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U.S. beef cattle stocks are among the most periodic economic time series. A theory of cattle cycles is constructed on the basis of breeding stock inventory decisions. The low fertility rate of cows and substantial lags and future feedback between fertility and consumption decisions cause the demographic structure of the herd to respond cyclically to exogenous shocks in demand and production costs. Known demographic parameters of cattle imply sharp numerical benchmarks for the resulting dynamic system and closely compare with independent econometric time-series estimates over the 1875–1990 period. The model fits extremely well.

I. Nature of the Problem

U.S. beef cattle stocks (fig. 1) are among the most periodic time series in economics. The fundamental reason for this is that cattle are both capital and consumption goods. Current breeding and consumption decisions have large effects on future stocks. Reproductive stock accounts for about 40 percent of total stocks, and substantial gestation and maturation lags stretch the stock adjustment process over lengthy intervals of time. Economists have recognized the importance of these lags in agricultural production (Ezekiel [1938] is the classic reference). However, biological lags must be combined with more comprehensive economic behavior to explain the persistence of the cycles in figure
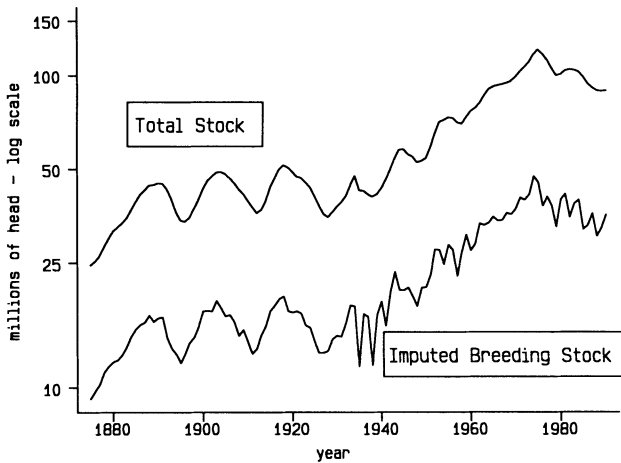
Fig. 1.—Stocks of beef cattle, 1875–1990

1. There are many empirical studies of the supply of beef.[1] Neverthe-less, more complete market analysis in terms of the modern econom-ics of renewable resources appears to be essential for this problem. The model presented here seems to account for much of the cyclical properties of these data.

Jarvis (1974) was the first to point out that capital investment as-pects of cattle can produce unusual supply responses and dynamics. An increase in demand for beef *reduces* current supply if younger animals are held back to achieve greater slaughter weight or if more females are withheld for reproduction to increase the future size of the herd (Jarvis 1974; Rosen 1987). We show how reproductive inventory decisions at the breeding margin combine with production lags to produce cattle cycles.

The model is based on a few elementary facts of beef cattle technol-ogy. Cattle typically are held on land that is below the margin of cultivation. Breeding stock and young animals forage in pastures, crop remnants, and rangeland. Once they are large enough, virtually all young males (steers) and about one-third of young females (heif-ers) are sent to corn feedlots for "finishing," to be slaughtered at approximately 2 years of age. Remaining females are added to the breeding stock and bred on an annual cycle, also beginning at age 2.

[1] Empirical studies of the supply of beef by Nerlove, Grether, and Carvalho (1979) and Nerlove, Fornari, and Tanizaki (1992) are the most comprehensive available. See also Trapp (1986) and Rucker, Burt, and LaFrance (1986), among others. Not much is available on the theory of market equilibrium for completely managed renewable resources. Recent work on annual storage of agricultural goods (Williams and Wright 1991; Deaton and Laroque 1992) is related but is inappropriate because there is no reproductive feedback to future stocks.

A cow has at most one calf after a 9-month gestation period, and an effective reproductive life of 8–10 years. The fertility rate of beef cattle is in the .80–.95 range, and the natural death rate is in the vicinity of 10 percent.

Gestation and birth delays embody natural "time-to-build" features (Kydland and Prescott 1982) in the age structure of stocks and cause cyclical feedback between current consumption and future reproduction decisions. Our work is related to recent developments that incorporate changing age distributions of stocks into agricultural dynamics (Chavas and Klemme 1986; Whipple and Menkhaus 1989; Foster and Burt 1992). Exogenous shocks have persistent effects by changing slaughter and breeding stock investment decisions. These decisions alter the age distribution of the herd and cause cyclical "echo" responses as the age distribution converges to its stable equilibrium value. The model has been specified with the minimum number of parameters necessary for the problem: the rates of fertility and natural deaths, the rate of interest, and demand and supply shocks. Beef cattle technology is so simple that these parameters are approximately known a priori and provide strong benchmarks for predicting the time path of cattle stocks and consumption. These predictions track the actual data extremely well and provide strong evidence that breeding stock inventory decisions are important for cattle cycles. Trends, including improvements in breeds and substitute food technologies, are interesting in their own right but are ignored because they do not contribute to understanding persistent cycles.

