AI1103-Assignment 4

Name: Avula Mohana Durga Dinesh Reddy , Roll Number: CS20BTECH11005

Download all python codes from

```
https://github.com/DineshAvulaMohanaDurga/
    AI1103/blob/main/assignment_4_new/codes/
    ai1103_assignment4 new.py
```

and latex codes from

```
https://github.com/DineshAvulaMohanaDurga/
    AI1103/blob/main/assignment_4_new/main.
    tex
```

1 Question

(CSIR-UGC-NET June 2015 Q 110)Suppose X has density $f\left(\frac{x}{\theta}\right) = \frac{1}{\theta}e^{-x/\theta}, x > 0$ where $\theta > 0$ is unknown.Define Y as follows:
Y=k if $k \leq X < k+1$, k=0,1,2 $\cdots$
Then the distribution of Y is :

1) Normal
2) Binomial
3) Poisson
4) Geometric

2 Answer

Lemma 2.1. *If the pdf of X is in the form of*

$$p(X = x) = (1-p)^x p, \ \ where \ \ (x = 0, 1, 2, \cdots) \tag{2.0.1}$$

Then the distribution is said to be in the form of geometric distribution.

Lemma 2.2. *pdf of X is*

$$\Pr(X) = \frac{1}{\theta}e^{-\frac{x}{\theta}} \tag{2.0.2}$$

Proof. Given pdf of X is

$$f(x|\theta) = \frac{1}{\theta}e^{-x/\theta}, x > 0, \text{where } \theta > 0 \text{ is unknown} \tag{2.0.3}$$

$$f(x) = \frac{1}{\theta}e^{-\frac{x}{\theta}} \text{since x and } \theta \text{ are independent} \tag{2.0.4}$$

Hence lemma 2.2 is proved. □

Lemma 2.3. *pdf of Y is*

$$p(Y = k) = e^{-\frac{k}{\theta}}\left(1 - e^{-\frac{k}{\theta}}\right) \tag{2.0.5}$$

and is in the form of geometric distribution.

Proof. Also given that Y=k if $k \leq X < k+1$ k=0,1,2,$\cdots$

$$p(Y = k) = \int_k^{k+1} p(X = x)\,dx \tag{2.0.6}$$

$$= \int_k^{k+1} \frac{1}{\theta}e^{-\frac{x}{\theta}}dx$$

$$= e^{-\frac{k}{\theta}}\left(1 - e^{-\frac{k}{\theta}}\right) \tag{2.0.7}$$

Hence using lemma 2.1 and (2.0.10) lemma 2.3 is proved. □

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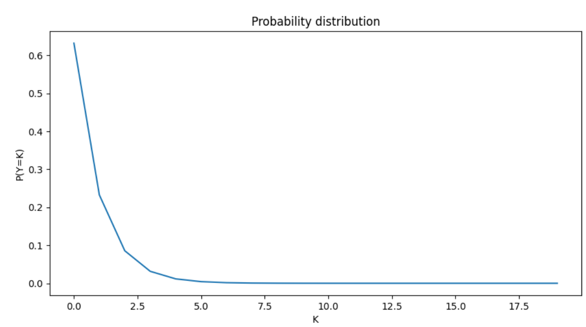

Fig. 4: Probability distribution of Y