Optimal Rangeland Stocking Decisions Under Stochastic and Climate-Impacted Weather

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A Stochastic Dynamic Programming (SDP) model is developed to analyze optimal stocking rates in the face of weather uncertainty and potential climate change projections. The model extends previous work modeling grazing as a predator-prey relationship. Attention is given to profit maximizing decisions when growing season precipitation is unknown. Comparisons are made across results from a model that utilizes constant growing season precipitation in all years. Results suggest that optimal stocking rates and profitability decrease in climate change scenarios with increased precipitation variability as compared to the historical stochastic weather scenario.

Key words: climate change, optimal stocking rate, precipitation, rangeland, stochastic dynamic programming (SDP).

JEL code: Q10.

Rangelands are an important resource that cover a large percentage of the world's landmass. Current estimates place rangelands at between 40% and 50% of the Earth's total land area, and rangelands provide around 70% of the global forage for domestic livestock (Derner, Boutton, and Briske 2006; Derner and Schuman 2007; Lund 2007; Brown and Thorpe 2008). Rangelands also provide important ecosystem services beyond forage for livestock, including biological diversity and ecological functions such as wildlife habitat, soil protection, and the ability to sequester greenhouse gases.

The ability of rangelands to provide forage resources for grazing, as well as ecosystem services, is dependent primarily on the condition of the range (Brown and Thorpe 2008). Proper range management, including the stocking decision, is crucial to long-term sustainability of rangelands, the resulting livestock production, and the provision of ecosystem services. Optimization of stocking decisions is complicated by variable range forage production caused largely by stochastic precipitation, and such decisions must often be made before growing season precipitation is realized.

Compounding this complexity, projected climate change is expected to impact the frequency, severity, and duration of drought on the world's rangelands (Intergovernmental Panel on Climate Change [IPCC] 2007a). Range managers must be able to respond to these ever-changing conditions if they want to ensure both stability and profitability of rangelands on a global scale (Brown and Thorpe 2008). Lund (2007) cites climate change and overuse as future threats to the world's rangelands. However, rangeland managers' limited ability to manage crucial inputs such as precipitation in forage production leaves them vulnerable to any changes in global climate. Therefore, adaptive management of rangeland resources will be a necessity, because climate change will potentially alter the impact of weather on rangelands.

Current policy recommendations for carbon sequestration on rangelands and literature related to range livestock production suggest that moderate stocking rates be maintained given uncertain or deficit precipitation. The Chicago Climate Exchange (CCX) Rangeland Soil Carbon Management Offset Program

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mandates, among other requirements, a stocking rate no higher than "moderate" as defined by the Natural Resources Conservation Service (NRCS) Field Office Technical Guide (Chicago Climate Exchange 2007, p. 4) in order to be eligible for benefits. Foran and Smith (1991) indicate that for droughts lasting two years or longer, maintaining a lower than average stocking rate is most profitable in the long run.

Unfortunately, given uncertain precipitation, strategies based on maintaining conservative stocking rates may also result in lower overall financial returns to livestock producers. If in fact "[m]anagement of the forage base, as well as the livestock, is the key to improved livestock performance" (Manley et al. 1997, p. 644), a producer must be aware of variability in precipitation impacts on forage production when making stocking decisions. Hart et al. (1988) report results of short-term grazing trials and indicate that the most profitable stocking rate in the short term is 60–80% higher than recommended rates. Derner et al. (2008) examine the long-term implications of the study by Hart et al. (1988). They note potential long-term decreases in range productivity due to higher stocking rates, but add that spring (growing season) precipitation has a major impact on this relationship. They go on to state that any modeling efforts in grazing management should incorporate this precipitation effect. Westoby, Walker, and Noy-Meir (1989) liken the need to update the stocking decision in response to precipitation impacts to a "continuing game, the object of which is to seize the opportunities and evade the hazards, so far as possible" (p. 266). Vetter (2005) states, "In grazing systems with very high climatic variability, forage availability varies to such a great degree with rainfall that herbivore population dynamics are driven by rainfall via its direct effect on forage availability in any given year" (p. 324).

The objective of our research is to develop a model that accounts for stochastic weather events, which may be exacerbated by climate change, and the interrelationships among forage productivity, animal performance, and stocking rate choice. We model this bioeconomic process in a framework of stochastic dynamic programming (SDP). Specifically, we use this modeling approach to compare and contrast the dynamics and economic implications of a range system under (*a*) a scenario of static "average" forage production across all years, (*b*) a scenario of stochastic weather in which an optimal stocking rate is determined from our model in the SDP framework, (*c*) a scenario of stochastic weather but with an imposed constant "moderate" stocking rate consistent with recommendations from CCX and the literature, and (*d*) a scenario that examines impacts of a subset of potential climate change scenarios.

Relevant Literature

Often in rangeland modeling efforts, forage production has been assumed to be constant across years. For example, Torell, Lyon, and Godfrey (1991) account for varying weather by using the average forage production every year. This approach, however, ignores potential updating of the stocking decision, which may be important in relation to the health of the range resource and long-term profitability. Smith (2007) states that if producers make the stocking decision based on average precipitation and ignore the fact that many years are below "average," they will tend to overstock the range and ultimately drive down both forage production and the returns to land.