Sections II and III sketch a simple market equilibrium model and discuss its dynamic properties. Empirical investigation of the model specification is presented in Section IV. Estimates and benchmark comparisons appear in Section V. Conclusions are presented in Section VI.

II.   The Model

We study the behavior of female breeding stock inventories, assuming a 1-year gestation-birth delay and a fixed 2-year maturation lag for both breeding and consumption (slaughter). With exogenously fixed lags, the interesting economic decision is whether to breed or consume a mature female. Since some heifers are always slaughtered and all steers are slaughtered at maturity, males are strictly inframarginal and can be treated as females marked for slaughter for present purposes.[2] A number of other simplifying assumptions are made: adult

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[2] The price of slaughter animals per unit weight is similar for steers and heifers. Beef quality is lower for cows, but since they are larger, gross value is not appreciably different. These equalizing differences are ignored to simplify the model.
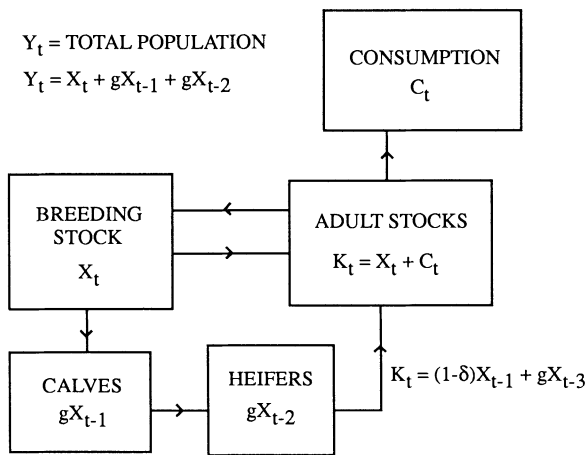
FIG. 2.—Population dynamics

cattle are homogeneous and undifferentiated, independent of age or prior fertility; the slaughter age is exogenously fixed at 2 years;[3] and there are no interactions between trends and cycles.

A. Population Mechanics

Population mechanics follow the schematic of figure 2. We assume a closed system with no imports or exports of live animals. Adult stocks are either held for breeding or slaughtered. After a 1-year lag, each animal reserved for breeding gives birth to g calves ($g < 1$). At birth, all surviving calves enter a "pipeline" and remain there for 2 years before merging into the adult stock at maturity. Assume that beef quality is invariant to age and breeding history. Then mature adult cows are consumed on equal terms with mature steers and heifers, and it is natural to treat all adults as neoclassical capital with exponential death rate δ. This simplification is required empirically because the stock data are total head counts, not classified by age, and it eliminates the need to keep track of the adult age distribution. The state space is compacted to only three capital goods: adults, yearlings,

[3] Making the slaughter age endogenous (the "Austrian margin") causes the order of the system describing market dynamics to vary with market conditions. This has proved too difficult to analyze and is omitted. It may not be important here, for three reasons. First, variations in slaughter age give only minor feedback on future population decisions compared to breeding stock inventory decisions. Second, veal is a very small proportion of U.S. beef consumption and is almost entirely produced by the male offspring of dairy cows (a distinct breed in the United States). Third, slaughter age seldom varies by as much as 1 year, yet the total stock cycles in fig. 1 are 9 years or more. The Austrian margin cannot affect much just as a matter of accounting.

and calves. Empirical evidence will be brought to bear on the realism of this specification below.

Population stock flow accounts implicit in figure 2 are very simple. Let x_t be the breeding stock, c_t the number consumed, and k_t the total number of mature animals at the beginning of period t. Then

$$k_t = x_t + c_t. \tag{1}$$

The gx_{t-1} calves born at t enter the pipeline, and the gx_{t-2} calves born one year earlier remain there as yearlings. The head count, y_t, of all stocks is the sum of adults, yearlings, and calves:

$$y_t = x_t + gx_{t-1} + gx_{t-2}. \tag{2}$$

Intertemporal constraints derive from standard demographic accounting because the U.S. beef cattle industry is essentially closed. Imports and exports of cooked beef products are trivial, and hardly any animals are shipped across international borders. Imports of fresh beef have been highly restricted in the United States over the years. Basically, none was imported until the 1960s, and at their highest levels today imports account for less than 10 percent of domestic consumption. Little beef is exported from the United States. We assume a closed system in which U.S. consumption is produced exclusively from domestic stock.

