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Source: *The Economic Journal*, Sep., 1982, Vol. 92, No. 367 (Sep., 1982), pp. 596-614

Published by: Oxford University Press on behalf of the Royal Economic Society

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THE ECONOMIC ROLE OF COMMODITY STORAGE*

Brian D. Wright and Jeffrey C. Williams

One of the earliest, and most successful, examples of economic policy is the oft-quoted Biblical account of Joseph's interpretation of the Pharaoh's dream. Joseph foretold that seven years of abundant harvests would precede seven years of drought, and recommended that the Pharaoh accumulate grain during the good years. Since that time the central role of storage in stabilising the economy in the face of exogenous disturbances has been obvious, but our understanding of the nature of that role has not greatly advanced.

Without divine assistance in forecasting stochastic production, the storage decision is considerably more complex than the one Joseph faced, and the role of storage quite different. In fact, several commonly held impressions about the role of storage of commodities such as grains are incorrect. Rather than stabilising production, storage actually accentuates its variability. Rather than causing a mean-price-preserving decrease or a mean-output-preserving decrease in the dispersion of price, storage generally causes a more complex modification of the distribution of price. Rather than being most effective at eliminating short-falls in consumption, storage actually is more effective at eliminating the incidence of exceedingly high consumption.

In this paper we explore the **role of storage in a model where production is stochastic and both production and storage are performed by competitive profit-maximisers who form rational expectations about the returns to their activities.** We derive the subtle but very important interactions among production, price expectations, and storage, which simpler models cannot capture. Finally, we make a comparative statics assessment of the distributional implications of storage. These results, while confirming the importance of the specification of the demand function and the supply elasticity identified in recent analytical studies (e.g. Wright (1979) and Newbery and Stiglitz (1979)), are surprisingly favourable to consumers, considering the asymmetric nature of the effects of storage on consumption and price.

Storage is simply a productive activity, one that transfers a commodity from one period to the next. Like other forms of production, it is costly. The total cost of storing an amount S_t from period t to period $t + 1$ is as of period $t + 1$

$$K(S_t) = kS_t + (1 - a)P_tS_t + r(P_tS_t + kS_t) \quad (1)$$

where $k \geq 0$ is the net unit cost of physical storage services, $(1 - a)$ is 'shrinkage' or wastage, r is the interest rate, and P is the price in period t . For commodities

* This material is based upon work supported by the National Science Foundation under Grant No. DAR-7910287. The authors would like to thank, with the usual caveat, David Newbery for several helpful comments and suggestions, and Nuong Brennan for professional, and patient, preparation of the manuscript.

such as grains, empirical evidence indicates that marginal physical storage costs are fairly constant (Paul (1970)).¹ Both a and r are, for simplicity, assumed constant over time, with $r > 0$ and $0 \leq a \leq 1$. All prices and costs are expressed in real terms.

We start with a closed competitive economy and assume that storers and all others in the model maximise expected profits and have rational expectations in the Muthian sense. Both the structure of the model and the distribution of the stochastic element in production are in the common information set Ω_{t-1} in period $t-1$. Storage, if positive, equates current period price with expected price in the next period, less the marginal cost of storage services, shrinkage, and interest on capital invested. That is to say,²

$$\begin{aligned} 0 &\geq (1+r)^{-1} a E P_{t+1} - (P_t + k), & S_t &= 0 \\ 0 &= (1+r)^{-1} a E P_{t+1} - (P_t + k), & S_t &> 0. \end{aligned} \quad (2)$$

These arbitrage conditions imply that when S_t is positive, expected price in period $t+1$, $E(P_{t+1})$, is $(1+r)$ times the current price P_t plus storage costs. This does not mean, however, that the price expected in period $t+2$, as of period t , is $(1+r)$ times $E(P_{t+1})$, for there is a positive probability that storage S_{t+1} will be zero. (This erroneous implication is contained in several models, for example that of Helmberger and Weaver (1977).)

The inverse consumption demand for the commodity in question is:

$$P = \alpha + \beta q^{1-C}, \quad \alpha \geq 0 \quad (3)$$

where q_t is the quantity consumed. This form includes the linear ($\alpha > 0$, $C = 0$, $\beta \leq 0$) and constant elasticity ($\alpha = 0$, $\beta > 0$, $C > 1$) as special cases. In what follows, the relative curvature of a given demand curve is measured by C ,

$$C \equiv - \frac{q P''(q)}{P'(q)}. \quad (4)$$

The quantity consumed is

$$q_t = x_t + a S_{t-1} - S_t = I_t - S_t \quad (5)$$

where x_t is production in period t , S_t is the amount stored to period $t+1$, a is the proportion of S_{t-1} available at time t , after shrinkage of $(1-a) S_{t-1}$, and I_t is the amount on hand.

Production in each period is subject to a random disturbance. Common

¹ Empirical research on the 'price of storage' relating grain stocks at the end of the crop year to the difference between the nearest futures price and the spot price shows that the net cost of storage includes an offsetting accessibility value or 'convenience yield' to users which makes the net marginal cost of storage negative at low levels of S . (See Working (1949), Brennan (1958), or Telser (1958).) This accessibility value, which is related to stochastic elements in distribution and demand, is discussed elsewhere at length by one of the authors (Williams (1980)). Here we assume away any accessibility value of stocks, and focus on the role of storage in mitigating the effects of aggregate production disturbances.

² Samuelson (1971) shows that, given an individualistic social welfare function and the appropriate transversality and regularity conditions, such as

$$\lim_{T \rightarrow \infty} (1+r)^{-T} S_T = \lim_{T \rightarrow \infty} (1+r)^{-T} E P_T = 0,$$

welfare-maximising storage in an undistorted economy with infinite horizon is a function of the amount available as in (11) below. The necessary conditions he derives are equivalent to the profit-maximising arbitrage conditions considered here.

sources of production instability are likely to have multiplicative effects on output, rather than the additive effects assumed in much of the literature on storage and market stabilisation. In grain production, for example, because weather determines the yield of a particular acre, the more acres planted the greater will be the variation in total output. Accordingly, the supply function is

$$x_t = \hat{x}(P_t^r) [1 + v_t] \quad (6)$$

where v_{t+1} is the random production disturbance with a probability density function $f(v)$ of finite variance. The disturbance is assumed to be serially uncorrelated and is the same for each producer.¹ P_t^r is the action certainty equivalent price at time $t-1$, when planned production, $\hat{x}(P_t^r)$, must be selected for time t . Under this specification short-run (same period) production is perfectly inelastic.

