Monod kinetics

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Monod kinetics

- Monod kinetics examines the effect of different substrate concentrations on the specific growth rate (μ) of bacteria.
- The effect of substrate on each growth phase has been discussed.
- There is a possibility of defining an overall equation for relating the specific growth rate at different phases with the substrate concentration.
- Monod kinetics is one such formula that correlates the specific growth rate and the substrate concentration.
- The Monod equation is semiempirical; it derives from the basis that a single enzyme system with Michaelis–Menten kinetics is responsible for uptake of *S*, and the amount of that enzyme or its catalytic activity is sufficiently low to be growth-rate limiting.
- The nature of the equation tells us that at low substrate concentrations, the specific growth rate (and in turn the growth rate) is a function of the substrate concentration.
- At high concentrations of substrate, the specific growth rate is equal to the maximum specific growth rate

Monod kinetics

$$\mu = \frac{\mu_{max} S}{K_s + S}$$

Where, μ → specific growth rate (Unit: time^{-1})

μ_{max} → maximum specific growth rate (Unit: time^{-1})

S → substrate concentration (Unit: mass . vol $^{-1}$)

K_s → half saturation constant (Unit: mass . vol $^{-1}$)

When $\mu = \frac{1}{2}\mu_{max}$; $K_s = S$

Hence K_s can be defined as the substrate concentration when the specific growth rate is half of the maximum specific growth rate.

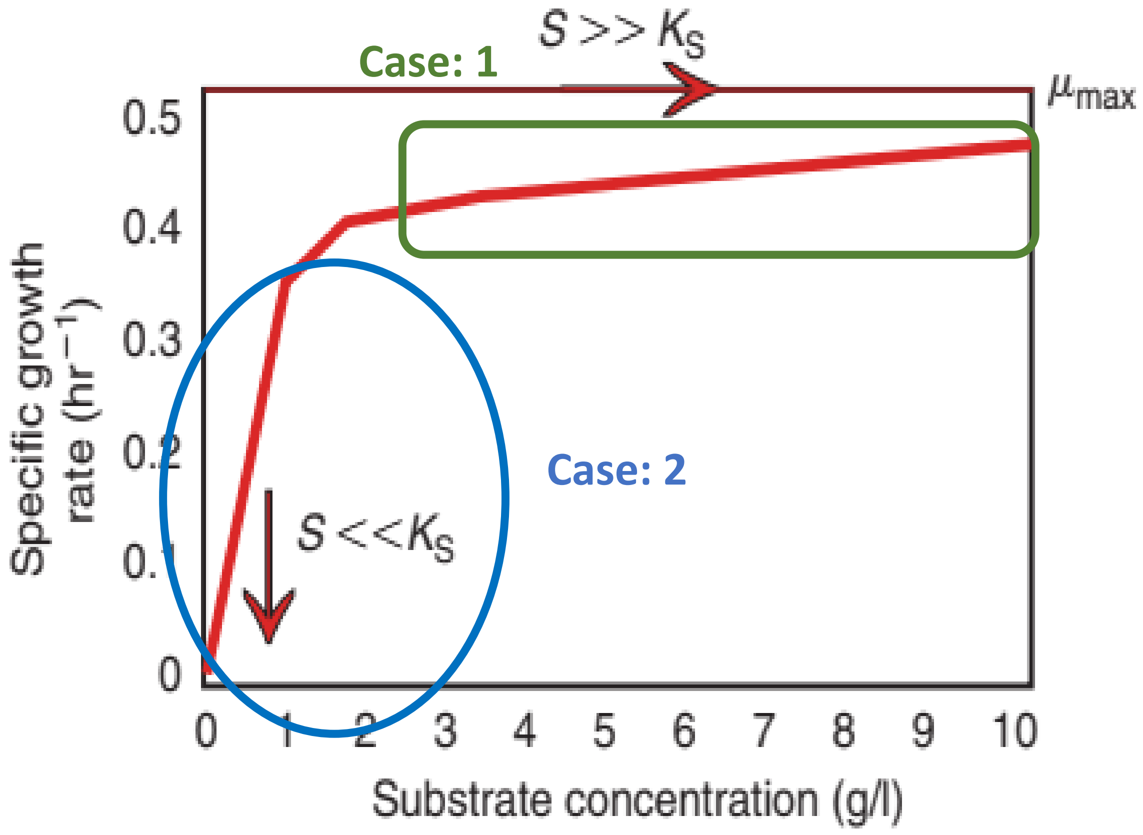
Specific cases:

Case 1: $S >> K_s$

$$\mu = \frac{\mu_{max} S}{K_s + S} \approx \frac{\mu_{max} S}{S} = \mu_{max}$$

Case 2: $S << K_s$

$$\mu = \frac{\mu_{max} S}{K_s + S} \approx \frac{\mu_{max}}{K_s} S$$



Inhibition in growth rates

- Growth inhibition in bioreactors can be owing to several reasons
  1. Introduction of inhibitory compounds.
  2. Higher concentrations of substrate or product.
  3. Osmotic issues.
  4. Viscosity of the medium.
- Similar to enzyme kinetics equations, inhibition can be both competitive as well as non-competitive. For the first three categories of inhibition, modified specific growth rate model versus substrate models can be formulated.
- The inhibition is thus broadly divided into three categories: High concentrations of substrate, high concentrations of product, presence of toxic compounds.