Some range studies have focused on optimal improvement strategies to preserve range condition (see Bernardo 1989; Pope and McBryde 1984) rather than optimal herd management to control range productivity. Carande, Bartlett, and Gutierrez (1995) understand the need for dynamic analysis, but they limit their approach to analyzing optimal sale dates across a fixed number of stocking rates and precipitation outcomes. While these studies address potential range resource implications, they do not directly address the interaction of precipitation and stocking decisions in a dynamic framework within the model.

Some recent work that has addressed agricultural issues or scenarios in an SDP framework includes studies by Couture (2008) and Moore and Conroy (2006), which examine forest management. Kobayashi et al. (2007) use a sophisticated model with multiple state variables that also includes stochastic weather impacts to analyze the effect of limited capital on sheep stocking strategies in Kazakhstan, concluding that there is a need for improving localized capital markets in that region. Passmore and Brown (1991) evaluate the idea of proper rangeland management in a stochastic setting, examining specifically (*a*) the use of biomass as the best proxy for range condition and (*b*) stocking rate as the decision most likely to impact this condition. They model sheep operations in the arid rangelands of Australia. The authors indicate that although producers often lack the technical expertise needed to fully take advantage of such models, dynamic programming in a stochastic setting is valuable in providing useful insight into the "intertemporal nature of the rangeland setting" (Passmore and Brown 1991, p. 154). These articles demonstrate the merits of modeling range livestock operations using SDP.

Model Development

The model developed here extends the work of Noy-Meir (1975, 1976) by incorporating the economic relationships of grazing management. Noy-Meir (1975) models cattle grazing as a predator–prey relationship between cattle and forage. He investigates the impacts of different stocking densities on range plant steady states given constant forage growth. Our model accounts for the economic relationships among forage growth, stocking decision, animal performance, price differentials associated with weight, and financial returns over time. We further extend this model by incorporating stochastic precipitation as it relates to forage growth. From this model, we analyze the optimal stocking rates associated with a producer whose aim is to maximize an infinite stream of returns to rangeland.

The biological model as described by Noy-Meir utilizes a logistic forage growth equation:

$$(1) \qquad G(V) = \gamma V \left(1 - \frac{V}{V_m}\right)$$

where γ is maximum growth rate per unit of time, V is vegetation density per unit of land (standing pounds of forage per acre in the current study), and V_m is the maximum plant biomass for a unit of land (carrying capacity). Growth is dependent upon initial standing forage, V, and is calculated on a seasonal basis.

Realistically, however, producers face uncertain forage production in each year. Loehle (1985) incorporates random weather effects into the Noy-Meir (1975) construct by altering the carrying capacity variable on the premise that the carrying capacity of the land will vary year to year based on fluctuations in forage production as driven by precipitation. However, if carrying capacity is in fact the amount of plant biomass a given ecosystem can support, this variable should not change based solely on precipitation.

More appropriately, the uncertainty in forage production should be driven by the impact of stochastic weather on plant growth. The model utilized in this article will incorporate variable forage production through equation (1), but the parameter affected by precipitation is the relative growth rate, γ. The Noy-Meir (1975) model is updated to account for variable weather by making the growth parameter, γ, explicitly a function of weather (ω), which is stochastic. The result is that annual forage growth can differ across years with identical beginning standing forage. The modified equation becomes:

$$(2) \qquad G(V, \omega) = \gamma(\omega) V \left(1 - \frac{V}{V_m}\right).$$

As with equation (1), initial standing forage, V, influences the growth, which is calculated on a seasonal basis once the value of γ is realized.

On the animal side of the problem, total consumption of plant biomass per area of land by cattle as modeled by Noy-Meir (1975) is of the Michaelis-Menten form, with:

$$(3) \qquad C = cS = c_m \left\{ \frac{(V - V_k)}{[(V - V_r) + V_k]} \right\} S$$

where C is total consumption per unit of land, c is consumption per animal per unit of land, S is stocking density per unit of land, c_m is the level of daily consumption associated with satiation, V_r is any ungrazeable residual or mandatory carryover biomass, and V_k is the plant biomass at which consumption equals half of satiation, also known as the Michaelis constant. This function has the properties over the relevant range of $C'(V) > 0$, and $C''(V) < 0$.