Production, like private storage, is assumed to be a competitive, expected profit-maximising activity. Expected profits of producer i are

$$E(\Pi_{it}) = E[x_{it}P(q_t)] - H(\hat{x}_{it}) = E(r_{it}) - H(\hat{x}_{it}) \quad (7)$$

where H is total cost, E denotes the conditional expectation given Ω_{t-1} , and r_{it} is revenue of producer i from realised output x_{it} . Under atomistic competition, each producer is a price-taker, but he recognises the perfect correlation between the disturbance in his own production and the disturbance in aggregate production. Hence the first order condition for competitive profit maximisation is

$$\frac{\partial E(\Pi_{it})}{\partial \hat{x}_{it}} = \frac{\partial E(r_{it})}{\partial \hat{x}_{it}} - H'(\hat{x}_{it}). \quad (8)$$

Thus a producer's action certainty equivalent price is the marginal expected return per unit of planned production, P_t^r , where (remembering that he is a price-taker),

$$P_t^r = \frac{\partial E(r_{it})}{\partial \hat{x}_{it}} = \frac{\partial}{\partial \hat{x}_{it}} E\{(1 + v_t) \hat{x}_{it} P[(1 + v_t) \hat{x}(P_t^r) + aS_{t-1} - S_t]\} = \frac{E(P_t x_{it})}{\hat{x}_{it}}. \quad (9)$$

The assumption of risk neutrality for producers allows us to concentrate on other important nonlinearities in the model. Obviously allocative results would be altered by producer risk aversion only if planned supply is not fixed; even then investigations of this topic by Newbery and Stiglitz (1979) suggest that the allocative implications of risk aversion are generally not very drastic. The distributional results discussed below may be more sensitive to producers' attitudes toward risk. However, welfare comparisons in a world of incomplete markets are, as the same duo have shown (Newbery and Stiglitz (1982)), rather tricky, and are not dealt with in this paper.

I. THE COMPETITIVE PROFIT-MAXIMISING STORAGE RULE

Private storers hardly operate in isolation. The price they receive depends on the size of the harvest, which in turn depends on the amount planted. The amount stored influences what producers expect to receive, so that the amount planted

¹ The presence of additional individual disturbances uncorrelated with aggregate production would not alter the results of this paper.

is a function of current storage. Likewise, the price of what is put into store depends on current storage, because what is not stored out of the amount available is consumed. In short, the arbitrage conditions for private storage can be expanded into:

$$\begin{aligned} 0 &\geq (1+r)^{-1} aEP_{t+1}\{\hat{x}[P_{t+1}^r(S_t)], S_t, S_{t+1}\} - P(I_t - S_t) - k, & S_t = 0 \\ 0 &= (1+r)^{-1} aEP_{t+1}\{\hat{x}[P_{t+1}^r(S_t)], S_t, S_{t+1}\} - P(I_t - S_t) - k, & S_t > 0. \end{aligned} \quad (10)$$

The competitive arbitrage conditions implicitly determine the amount of current storage as a function of the amount available.

$$S_t = f(I_t), \quad 0 \leq f' \leq 1. \quad (11)$$

Given current inventory, and appropriate transversality and regularity conditions (see footnote 2, page 597), optimal storage in the current period is the solution to a stochastic dynamic programming problem, in the tradition of Gustafson (1958*a, b*). (See also Johnson and Summer (1976), Newbery and Stiglitz (1981*a, b*).) Derivation of this competitive storage rule is analytically intractable in general, due to a fundamental discontinuity in the storage rule.¹ Although it is possible to store for the future, it is physically impossible for the market to borrow from the future. This fundamental asymmetry, which has been ignored in many previous analytical models of stabilisation by storage from the path-finding studies of Muth (1961) and Massell (1969) to the present (a recent example is Turnovsky, 1979), has crucial implications for the effect of storage on consumption and price.

The numerical method Gustafson followed to obtain the storage rule under the non-negativity constraint is applicable only when planned supply is perfectly inelastic. Until recently, models where both production and storage are rationally responsive to economic incentives have not been analysed. Wright and Williams (forthcoming) describe a stochastic dynamic programming technique for an infinite horizon which can deduce competitive storage behaviour when production is elastic, using a method of successive approximations to derive simultaneous equilibria in storage and current production for any given level of availability I in the previous period.²

The rule for profit-maximising storage depends upon the particular specifications of supply and consumption demand, as well as on the degree of shrinkage, the cost of storage services, and the interest rate. Fig. 1 shows two storage rules. The solid line represents the case where the elasticity of demand η^D is -0.2 , the elasticity of supply η^S is 0, the interest rate is 0.05, and supply in the absence of the stochastic disturbance (i.e. at $v_t \equiv 0$) is 100. The dashed line is the storage rule when η^S is 1.0 instead of 0. The distribution of the multiplicative disturbance is a discrete approximation to a normal density function with a mean of zero and a standard deviation of 0.05.³ Physical storage costs are set at zero, and shrinkage is assumed to be zero. Notice that when a quantity less than an amount I^* (equal

¹ A linear storage rule is derived analytically in a starkly simplified model in Aiyagari *et al.* (1980).

² A description of the numerical approach is available from the authors.

³ The discrete approximation consists of eighty equal intervals from minus four standard deviations to plus four standard deviations from the mean.

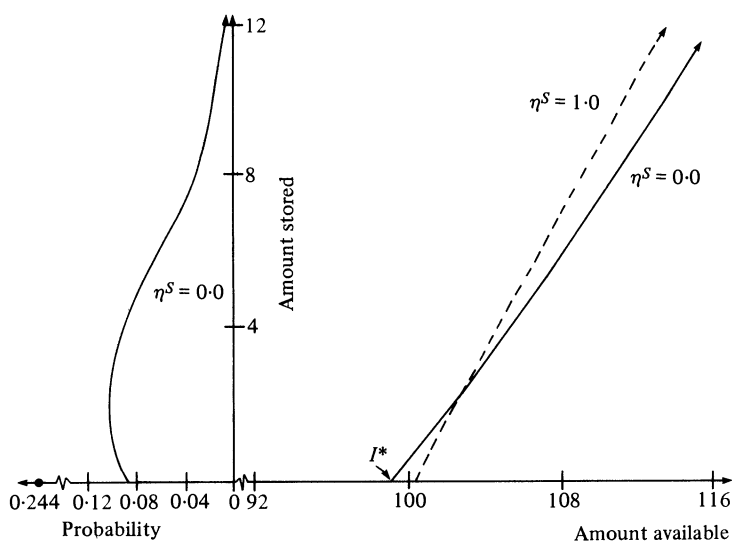


Fig. 1. Storage rules and distribution.

