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Solving Stochastic Models of Competitive Storage and Trade by Chebychev Collocation Methods

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We show how to solve the stochastic spatial-temporal price equilibrium model numerically using the Chebychev collocation method. We then use the model to analyze the joint and interactive stabilizing effects of competitive storage and trade.

Although the competitive spatial-temporal price equilibrium model has been widely used to study commodity markets, applications have historically ignored output and price uncertainty (Takayama and Judge). Uncertainty, however, arises naturally in commodity markets and, in many instances, is of fundamental economic interest. This is especially true of agricultural commodity markets, where production is subject to the profound and unpredictable effects of weather, blight, and other natural phenomena.

Commodity price, income, and supply instability issues are central to agricultural policy and food security debates (Bigman; Newbery and Stiglitz). Numerous studies have examined the stabilizing effects of trade in a static open economy under uncertainty (e.g., Bale and Lutz; Grinois). Other studies have examined the stabilizing effects of storage in a dynamic closed economy under uncertainty (e.g., Gardner; Wright and Williams). Only recently, however, have researchers attempted to integrate time, space, and uncertainty into a unified framework capable of explaining how trade and storage interact to affect commodity market stability.

The major obstacle to analyzing models of trade and storage under uncertainty is that such models typically do not possess a closed-form solution, rendering conventional algebraic methods useless. Also, quadratic and nonlinear programming methods for solving deterministic spatial-temporal equilibrium models are not applicable to stochastic models. Recent efforts to solve models of trade and storage under uncertainty have employed numerical dynamic programming strategies (Williams and Wright). These efforts, however, have also relied on ad-hoc adaptations of the curve-fitting techniques commonly used in econometrics. Such approaches are known to be inefficient and can often generate highly inaccurate results (Miranda; Judd).

In this paper, we employ established methods of numerical analysis to solve the stochastic spatial-temporal price equilibrium model accurately and efficiently. Our approach is based on direct solution of the stochastic functional equation that characterizes the market equilibrium. To solve the functional equation, we employ the Chebychev collocation method. Chebychev collocation has been used widely by engineers to solve the functional equations that arise in the analysis of dynamic physical systems. The adaptation of these techniques to the study of dynamic economic systems, however, has been only a recent development (Judd).

In the next section, we formulate a model of competitive spatial-temporal price equilibrium under uncertainty. In the subsequent section, we discuss how to solve the model using the Chebychev collection method. We conclude with an application of the method to the analysis of the interactive stabilizing effects of competitive storage and trade. Throughout the paper, we limit our discussion to a two-region world. This is done solely to reduce the notation burden on the reader, particularly in the presentation of the computational methods. The theory and methods are easily generalized to more than two regions.

Stochastic Spatial-Temporal Equilibrium

Consider a two-region commodity market comprising competitive interregional trade, competi-

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tive intertemporal storage, lagged production decisions, and output and price uncertainty. For each period t and region $i = 1, 2$, denote market price by p_{it}, initial supply by s_{it}, consumption by c_{it}, exports by x_{it}, ending stocks by v_{it}, acreage planted by a_{it}, and per-acre yield by $\tilde{y}_{it}$. Also, given region $i = 1, 2$, denote the other region by i'.

In any period t, the supply initially available in each region is composed of carryover from the preceding period and new production, which is determined by an exogenous random yield on the acreage planted the preceding period:

$$(1) \qquad s_{it} = a_{it-1} \cdot \tilde{y}_{it} + v_{it-1}.$$

Initial supply may be supplemented by imports, and must be either consumed, exported, or stored:

$$(2) \qquad s_{it} + x_{i't} = c_{it} + x_{it} + v_{it};$$

here, $x_{i't}$, denotes the other region's exports. The market clearing price is a strictly decreasing function of the consumption level:

$$(3) \qquad p_{it} = \pi_i(c_{it}).$$

And acreage planted is a strictly increasing function of the expected per-acre revenue[1]:

$$(4) \qquad a_{it} = a_i(E_t[p_{it+1} \cdot \tilde{y}_{it+1}]).$$

Competition among profit-maximizing traders precludes the existence of economic profits from trade in equilibrium. Specifically, one region's price cannot exceed the other region's price by more than the unit cost of transportation. Otherwise, competitive traders would exploit profit opportunities by exporting the commodity from the low price region to the high price region, raising the price in the exporting region and lowering it in the importing region. Of course, the commodity is not exported if economic losses would be incurred by doing so. This gives rise to the spatial arbitrage complementary slackness conditions:

$$(5) \qquad x_{it} \geq 0 \quad \perp \quad p_{i't} \leq p_{it} + \tau_i;$$

here, $p_{i't}$ is the other region's price and τ_i is the unit cost of transporting the commodity from region i to region i'.[2]