All stocks in the pipeline in figure 2 move up one place each year. A total of c_t adults are sent to slaughter (all steers and a strictly positive fraction of heifers), and δk die of natural causes. The gx_{t-3} yearlings in the pipeline at $t - 1$ enter the adult herd in period t, so breeding stock evolves as

$$x_t = (1 - \delta)x_{t-1} + gx_{t-3} - c_t. \tag{3}$$

The three variables $[x_{t-1}, x_{t-2}, x_{t-3}]$ conveniently represent the state of the population because current stocks of calves, yearlings, and adults are readily calculated from them.

B. Breeding Stock Inventory Policy

Ranchers choose between breeding and slaughtering adults. In an ongoing market equilibrium, they do some of each, and profits are equated at both margins. With constant returns to scale, a typical rancher's calculation is made independent of herd size, and profits are equated on the average as well as on the margin. Let p_t be the price of an animal on the hoof and m_t the feeding cost of preparing it for slaughter. Define $q_t = p_t - m_t$ as the net return from selling an animal at t for consumption.

The return from breeding an adult animal is the opportunity to sell it and its progeny later minus holding costs and forgone interest. A cow survives with probability $1 - \delta$ and is worth $E_t(1 - \delta)q_{t+1}$ next year, where E_t is expectations given period t information. Its g progeny sit in the pipeline for 2 years and have value $E_t g q_{t+3}$ when potentially sold 3 years from now. Let ι be the rate of interest and $\beta = 1/(1 + \iota)$ the discount factor. Then the gross return from breeding is $E_t[\beta(1 - \delta)q_{t+1} + \beta^3 g q_{t+3}]$. Net return subtracts expected holding costs. Let h_t be the unit holding cost of an adult and assume proportional adult equivalent holding costs of $\gamma_0 h_t$ and $\gamma_1 h_t$ for calves and yearlings. Then discounted holding costs are

$$z_t = h_t + \beta g \gamma_0 h_{t+1} + \beta^2 g \gamma_1 h_{t+2}. \tag{4}$$

If there is strictly positive supply (and we ignore nonnegativity constraints), market equilibrium requires equal value to both slaughter and breeding, or

$$q_t = E_t[\beta(1 - \delta)q_{t+1} + g\beta^3 q_{t+3} - z_t]. \tag{5}$$

Note that the coefficients on the leading terms in q in equation (5) are the discounted coefficients of the lagged terms in x in equation (3). This duality property arises because equation (5) is recognized as the Euler equation from maximizing the discounted expected value of beef production in a perfectly competitive market.

Demand and boundary conditions complete the description of the model. Specifying a particular demand function for such a lengthy period is problematic, but since population dynamics are linear, we take demand as approximately linear too:

$$c_t = \alpha_0 - \alpha p_t + d_t = \alpha_0 - \alpha q_t - \alpha m_t + d_t, \tag{6}$$

where $p - m = q$ and d_t is a demand shifter (scaled so its coefficient is unity). Initial stocks are exogenously given, and at least one of them is strictly positive. Limiting values of capital in the future are bounded if the average rates of growth of all cost and demand shifters are less than the rate of interest.[4]

Equations (1)–(6) reduce to two linear third-order difference equations in x_t and q_t:

$$x_t - (1 - \delta)x_{t-1} - gx_{t-3} = -\alpha_0 + \alpha q_t + \alpha m_t - d_t \tag{7}$$

[4] We impose upper-bound constraints on the amounts of cattle and money producers can borrow to rule out pyramiding schemes that would cause future capital values to be unbounded. Once such schemes are ruled out, it can be shown that the proposed solution is the market equilibrium (Scheinkman 1986).

and

$$E_t[q_t - \beta(1 - \delta)q_{t+1} - \beta^3 g q_{t+3} + h_t$$
$$+ \beta g \gamma_0 h_{t+1} + \beta^2 g \gamma_1 h_{t+2}] = 0. \tag{8}$$

III. Market Solution

We concentrate on the case in which feed, forage, and land are elastically supplied to the industry. Then holding costs h_t and finishing costs m_t are exogenous, independent of x_t or c_t, and the model is recursive. The supply of beef is elastic at constant supply price q_t determined from equation (8), independent of x_t; consumption is demand determined at that price; and stocks evolve according to equation (7).