Table 1
Characteristics of Storage Rules

Case (C, η^D, η^S)†	Intercept of storage rule I^*	Marginal propensity to store at	
		I^*	$I + \sigma_I$
1. (0.0, -0.2, 0.0)‡	99.6	0.62	0.76
2. (0.0, -0.2, 1.0)‡	100.5	0.88	0.90
3. (6.0, -0.2, 0.0)§	99.1	0.60	0.74
4. (6.0, -0.2, 1.0)§	100.4	0.87	0.90
5. (6.0, -0.5, 0.0)	101.2	0.52	0.56
6. (6.0, -0.5, 1.0)	101.9	0.75	0.75
7. (6.0, -1.0, 0.0)	104.5	0.44	0.45
8. (6.0, -1.0, 1.0)	104.9	0.61	0.61
9. (6.0, -0.2, 0.0, $\sigma_v = 0.025$)§	100.4	0.56	0.63
10. (6.0, -0.2, 1.0, $\sigma_v = 0.025$)§	100.8	0.87	0.88
11. (6.0, -0.2, 0.0, $\sigma_v = 0.075$)	97.7	0.61	0.79
12. (6.0, -0.2, 1.0, $\sigma_v = 0.075$)	100	0.87	0.90
13. (6.0, -0.2, 0.0, $r = 0.075$)	99.8	0.58	0.58
14. (6.0, -0.2, 0.0, $a = 0.93$)	101.1	0.56	0.62
15. (6.0, -0.2, 0.0, $k = 5.0$)	100.6	0.54	0.60
16. (6.0, -0.2, 0.2)	99.8	0.73	0.80

† The symbols (C, η^D, η^S) denote the measure of demand curvature ($\equiv -q p''(q)/p'(q)$), the elasticity of consumption demand, and the (one period lagged) elasticity of supply, respectively. Both elasticities are measured at the point (quantity = 100, price = 100). Except where otherwise stated, $\sigma_v = 0.05$, interest rate $r = 0.05$, marginal and average physical storage cost $k = 0$, and the shrinkage factor $a = 1$.

‡ Linear demand curve.

§ Constant elasticity demand curve.

|| I and σ_I are mean and standard deviation of I , estimated from a sample of 10,000 periods.

to 99.09 in this example) is on hand, from current output and previous storage, all of the available commodity is consumed. When $\eta^S = 0$, any excess above I^* is divided between current consumption and storage. The marginal propensity to store increases with I , for $I > I^*$.

The nonlinearity of storage rules for cases where $\eta^S = 0$ is more clearly seen by referring to Table 1. The first column in Table 1 shows how the intercept I^* , the availability above which storage occurs, is shifted by changes in each of the parameters. Lines 1, 3, 11, 13 and 14 show that at low η^S , I^* may be below mean production (which is 100 when $\eta^S = 0$), if demand is sufficiently inelastic. The intercept is notably higher at higher demand elasticities (lines 5 and 7). Lines 9 and 11 show that when the standard deviation of the multiplicative disturbance in production falls (rises), the intercept shifts out (in) – storage becomes less frequent as the underlying variability decreases, and vice versa. Line 13 shows that a higher interest rate shifts the storage rule to the right. A qualitatively similar effect is associated with a spoilage rate of 7 % (line 14), and with a storage cost of 5.0 per unit (line 15).

The second column shows that the marginal propensity to store at I^* increases with σ_v , and decreases markedly as demand becomes more elastic; the marginal propensity to consume at I^* differs by 40 % between lines 3 and 7. When supply is inelastic, the storage functions are quite non-linear. In lines 1 and 3, for example, the marginal propensity to consume falls by about one third between I^* and $\bar{I} + \sigma_I$, where \bar{I} is mean availability and σ_I is the standard deviation of I . At higher demand elasticities the curvature of the storage rules is less pronounced.¹ The results in the zero supply elasticity lines in Table 1 are qualitatively similar to those Gustafson (1958*a*, Table 1) reported for various parameterisations of a model with zero supply elasticity and linear demand, though a casual perusal of his Figs. 1–5 and his text may leave the impression that only the intercept I^* is sensitive to changes in demand elasticity and production variability.

Using our storage model, we can move beyond previous work to consider the effects of a rational production response on the profit-maximising storage rules.² As Table 1 shows, an increase in η^S from 0 to 1.0 increases the marginal propensity to store by as much as 50 %. (Line 16, with $\eta^S = 0.2$, indicates that this effect is nonlinear in η^S .) When supply is elastic and demand is inelastic (lines 2, 4, 10, and 12) the marginal propensity to consume is only around 10–13 %. This higher marginal propensity to store, however, does not imply that mean storage is much greater under responsive supply, for two reasons. The intercept I^* is also higher, and accumulation of stocks is damped by a compensating production response, as discussed further below.

Responsive supply renders the storage rules virtually linear. It also removes most of the effects of changes in the multiplicative production disturbance σ_v . Finally the intercept I^* is increased somewhat in each case. A rough rule of thumb inferred from previous exercises assuming no supply response is that when

¹ The difference in curvature cannot be inferred immediately from Table 1, since the range $(\bar{I} + \sigma_I - I^*)$ is smaller at higher demand elasticities.

² Gardner (1979) presents some implications of responsive supply in a model with integer storage and additive disturbances in production.

storage occurs, one unit is stored for every unit consumed (see, for example, Newbery and Stiglitz (1981*a*)). The marginal propensities to store seen in Table 1 show how inaccurate that rule can be, especially when supply is elastic.