Rangeland productivity responds to both grazing pressure and natural fluctuations in weather. A producer therefore must make grazing decisions in response to the current state of the range and expectations about nature, yet before current year precipitation is realized. Animal consumption is based on available forage, but it is not directly impacted by weather outcomes. The equation of motion must account for the dependence on annual forage growth:

$$\text{(4)} \quad \dot{V} = G(V, \omega) - C(V, S) = G(V, \omega) - c(V)S.$$

As ω varies across years, seasonal growth depends explicitly on random weather events. With vegetation biomass growth and animal consumption fully represented, an equation relating animal performance to consumption of biomass is needed. To convert animal consumption to animal gain, Huffaker and Wilen (1991) utilize a forage conversion factor of 0.096 pounds of gain for every pound of consumption. This factor is similar to results seen from stocker grazing studies in Wyoming (see e.g., Derner et al. 2008; Manley et al. 1997). Thus for our model, total animal gain over the season is 0.096**Average Daily Consumption** *Days on Pasture*. Weights for animals coming off rangelands are the sum of total gain plus initial weight at the beginning of the season, as given explicitly as:

$$\text{(5)} \quad W_{end}(W_{init}, V) = W_{init} + Gain(c(V)) = W_{init} + (0.096^* c(V)).$$

Producers are faced with declining prices per hundredweight (cwt) as weight per animal is increased. Cooper and Huffaker (1997) acknowledge this price slide effect and model a system where animals are purchased at 600 lb at \$0.78/lb and sold at the end of the season for only \$0.65/lb. In order to account for the price slide effect in the current model, an equation forecasting prices was generated from data available for the Torrington, Wyoming, auction. This allowed for a continuous slide over the relevant range of potential weight gain. It was hypothesized that grain prices would also affect the price slide. The relevant livestock and grain price data were obtained from the Livestock Marketing Information Center (LMIC) (unpublished data supplied by Jim Robb, LMIC, Lakewood, Colorado, June 22, 2007). Weekly prices from 1992 through 2006 were collected and converted to 2008 dollars. Ordinary least squares were used to estimate steer price per cwt as a function of weight and corn price. While many studies estimate the price slide utilizing a quadratic weight relationship (see e.g., Dhuyvetter and Schroeder 2000), the data from the Torrington auction suggested that a cubic relationship was better suited. The estimated equation is:

$$\text{(6)} \quad P(W, P_{corn}) = \beta_0 + \beta_1 * W + \beta_2 * W^2 + \beta_3 * W^3 + \beta_4 * P_{corn} + \beta_5 * W * P_{corn}.$$

The regression returned $R^2 = 0.4717$ and $F = 20.357$ for the joint significance of the regression coefficients. The corresponding coefficient estimates are reported in table 1. This equation was used to determine initial price per pound (P_i) and price per pound when the animals were removed from the range at season's end (P_e).

Producers are not expected to make decisions strictly to maximize single-season returns, so the model is explicitly dynamic in nature. Producers should account for forage dynamics when making stocking rate decisions. Therefore, the intertemporal decision model is represented as[1]

$$\text{(7)} \quad \underset{S}{Max} \int_0^T \beta^t * \{[P_e(W_{end}(W_{init}, V), P_{corn}) * W_{End}(W_{init}, V) * \times (1 - deathloss) - P_i(W_{init}, P_{corn}) * W_{Init} - C] * S\}dt$$

$$\text{(8)} \quad \text{subject to}: \dot{V} = G(V, \omega) - c(V)S$$

where P_e is price per pound of the animal at ending weight (W_{end}), P_i is weight per pound at the initial weight (W_{init}), P_{corn} is price per bushel of corn, C is seasonal carrying cost per animal, β is the discount factor, and *deathloss* is the percentage loss of animals due to death over the grazing season.

Optimization Technique—Bellman Approach

In order to solve this maximization problem, the model utilizes the Bellman equation. The Bellman equation for this model is:

$$\text{(9)} \quad U_t(V) = \max_{s \in S(v)} \{f(V, S, P_i, P_e, W_{init}, W_{end}) + \beta^* E_\omega * U_{t+1} f(g(V, S, \omega))\}, \quad v \in V, t = 1, 2, \ldots T$$

[1] We recognize that all producers are not in fact risk neutral, and results presented here may not hold for other risk assumptions. However, by imposing the risk neutrality assumption, we allow for comparisons with previous studies utilizing the same assumption.

Table 1. Price Slide Regression Output

	Coefficients	t-Statistic	
Intercept	589.339274	1.618577	
Weight	−1.48295815	−0.929263	
Weight^2	0.00191277	0.828557	
Weight^3	−8.822E−07	−0.800675	
Corn price	−32.24591	−2.872251	
Corn Price * Weight	0.0227388	1.436657	($R^2 = 0.4717$)

where U_t is the value function (namely the maximum of current and future returns), $f(V, S, P_i, P_e, W_{init}, W_{end})$ is the per season return function, E_ω is expectations of weather realizations, and $g(V, S, \omega)$ represents standing forage in period $t+1$. The trade-off between current returns and all future expected returns is explicitly embedded in this equation. It is expected that producers will attempt to maximize the present value of their land, which is a nondepreciable resource with a life span well beyond any individual's planning horizon; therefore, the model is solved with $T = \infty$. Rational producers are expected to find the most profitable long-run steady state, as well as an optimal approach to reach that state in the short run.