A. Renewal Conditions

The primary task is to characterize the stochastic processes governing $\{x_t, c_t\}$. Renewal requirements give rise to one unusual detail. Suppose that m, h, and d were constant and x and q settled down to steady-state values. Inspection of equations (7) and (8) reveals two necessary *renewal conditions* for this problem:

$$g - \delta > 0 \tag{9}$$

and

$$\beta^3 g + \beta(1 - \delta) - 1 > 0. \tag{10}$$

Obviously, births must exceed natural deaths for renewal in equation (9). In addition, the gross return from reproduction must exceed the return to slaughter at the steady-state price in equation (10). Otherwise holding costs could not be covered and the herd rationally would be driven to extinction.

B. Market Equilibrium

For illustrative purposes, assume AR(1) processes for the shocks $\{d_t, m_t, h_t\}$:

$$u_t = \rho_u u_{t-1} + \epsilon_t^u \quad \text{for } u = d, m, \text{ and } h, \tag{11}$$

where the ρ_u's are serial correlation parameters with $|\rho_u| \leq 1$, and ϵ_t^u are independently and identically distributed (i.i.d.) innovations with zero means, constant variances, and zero cross-correlations at all leads and lags. When we substitute equation (11) into equations (7) and (8), rescale ϵ^m to include the multiplicative effect of α, and sup-

press constants, the full model is, in operator notation,

$$(1 - \phi_1 L)(1 - \phi_2 L)(1 - \phi_3 L)x_t = \alpha q_t + \frac{\epsilon_t^m}{1 - \rho_m L} - \frac{\epsilon_t^d}{1 - \rho_d L} \quad (12)$$

and

$$E_t\left[(1 - \lambda_1^{-1}L^{-1})(1 - \lambda_2^{-1}L^{-1})(1 - \lambda_3^{-1}L^{-1})q_t \right.$$
$$\left. + (1 + \beta g\lambda_0 L^{-1} + \beta^2 g\lambda_1 L^{-2})\frac{\epsilon_t^h}{1 - \rho_h L}\right] = 0, \quad (13)$$

where ϕ_i and λ_i are the roots of the two cubic characteristic equations[5]

$$\phi^3 - (1 - \delta)\phi^2 - g = 0 \quad (14)$$

and

$$g\beta^3\lambda^3 + (1 - \delta)\beta\lambda - 1 = 0. \quad (15)$$

The characteristic roots have three important properties.[6]

i) Equations (14) and (15) each have one real root and two imaginary roots. The complex roots produce cyclical responses to shocks. Let $[\phi_1, \lambda_1]$ be the real roots and let $[\phi_2, \phi_3, \lambda_2, \lambda_3]$ be the imaginary roots.

ii) The product of complementary roots across equations equals $1 + \iota$: $\phi_i\lambda_i = 1/\beta$, for $i = 1, 2, 3$. This familiar property results from the symmetry of leading and lagging coefficients in equations (3) and (5) mentioned above.

iii) Three roots are explosive and three are stable. The real root of equation (14) exceeds $1 + \iota$, $\phi_1 > 1/\beta$, so the real root of equation (15) is stable, from property ii: $\lambda_1 < 1$. The complex roots of equation (14) are stable and the complex roots of equation (15) are explosive: $|\phi_j| < 1$ and $|\lambda_j| > 1/\beta$, for $j = 2, 3$.

Roughly speaking, the stable real root in equation (15) is associated with the overall speed of convergence of the system. The two complex

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[5] The sixth-order characteristic equation of (12) and (13) factors into these two cubics in the recursive case. If h_t or m_t is increasing in x_t and c_t, there is a rising supply price of beef and the structural price and quantity system is fully simultaneous. The characteristic equation remains of order six but has no cubic factors. The recursive form has proved useful in uncovering the essential economics of the problem. Numerical calculations reveal that the characteristic roots are fairly insensitive to rising supply price specifications in the relevant parameter range, so stock dynamics are not affected much. This class of models easily accommodates both a rising supply price and shock processes of any order. Note that a rising supply price would cause cycles in prices and consumption that are not observed. See Secs. V and VI.