II. THE EFFECTS OF STORAGE ON MARKET DEMAND

Under profit-maximising storage, current price can be expressed as a function of the amount in store. Using equation (3), the inverse consumption demand function, and equations (5) and (11),

$$P_t = P[f^{-1}(S_t) - S_t] = \phi(S_t). \quad (12)$$

This expression is the inverse demand function for storage. More precisely, it is the inverse derived demand for the input of the commodity into the storage process; accordingly, it is a function of the costs of the other inputs into that process, including the costs of shrinkage, storage services, and capital. The derived demand for storage corresponding to the storage function with zero supply elasticity in Fig. 1 meets the price axis at $P^* = 104.7$. When current price exceeds P^* , expected future price net of all storage costs is less than current price, so that there is no profit in even the first unit of storage.

Horizontal addition of the storage demand function to the consumption demand function yields the market demand function shown in Fig. 2. At price

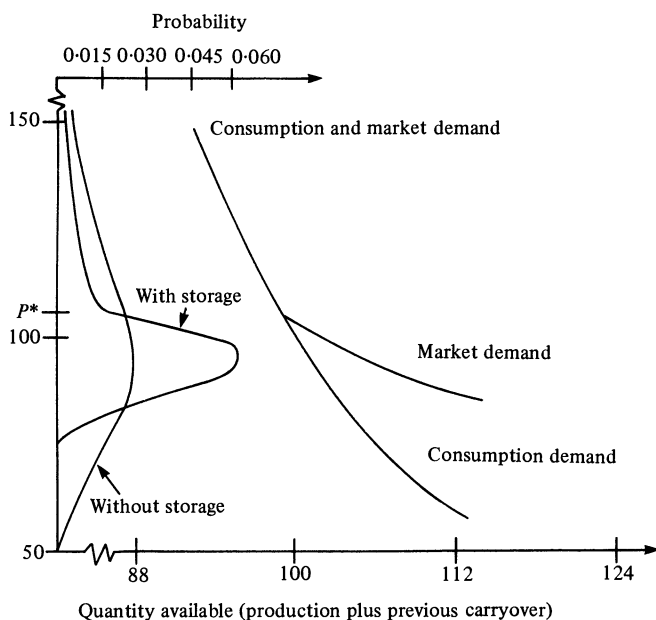


Fig. 2. Demand curves and price distributions.

P^* the elasticity of market demand changes from 0.20 to 0.48. This augmentation of consumption demand below P^* by the storage demand function may explain the (admittedly tentative) conclusion of Hillman *et al.* (1975) that the demand

curve for corn is highly nonlinear, being much less elastic at high prices than at lower prices. Their measurements, relating price changes to changes in availability rather than in consumption, may reflect the demand for storage, rather than any nonlinearity in the underlying consumption demand curve. This distinction is important, because the welfare effects of stabilisation are crucially dependent on the curvature of the consumption demand curve, not of the market demand curve. The failure of Hillman *et al.* to draw this distinction is shared by several studies of price stabilisation that quote their conclusion, including Reutlinger (1976) and Just *et al.* (1978).

III. THE EFFECTS OF STORAGE WITH ZERO SUPPLY ELASTICITY

As observed above, the rule for optimal storage has the property that below some level of availability I^* , no storage is carried over from one production period to the next. Above I^* , consumption and storage both increase as I^* increases, and when $\eta^S = 0$ the marginal propensity to store also rises.

These simple qualities of the storage rule actually have strong implications for the effects of storage. To show this, we used the example behind the storage rule illustrated above, with constant elasticity of demand $\eta^D = -0.2$ ($C = 6$), and elasticity of supply $\eta^S = 0.0$. Starting with nothing in store, we applied the storage rule in a simulation of 10,000 periods, drawing from the random distribution of the production disturbance, and saved all market data beyond the fourth period. (For a sample of this size, the distributions of the variables of interest should closely follow the population distributions.)

The distribution of storage is shown as part of Fig. 1. It is clearly bi-modal and highly skewed. No storage occurs 24.4 % of the time. Mean storage is 3.4 and the standard deviation is 3.5. (Sample means and standard deviations for the distributions discussed in this and the following sections are displayed in Table 2.) We also simulated the same number of periods with the identical string of random numbers holding storage at zero. A comparison of these two simulations provides an instructive illustration of the effects of storage on market variables.

III. 1 *Effects of Storage on Price*

Storage causes a large, asymmetric and possibly counter-intuitive change in the distribution of price. The distribution of price in the absence of storage is shown as part of Fig. 2. Although the production disturbance is symmetric, the distribution of price is not, because of the nonlinearity of the constant elasticity demand curve. Fig. 2 also shows the distribution of price for the same production sequence when storage is possible. Table 2 (lines 3 and 4) shows that storage lowers the mean price in this example by 2.4 %. Because the mean changes, the total effect of storage is not a mean-price-preserving decrease in the dispersion of price, in the terminology of Rothschild and Stiglitz (1970 and 1971).

To isolate the changes in dispersion from this change in the mean, we shifted the distribution of price with storage in Fig. 2 by the difference in the mean price and subtracted the densities without storage from those with storage to obtain

Table 2
Examples of the Market Effects of Storage

Case (C, η^D, η^S) †	Sample means				Action certainty equivalent price without storage
	Production and consumption without storage	Production and consumption with storage	Storage	Price	Price without storage
1. (0.0, -0.2, 0.0) ‡	100.0	100.0	3.2	100.1	100.1
2. (0.0, -0.2, 1.0) ‡	99.9	99.9	3.2	100.7	100.7
3. (6.0, -0.2, 0.0) §	100.0	100.0	3.4	101.6	101.0
4. (6.0, -0.2, 1.0) §	100.4	100.0	3.4	100.9	102.4
5. (6.0, -0.5, 0.0)	100.0	100.0	1.1	101.1	102.0
6. (6.0, -0.5, 1.0)	100.3	100.1	1.4	100.7	101.6
7. (6.0, -1.0, 0.0)	100.0	100.0	0.25	100.7	100.9
8. (6.0, -1.0, 1.0)	100.2	100.2	0.31	100.5	100.8
9. (6.0, -0.2, 0.0, $\sigma_\theta = 0.25$) §	100.0	100.0	0.76	100.6	100.5
10. (6.0, -0.2, 1.0, $\sigma_\theta = 0.25$) §	100.1	100.0	0.91	100.4	101.0
					100.3
					100.1
Case (C, η^D, η^S) †	Sample standard deviations				Price without storage
	Production and consumption without storage	Planned production	Production	Consumption	Storage
1. (0.0, -0.2, 0.0) ‡	5.0	0.0	5.0	3.1	3.5
2. (0.0, -0.2, 1.0) ‡	5.0	2.1	5.5	2.4	3.4
3. (6.0, -0.2, 0.0) §	5.0	0.0	5.0	3.0	3.5
4. (6.0, -0.2, 1.0) §	5.0	2.2	5.5	2.3	3.5
5. (6.0, -0.5, 0.0)	5.0	0.0	5.0	3.9	1.7
6. (6.0, -0.5, 1.0)	5.0	1.2	5.2	3.5	2.1
7. (6.0, -1.0, 0.0)	5.0	0.0	5.0	4.6	0.7
8. (6.0, -1.0, 1.0)	5.0	0.4	5.1	4.5	0.9
9. (6.0, -0.2, 0.0, $\sigma_\theta = 0.25$) §	2.5	0.0	2.5	1.9	1.1
10. (6.0, -0.2, 1.0, $\sigma_\theta = 0.25$) §	2.5	0.0	2.7	1.6	1.3

Footnotes: See Table 1.