Competition among expected-profit-maximizing storers precludes the existence of expected economic profits from storage in equilibrium. Specifically, the discounted expected future price in any region cannot exceed the current price by more than the unit carrying cost. Otherwise, storers would exploit expected profit opportunities by purchasing and storing the commodity, raising the current price and lowering the expected future price. Of course, the commodity is not stored if economic losses would be expected from doing so. This gives rise to the temporal arbitrage complementary slackness conditions:

$$(6) \qquad v_{it} \geq 0 \quad \perp \quad \delta E_t p_{it+1} \leq p_{it} + \kappa_i,$$

here, $\delta < 1$ is the discount rate and κ_i is the unit cost of storage in region i.

Model closure requires a theory of how expectations are formed. To this end, we generalize the perfect foresight assumption of the deterministic spatial-temporal equilibrium model by assuming that the expectations in the stochastic model are rational in the sense of Muth. The rationality assumption simply asserts that the price and revenue expectations formed by storers and producers in the model are consistent with the stochastic price distributions implied by the model.

Numerical Solution Methods

The stochastic spatial-temporal price equilibrium model cannot be solved using standard algebraic techniques. To see why this is so, note that in each period t and region $i = 1, 2$, there are seven contemporary endogenous variables, p_{it}, c_{it}, a_{it}, v_{it}, $E_t p_{it+1}$, and $E_t p_{it+1} \tilde{y}_{it+1}$, and one predetermined endogenous variable, s_{it}. Of the fourteen conditions that determine the values of all fourteen endogenous variables, only ten, namely (2)–(6), are conventional algebraic expressions. The remaining four conditions, that price and revenue expectations for each region be rational, are not.

One might hope to capture the rationality assumption algebraically. For example, let λ_i denote the function that gives the equilibrium price in region i in terms of the initial supplies s_1 and s_2 in regions 1 and 2. Having the equilibrium price functions for both regions, λ_1 and λ_2, the expected prices and per-acre revenues implied by the model could be computed by integrating over the yield distributions:

$$(7) \qquad E_t p_{it+1} = E_{\tilde{y}} \lambda_i(a_{1t} \cdot \tilde{y}_{1t+1} + v_{1t}, a_{2t} \cdot \tilde{y}_{2t+1} + v_{2t})$$

and

$$(8) \qquad E_t p_{it+1} \tilde{y}_{it+1} = E_{\tilde{y}} \lambda_i(a_{1t} \cdot \tilde{y}_{1t+1} + v_{1t}, a_{2t+1} + v_{2t}) \cdot \tilde{y}_{it+1}.$$

[1] The assumptions of deterministic consumption demand and acreage supply have been made solely to simplify the exposition and can easily be relaxed. Lagged production could also be replaced with contemporaneous production to accommodate nonagricultural commodities.

[2] Given a real number x and a real-valued function $f(x)$, we write $x \geq 0 \perp f(x) \leq 0$ to denote that the inequalities are complementary. That is, both hold and either $x = 0$ or $f(x) = 0$.

Appending these equations to (2)–(6) would result in fourteen conditions that could be solved for the fourteen unknowns.

Unfortunately, the equilibrium price functions λ_i are not known a priori and deriving them, the key to solving the stochastic spatial-temporal price equilibrium model, is a nontrivial functional equation problem. Computing the equilibrium price functions is a functional equation problem because the unknowns are real-valued functions that are characterized by an infinite number of conditions. Specifically, the equilibrium price functions λ_i are characterized by the conditions that for every possible pair of initial supplies s_{1t} and s_{2t}, $\lambda_i(s_{1t}, s_{2t}) = p_{it}$, where p_{it} is the equilibrium price that solves (2)–(6).

Functional equation problems typically lack closed-form solution and cannot be solved exactly using standard algebraic or mathematical programming methods. However, approximate solutions of arbitrary accuracy can be computed using appropriate numerical techniques. In what follows, we discuss the method of Chebychev collocation. The Chebychev collocation method is a highly accurate and efficient technique for solving the functional equation problems that arise in dynamic economic analysis (Judd).