[6] The graphs of eqq. (14) and (15) and renewal condition (10) prove properties i and iii. Substituting $\phi_i\lambda_i = 1/\beta$ into either equation verifies property ii.

stable roots in equation (14) are associated with convergence of the age structure of the herd to its stable distribution. Cycles arise from demographic "echo effects" because the current age distribution constrains future reproductive capacity. Demand and supply shocks cause ranchers to change their breeding stock inventories, and these decisions percolate through all future birth cohorts. The percolations oscillate unless initial stocks are accidentally lined up just right.

The model is solved by taking the unstable roots forward and the stable roots backward (Sargent 1979; Hansen and Sargent 1990).[7] The solution in terms of the state variables $[x_{t-1}, x_{t-2}, x_{t-3}]$ and current and past innovations $\{\epsilon_t^d, \epsilon_t^m, \epsilon_t^h\}$ can be shown to be

$$(1 - \lambda_1 L)(1 - \phi_2 L)(1 - \phi_3 L)x_t = \Omega_t \qquad (16)$$

and

$$c_t = (\phi_1 - \lambda_1)(1 - \phi_2 L)(1 - \phi_3 L)x_t - \Omega_t, \qquad (17)$$

with

$$\Omega_t = \frac{(\rho_d - \lambda_1)\epsilon_t^d}{(\phi_1 - \rho_d)(1 - \rho_d L)} - \frac{(\rho_m - \lambda_1)\epsilon_t^m}{(\phi_1 - \rho_m)(1 - \rho_m L)}$$
$$- \left[\frac{\alpha \Gamma \lambda_1}{(\phi_1 - \rho_h)(1 + r^2 \rho_h^2 + 2r\rho_h \cos \theta)} \right] \left(\frac{\epsilon_t^h}{1 - \rho_h L} \right), \qquad (18)$$

where Γ and θ are defined by $\Gamma = 1 + \beta \rho_h \gamma_0 g + \beta^2 \rho_h^2 \gamma_1 g$ and $\phi_j = re^{\pm i\theta}$, for $j = 2, 3$. The formula for prices in terms of the state variables is similar to equation (17). Finally, the autonomous form for c_t results from substituting equation (16) into law of motion equation (3):

$$(1 - \lambda_1 L)c_t = -(1 - \phi_1 L)\Omega_t. \qquad (19)$$

Autonomous equations for p_t or q_t look similar to equation (19).

In this example, the c_t (slaughter) process in equation (19), and by implication the price process, is of lower order than the process for x_t (breeding stock) in equation (16) and contains only real roots. Consumption and prices do not oscillate, only stocks do. This strong restriction arises from the recursive specification—constant returns to scale and elastic factor supplies to the industry.[8] If factor supplies are

[7] Some technical details, including possible corner solutions for this class of models, are presented in Rosen, Murphy, and Scheinkman (1989).

[8] Favarro (1989) structured a related model for Uruguayan cattle, allowing for rising supply price of land and grazing competition between cattle and sheep. Another intriguing possibility is to include rising supply price of feed grains and competition for them among cattle, hogs, and poultry. Some experimentation along those lines suggests that the dynamic effects on cattle of shocks in competing industries are second-order relative to what we focus on here, but these possibilities deserve more investigation than we have given them.
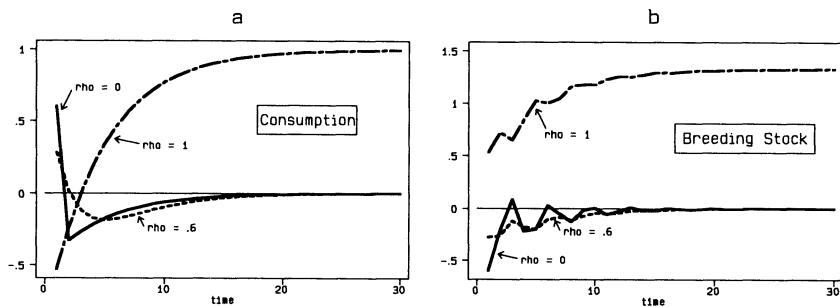
Fig. 3.—Consumption and breeding stock distributed lags

not elastic, the stable complex roots of the characteristic equation appear in both autonomous consumption and autonomous breeding stock equations. Prices have predictable cycles in that case (though no profit can be made of it). Nevertheless, the recursive structure suggests that there is a sense in which cycles are more important for breeding stock than for prices or consumption. Furthermore, consumption cycles are not detected in the historical time series (see Sec. IV), so the recursive specification has empirical justification.