Fig. 3. With the help of Fig. 3, we can see that storage affects price dispersion mainly by shifting probability mass from the lower tail towards the mean. Thus the effects of storage on the price distribution are asymmetric in a fashion that contradicts popular notions about storage. We tend to think of storage primarily as protection of consumers against commodity shortages and high prices. But the type of storage considered here is much more dependable in precluding commodity gluts and low prices. The greater the inconvenience to consumers of a shortage (reflected in the demand curve) the higher will be expected price and the larger the incentive to store. Even so, profit-maximising storage will not be large enough to ensure that there will never be a shortage. Since in the solution consumption may with positive probability exceed production in any given period, the amount stored will fall to zero with probability one in finite time, as shown by Townsend (1977).

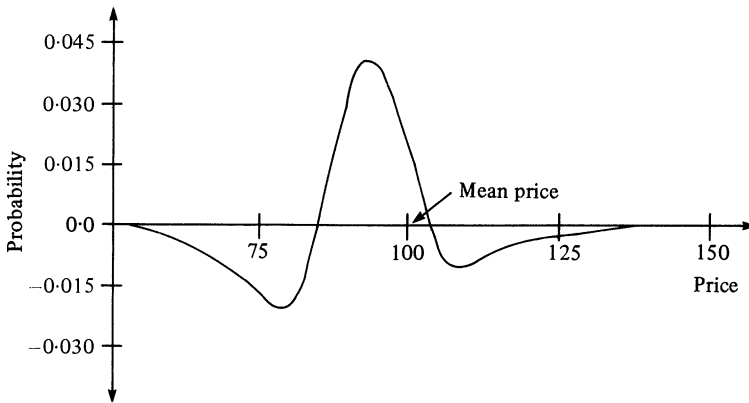


Fig. 3. Effect of storage on price (mean adjusted).

III. 2 *Effects of Storage on Consumption*

Because the elasticity of supply is zero in this example, storage does not affect average consumption. Without storage, consumption has the same distribution as production. Storage causes a large mean-preserving decrease in the dispersion of consumption. In the sample of 10,000 periods, storage reduces the standard deviation of consumption from 5.0 to 3.0; by that measure it goes forty percent of the distance to complete stabilisation. But this decrease in dispersion is clearly asymmetric, as the resulting distribution is significantly skewed to the left. Fig. 4 shows the difference between the frequency of consumption with and without storage.

IV. THE IMPLICATIONS OF RESPONSIVE SUPPLY

IV. 1 *Effects of Storage on Planned Production*

Once storage is introduced in the model, the assumption of perfectly inelastic supply becomes a very important restriction. In the absence of storage, the elasticity of supply is in fact irrelevant. Because there is no serial correlation in the

production disturbances, a shortage or glut in one season has no effect on price in the next. Hence P_t^r , the action certainty equivalent price for production in year t as of year $t-1$, when production must be planned, is constant from year to year.

Storage effects P_t^r in a given period in two ways. First, for a given current output of the commodity, the demand for current storage increases price by augmenting the consumer demand curve, as was shown in Fig. 2 for a case with supply elasticity of zero. Second, for a given output, any carryover from the previous year depresses the realised price. The relative strength of these two effects on the incentive to produce varies from period to period, so that P_t^r , and hence planned production, are sometimes higher, sometimes lower than they would be without storage. This interaction of storage and production is quite important to the effects of storage, a fact missed by other models in the tradition of Gustafson (1958*a, b*) in which elasticity of supply is fixed at zero. We illustrate

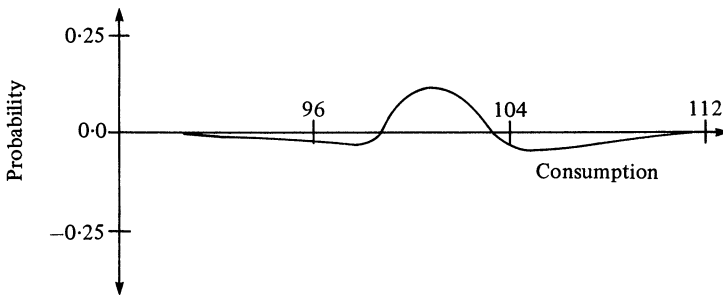


Fig. 4. Effect of storage on consumption.

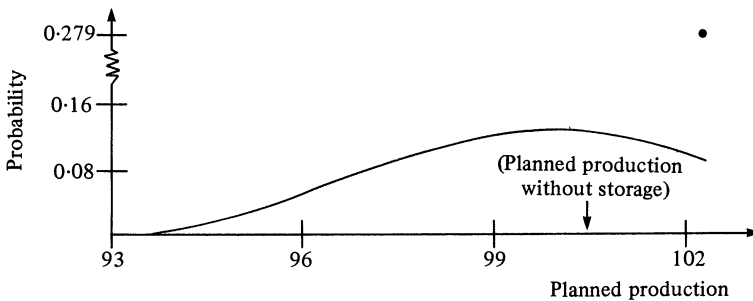


Fig. 5. Distribution of planned production with storage.

the net effect of storage on P_t^r using the previous example modified so that supply elasticity η^s is constant at 1.0 and supply is linear within the observed range of planned production. Although storage is generally thought of as a market-stabilising mechanism, it clearly destabilises planned production, as can be seen in the distribution of planned production under storage shown in Fig. 5. In fact, the coefficient of variation of planned production rises from 0 % under completely inelastic supply to 41 % of that of realised production, which is in turn 10 % higher than the coefficient of variation of production without storage. It has been a common practice of papers in the tradition of Massell (1969) to model the

effect of storage as a decrease in the dispersion of production; these results show that storage has the opposite effect. It is obvious that the derived demands for production inputs (which are not explicitly considered here) are also destabilised by storage. Rather than being regarded as a means of stabilising production, competitive storage should be thought of as a way of efficiently dispersing the effects of a disturbance throughout an (undistorted) economy.