Mathematical techniques are now available to overcome the hurdle of continuous state problems by allowing the optimal policy function to be approximated. The approximation technique allows for an optimal policy rule for any possible state, while leaving little error from the true unknown policy function.[2] While it is impossible to calculate the value function for every possible state given a continuous state function, many mathematical techniques have been devised for approximating the value function.[3] According to Miranda and Fackler (2002), we can approximate this value function through contraction mapping, basically finding a map T that satisfies $V = TV$.

Howitt et al. (2002) have formulated a numerical approximation to this technique for infinite time horizon problems.[4] They prescribe the use of Chebyshev polynomials for their orthogonal nature. This orthogonality is due to the terms of the Chebyshev polynomials being sinusoidal in nature. They utilize the value iteration method to approximate the intractable value function. They suggest an interpolation scheme (following Judd 1989) utilizing a functional form consisting of a linear combination of n linearly independent basis functions $\Phi_1, \Phi_2, \ldots, \Phi_n$, in order to perform a regression across the basis coefficients to interpolate the value function.

The Bellman equation is represented with the following system of n equations and n unknowns, allowing computation of the basis coefficients, based on the expectations of weather events:

$$(10) \quad U(V) = \sum_{j=1}^{n} c_j \Phi_j(V_j) = \max_{s \in S(x_j)} \left\{ f(V_j, S, P_i, P_e, W_{init}, W_{end}) + \beta E_\omega \sum_{j=1}^{n} c_j \Phi_j(g(V_j, S, \omega)) \right\}.$$

As with Howitt et al. (2002), we approach this by iteratively solving the Bellman equation at each of these user-defined nodes, maximizing the value of:

$$(11) \quad U^i(V_t) = \max_{V_{t+1}} \left\{ f(V_t, g^{-1}(V_t, V_{t+1})) + \beta U^{(i-1)}(V_{t+1}) \right\}$$

where i represents the iteration, g^{-1} is used to represent the inverse of the equation of motion as a function of current and future values of standing forage, and $U^{(i-1)}$ is the approximated value function of the previous iteration.

In this case the problem is stochastic, and attention must be given to computing the expectations. Miranda and Fackler (2002) state

[2] For the value iteration method used here, errors are measured as residuals calculated at user specified nodes.

[3] "[T]he Bellman equation that characterizes the solution of an infinite horizon dynamic optimization model is a function fixed-point equation" (Miranda and Fackler 2002, p. 115).

[4] The idea of mapping can be accomplished by the following two propositions: (*a*) any function can be approximated by a polynomial of sufficient order and (*b*) such a function can be found within some finite number of iterations (Howitt et al. 2002, p. 4). With these holding, the authors show an approach to approximate a function that in fact maximizes the total value function.

that "the continuous random variable (ω) in the state transition function is replaced with a discrete approximant, say, one that assumes values $\omega_1, \omega_2, \ldots, \omega_k$ with probabilities $p_1, p_2, \ldots, p_k$, respectively" (p. 229). This alters the collocation function for the Bellman equation:

$$(12)\ U(V_i) = \max_{s \in S(V_i)} \left\{ f(V_i, S, P_i, P_e, W_{init}, W_{end}) + \beta \sum_{k=1}^{n} p_k \left[U\left(\sum_{j=1}^{n} c_j \Phi_j (g(V_i, S, \omega_k)) \right) \right] \right\}.$$

Here p_k represents the probabilities associated with each distinct weather outcome. Therefore, the interpolation process solves for the coefficients that maximize the expectation of the value function. The producer is assumed to have a constant expected forage production for any given state across years without regard to any previous realized forage production. The model therefore has the same forage production probabilities regardless of prior-year weather outcomes. The Bellman equation is solved as stated above, resulting in optimal decisions to maximize an infinite stream of values for any initial forage condition.

Model Parameterization

The model is parameterized to represent a stocker operation in central Wyoming, a state with over 27 million acres of permanent pasture and rangeland (National Agricultural Statistics Service 2007). Producers determine their stocking rate in early summer and sell all animals in the fall. Estimated parameters are for a representative acre of land, and are given in table 2. The parameter representing the maximum plant biomass for this area (V_m) is based on an estimate of 0.39 animal unit months (AUMs) per acre productivity for Fremont County, Wyoming, from Bastian, Freeburn, and Hewlett (2005) with an AUM representing 800 pounds of grazeable forage. Average daily consumption is set at 15.6 pounds of dry matter per day over the grazing season. This consumption is 2.5% of the average body weight over the growing season for stockers put on the range at 550 pounds and taken off near 700 pounds. The weight of stockers going on and coming off the range, as well as the 120-day grazing season, is in line with studies in Wyoming by Bastian et al. (1991) and Derner et al. (2008). Over a 120-day grazing season, this translated into 1,872 pounds of dry forage consumption per animal. Huffaker and Wilen (1991), based on Noy-Meir (1976), also utilize 20% of carrying capacity for the Michaelis constant for consumption, translating here into 62.4. The initial price per cwt represents the mean of the Torrington auction price from the price function data at the 550-weight interval. Likewise, the price of corn is initially set as the mean value from the LMIC data over the time period used in estimating the price function. Van Tassell et al. (1997) calculate animal costs per AUM. The sum of association fees, veterinary, moving, herding, miscellaneous labor and mileage, salt and feed, water, horse, and improvement maintenance costs are \$9.08 per AUM. Inflating these animal costs to 2008 dollars results in animal costs of \$15.42 per AUM. This translates to animal carrying costs of \$43.17 (\$15.42*4 months*0.7 AUM equivalent) based on average animal weights over the season in this study.

