CSIR-UGC NET-June 2015-Question-110

Avula Mohana Durga Dinesh Reddy

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Question

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Suppose X has density $f\left(\frac{x}{\theta}\right) = \frac{1}{\theta}e^{-x/\theta}, x > 0$ where $\theta > 0$ is unknown.Define Y as follows:

$$Y = k \text{ if } k \leq X < k+1, k = 0, 1, 2 \cdots \tag{1}$$

Then the distribution of Y is :

1. Normal
2. Binomial
3. Poisson
4. Geometric

Definitions

Definition of Normal Distribution

The distributions which have probability distribution function in the form of

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \tag{2}$$

are known as normal distribution.
Where μ is mean(and also median and mode) and σ is standard distribution of *PDF*.

Other Properties of Normal Distribution

1. Normal distribution is also known as Gauss or Gaussian or Laplace-Gauss distribution.
2. It is a symmetric distribution function about its mean.

Definitions Contd.

Binomial Distribution

Binomial distribution is a common probability distribution that models the probability of obtaining one of two outcomes under a given number of parameters.It's probability distribution function is in the form of

$$f(k,n,p) = \Pr(k,n,p) = \Pr(X=k) = \binom{n}{k} p^k (1-p)^{n-k} \tag{3}$$

Where

- k is the number of occurences.
- p is the probability of an outcome being true
- n is total number of trials

Definitions Contd.

Poisson distribution

A random variable X is said to have poisson distribution with parameter $\lambda > 0$,if it has probability distribution function in the form of

$$f(k,\lambda) = \Pr(k,\lambda) = \frac{\lambda^k e^{-\lambda}}{k!} \tag{4}$$

Where

- k is the number of occurences$(k = 0, 1, 2..)$.
- e is the eulers number.$(e = 2.71828)$

Definitions Contd.

Geometric distribiution

A distribution is said to be a geometric distribution if it is one of the two following distributions

1. The probability distribution of X number of bernoulli trials needed to get one success, supported on the set $(1, 2, 3..)$

$$\Pr(X = k) = (1 - p)^{k-1} p, (k = 1, 2, 3, \cdots) \tag{5}$$

2. The probability distribution of number Y=X-1 of failures before the first success, supported on the set $(0, 1, 2, 3..)$

$$\Pr(Y = k) = \Pr(X = k + 1) = (1 - p)^{k} p, (k = 0, 1, 2, \cdots) \tag{6}$$

Solution

Lemma

PDF of X is

$$P\left(X = x\right) = e^{-x} \tag{7}$$

Proof.

Given *PDF* of X is

$$f\left(\frac{x}{\theta}\right) = \frac{1}{\theta}e^{-x/\theta}, x > 0, \text{where } \theta > 0 \text{ is unknown} \tag{8}$$

$$f\left(X\right) = \frac{1}{\theta}e^{-X}, X > 0 \text{ as } x > 0, \theta > 0 \tag{9}$$

Solution Contd.

Proof (Contd.)

$\because$ The total probability is 1

$$\int_0^\infty f(X)\,dX = 1 \tag{10}$$

$$\int_0^\infty \frac{1}{\theta} e^{-X} dX = 1$$

$$\frac{1}{\theta} = 1$$

$$\Rightarrow \theta = 1 \tag{11}$$

$$\text{So } f(X) = e^{-X} \tag{12}$$

Hence lemma 2.2 is proved. $\square$

Solution Contd.

Lemma

PDF of Y is

$$p\left(Y=k\right)=e^{-k}\left(1-e^{-1}\right) \tag{13}$$

and is in the form of geometric distribution.

Proof.

Also given that Y=k if $k \leq X < k+1$ k=0,1,2,$\cdots$

$$\begin{aligned} p\left(Y=k\right) &= \int_{k}^{k+1} p\left(X=x\right)dx & (14) \\ &= \int_{k}^{k+1} e^{-x}dx \\ &= e^{-k}\left(1-e^{-1}\right) & (15) \end{aligned}$$

Hence using lemma 2.1 and (15) lemma 2.3 is proved. □

Example figure

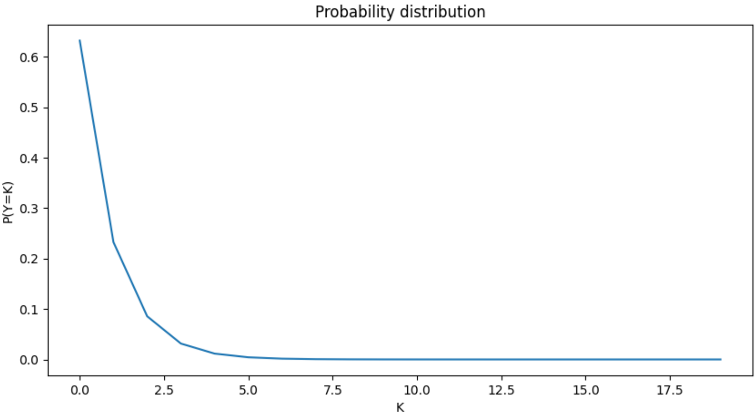

Figure: Probability distribution of Y