Chebychev collocation is a special case of polynomial collocation. Polynomial collocation calls for each equilibrium price function λ_i to be approximated using 2-dimensional n^{th} degree polynomial. The approximating polynomials are expressed as linear combinations of the tensor product of 1-dimensional basis polynomials ϕ_j selected by the analyst:

$$\text{(9)} \qquad \lambda_i(s_1,s_2) \approx \sum_{j_1=0}^{n} \sum_{j_2=0}^{n} b_{ij_1j_2}\phi_{j_1}(s_1)\phi_{j_2}(s_2).$$

Here, each basis polynomial ϕ_j is a 1-dimensional polynomial of order j. In the polynomial collocation method, the $2(n+1)^2$ unknown coefficients $b_{ij_1j_2}$ are fixed by imposing $2(n+1)^2$ conditions that the polynomial approximants of λ_i exactly fit the prices implied by expressions (2)–(8) at a specified grid of $(n+1)^2$ collocation nodes (s_{1k_1}, s_{2k_2}), where $k_i = 0, 1, 2, \ldots, n$.

Chebychev collocation is distinguished from other polynomial collocation methods in how the collocation nodes and polynomial basis functions are specified. Chebychev collocation calls for the nodes to be selected so as to minimize the maximum polynomial approximation error. A well-known result from numerical analysis theory is that polynomial approximation error is minimized by selecting the Chebychev nodes (Atkinson):

$$s_{ik} = 0.5(\underline{s}_i + \bar{s}_i) + 0.5(\bar{s}_i - \underline{s}_i)\cos\left(\frac{k+0.5}{n+1}\right). \qquad \text{(10)}$$

Here, $\underline{s}_i$ and $\bar{s}_i$ are lower and upper bounds on the initial supplies that can be realized in region i. An important property of Chebychev node approximation is that the approximation error is guaranteed to go to zero as the number of nodes rises (Atkinson).

Chebychev collocation further calls for the basis polynomials to be selected to as to minimize the rounding error and computational cost associated with computing the coefficients $b_{ij_1j_2}$ of the polynomial approximants. Ideally, the basis polynomials ϕ_j are mutually orthogonal at the collocation nodes. The polynomials that satisfy this condition at the Chebychev nodes are the Chebychev polynomials (Atkinson). The Chebychev polynomials are recursively defined by $\phi_0(s) = 1$, $\phi_1(s) = s$, and

$$\text{(11)} \qquad \phi_{j+1}(s) = 2s\phi_j(s) - \phi_{j-1}(s).$$

In the vernacular of econometrics, the choice of Chebychev polynomials for basis functions completely eliminates "multicolinearity" among the "regressors".

The Chebychev collocation method replaces the original functional equation problem that characterizes stochastic spatial-temporal price equilibrium with a nonlinear complementarity problem. Although various methods may be used to solve for the equilibrium price function approximants, the following successive approximation algorithm strikes a nice balance between ease of implementation and computational efficiency:

0. Initial Step: Select the degree of approximation n; for $i = 1, 2$, select the supply bounds $\underline{s}_i$ and $\bar{s}_i$ and compute the Chebychev collocation nodes s_{ik_i} for $k_i = 0, 1, \ldots, n$; and for $i = 1, 2$; and for $j_i = 0, 1, \ldots, n$, make initial guesses for the coefficients of the approximating polynomial $b_{ij_1j_2}$.
1. Solution Step: For $i = 1, 2$ and $s_i \in [\underline{s}_i, \bar{s}_i]$, let

$$\lambda_i(s_1,s_2) \equiv \sum_{j_1=0}^{n} \sum_{j_2=0}^{n} b_{ij_1j_2}\phi_{j_1}(s_1)\phi_{j2}(s_2).$$

