

CSIR-UGC NET-June 2015-Question-110

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Question

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Suppose X has density $f\left(\frac{x}{\theta}\right) = \frac{1}{\theta}e^{-x/\theta}$, $x > 0$ where $\theta > 0$ is unknown. Define Y as follows:

$$Y = k \text{ if } k \leq X < k + 1, k = 0, 1, 2 \dots \quad (1)$$

Then the distribution of Y is :

- ① Normal
- ② Binomial
- ③ Poisson
- ④ Geometric

Definitions

Definition of Normal Distribution

The distributions which have probability distribution function in the form of

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad (2)$$

are known as normal distribution.

Where μ is mean (and also median and mode) and σ is standard distribution of *PDF*.

Other Properties of Normal Distribution

- 1 Normal distribution is also known as Gauss or Gaussian or Laplace-Gauss distribution.
- 2 It is a symmetric distribution function about its mean.

Definitions Contd.

Binomial Distribution

Binomial distribution is a common probability distribution that models the probability of obtaining one of two outcomes under a given number of parameters. Its probability distribution function is in the form of

$$f(k, n, p) = \Pr(k, n, p) = \Pr(X = k) = \binom{n}{k} p^k (1 - p)^{n-k} \quad (3)$$

Where

- k is the number of occurrences.
- p is the probability of an outcome being true
- n is total number of trials

Definitions Contd.

Poisson distribution

A random variable X is said to have poisson distribution with parameter $\lambda > 0$, if it has probability distribution function in the form of

$$f(k, \lambda) = \Pr(k, \lambda) = \frac{\lambda^k e^{-\lambda}}{k!} \quad (4)$$

Where

- k is the number of occurrences ($k = 0, 1, 2, \dots$).
- e is the eulers number. ($e = 2.71828$)

Definitions Contd.

Geometric distribution

A distribution is said to be a geometric distribution if it is one of the two following distributions

- 1 The probability distribution of X number of bernoulli trials needed to get one success, supported on the set $(1, 2, 3..)$

$$\Pr(X = k) = (1 - p)^{k-1} p, (k = 1, 2, 3, \dots) \quad (5)$$

- 2 The probability distribution of number $Y=X-1$ of failures before the first success, supported on the set $(0, 1, 2, 3..)$

$$\Pr(Y = k) = \Pr(X = k + 1) = (1 - p)^k p, (k = 0, 1, 2, \dots) \quad (6)$$

Solution

Lemma

PDF of X is

$$P(X = x) = e^{-x} \quad (7)$$

Proof.

Given *PDF* of X is

$$f\left(\frac{x}{\theta}\right) = \frac{1}{\theta} e^{-x/\theta}, x > 0, \text{ where } \theta > 0 \text{ is unknown} \quad (8)$$

$$f(X) = \frac{1}{\theta} e^{-X}, X > 0 \text{ as } x > 0, \theta > 0 \quad (9)$$

Solution Contd.

Proof (Contd.)

∴ The total probability is 1

$$\int_0^{\infty} f(X) dX = 1 \quad (10)$$

$$\int_0^{\infty} \frac{1}{\theta} e^{-X} dX = 1$$

$$\frac{1}{\theta} = 1$$

$$\Rightarrow \theta = 1 \quad (11)$$

$$\text{So } f(X) = e^{-X} \quad (12)$$

Hence lemma 2.1 is proved. □

Solution Contd.

Lemma

PDF of Y is

$$p(Y = k) = e^{-k} (1 - e^{-1}) \quad (13)$$

and is in the form of geometric distribution.

Proof.

Also given that $Y=k$ if $k \leq X < k+1$ $k=0,1,2,\dots$

$$p(Y = k) = \int_k^{k+1} p(X = x) dx \quad (14)$$

$$\begin{aligned} &= \int_k^{k+1} e^{-x} dx \\ &= e^{-k} (1 - e^{-1}) \end{aligned} \quad (15)$$

Hence using definition of geometric distribution and (15) lemma 2.2 is

Example figure

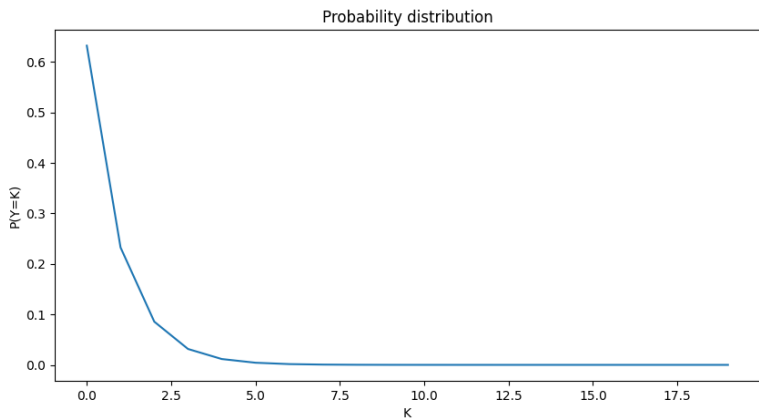


Figure: Probability distribution of Y