AI1103-Assignment 4

Name: Avula Mohana Durga Dinesh Reddy , Roll Number: CS20BTECH11005

Download all Latex codes from

https://github.com/DineshAvulaMohanaDurga/AI1103/blob/main/assignment_4/main.tex

1 Question

(CSIR-UGC-NET June 2015 Q 104)Let X and Y be random variables with joint cumulative distribution function F(x, y). Then which of the following cnditions are sufficient for $(x, y)\in R^2$ to be a point of continuity of F?

1) P$(X = x, Y = y)$=0
2) Either P$(X = x)$=0 or P$(Y = y)$=0.
3) P$(X = x)$=0 and P$(Y = y)$=0.
4) P$(X = x, Y \leq y)$=0 and P$(X \leq x, Y = y)$=0.

2 Answer

Let F(x, y) be joint cumulative distribution function and P(x, y) be joint probability distribution function. From the definitoion of cummulative distribution function

$$pdf = \frac{d}{dx}cdf$$
$$P(X) = \frac{d}{dx}F(X)$$
$$or$$
$$F(x) = \int_{X=-\infty}^{x} P(X)\,dX \tag{2.0.1}$$
$$\Rightarrow F(x, y) = \int_{Y=-\infty}^{y}\int_{X=-\infty}^{x} P(X, Y)\,dXdY \tag{2.0.2}$$

For F to be continuous at (x, y)

$$\lim_{k\to 0}\lim_{h\to 0}\left(F(x+h, y+k) - F(x, y)\right) = 0 \tag{2.0.3}$$
$$\lim_{k\to 0}\lim_{h\to 0}\int_{Y=-\infty}^{y+k}\int_{X=-\infty}^{x+h} P(X, Y)\,dXdY - \int_{Y=-\infty}^{y}\int_{X=-\infty}^{x} P(X, Y)\,dXdY = 0 \tag{2.0.4}$$

On expanding the limits of integrals we get

$$\lim_{k\to 0}\lim_{h\to 0}\int_{Y=y}^{y+k}\int_{X=-\infty}^{x} P(X, Y)\,dXdY + \int_{Y=-\infty}^{y}\int_{X=x}^{x+h} P(X, Y)\,dXdY + \int_{Y=y}^{y+k}\int_{X=x}^{x+h} P(X, Y)\,dXdY = 0 \tag{2.0.5}$$

Which is zero if P(X, Y) is defined over the integrating region i.e P(X, Y) should be defined over the intergrating region of

1) $$\lim_{k\to 0}\lim_{h\to 0}\int_{Y=y}^{y+k}\int_{X=-\infty}^{x} P(X, Y)\,dXdY$$
2) $$\lim_{k\to 0}\lim_{h\to 0}\int_{Y=-\infty}^{y}\int_{X=x}^{x+h} P(X, Y)\,dXdY$$
3) $$\lim_{k\to 0}\lim_{h\to 0}\int_{Y=y}^{y+k}\int_{X=x}^{x+h} P(X, Y)\,dXdY$$

The integrating regions tend to be equal to

1) $-\infty < X < x, Y = y$
2) X=x,$-\infty < Y < y$
3) point (x, y)

Let these be regions 1,2,3 respectively
$\Rightarrow$ P(X, Y) should be defined over regions 1,2,3. i.e

1) P$(-\infty < X < x, Y = y)$
2) P$(X = x, -\infty < Y < y)$
3) P(x, y) should be defined

Now lets verify the options

1) Option 1 defines P(X) only in region 3 but not 1,2.
   Lets disprove this option using a counter-example.
   Let's consider an example similar to dirac-delta function.

$$G(X,Y) = \infty \qquad (X,Y) = (x, y_1) \text{ where } y_1 < y$$
$$= 0 \qquad \text{otherwise} \qquad (2.0.6)$$

   Here G(x, y) is 0 still F is not continuous at (x, y) parellel to X-axis

$$F(X,Y) = 1 \qquad X > x, Y > y_1$$
$$= 0 \qquad otherwise \qquad (2.0.7)$$

   So this option is false
2) Option 2 defines P(X) only in region 3 and either region 1 or region 2.
   If we take the same example in equation 2.0.6 it satisfies G($Y = y$)=0 i.e option 2 but F is not continuous at (x, y).
   So this option is also false
3) Option 3 defines p(X) in all three regions.
   So option 3 is true
4) Option 4 also defines p(X) in all three regions.
   So option 4 is also true

Hence correct options are 3,4.