For $i = 1, 2$ and $k_i = 0, 1, \ldots, n$, solve the nonlinear complementarity problem

$$s_{ik_1} + x_{i'k_1k_2} = c_{ik_1k_2} + x_{ik_1k_2} + v_{ik_1k_2},$$

$$a_{ik_1k_2} = \alpha_i(r^e_{ik_1k_2}),$$

$$p_{ik_1k_2} = \pi_i(c_{ik_1k_2}),$$

$$x_{ik_1k_2} \geq 0 \quad \perp \quad p_{i'k_1k_2} \leq p_{ik_1k_2} + \tau_i,$$

$$v_{ik_1k_2} \geq 0 \quad \perp \quad \delta p^e_{ik_1k_2} \leq p_{ik_1k_2} + \kappa_i,$$

$$p^e_{ik_1k_2} = E_{\tilde{y}}\lambda_i(a_{1k_1k_2} \cdot \tilde{y}_1 + v_{1k_1k_2}, a_{2k_1k_2} \cdot \tilde{y}_2 + v_{2k_1k_2}),$$

$$r^e_{ik_1k_2} = E_{\tilde{y}}\lambda_i(a_{1k_1k_2} \cdot \tilde{y}_1 + v_{1k_1k_2}, a_{2k_1k_2} \cdot \tilde{y}_2 + v_{2k_1k_2})\tilde{y}_i,$$

for $p_{ik_1k_2}$, $x_{ik_1k_2}$, $c_{ik_1k_2}$, $v_{ik_1k_2}$, $a_{ik_1k_2}$, $p^e_{ik_1k_2}$, and $r^e_{ik_1k_2}$, where $i = 1, 2$ and $i' \neq i$.

2. Update Step: Find the coefficient $b'_{ij_1j_2}$, $i = 1, 2$, $j_i = 0, 1, \ldots, n$, that solve the linear equation system

$$\sum_{j_1=0}^{n} \sum_{j_2=0}^{n} b'_{ij_1j_2}\phi_{j_1}(s_{k_1})\phi_{j_2}(s_{k_2}) = p_{ik_1k_2}.$$

3. Convergence Step: Convergence Check: If $|b'_{ij_1j_2} - b_{ij_1j_2}| < \in$ for $i = 1, 2$, $j_i = 0, 1, \ldots, n$, and some convergence tolerance $\in$, update the coefficients by setting $b_{ij_1j_2} \leftarrow b'_{ij_1j_2}$ and stop; otherwise update the coefficients and return to step 1.

We solved the embedded nonlinear complementarity problem using Newton's method and the embedded linear equation problem using L-U factorization (Josephy, Atkinson).[3]

The algorithm above differs from that employed by Williams and Wright in several critical respects. Williams and Wright employ curve-fitting techniques fashioned from misplaced econometric intuition, rather than from established numerical analysis theory. More specifically, Williams and Wright promote approximating λ_1 and λ_2 with low-order polynomials, using least-squares to fit the polynomials at a large number of equally spaced nodes. Williams and Wright further suggest increasing the number of nodes, but not the order of the polynomial, to improve accuracy.

Williams and Wright prescriptions are problematic. First, one must increase both the number of nodes and the degree of the polynomial to improve the accuracy of the approximation; if the true solution functions are not low-order polynomials, the approximation will be poor regardless of the number of nodes. Second, uniform nodes are notoriously poor choices for polynomial approximation. There are a number of well-known examples of uniform node polynomial approximation schemes that actually lead to explosive, rather than convergent, approximation error (Atkinson). Finally, approximation based on fitting low-order polynomials to many nodes using least squares is actually slower than fitting the higher-order interpolating polynomial, particularly if orthogonal polynomials are used as basis functions.

Storage-Trade Interactions

To demonstrate the use of the methods and model presented in the preceding sections, we now analyze the joint and interactive effects of intertemporal storage and interregional trade under uncertainty. No specific commodity or pair of regions is assumed. However, market parameter values are varied over ranges sufficiently broad to contain values representative of world markets for major feedgrains and oilseeds. The base-case values of the market parameters are given in Table 1.

Three trade-storage regimes are considered. Under the first regime, storage but not trade is allowed; under the second, trade but not storage is allowed; and, under the third, both storage and trade are allowed. The market effects of introducing trade in the presence of storage can be ascertained by comparing the results for the "storage" and "both" regimes; the market effects of introducing storage in the presence of trade can be as-

Table 1. Parameter Base Case Values[a]

Parameter	Base Value
Consumption demand elasticity	0.6
Acreage supply elasticity	0.8
Unit storage cost	10
Unit transport cost	15
Production variability	15
Annual interest rate	5

[a]Elasticities expressed in absolute value and production variability expressed in percent coefficient of variation.

