

$$= \left(1 - p + pe^{it}\right)^{r_1} \times \left(1 - p + pe^{it}\right)^{r_2} \tag{2.0.6}$$

$$= \left(1 - p + pe^{it}\right)^{r_1 + r_2} \tag{2.0.7}$$

$$= B(r_1 + r_2, p)$$

$$\therefore B(r_1, p) + B(r_2, p) = B(r_1 + r_2, p) \tag{2.0.8}$$

using this recursively we get

$$Y = B(rn, p) \tag{2.0.9}$$

$\Rightarrow$ using standard formulae

$$\text{mean of Y } \mu_Y = nrp$$

$$\text{and variance } \sigma_Y^2 = nrp(1 - p) \tag{2.0.10}$$

By central limit theorem(CLT)

$$Z_n = \sqrt{n}\left(\frac{\frac{Y}{n} - \mu_Y}{\sigma_Y}\right) = \frac{Y - n\mu_Y}{\sqrt{n}\sigma_Y} \tag{2.0.11}$$

$\lim_{n\to\infty} Z_n \sim N(0, 1)$
Which is a normal distribution
$\therefore$ the correct answer is option D

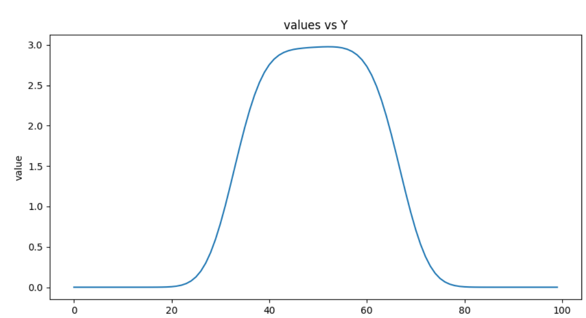

Fig. 4: distribution of Y