C. Distributed Lags

Figure 3 shows how positive demand innovations ϵ_t^d work their way through the market. Cost innovations would look similar except for sign. Figure 3 graphs the power series expansion of $[(1 - \lambda_1 L)(1 - \phi_2 L)(1 - \phi_3 L)(1 - \rho_d L)]^{-1}$ for breeding stock x_t from equation (16) and of $(1 - \phi_1 L)[(1 - \lambda_1 L)(1 - \rho_d L)]^{-1}$ for slaughter c_t from equation (19), multiplied by the coefficient on ϵ^d in equation (18), all calculated at realistic cattle parameters $g = .85$, $\beta = .909$, and $\delta = .1$. Figure 3 indicates different market responses to transitory ($\rho_d = 0$), persistent ($\rho_d = 1$), and intermediate ($\rho_d = .6$) shocks. The coefficients on the innovation terms in equation (18) imply that innovations are transitory when $\rho_d < \lambda_1$: the shock dies out quicker than the market's overall response speed. As illustrated with $\rho_d = 0$, adult stocks are sold off and consumption is increased in the period in which a positive shock occurs. Consumption is reduced later so that stocks can be built back up to sustain normal demand. The initial sell-off of adult stocks disturbs the age distribution of young animals in the pipeline and causes the oscillatory responses of stocks. Ranchers behave opportunistically and sell more when demand is high because they anticipate that it is likely to fall back to normal later.
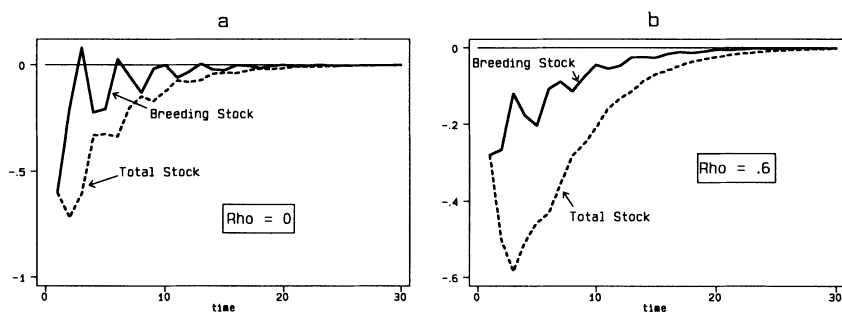
FIG. 4.—Breeding stock and total stock response comparisons

Innovations are permanent in an economic sense when $\rho_d > \lambda_1$ because then the shock is more persistent than the overall response speed of the system. Here intertemporal dynamics are quite different from what happens when shocks are transitory. For the $\rho_d = 1$ case illustrated in figure 3, consumption initially *falls* when demand increases. Ranchers know that increased demand is likely to be sustained for a long time into the future. More than the average number of females must be bred initially to achieve the greater herd size necessary to provide higher future demand. Consumption turns positive when stocks get large enough and smoothly converges to its new steady state. Breeding stocks increase initially and then grow cyclically to their new steady state for the same reasons as before. Figure 3 reveals a kind of backward-bending initial supply response to permanent shocks and a normal rising supply price response to transitory shocks (Rosen 1987). An intermediate case in which $\rho_d = .6$ works out to be slightly on the transitory side of this example. Its responses are smaller because the two effects tend to cancel each other.

Figure 4 contrasts responses of breeding stocks to total stocks for $\rho_d = 0$ and $\rho_d = .6$. Notice how the moving average in equation (2) redistributes the y responses and stretches them out over a longer time interval. This is an important aspect of understanding the cyclicity of total stocks in figure 1.

IV. Empirical Comparisons

A. Estimating the Breeding Stock

The stock data[9] in figure 1 are total head counts of all beef cattle in the United States at the end of each year, not only the breeding stock

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[9] All data are taken from *Historical Statistics of the United States: Colonial Times to 1970* (U.S. Department of Commerce, Bureau of the Census) and annual issues of *Agricultural Statistics* (U.S. Department of Agriculture). Beef cattle are estimated by subtracting dairy cattle from total cattle reported on farms. Oxen may be included in the earliest years, but further adjustments for this are not possible.

portion in equation (16). However, equation (2) implies that breeding stocks x_t can be replaced in equation (16) by inverting total stock y_t:

$$x_t = \frac{y_t}{1 + gL + gL^2} \equiv \frac{y_t}{G(L)}. \tag{20}$$

We use equation (20) to impute a breeding stock x_t from the total herd count data y_t. The construction is important because it bears on the empirical relevance of the simplifying assumptions used to compact the state space.