In effect, storage acts as a substitute for production. When current supplies are abundant and the price of the commodity put into storage is low, it is more economical to deliver supplies next period by expanding storage and contracting production. On the other hand, if current supplies are expensive, production is relatively more attractive. When production is more responsive, these two substitutes each display greater variation, but their combined action results in more stable consumption.

Besides increasing the dispersion of production, storage also changes its mean, by -0.4% in the case illustrated in Fig. 5. The direction of this change is related to the curvature of the demand curve, measured by the relative commodity risk aversion parameter C . If the example is changed so that C equals 0.0 , its value when demand is linear, while the demand elasticity (at the equilibrium consumption in the non-stochastic case) remains -0.2 , storage marginally increases mean planned production when $\eta^S = 1$. For demand curves with intermediate values of C , but the same elasticity, the direction of change of mean planned production when $\eta^S = 1$ depends on the degree of stabilisation of consumption effected by storage, which is itself a function of the cost of storage.¹

The contribution of responsive production to this process is asymmetric. Maximum planned production, at 102.3 , is the level of planned production whenever storage is zero, which occurs in 27.9% of all periods. Therefore responsive production is poor insurance against a run of particularly bad harvests, since it provides a maximum offsetting increase in expected availability of only 2.3% . Production response is much more flexible in compensating for abnormally good years; minimum planned production in the sample is 9.2% below the mean. This may explain why Gustafson (1958*b*) indicates observed yields per acre of field crops are significantly skewed to the left, while Day (1965) concludes yields in controlled experiments are skewed to the right, if at all. Through its effects on economic incentives, storage may alter realised production asymmetrically not only through acres planted but through yields.

IV. 2 Effects of Responsive Supply on Storage, Price, and Consumption

Responsive supply greatly accentuates the effects of storage on price and consumption, though it scarcely changes the first two moments of the storage distribution. In the standard example ($\eta^D = -0.2$, $C = 6$), mean storage is higher by only 1.3% for $\eta^S = 1$ relative to $\eta^S = 0$, while the standard deviation is virtually unaltered. But under the more responsive supply the distribution is much less skewed, and the maximum amount in store in the sample is reduced

¹ For example, in the case where $C = 1.95$, $\eta^D = -0.2$, $\eta^S = 1$, mean planned production is less under storage. However, if the market is completely stabilised (i.e. if v is fixed at zero), planned production is slightly higher than in a stochastic market without storage.

from 24.8 to 20.3. Reductions in planned production moderate the build-up of storage in a string of good years.

The existence of responsive supply greatly enhances the decrease in the price dispersion caused by storage. The coefficient of variation is lower by 22.4 % compared to the case with storage illustrated in Fig. 2. The distribution is much more highly skewed and minimum price is more than doubled in this sample.

The dramatic effect of responsive supply on the dispersion of consumption under storage is shown in Fig. 6. The clustering effected by storage is greatly accentuated by a transfer of probability mass from areas both above and below the mean, reducing the coefficient of variation by 24 %. The difference in mean consumption is negligible, but the new distribution is much more skewed and has much higher kurtosis. The most striking effect is that maximum consumption in the sample is reduced by 12.5 %, even though maximum production is actually increased. Maximum consumption is in fact only a miniscule 0.35 % higher than maximum planned production, which occurs whenever storage is zero.

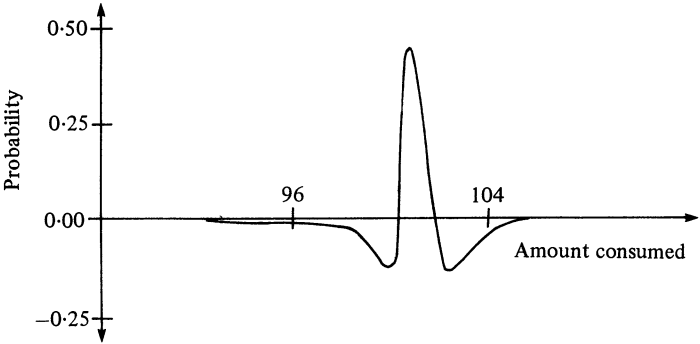


Fig. 6. Effect of responsive supply on consumption.

The effect of responsive supply on maximum consumption can be explained as follows. When supply is perfectly inelastic, very high consumption levels occur after consecutive years of very high production. When supply is elastic, planned production is reduced after a good year, and storage is increased; the net effect is a lower level of consumption in the current year and the next, relative to the situation with fixed long-run supply. The same kind of compensation does not occur in a string of very bad years, however, because below the level of availability at which storage is zero, further marginal shortfalls do not increase the action certainty equivalent price P^r . This explains why minimum consumption in the sample is higher by only 0.7 % under elastic supply, even though maximum consumption is so drastically reduced.

V. THE RELEVANCE OF DEMAND SPECIFICATION

Both the slope and curvature of the demand curve affect storage behaviour. The less steep is the demand curve, the lower is average storage, and the less frequent is the occurrence of storage. For example, line 5 of Table 2 shows that at $\eta^D = -0.5$, $C = 6$, and $\eta^S = 0$, mean storage is 1.1 (compared to 3.4 in line 3 for $\eta^D = -0.2$,

$C = 6$, $\eta^S = 0$); storage occurs 49.0 % of the time (compared to 75.6 %) and the standard deviation of consumption is reduced by only 21 % (compared to 40 %). Indeed, further examples, such as lines 7 and 8, show that for higher demand elasticities the effects of storage become negligible.

The effects of demand curvature, measured by C , the degree of 'relative commodity risk aversion', can be inferred from the cases summarised in Table 2. The higher risk aversion at $C = 6$ is reflected in somewhat higher storage and lower variance of consumption. But the dispersion of prices is greater for $C = 6$, whether or not storage is possible. Although the magnitudes of these effects of demand curvature are not very great, the distributional implications are very important, as we shall now show.