Table 2. Parameters Used in Stochastic Dynamic Model

Parameter	Value
γ (relative growth rate of forage)	Stochastic
V_m (maximum standing vegetation)	312 lb/acre
C_m (maximum daily consumption)	15.6 lb/animal/day
V_r (mandatory forage residual)	0 lb/ acre
V_k (Michaelis constant)	62.4 lb/ acre
W_{init} (initial weight)	550 lb
P_c (price of corn)	\$3.31
β (discount factor)	0.909091
CC (carrying cost per animal)	\$43.17/animal
Days on Pasture	120
Deathloss	0.02

Weather Parameterization

In order to utilize the Chebyshev collocation process, the weather impact on growth must be in discrete form. The model is parameterized for central Wyoming, where Smith (2005) has previously ascertained that spring precipitation, specifically from early March through late May, is the strongest single predictor of yearly forage production:

$$(13)\quad \textit{Predicted Forage}(\text{lb/acre}) = 216.0464 + 48.4887X \quad (R^2 = .32)$$

Table 3. Frequency of Forage Production (in lbs/acre)

	Median Value	Frequency	Cumulative %
[250,315)	282.43	9	10.34
[315,380)	347.30	28	42.53
[380,445)	412.18	28	74.71
[445,510)	477.06	15	91.95
[510,575)	541.93	3	95.40
[575,640)	606.81	3	98.85
[640,704)	671.69	1	100.00

Note: The square bracket, [, indicates inclusiveness, while the round bracket,), indicates exclusiveness.

where X is total precipitation (in inches) occurring from March 5 through May 25. As the model has been parameterized for Fremont County, Wyoming, weather data from the Riverton Weather Station and obtained from the National Climatic Data Center of the National Oceanic Atmospheric Administration (http://www.ncdc.noaa.gov/oa/ncdc.html) were used in the forage prediction regression equation. Precipitation data from 1921 through 2006, 86 years of observations, were used in our calculations. These weather data were used to estimate forage production over this same time horizon for the study area. Predicted forage production was discretized into seven equal range outcomes over the relevant range, as reported in table 3.

Climate Change Scenarios

Given the potential for climate change to have an impact on rangeland productivity and the economic viability of many stock operations, we also investigated how the system would react to an array of projected changes in precipitation. Projections for future precipitation were obtained from the IPCC's Fourth Assessment Report (IPCC 2007b) and represent modeled spring (March-May) conditions in 2050. The specific projections chosen represent the 5th and 95th percentiles for change in average precipitation, as derived from a suite of twenty-one independent models running under the same initial conditions. All models were also run under the A1B1 emissions scenario, which likely represents a conservative trajectory for the consumption of fossil fuels and global economic growth (IPCC 2007b). The projections themselves are shown in table 4. In order to incorporate these changes

Table 4. Percentiles of Probability Distributions for Precipitation Change in Year 2050

Percentile	5	25	50	75	95
Percent change in mean precipitation	−10.25	−5.5	−2.75	0	5

Note: Projections were derived from a suite of 21 climate models running under the IPCC [19, (2007b)] A1B1 emission scenario. Our analyses focused on the 5th and 95th percentile projections. The 25th, 50th, and 75th percentiles are shown for reference.

into our model, we shifted our existing precipitation profile by the corresponding percentage changes in mean precipitation (table 4).

A growing number of studies also suggest that climate change may bring increased variability in precipitation that may be manifested as changes in storm frequency and severity (Trapp et al. 2007), as well as enhanced interseasonal and interannual variability (Leung et al. 2004). However, significant model uncertainty remains (IPCC 2007b), and the potential magnitude of such changes is likely to vary by geographic region (Trapp, Diffenbaugh, and Gluhovsky 2009). As a result, we made an initial assessment of the impacts of enhanced precipitation variability by increasing the distribution of our precipitation profiles by 1 *SD*. Such an increase is well within the range of IPCC (2007b) model predictions. Moreover, analyses of tree-ring records from central Wyoming show that even greater levels of precipitation variability have occurred within the past 1,000 years (e.g., Gray et al. 2004), thereby suggesting that a 1 *SD* shift is well within the range of physical plausibility. Once the climate change scenarios were estimated for our study area, the resulting forage production was predicted and likewise discretized into seven ranges for use in the model.