Two initial conditions are needed to iterate x_t from y_t in equation (20). The average annual growth rate of 4.5 percent for the U.S. cattle population over the 1875–1990 period is extrapolated back to 1875 and 1876 for this purpose.[10] The following argument shows that errors do not accumulate in the iteration. The solution to equation (20) is

$$x_t = \frac{y_t}{(1 - \xi_1 L)(1 - \xi_2 L)} + b_1 \xi_1^t + b_2 \xi_2^t, \tag{21}$$

where the b's are constants satisfying some initial conditions and the ξ's solve $\xi^2 + g\xi + g = 0$. The roots ξ_j are complex and stable ($\xi_1\xi_2 = g < 1$ and $|\xi_j| = \sqrt{g}$), so the effects of measurement error decline in damped oscillations in subsequent years. Some experimentation with different initial conditions showed convergence to the same constructed x series within 15 or 20 years.

B.   Validating the Three-State Approximation

Three alternative breeding stock (x_t) series have been calculated for values of g approximating the net fertility rate of U.S. cattle under modern conditions: .80, .85, and .90. Adding consumption to both sides of equation (3), we get

$$k_t = c_t + x_t = (1 - \delta)x_{t-1} + gx_{t-3}. \tag{22}$$

Given the assumptions in figure 2, equation (22) is an identity in the theoretical model, though certainly not in the actual data. Its empiri-

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[10] In a deterministic system with a constant 4.5 percent growth rate, the ratio of breeding stock to total population, x/y, is fixed at $[1 + (g/1.045) + (g/1.045^2)]^{-1}$, from eq. (20). Given g, this factor is applied to y_{1875} to get an initial condition for x_{1875}. An initial condition for $t = 1876$ is obtained by applying $x_{1876} = 1.045x_{1876}$. Recall that g is interpreted as the number of live births. The recursion formula in eq. (20) assumes that all calves survive into their second year. Calves survive at a lower rate than yearlings, but we have not experimented with alternative assumptions because this is likely to be an unimportant source of error.

TABLE 1

REGRESSION ESTIMATES:

$$k_t = x_t + c_t = b_0 + b_1 x_{t-1} + b_2 x_{t-2} + b_3 x_{t-3}$$

| | REGRESSION COEFFICIENT | | | | |
PERIOD	b_0	b_1	b_2	b_3	$\overline{R}^2$
			$g = .80$		
1900–1990	−1,544	1.11	.04	.83	.978
	(849)	(.12)	(.15)	(.12)	
1930–90	1,785	.97	.04	.88	.977
	(1,153)	(.12)	(.15)	(.12)	
			$g = .85$		
1900–1990	−1,524	1.01	.09	.92	.976
	(859)	(.11)	(.14)	(.11)	
1930–90	1,855	.87	.08	.96	.976
	(1,162)	(.11)	(.13)	(.11)	
			$g = .90$		
1900–1990	−1,509	.91	.14	1.00	.977
	(863)	(.11)	(.13)	(.10)	
1930–90	1,895	.80	.13	1.03	.975
	(1,160)	(.11)	(.12)	(.10)	

NOTE.—Standard errors are in parentheses. x_t is constructed for the indicated value of g from total stock series by methods described in the text; c_t is slaughter numbers (from U.S. Dept. of Agriculture sources).

cal validity can be verified by adding an intercept, a second lag in x_{t-2}, and a disturbance term to the right-hand side and treating it as a regression. Actual slaughter numbers (available only since 1900) are added to the constructed x_t series to estimate k_t in equation (1), and k_t is regressed on x_{t-1}, x_{t-2}, and x_{t-3}. If the assumed population-dynamic specification in the model is valid, the regression coefficient on x_{t-1} should estimate $1 - \delta$ in the range [.9, .95] for cattle, both the intercept and the regression coefficient on x_{t-2} should be zero, and the coefficient on x_{t-3} should give back the value of g used to construct x_t.