Inhibition in growth rates

Toxic compound inhibition

Competitive inhibition:

$$\mu = \frac{\mu_{max} S}{K_s(1 + \frac{I}{K_I}) + S}$$

Non-competitive inhibition:

$$\mu = \frac{\mu_{max} S}{(K_s + S)(1 + \frac{I}{K_I})}$$

Un-competitive inhibition:

$$\mu = \frac{\mu_{max} S}{(\frac{K_s}{(1 + \frac{I}{K_I})} + S)(1 + \frac{I}{K_I})}$$

Inhibition in growth rates

Substrate inhibition: At high concentrations of substrate, the enzyme activity for a single enzyme catalyzed single substrate reaction is rate limiting for growth. Under these circumstances, the inhibition of the enzyme by high substrate concentration can reduce the microbial growth in similar inhibitory pattern

Competitive inhibition:

$$\mu = \frac{\mu_{max} S}{K_s(1 + \frac{S}{K_I}) + S}$$

Non-competitive inhibition:

$$\mu = \frac{\mu_{max} S}{(K_s + S)(1 + \frac{S}{K_I})}$$

When $K_I >> K_S$

$$\mu = \frac{\mu_{max} S}{(K_s + S + \frac{S^2}{K_I})}$$

Inhibition in growth rates

Product inhibition: Higher concentrations of product, the growth rate might be inhibited.

Competitive inhibition:

$$\mu = \frac{\mu_{max} S}{K_s(1 + \frac{P}{K_P}) + S}$$

Non-competitive inhibition:

$$\mu = \frac{\mu_{max} S}{(K_s + S)(1 + \frac{P}{K_I})}$$

Monod kinetics numerical

Question 1: A bioreactor is operating with a μ_{max} of 0.8 h^{-1} and a K_s of 3 mg/L. If the substrate concentration (S) is 6 mg/L, find the growth rate of the microorganisms and determine whether the culture is substrate-limited.

Answer:

Using the Monod equation:

$$\mu = \mu_{max} \frac{S}{K_s + S}$$

$$\mu = \frac{0.8 \times 6}{3 + 6}$$

= 0.533 h^{-1}

- Since μ (0.533 h^{-1}) is less than μ_{max} (0.8 h^{-1}), the culture is substrate-limited.

Question 2: A bioreactor is growing a microorganism in a medium with a substrate concentration of 10 g/L. The maximum specific growth rate (μ_{max}) is 0.4 h^{-1}, and the half-saturation constant (K_s) is 2 g/L. A non competitive inhibitor is introduced into the system with a concentration of 4 g/L, and the inhibition constant (K_I) is 3 g/L. Calculate the specific growth rate (μ) of the microorganism under these conditions.

Answer:

$$\mu = \frac{\mu_{max} S}{(Ks+S)(1+\frac{I}{K_I})}$$

$$\mu = \frac{0.4 \times 10}{(2 + 10) \times (1 + \frac{4}{3})}$$

$$\boldsymbol{\mu = 0.1428\ h^{-1}}$$

Monod kinetics numerical

Question 3: Repeat Question 2 for competitive inhibitor using same data.

Solution: For competitive inhibition:

$$\mu = \frac{\mu_{max} S}{K_s(1+\frac{I}{K_I})+S}$$

$$\mu = \frac{4\times 10}{2\times\left(1+\frac{4}{3}\right)+10} = \frac{40}{14.67} = 2.72\ \mathrm{h}^{-1}$$

Question 4: The substrate concentration at maximum growth rate of 0.31 h^{-1} is 2 g/L for an experiment. Estimate (a) The growth rate when substrate concentration is 50 g/L (b) when the substrate concentration is 0.025 g/L.

K_s = 2/2 = 1 g/L

(a) Substrate concentration is 50 g/L: This means the growth rate is nearly equal to μ_{max} as the substrate concentration is higher

$$\mu = \mu_{max} \times \frac{S}{K_s+S} = 0.31 \times \frac{50}{1+50} = 0.303\ \mathrm{h}^{-1}$$

(b) Substrate concentration is 0.025 g/L: This means growth rate is directly proportional to $\mu_{max} \times \frac{S}{K_s}$

Therefore $\mu = \mu_{max} \times \frac{S}{K_s} = 0.31 \times \frac{0.025}{1} = 0.0075\ \mathrm{h}^{-1}$

(if the general formula of Monod equation is used $\mu = \mu_{max} \times \frac{S}{K_s+S} = 0.31 \times \frac{0.025}{1.025} = 0.0756\ \mathrm{h}^{-1}$