Results

Model Simulation

The model was solved with GAMS (Generalized Algebraic Modeling Systems) software utilizing the value iteration method (Howitt et al. 2002). This solution procedure determines the optimal decision rule for any given state value. However, when weather is stochastic, the forage level will not converge to a single value over time, so in order to show optimal behavior, a time path was simulated. A

100-year horizon was evaluated with a probabilistic draw using a random number generator to choose the appropriate discretized forage production range, as reported in table 4. In order to determine whether outcomes forecasted with constant "average" weather (as some previous efforts have done) are attainable, the model was initially simulated for a scenario that includes constant static weather in all years, with the growth parameter set at the expected value seen in the stochastic scenario (0.042837746). Then, the simulated time path of outcomes based on the already determined decision rules from the SDP model was generated over this stochastic time path of weather outcomes to map optimal decisions when faced with realized weather impacts. For comparison, the model was also solved for a 100-year planning horizon (given the base stochastic weather scenario) that imposes a fixed stocking rate of 0.1773 head/acre. This stocking rate is H_s, the "safe carrying capacity," or the stocking rate that ensures no danger of crashing; from Noy-Meir (1976, pp. 93–94):

$$(14) \quad H_s = \frac{\gamma V_k}{c_m}.$$

Noy-Meir's (1976) study did not include stochastic growth, so the growth parameter used to calculate H_s is the expected value of the growth parameter (0.042837746). The value generated as the "safe" capacity is also similar to the moderate stocking rate utilized by Manley et al. (1997) and Derner et al. (2008) for the study area,[5] which would currently be the highest rate allowed under the CCX Rangeland Soil Carbon Management Offset Program.

For the scenarios that solve for optimal stocking, given initial forage, the control is set according to the policy function in order to reach a desired expected state in the subsequent period. After the management decision is made based on the expected weather outcome, for the scenarios that include stochastic weather impacts, the system is shocked with a realized weather impact, which ultimately determines the ending state of forage. In the next period, the decision is again made according to the policy function at the realized new forage state, and the process is continued over the planning horizon.

As seen in table 5, when weather impacts are static and based on "average" weather outcomes, producers optimally leave 173 pounds of forage each year. This is in line with the traditional range management view of "take half, leave half" (Bastian et al. 1991), suggesting that the model predicts reasonable behavior in the face of constant precipitation. With stochastic weather impacts, producers generally leave less forage; however, they also stock at a more conservative rate. This implies that producers who plan on "average" growth each year will tend to overstock compared with a producer who is aware of stochastic weather impacts on rangeland production. Also, the average per acre return to land under the static "average" weather scenario is much higher ($20.93) compared with the average per acre return given optimal decisions when weather outcomes are stochastic ($16.57). Therefore, producers' expectations of returns are likely to be overly optimistic if based on the projected outcomes of the static system.

In the stochastic scenario, producers are unaware of current-year weather realizations when decisions are made; they can make decisions based on only expected outcomes of weather. Producers, in fact, try to leave more standing forage to ensure future productivity, yet poor weather outcomes often force the ending state below the desired state seen in the static case. In years with favorable weather outcomes, producers leave more forage than the desirable amount. The average stocking rate under the scenario with stochastic weather (0.2528 steers/acre) is only slightly higher than a stocking rate (0.2266) set in a Wyoming rangeland study by Schuman et al. (1999) designed for slightly less than 50% utilization of annual forage production.

The result for the stochastic weather scenario suggests that the model predicts reasonable behavior compared with actual stocking rates seen in the literature for Wyoming. It is important to note that while our model predicts a stocking rate that is higher than some literature would suggest as being moderate, the SDP formulation here utilizes an adaptive approach which allows lower stocking rates as range production declines due to unfavorable weather. This is juxtaposed with the study by Hart et al. (1988), who state that the most profitable stocking rate seen during their study was between 60% and 80% above recommended (moderate) rates. Although further analysis showed

[5] Manley et al. (1997) cite a moderate stocking rate as 0.42 steers/ha, which equates to 0.1699 steers/acre, and Derner et al. (2008) cite a moderate stocking rate in the range of 0.1686 steers/acre.

Table 5. Comparison of Outcomes Across Various Weather and Stocking Scenarios

	Average Ending State (lb/acre)	*SD*	Average Stocking Rate (head/acre)	*SD*	Average Returns ($/acre)	*SD*	Average Ending Weights (lb/head)	*SD*
Static weather—optimal stocking	173	–	0.2903	–	$20.93	–	681	–
Stochastic Weather								
Base weather—optimal stocking	160	60	0.2528	0.0530	$16.57	$5.02	675	15
Base weather—fixed stocking	188	92	0.1773	–	$11.65	$5.37	675	28
Climate Change Projection Scenarios								
5th percentile—optimal stocking	160	60	0.2418	0.0511	$15.89	$4.85	675	15
95th percentile—optimal stocking	160	60	0.2582	0.0540	$16.90	$5.11	675	15
5th percentile, increased variability—optimal stocking	147	79	0.2064	0.0741	$12.69	$6.95	667	23
Base weather, increased variability—optimal stocking	147	81	0.2047	0.0941	$13.12	$7.67	666	26
95th percentile, increased variability—optimal stocking	147	82	0.2007	0.1090	$13.34	$8.03	665	28

Table 6. Sensitivity of Outcomes Across Parameter Values for Base Weather Given Optimal Stocking