[3] Using the successive approximation algorithm, the two-region model discusses in the following section can be solved on an 80486 50 mega-hertz Gateway 2000 personal microcomputer using the Lahey Professional FORTRAN compiler 5.1 under MS-DOS 6.0. Solving the model took less than 5 seconds. We will make the FORTRAN code developed for the paper available to academic researchers. I will try to honor all requests placed via electronic mail to miranda.4@osu.edu.

Table 2. Base Case Simulation Results. Steady-State Levels of Selected Country-Specific Market Variables Under Different Scenarios and Regimes

		Scenario		
Variable	Regime	Surplus Producer	Balanced Producer	Deficit Producer
Stocks	Storage only	1.24	1.78	2.33
	Storage and trade	1.30	0.98	0.00
Exports	Trade only	14.66	1.28	0.00
	Storage and trade	14.67	1.07	0.00
Price standard deviation	Storage only	16.54	20.73	25.79
	Trade only	18.98	19.57	18.98
	Storage and trade	16.69	16.54	16.69

certained by comparing the results for the "trade" and "both" regimes.

In addition, three regional production scenarios are examined. A balanced producer is assumed to account for 50% of world supply and 50% of world demand. A surplus producer is assumed to account for 60% of world supply but only 40% of world demand. A deficit producer is assumed to account for 40% of world supply but 60% of world demand. By definition, the world consists of either two balanced producers or two unbalanced producers, one surplus and one deficit.

Quantity and price are normalized such that mean annual world production equals 100 quantity units and the world market clears, on average, at a price of 100 currency units per quantity unit. Each region is assumed to have a Cobb-Douglas, constant elasticity demand curve with the constant term calibrated to assure that the quantity demanded at the reference price of 100 equals the region's share of world demand under the given scenario. The random yield is assumed to be serially and spatially independent and, for each region, to follow a uniform three-point distribution (poor, average, and good harvest) symmetric about its mean.

Table 2 gives the results of stochastic simulations of the market model with parameters set equal to their base case values. The table provides steady-state estimates of selected market variables for the three trade-storage regimes and the three market scenarios. The results where generated by a single Monte Carlo simulation of 60,000 periods in length.

As seen in Table 2, the direction of trade reflects the relative supply-demand imbalance between the two trading partners. If the two regions are balanced, trade can flow in either direction in any given year, though never in both directions simultaneously. In the longrun, exports equal expected imports for a balanced producer, with (1.28) or without (1.07) storage. If the two regions are unbalanced, on the other hand, trade flows exclusively from the surplus to the deficit producer.[4]

Total trade between two balanced producers (2.56 without storage, 2.14 with storage) is substantially less than total trade between two unbalanced producers (14.66 without storage, 14.67 with storage). Trade occurs between two balanced producers only if there is some combination of production shortfall in one region and a bumper crop or excess stocks in the other. That is, trade between balanced producers arises from the temporary supply imbalances created by random differences in output between the two regions. Trade between unbalanced producers, on the other hand, arises mainly from the permanent structural supply-demand imbalance between the two regions, which is unrelated to random output variations, per se.

Total storage by two unbalanced producers in the presence of trade (1.30) is substantially less than total storage by two balanced producers (1.96). In an unbalanced scenario, the two regions are rigidly linked through trade and prices in the two regions rise and fall in tandem, always differing exactly by the cost of transportation.[5] Trade allows supply shocks to be easily transmitted between regions, effectively pooling the supply risks of the two regions and causing the role of storage in stabilizing supplies to diminish in each region. In the balanced scenario, on the other hand, actual

[4] This is true for the present parameterization of the model but not in general. If the regions were more balanced, or transportation costs were lower, or outputs were more variable, trade flow from the deficit to the surplus producer could occasionally arise. We purposely chose the present parameterization to represent the many one-sided trade relationships that exist in primary commodities.

[5] Under these circumstances, all storage will shift to one region. It follows from the arbitrage conditions (5)–(6) that storage will never be profitable in the deficit region, say region 1, if $(1 - \delta) \cdot \tau_1 > \kappa_1 - \kappa_2$; storage will never be profitable in the surplus region if the inequality holds strictly in the opposite direction. In particular if transportation costs are positive and storage costs are the same in both regions, the deficient region will never store.

trade is infrequent and the two regions are only loosely linked through trade. Trade is less effective at pooling supply risks and each region places greater reliance on storage as a means of stabilizing supplies.

