AI1103-Assignment 2

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Download all python codes from

```
https://github.com/DineshAvulaMohanaDurga/
    AI1103/blob/main/assignment_3/codes/
    ai1103_assignment3.py
```

and latex codes from

```
https://github.com/DineshAvulaMohanaDurga/
    AI1103/blob/main/assignment_3/main.tex
```

1 Question

$(GATE-2021\,(ME, set-1)\,problem-5)$
Consider a binomial random variable X. If $X_1, X_2, ..., X_n$ are independent and identically distributed samples from the distribution of **X** with sum Y=$\Sigma_{i=1}^{n} X_i$, then the distribution of **Y** as $n \to \infty$ can be approximated as

1) Exponential
2) Bernoulli
3) Binomial
4) Normal

2 Answer

Given a binomial random variable **X**

$$\Rightarrow X \sim B(r, p) \tag{2.0.1}$$

also given that $X_1, X_2, ..., X_n$ are independent and identically distributed samples

$$\Rightarrow X_1 = X_2 = ... = X_n = X \sim B(r, p) \tag{2.0.2}$$

also given that

$$Y = \Sigma_{i=1}^{n} X_i \tag{2.0.3}$$

consider two sets of Bernoulli trials with one set containing r elements and other one containing one element where both trials have the same probability 'p'. Now considering both as a whole set

$$\Pr(X = k) = B(r, p) + B(1, p) \tag{2.0.4}$$

$$= \left(\binom{r}{k} p^k q^{r-k} \times \binom{1}{0} q\right) + \left(\binom{r}{k-1} p^{k-1} q^{r+1-k} \times \binom{1}{1} p\right)$$

$$= \binom{r+1}{k} p^k q^{r+1-k}$$

$$= B(r+1, p)$$

$$\therefore B(r, p) + B(1, p) = B(r+1, p) \tag{2.0.5}$$

applying this recursively we get

$$B(n_1, p) + B(n_2, p) = B(n_1 + n_2, p) \tag{2.0.6}$$

using this recursively we get

$$Y = B(rn, p) \tag{2.0.7}$$

$\Rightarrow$ using standard formulae

$$\text{mean of Y } \mu_Y = nrp$$

$$\text{and variance } \sigma_Y^2 = nrp(1-p) \tag{2.0.8}$$

By central limit theorem(CLT)

$$Z_n = \sqrt{n}\left(\frac{\frac{Y}{n} - \mu_Y}{\sigma_Y}\right)$$

$$= \frac{Y - n\mu_Y}{\sqrt{n}\sigma_Y} \tag{2.0.9}$$

$\lim_{n\to\infty} Z_n \sim N(0, 1)$
Which is a normal distribution
$\therefore$ the correct answer is option D

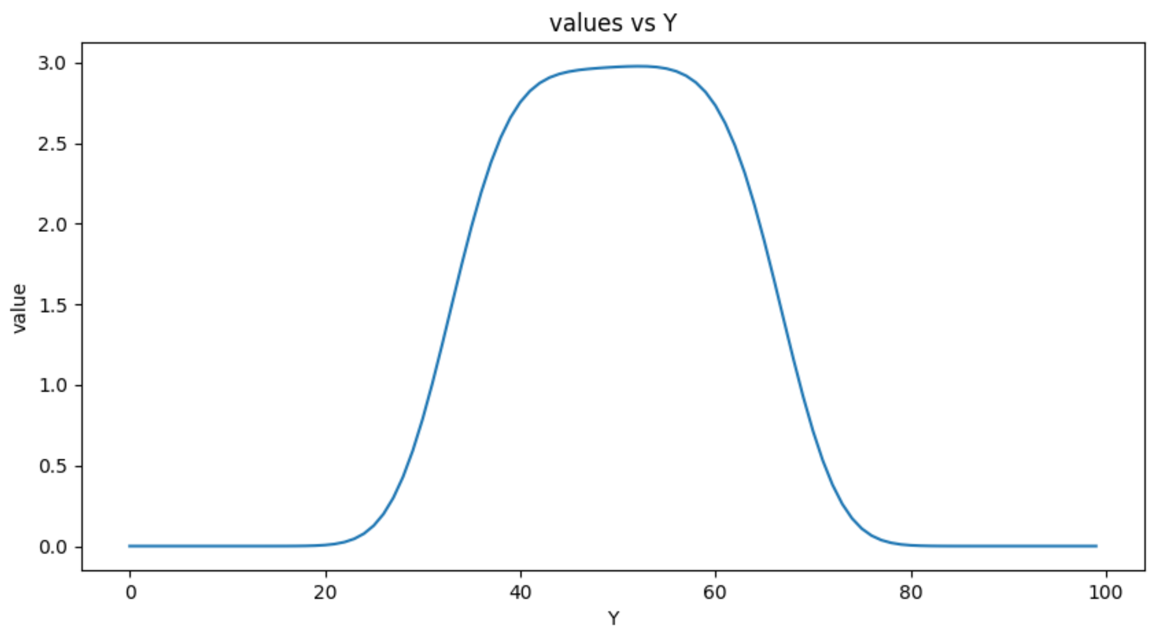

Fig. 4: distribution of Y