AI1103-Assignment 2

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1 QUESTION

(GATE-1999 problem-1.31) The joint probability density function of the random variables X, Y and Z is

$$\begin{cases} f(x,y,z) &= 8xyz, 0 < x, y, z < 1 \\ &= 0 \text{ otherwise} \end{cases}$$

Then P(X < Y < Z) is

1) $\frac{1}{8}$
2) $\frac{1}{3}$
3) $\frac{1}{6}$
4) $\frac{3}{8}$

2 ANSWER

Given joint probability density function j.d.f

$$\begin{cases} f(x,y,z) &= 8xyz, 0 < x, y, z < 1 \\ &= 0 \text{ otherwise} \end{cases}$$

we know that if probability distribution function (p.d.f) = f(X) then

$$\Pr(X \le x) = \int_{-\infty}^{x} f(x) \ dx \qquad (2.0.1)$$

from the given probability function we can say that $0 < x, y, z < 1$ as the probability is zero otherwise,So

$$\Pr(x < y < z) = \int_{z=0}^{1} \int_{y=0}^{z} \int_{x=0}^{y} f(x,y,z) \ dx \ dy \ dz$$
$$(2.0.2)$$

$$= \int_{z=0}^{1} \int_{y=0}^{z} 8yz \left(\int_{x=0}^{y} x \ dx \right) \ dy \ dz$$

$$= \int_{z=0}^{1} 4z \left(\int_{y=0}^{z} y^3 \ dy \right) \ dz$$

$$= \int_{z=0}^{1} z^5 dz$$

$$= \frac{1}{6} \qquad (2.0.3)$$
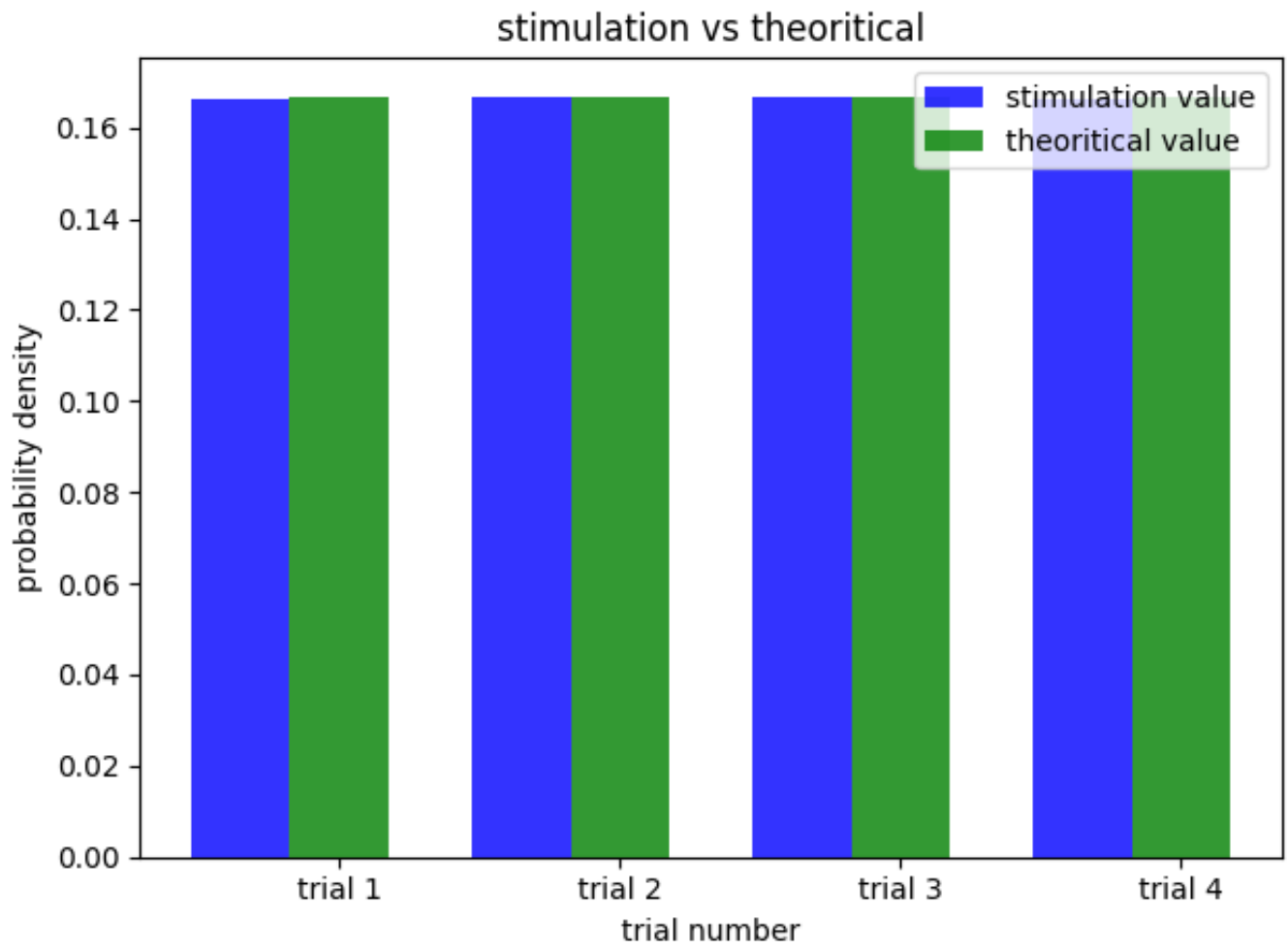
$\therefore$ The value of $\Pr(X < Y < Z)$ is $\frac{1}{6}$

$\therefore$ option C is correct

Fig. 4: Simulation vs Theoritical