The introduction of trade reduces global stockholding significantly. Total stockholding by two balanced regions falls 45% from 3.56 to 1.96; total stockholding by two unbalanced regions falls 63% from 3.57 to 1.30. In contrast, introducing storage has limited effect on global trade flow. The decline in the balanced scenario is 16% from 2.56 to 2.14 and is negligible in an unbalanced scenario.

To understand how trade and storage interact on a global scale, consider first the balanced scenario. In the balanced scenario, storage and trade both help correct temporary supply-demand imbalances arising from random output variations by distributing supplies over time and space, respectively. Trade and storage perform similar functions and thus, introducing trade will reduce stockholding and introducing storage will reduce trade flow. Trade, however, has a more substantial impact on storage than conversely because trade can correct temporary supply-demand imbalances with an immediacy not possible through storage: in the event of production shortfall, a nontrading region without sufficient stocks is helpless, while a trading region can always import to supplement short supplies.

In an unbalanced scenario, on the other hand, storage and trade perform different functions and interact quite differently. In an unbalanced scenario, unlike in the balanced scenario, the predominant function of trade is to help rectify the permanent supply-demand imbalance between the surplus and deficit producers. Storage can only distribute supplies over time within a region and thus cannot alleviate the chronic supply-demand imbalance between regions. Thus, while trade is at least a partial substitute for storage, the converse is not true. Accordingly, in an unbalanced scenario, trade significantly reduces global stocks but storage barely affects global trade flow.

As seen in Table 2, introducing storage in the presence of trade stabilizes price. For either a surplus or deficit producer, storage reduces price variability from 18.98 to 16.69; for a balanced producer, storage reduces price variability from 19.57 to 16.54. Trade, on the other hand, allows market instability to be transmitted from one region to another and can potentially destabilize a region's price if its trading partner's market is substantially more unstable. In the base case parameterization, this is true for a surplus producer, where introducing trade raises price variability from 16.54 to 16.69. In contrast, introducing trade reduces a deficit producer's price variability from 25.79 to 16.69 and a balanced producer's price variability from 20.73 to 16.54.

Let us now examine how the joint and interactive market effects of storage and trade vary under alternative assumptions regarding the values of the market parameters. For this phase of the analysis, we consider only the regime in which both storage and trade are allowed and focus on the longrun levels of just four variables: stocks, exports, acreage, and price variability. Each of the parameters in Table 1 is perturbed individually to compute the elasticity of the endogenous variables with respect to the parameter, assuming all other parameters are held constant at their base case values. The general results of the sensitivity analysis appear in Table 3.

As seen in Table 3, a larger domestic or foreign demand elasticity leads to a more stable price under all scenarios. The fall in price volatility reduces the incentive for storage and trade. Exports, imports, and stocks all fall. One notable exception is that the deficit producer, who holds no stocks regardless of demand elasticity changes.

Perhaps surprisingly, the longrun market equilibrium levels of all the endogenous variables are insensitive to changes in the acreage supply elasticity. Also, acreage planted appears to be insensitive to changes in all of the market parameters. The primary explanation of this is that while price variability may be substantially affected by changes in demand elasticity and in transportation and storage costs, the overall price level is not.

Raising the domestic storage cost discourages domestic stockholding and encourages foreign stockholding to a nearly equal extent. Since storage is generally price stabilizing, increases in the storage cost also destabilize the domestic price. In an unbalanced trade scenario, where the prices in the two regions are rigidly linked, increases in the storage cost will also destabilize the trading partner's price. In a balanced trade scenario, increases in storage costs have minimal effects.

For an unbalanced scenario, trade is driven by the fundamental supply-demand imbalance and is unaffected by changes in storage costs. For the balanced scenario, however, imports are sensitive to domestic storage costs while exports are not. Higher storage costs implies lower stock levels, undermining the market's ability to deal with high prices and short supplies. In compensation, the market places greater reliance on importation as a means of dealing with supply shortfalls.

