AI1103-Assignment 4

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Download all Latex codes from

https://github.com/DineshAvulaMohanaDurga/AI1103/blob/main/assignment_4 new/main.tex

1 Question

(CSIR-UGC-NET June 2015 Q 110)Suppose X has density $f\left(\frac{x}{\theta}\right) = \frac{1}{\theta}e^{-x/\theta}, x > 0$ where $\theta > 0$ is unknown.Define Y as follows:
Y=k if $k \leq X < k+1$, k=0,1,2 $\cdots$
Then the distribution of Y is :

1) Normal
2) Binomial
3) Poisson
4) Geometric

2 Answer

Given pdf of X is

$$f\left(\frac{x}{\theta}\right) = \frac{1}{\theta}e^{-x/\theta}, x > 0, \text{where } \theta > 0 \text{ is unknown} \tag{2.0.1}$$

$$f(X) = \frac{1}{\theta}e^{-X}, X > 0 \text{ as } x > 0, \theta > 0 \tag{2.0.2}$$

$\because$ The total probability is 1

$$\int_0^\infty f(X)\,dX = 1 \tag{2.0.3}$$

$$\int_0^\infty \frac{1}{\theta}e^{-X}dX = 1$$

$$\frac{1}{\theta} = 1$$

$$\Rightarrow \theta = 1 \tag{2.0.4}$$

$$\text{So } f(X) = e^{-X} \tag{2.0.5}$$

Also given that Y=k if $k \leq X < k+1$ k=0,1,2,$\cdots$

$$p(Y = k) = \int_k^{k+1} e^{-X}dX \tag{2.0.6}$$

$$= e^{-k}\left(1 - e^{-1}\right) \tag{2.0.7}$$

Which is of the form of geometric distribution $p(X = x) = (1-p)^{x-1}p$ where $p = e^{-1}$ and $\because k \in (0, 1, \cdots, \infty)$ the forward shift by one unit doesn't matter.
Hence the distribution is geometric distribution.

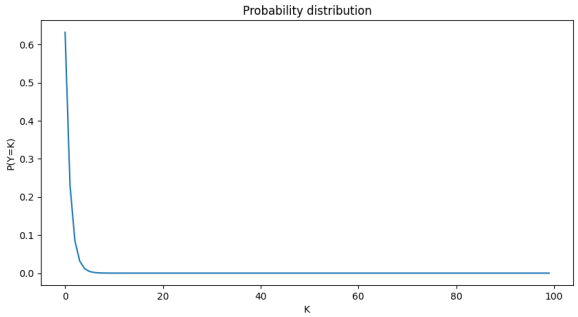

Fig. 4: Probability distribution of Y