		Average Ending State (lb/acre)	*SD*	Average Stocking Rate (head/acre)	*SD*	Average Returns ($/acre)	*SD*	Average Ending Weights (lb/head)	*SD*
Discount rate	1%	162	60	0.2523	0.0523	$16.62	$4.92	675	14
	5%	161	60	0.2525	0.0526	$16.60	$4.96	675	15
	10%	160	60	0.2528	0.0530	$16.57	$5.02	675	15
	20%	158	60	0.2534	0.0541	$16.50	$5.16	674	15
Michaelis constant	Less 50%	150	61	0.2128	0.0509	$19.38	$5.79	696	12
	Base	160	60	0.2528	0.0530	$16.57	$5.02	675	15
	Plus 50%	171	58	0.2878	0.0587	$13.95	$4.28	660	15
Corn price	$2.21	159	60	0.2533	0.0534	$17.41	$5.28	674	15
	$3.31	160	60	0.2528	0.0530	$16.57	$5.02	675	15
	$4.51	161	60	0.2523	0.0526	$15.65	$4.74	675	15
Cattle price	Less 20%	163	59	0.2501	0.0525	$11.16	$3.57	675	14
	Average	160	60	0.2528	0.0530	$16.57	$5.02	675	15
	Plus 20%	154	59	0.2537	0.0545	$21.82	$6.84	673	16

that some reduction in range quality could occur due to this high stocking rate, especially in combination with lower growing season precipitation (Derner et al. 2008), the study initiated by Hart et al. (1988) held a fixed stocking rate in all years.

Comparison Across Parameter Values

Sensitivity analysis was conducted across various parameter values for the base stochastic weather scenario, with results reported in table 6. Parameter values analyzed included discount rate, Michaelis constant, cattle prices, and grain prices. Specifically, the model was solved across a range of discount rates, as well as with values of the Michaelis constant set 50% above and below the initial value, with cattle prices 20% above and below the mean observations and the extreme prices of corn observed in the LMIC data.

Modifying the discount rate utilized in the model qualitatively showed expected impacts on model results. As the discount rate was increased, so too was the realized stocking rate for this weather scenario. However, quantitatively the differences in optimal decisions were not large. As this is an infinite time horizon, producers will generally respond to variable weather in the same manner. Table 6 shows that the discount rate did not heavily impact optimal stocking rate. As seen in table 6, there is slight variation in the occurrence of different ending weights for each animal, as well as yearly returns to land, but the overall pattern remains steady for different discount rates.

The outcomes are very sensitive to alternate values of the Michaelis constant. If animals are more efficient grazers (lower Michaelis constant), producers should stock lighter and allow animals to add more weight. Efficient grazers also imply that producers will leave less standing forage each year (similar to Noy Meir's [1976] results), and annual returns will be higher. However, producers with less efficient grazers (higher Michaelis constant) must stock at higher rates, but poor animal performance results in leaving more standing forage. The result is lighter animals and lower annual returns.

Both corn and cattle prices have an impact on annual returns, but little impact on optimal decisions (stocking rate and standing forage left). Higher corn prices imply that feedlots will incur higher costs to finish animals and generally pay less for stocker animals, negatively affecting annual returns to stocker producers, with the opposite true for low corn prices.

Comparison with Fixed Stocking Rate

Table 5 compares the optimal stocking decisions with a simulated path that utilized a fixed stocking rate in all years. As stated previously, this stocking rate was determined by Noy-Meir's (1976) idea of a safe carrying capacity but is also in line with the moderate stocking rate used by Manley et al. (1997) and Derner et al. (2008). While the fixed stocking rate scenario does leave more standing forage on average, this result is more variable than under the optimal stocking rule. While both scenarios tend to add the same amount of gain per animal, the optimal stocking rule, by stocking heavier on average, will tend to have a higher gain per land area, thereby increasing annual returns to land. The optimal stocking rule produces on average over 40% higher returns than the fixed stocking scenario. This could have implications for producers who are considering stocking no higher than moderate rates in response to variable weather.

Impacts of Climate Change

Table 5 shows the results across our various climate change predictions. As expected, a simple shift in distribution of precipitation (5th and 95th percentile scenarios) has an impact on stocking rate and therefore per acre revenues. However, there is no impact on either average ending states or animal weights. The major impacts come from increased variability in precipitation. Even in the scenario (95th percentile) analyzing higher mean precipitation, increased variability has drastic impacts on ending states, stocking rates, profitability, and animal weights compared with the base weather scenario. In all of the scenarios that include increased precipitation variability, there was a significant drop in average ending state values at the same time that variability of standing forage increased. This is also true for stocking rates. Increasing weather variability forces producers to stock less on average but to alter stocking more drastically in response to variability in precipitation. Per acre returns also decrease due to the reduction in stocking rates, and also experience greater variation across years. Ending animal weights are slightly reduced, again exhibiting wider fluctuations