Higher import costs reduce imports but do not affect exports. In the balanced scenario, higher import costs impede trade and lead to price instability

Table 3. Elasticity of Steady-State Values of Selected Endogenous Variables With Respect to Parameters Under Alternative Scenarios and Regimes

Variable/	Demand Elasticity		Acreage Supply Elasticity		Cost of Storage		Cost of Transportation		Production Variability	
Scenario	Domestic	Foreign	Domestic	Foreign	Domestic	Foreign	Export	Import	Domestic	Foreign
Storage										
Surplus	−1.01	−1.22	0.12	0.04	−2.00	2.00	0.05	0.00	1.60	0.99
Deficit	0.00	0.00	0.00	0.00	−2.00	2.00	0.00	0.00	0.00	0.00
Balanced	−1.36	−0.15	0.12	0.00	−1.17	0.02	0.00	0.32	1.94	0.00
Exports										
Surplus	−0.04	−0.12	−0.13	−0.07	0.00	0.00	−0.34	0.00	−0.05	0.03
Balanced	−0.12	−0.29	0.01	−0.02	−0.01	0.22	−0.73	0.14	0.35	1.02
Acreage										
Surplus	0.01	−0.02	−0.05	−0.01	0.00	0.00	−0.06	0.00	−0.01	0.01
Deficit	−0.02	0.00	0.04	0.02	0.00	0.00	0.00	0.06	−0.01	0.02
Balanced	0.00	0.00	0.00	0.00	0.00	0.00	−0.01	0.01	−0.01	0.01
Price variability										
Surplus	−0.34	−0.45	0.00	−0.02	0.14	0.10	0.00	0.00	0.49	0.31
Deficit	−0.45	−0.34	−0.02	0.00	0.10	0.14	0.00	0.00	0.31	0.49
Balanced	−0.56	−0.11	−0.03	0.00	0.20	0.02	0.04	0.03	0.70	0.04

and increased dependence on storage as a stabilization mechanism. For a deficit producer, however, trade is price destabilizing and increases in the import cost stabilize price by discouraging trade. This does not, however, lead to significant reduction in stocks. Higher export costs reduce exports but do not affect imports. In the balanced scenario, higher export costs impede trade and lead to price instability but have ambiguous effects on storage. For a surplus producer, higher export costs stimulate stockholding and stabilize price by insulating the market from the more unstable market of the trading partner.

Increased production variability, whether domestic or foreign, generally leads to increased price volatility. It also leads to increased storage, save for a deficit producer, who does not store. In balanced scenario, where market volatility is the main cause of trade, increased production variability leads to more trade in both directions. In unbalanced scenarios, however, where the chronic trade imbalance and not market volatility is the main cause of trade, increased production variability has only negligible effects on the volume and composition of trade.

Finally, we close with a qualitative comparison between the implications of the stochastic and deterministic spatial-temporal equilibrium models. First, in a stationary model, such as the one developed above, there would be no storage in the steadstate in the deterministic model—the price volatility necessary to drive storage would not exist. Second, in a deterministic world, two balanced producers would have no reason to ever trade—the random short-run supply-demand imbalances that drive trade between two otherwise identical regions would not exist. Third, a deterministic model is inherently incapable of generating price and supply instability. As such the deterministic model would be useless in investigating market instability issues.

Summary

In this paper, we have developed a model of competitive spatial-temporal price equilibrium under uncertainty and have shown how to solve it quickly and accurately using Chebychev collocation methods. Chebychev collocation replaces the underlying functional equation that characterizes the stochastic spatial-temporal equilibrium with a system of nonlinear complementarity conditions that can be solved using a combination of successive approximation and Newton-like methods. These use of Chebychev interpolation nodes and polynomial basis functions minimize the approximation error and the cost of solution. Chebychev collocation methods are superior to econometric curve-fitting strategies that have been promoted in the past, which can produce inaccurate approximations.

By performing stylized simulations of a generic two-region commodity market model we examined the joint and interactive effects of competitive commodity storage and trade. We found that where no chronic supply-demand imbalance between regions exist, trade and storage are both driven by the temporary supply-demand imbalances caused by random output variability. In this instance, trade and storage have similar market effects. Where a chronic supply-demand imbalance exists, on the other hand, trade is driven predominantly by the structural supply-demand imbalance and an asymmetry between trade and storage arises. In this instance, trade will profoundly undermine storage activity but storage will have little effect on trade.

Due to the computational effort required, the stochastic spatial-temporal equilibrium model may never be implemented on the same scale as its deterministic counterpart. Nonetheless, given the development of supercomputers and the use of appropriate numerical techniques the stochastic spatial-temporal equilibrium model should ultimately prove useful in applied studies of international commodity market stabilization issues.

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