VI. THE DISTRIBUTIONAL IMPLICATIONS OF STORAGE

So far we have considered the effects of storage on prices and quantities. Many studies of storage consider nothing else. But the ultimate interest of the results depicted in the figures and in Table 2 lies in their implications for human welfare. There is a large analytical literature in the tradition of Waugh (1944), Oi (1964) and Massell (1969) which attempts to model the welfare effects of storage as a symmetric reduction in the dispersion of the production disturbance, implicitly or explicitly assuming away the non-negativity constraint on storage, which, as noted above, makes the problem analytically intractable. (See Turnovsky (1978) for a survey of this literature on stabilisation. More recent work includes Newbery and Stiglitz (1979) and Wright (1979).) These studies, like this paper, take a comparative statics approach. That is, they compare average conditions in economies with and without a long history of storage; they do not consider the welfare implications of the initial buildup of stocks upon the introduction of storage.¹ Because storage is much more reliable in eliminating gluts than in alleviating shortages, it might seem likely that the share of the allocative benefits accruing to consumers might be significantly lower than under the symmetric reduction in the dispersion of consumption effected by ideal stabilisation. Further, from Table 2 one might guess that storage favours consumers most when it lowers consumption variance the most (line 4), or when it lowers price variance the most (line 2). In fact none of these deductions from the information presented thus far is correct.

To assess the comparative statics distributional implications of storage, we measured the mean changes in the present value of producer rents at the time of harvest (denoted by the shorthand term 'land value') and the mean changes in present value of consumer surplus.² To make these measures meaningful, we

¹ Such dynamic effects are considered in Wright and Williams (forthcoming). The dynamic effects favour producers at the expense of consumers, the strength of this effect depending on the specification of producer anticipations.

² Assuming an individualistic social welfare function and identical consumers, the change in the area under the uncompensated consumption demand curve is an exact measure of the change in welfare only if the marginal utility of income is constant over the relevant range of price. This is true if $R = \eta^Y$ over the range of prices considered, where R is the coefficient of relative risk aversion with respect to income, and η^Y is the income elasticity of demand. This condition is fulfilled if, for example, R is constant and the indirect utility function has the additively separable form in each time period:

$$V = A(P) + F(Y)$$

[Footnote continued on next page]

expressed them as percentages of a common base, the expected annual value of production in a market without storage. This base was preferred to land value without storage, because land value is dependent on the specification of the entire supply function from zero to maximum production, that is, well beyond the relevant range here.¹

Table 3

Distributional Implications of Storage and Ideal Stabilisation
Difference from Situation without Storage

(% of expected annual revenue without storage)||

Case (C, η^D, η^S)†	Land values		Present value of consumer surplus	
	Storage (Sample means)	Ideal stabilisation	Storage (Sample means)	Ideal stabilisation
1. (0.0, -0.2, 0.0)‡	12.2	24.7	-7.1	-12.4
2. (0.0, -0.2, 1.0)‡	2.2	4.1	4.2	8.6
3. (6.0, -0.2, 0.0)§	-33.6	-49.1	38.0	61.4
4. (6.0, -0.2, 1.0)§	-5.8	-8.3	11.6	20.6
5. (6.0, -0.5, 0.0)	-8.0	-19.9	8.7	24.9
6. (6.0, -0.5, 1.0)	-3.4	-6.6	4.3	11.6
7. (6.0, -1.0, 0.0)	-1.4	-10.0	1.5	12.5
8. (6.0, -1.0, 1.0)	-0.9	-5.0	1.0	7.4
9. (1.95, -0.2, 0.0)	-3.1	0.6	7.9	11.7
10. (1.95, -0.2, 1.0)	-0.6	0.1	6.6	12.2

Footnotes: See Table 1.

The results for ten cases are presented in Table 3. It is immediately clear that the distributional effects are heavily dependent on the three parameters C , η^D , and η^S . The direction of the effects depends largely on C . Consider first the cases where $\eta^S = 0$. Under linear demand ($C = 0$), storage favours land holders at the expense of consumers, but under constant elasticity of demand ($C = 6$ when $\eta^D = -0.2$), the opposite is true. In the intermediate case added here in lines 9 and 10 ($C = 1.95$, which approximates the hyperbolic demand specification ($P = a + bq^{-1}$)), storage has only a minor distributional impact. These distri-

where $F(Y)$ is linear in $\log(Y)$ (in which case $R = \eta^Y = 1$), or in Y (risk neutral, $\eta^Y = 0$). More generally, the error involved in using the Marshallian demand curve is small if the commodity in question has a low share of the consumer budget or a low income elasticity of demand (Willig, 1976). Under these conditions, the measure of demand curvature, C , is at least approximately independent of R .

For producers and storers, we have assumed either that $R = 0$ or that they behave in a risk-neutral fashion because they have access to a competitive capital market, and because the coefficient of variation of the land price is very small. (The coefficient of variation of the land prices in the examples in Table 2 is always below 0.03, assuming the land share is at least 0.3.) The relaxation of the assumption of risk neutrality with respect to income is an obvious topic for further research. The implications of risk averse behaviour on the part of producers is investigated in an analytical model of price stabilisation by Newbery and Stiglitz (1979).

¹ In the 10,000 sample observations for the set of cases considered, planned production ranged from 2% above to about 8% below the equilibrium output under ideal stabilisation. Since nonlinearities in supply outside this range would have virtually no effect on the derivation of the storage rule or on the calculation of changes in land value, we have chosen a yardstick that does not impose an unnecessary restriction on the supply function outside the relevant range.

butional results for $\eta^S = 0$ reflect the fact that consumer surplus is convex (concave) in equilibrium consumption for C less than (greater than) 1. Thus if the consumption demand curve has C greater than 1, the representative consumer is 'commodity market risk averse', in the sense that he would pay for a mean-preserving decrease in the dispersion of his equilibrium amount of the commodity he purchases and consumes. Likewise if C is less than 1, the consumer is 'commodity market risk preferring'.