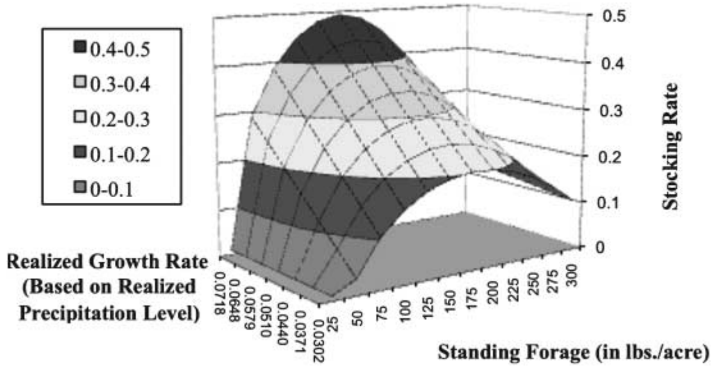

Figure 1. Optimal stocking rates across standing forage and weather realizations.

Note: As standing forage exceeds 200 lbs., optimal stocking rate is decreased, which may be counterintuitive. This is in part due to the logistic growth of forage, resulting in daily forage growth declining at either end of the state space. Second, individual animal consumption is a function of standing forage, and at the upper end of the state space, individual animal intake is increased more than the reduction in stocking rate. Even as stocking rate declines toward the upper end, total consumption does increase over the entire state space.

across time. While changes in mean precipitation have the expected results on model outcomes, it appears as though variability in precipitation has a much more drastic impact on outcomes.

Optimal Stocking Rate

The outcomes discussed previously were based on producer who have knowledge of only long-term expected outcomes of random weather events. But what should producers do if they are able to observe, or accurately predict, precipitation before the stocking rate is set? The value function is able to help determine the optimal policy function as well. Figure 1 shows what stocking rate should be set according to observed standing forage and observed (or predicted) current-year growing season weather for the base stochastic weather scenario. The graph is based on the base scenario of a 10% discount rate, average corn prices, and a Michaelis constant of 64.2. As standing forage reaches the desired level from either direction, optimal stocking rates also increase. As expected, as precipitation increases, optimal stocking rate also increases. This follows Vetter's (2005) conclusion that basing herd numbers conservatively on the idea that maintaining a constant herd size is important is inappropriate and that "[d]rought risks are minimized not by maintaining conservative stocking rates, but rather by allowing livestock numbers to increase in wet years" (p. 330). Optimal management includes increasing stocking rates in response to favorable conditions and reducing stocking rates in poor years. Management decisions must account for current conditions and expected outcomes to ensure future productivity of rangeland.

Conclusion

With increasing global attention to rangelands and the services they provide, a dynamic model maximizing returns to land over stochastic weather events is evaluated in terms of optimal stocking decisions. This model extends the biological foundations of Noy-Meir (1975, 1976) in order to examine optimal stocking decisions given scenarios of stochastic and climate-impacted weather. Our results suggest that basing the yearly stocking rate on average forage production could lead to overstocking of rangelands.

The dynamic model optimized over conditional expectations, as compared with a model that uses average forage production, ends typically with less standing forage, a more conservative stocking rate, and lower financial returns. Results were also compared with a fixed stocking rate simulation. Results suggest that setting a fixed "moderate" stocking rate, even if average standing forage is set to be higher, leads to more variable standing forage and lower annual returns when compared with an adaptive stocking strategy. This implies that producers must constantly monitor the state of the range and make corresponding changes to stocking rates if they are to maximize financial returns. This is especially true for climate change predictions that increase variability in growing season precipitation. Climate change predictions suggest that while changes in mean precipitation will impact optimal stocking and financial returns, increased variability in precipitation has major impacts on model results, decreasing profitability while requiring more adaptive management.

Overall, the results suggest that the model does predict reasonable outcomes. Moreover, the results suggest that producers can improve financial returns by adapting their stocking decisions with updated information related to standing forage and precipitation expectations. This approach addresses, in part, overstocking and eventual decline in the range resource, provided that standing forage provides a reasonable proxy for range sustainability. These results also suggest that some recommendations related to stocking rates, such as those suggesting conservative stocking in response to variable weather or mandated rates for carbon sequestration, may not be optimal.

One limitation of the approach used here is the fact that the biophysical parameters of the Michaelis constant and the forage growth rate and carrying capacity are unlikely to remain constant across breeds or rangelands, making applications of these particular results to other systems more challenging. Another issue is that the proper value of the Michaelis constant is difficult to ascertain. Research addressing the appropriate value of this parameter for varying breeds and forage diets would greatly improve the accuracy of this type of modeling effort. Another limitation of the current model is the assumption that forage growth is dependent upon only precipitation and current standing forage. The state of range health as measured by criteria other than standing forage may also affect forage growth potential. Future modeling efforts that address this issue also would make a valuable contribution. Given the impact of increased precipitation variability on the model results, more detailed investigations of how the magnitude of swings between drought/wet events, as well as the frequency and duration of droughts, may be particularly fruitful. Such knowledge is even more important given some projections regarding climate change and its impacts on both variability of precipitation and drought events.

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