AI1103-Assignment 2

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Download all python codes from

```
https://github.com/DineshAvulaMohanaDurga/
    AI1103/blob/main/assignment_2/codes/
    ai1103_assignment1.py
```

and latex codes from

```
https://github.com/DineshAvulaMohanaDurga/
    AI1103/blob/main/assignment_2/main.tex
```

1 Question

(GATE-1999 problem-1.31) The joint probability density function of the random variables X, Y and Z is

$$\begin{aligned} f(x,y,z) &= 8xyz, 0 < x,y,z < 1 \\ &= 0 \text{ otherwise} \end{aligned} \tag{1.0.1}$$

Then $\Pr(X < Y < Z)$ is

(A) $\frac{1}{8}$ (B) $\frac{1}{3}$ (C) $\frac{1}{6}$ (D) $\frac{3}{8}$

2 Answer

Given joint probability density function j.d.f

$$\begin{aligned} f(x,y,z) &= 8xyz, 0 < x,y,z < 1 \\ &= 0 \text{ otherwise} \end{aligned} \tag{2.0.1}$$

we know that probability distribution function

$$f(x) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \text{f(x,y,z) dy dz} \tag{2.0.2}$$

When 0<x<1

$$= \int_{-\infty}^{\infty}\left(\int_{-\infty}^{0} \text{0 dy} + \int_{0}^{1} \text{8xyz dy} + \int_{1}^{\infty} \text{0 dy}\right)dz \tag{2.0.3}$$

$$\begin{aligned} &= \int_{-\infty}^{\infty}\left(0 + 8xz\int_{0}^{1} \text{y dy} + 0\right)dz \\ &= \int_{-\infty}^{\infty}\left(8xz \times \frac{1}{2}\right)dz \\ &= 4x\int_{-\infty}^{\infty} \text{z dz} \\ &= 2x \end{aligned} \tag{2.0.4}$$

When x<0 or x> 1:-

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \text{0 dy dz} \tag{2.0.5}$$

$$= 0 \tag{2.0.6}$$

$$\tag{2.0.7}$$

$$\begin{aligned} \Rightarrow f(x) &= 2x \qquad 0 < x < 1 \\ &= 0 \qquad x < 0 \text{ or } x > 1 \end{aligned} \tag{2.0.8}$$

similarly

$$\begin{aligned} f(y) &= 2y \qquad 0 < y < 1 \\ &= 0 \qquad y < 0 \text{ or } y > 1 \end{aligned} \tag{2.0.9}$$

and

$$\begin{aligned} f(z) &= 2z \qquad 0 < z < 1 \\ &= 0 \qquad z < 0 \text{ or } z > 1 \end{aligned} \tag{2.0.10}$$

assuming $0 < x,y,z < 1$ as the pdf is 0

$$\Pr(x < y) = \int_{-\infty}^{y} f(x)\,dx \tag{2.0.11}$$

$$= \int_{-\infty}^{0} \text{0 dx} + \int_{0}^{y} \text{2x dx} \tag{2.0.12}$$

$$= y^2 \tag{2.0.13}$$

$$\Pr(x<y<z) = \int_{-\infty}^{\infty} f(z)\left(\int_{-\infty}^{z} \Pr(x<y)\times f(y)\,dy\right)dz \tag{2.0.14}$$

$$= \int_{-\infty}^{\infty} f(z)\left(\int_{-\infty}^{0} 0 \text{ dy } + \int_{0}^{z} 2y^3 dy\right)dz$$

$$= \int_{-\infty}^{\infty} f(z)\frac{z^4}{2}dz$$

$$= \int_{-\infty}^{0} 0 \text{ dz} + \int_{0}^{1} z^5 \text{ dz} + \int_{1}^{\infty} 0 \text{ dz}$$

$$= \frac{1}{6} \tag{2.0.15}$$

$\therefore$ The value of $\Pr(X<Y<Z)$ is $\frac{1}{6}$
$\therefore$ option C is correct

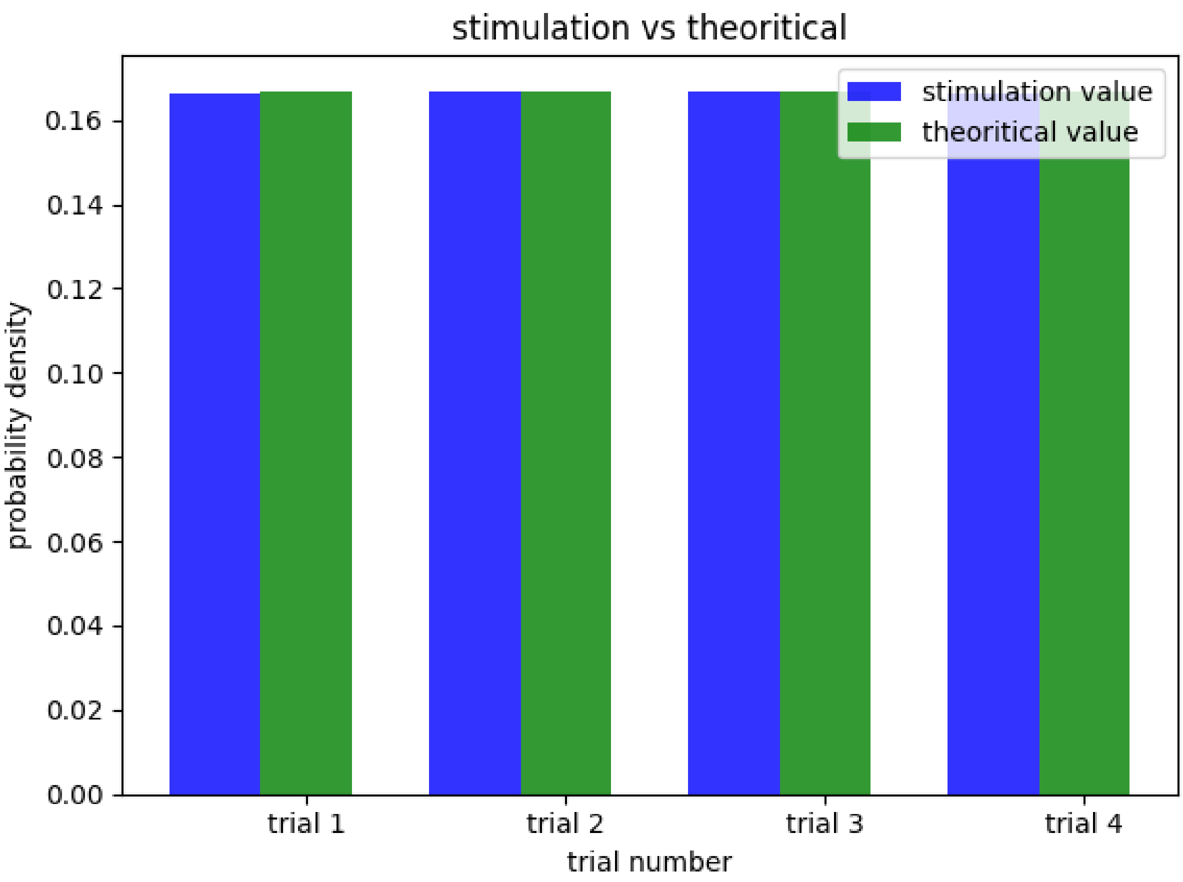

Fig. 4: Simulation vs Theoritical