If $\eta^S = 1$, storage always increases the expected welfare of consumers in Table 3, even if they are 'commodity market risk preferring' ($C = 0$). It is also evident that responsive production greatly moderates the distributional impact of storage. Therefore, the assumption in most previous studies of either $\eta^S = 0$ or an 'irrational' response (e.g. adaptive expectations) in supply may result in misleading distributional inferences. Note also that responsive supply increases the sum of the changes in the expected present value of producer and consumer surplus so that, as the adverse distributional effects decline, the increase in net welfare is greater. The case in line 2 of Table 3 in which the reduction in the standard deviation of price is greatest (see Table 2) actually has the greatest net increase in welfare (in the comparative statics sense), but certainly does not confer the greatest benefit on consumers, as intuition might suggest. Two other perhaps counter-intuitive results are that the net gains are largest in the case when consumers have commodity market risk preference, and that the reduction in the variance of consumption is not greatest when the net gains are largest.

Lines 5 to 8 in Table 3 show that storage has much less significance to welfare at higher elasticities of demand, in line with the less pronounced effects on price and consumption shown for such elasticities in Table 2. At higher elasticities of demand, consumers can more easily substitute other goods for the commodity in question during a shortage, so storage is of less importance.¹

The second and fourth columns of Table 3 display the differential effects of ideal stabilisation, that is, the complete absence of the production disturbance itself. Even though the storage modelled here has a very low cost (an interest rate of 5 % being the only carrying charge), ideal stabilisation has much greater distributional effects and net benefits. Furthermore, lines 9 and 10 indicate that the sign of the effect on land value reverses at a higher value of C under ideal

¹ Further simulations (not reported here) show that the effect of storage on land value has a nonlinear relation to η^D and η^S . This can be shown analytically for ideal stabilisation. From Wright (1979, p. 1025, equation 36) the annual expected gain in producer surplus relative to $\bar{P}\bar{q}$ is approximated by

$$G_R = \frac{\sigma^2(1-C/2)}{|\eta^D| + \eta^S}$$

where $|\eta^D|$ is the absolute value of η^D .

Thus

$$\frac{\partial G_R}{\partial |\eta^D|} = \frac{\partial G_R}{\partial \eta^S} = -\frac{\sigma^2(1-C/2)}{(|\eta^D| + \eta^S)^2}$$

and

$$\frac{\partial^2 G_R}{\partial (|\eta^D|)^2} = \frac{\partial^2 G_R}{\partial (\eta^S)^2} = \frac{2\sigma^2(1-C/2)}{(|\eta^D| + \eta^S)^3}.$$

The numerical results for storage are qualitatively similar. They show that marginal increases in $|\eta^D|$ or η^S moderate the positive or negative effects of storage on producer surplus, the effect decreasing as the absolute value of the elasticity in question increases.

stabilisation than under storage, so that over a certain range, ideal stabilisation has an effect opposite to that of storage. In both storage and ideal stabilisation, the distributive effects are approximately linearly related to C .

The most noteworthy lesson to be drawn from Table 3 is that the asymmetric effects of storage, emphasised in previous sections, do not result in a greater share of the allocative benefits accruing to producers. Relative to the net gain, the differential gain to consumers is even greater under storage than under ideal stabilisation, except in line 1. The explanation lies in the incompleteness of the stabilisation effected by storage. Symmetric reductions in variance always favour consumers for $C > 1$. Sufficiently large reductions also favour producers.¹ Given C , the total distributive outcome depends on the extent of storage, which is a function of the cost of storage, the consumer demand elasticity, and the supply elasticity.

CONCLUSION

Competitive storage of commodities subject to stochastic production disturbances is much more effective in eliminating excessive levels of consumption and low prices than in preventing low levels of consumption and high prices. This asymmetry stems from the constraint that storage must not be negative, and is greatly accentuated when producers, as well as storers, respond to incentives with rational expectations. When this is the case, the interaction between storage and responsive production is subtle and complex. Responsive production significantly increases the slope of the storage rule while it renders the rule virtually linear. Responsive production generally has relatively little effect on mean storage, and vice versa, so in this sense it is not clear whether the two activities are substitutes or complements. Yet when combined, they stabilise consumption and market price in a highly complementary way, even though storage destabilises planned and realised production.

The comparative statics implications storage for producers and consumers

¹ A parallel result can be shown using a simplified analytical model of storage. The easiest cases to consider are those for which $\eta^S = 0$. Ignoring the non-negativity constraint on S , and assuming a constant marginal propensity to store s , $0 \leq s \leq 1$, the action certainty equivalent price P^r becomes, using a second order approximation to the inverse demand function evaluated at mean consumption \bar{q} ,

$$\begin{aligned} P^r(s) &= E\{(1+v) [P(\bar{q}) + v\bar{q}(1-s) P'(\bar{q}) + (\tfrac{1}{2}) v^2 \bar{q}^2 (1-s)^2 P''(\bar{q})]\} \\ &= P(\bar{q}) + \sigma_v^2 \bar{q} (1-s) P'(\bar{q}) + (\tfrac{1}{2}) \sigma_v^2 \bar{q}^2 (1-s)^2 P''(\bar{q}). \end{aligned}$$

For any particular marginal propensity to store \tilde{s} the difference in mean producer surplus due to storage relative to expected revenue is $G(\tilde{s})$ where,

$$G(\tilde{s}) = \frac{\sigma_v^2}{\eta^D} \left\{ C \left[\left(\frac{\tilde{s}}{2} \right)^2 - \tilde{s} \right] + \tilde{s} \right\}.$$

The effect of a marginal increase in storage is given by

$$\frac{\partial G(\tilde{s})}{\partial \tilde{s}} = \frac{-\sigma_v^2}{\eta^D} [C(\tilde{s} - 1) + 1].$$

Thus when there is no storage ($\tilde{s} = 0$), the introduction of a small amount of storage reduces producer surplus (and, since the net benefits are positive, must favour consumers) if $C > 1$. Producers always gain from marginal storage in the neighbourhood of $\tilde{s} = 1$. For a related analytical derivation of the distributional effects of stabilisation, see Newbery and Stiglitz (1979).

cannot be directly inferred from its effects on the dispersion of consumption or price. A numerical welfare analysis shows that when demand is relatively inelastic the storage activity may have substantial effects on the expected present value of consumer surplus and on land values, the signs of such effects depending on the curvature of the demand curve and the supply elasticity. Given the asymmetric effects of storage on consumption and price, which would seem to favour producers, it is noteworthy that the differential effects on consumers who are commodity risk averse are more favourable, relative to the net social value of storage, than they are under the symmetric elimination of the disturbance defined as ideal stabilisation.

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Date of receipt of final typescript: March 1982

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