Regression coefficients for the entire 1900–1990 period for which k_t can be constructed and for the subperiod 1930–90 appear in table 1. Considering the drastic simplifications of reducing the age distribution to three state variables and ignoring known changes in technology over the period, such as corn feeding, the fit is slightly astonishing. A value of zero for the constant term cannot be statistically rejected, and its point estimate is unreliable. The other coefficients are more sharply estimated. In all cases the estimated coefficient on x_{t-2} is numerically small and not significantly different from zero.
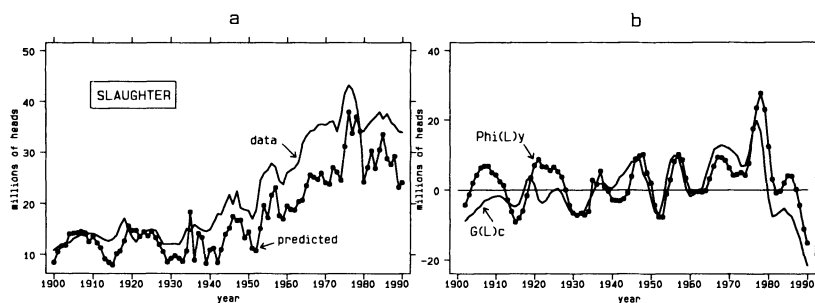
FIG. 5.—Population dynamics validity check

The estimates of $1 - \delta$ are decreasing in the assumed value of g and only mildly out of the .90–.95 range. The implied estimates of g in the last column are slightly too large, and increasingly so for larger assumed values. However, 95 percent confidence intervals cover the assumed value in every case. On the deficit side, F-tests formally reject the joint hypothesis ($b_0 = b_2 = 0$, $b_1 = .95$, $b_3 =$ assumed value of g) in table 1. And when additional lagged values of x are included in the regression, the coefficient on x_{t-4} is statistically significant (all larger lags are insignificant), indicating that the model's intertemporal specification is not strictly accurate.

Graphic evidence that our dynamic specification is good enough to warrant serious consideration is presented in figure 5. Equation (22) implies a consumption series corresponding to any given $\{x\}$ series. This completely artificial consumption series imputed from $\{x\}$, calculated with $\delta = .1$ and $g = .85$, is compared to actual slaughter numbers, a series that is collected independently of total stock counts, in figure 5a. Though the constructed series is noisier, its major cyclical variations compare very well in both timing and magnitude to actual consumption. It errs mostly in the trends. The actual productivity of U.S. cattle farmers in sustaining consumption is greater than equation (22) predicts, and increasingly so after 1940. However, the comparison is quite close if trends are eliminated. This can be shown in a different way as follows. Consumption $\{c\}$ and total stock $\{y\}$ are independent series. Equations (20) and (22) imply the following relationship between observable transformations of them:

$$G(L)c_t \equiv (1 + gL + gL^2)c_t$$
$$= -[1 - (1 - \delta)L - gL^3]y_t \quad (23)$$
$$\equiv \Phi(L)y_t.$$

These are plotted ($g = .85$ and $\delta = .1$) in figure 5b after trends are removed. The cyclical movements of $G(L)c$ and $\Phi(L)y$ are virtually identical.

Breeding stock $\{x\}$ imputed from $g = .85$ is plotted along with total herd size $\{y\}$ in figure 1. The y series is smoother and more periodic than the x series because $\{y\}$ is a three-term moving average of $\{x\}$. Both change their character after 1930, probably because of improvements in the quality of the data as well as changes in the dynamic structure of cattle technology caused by corn feeding and other agricultural innovations that are ignored here.

C. ARMA Formulation

It is pointless to test all the strong restrictions implied by this simple model: it is known to be wrong in its details, and formal statistical rejections of the null would tell us no more than we already know. The more interesting question is, How wrong is it? We proceed by comparing estimated autonomous autoregressive moving average (ARMA) forms for x_t and c_t in equations (16) and (19) with some theoretical benchmarks.

The benchmarks are found by multiplying both sides of equations (16) and (19) by $(1 - \rho_d L)(1 - \rho_m L)(1 - \rho_h L)$. The resulting composite error terms have zero autocorrelation after two lags, so the autonomous forms for $\{x_t, c_t\}$ can be represented as ARMA processes. The result is an ARMA(6, 2) for x_t:

$$(1 - \lambda_1 L)(1 - \phi_2 L)(1 - \phi_3 L)(1 - \rho_d L)(1 - \rho_m L)(1 - \rho_h L)x_t \\ = \omega_t + b_{x1}\omega_{t-1} + b_{x2}\omega_{t-2}, \tag{24}$$

where ω_t is an i.i.d. random variable with zero expectation defined by

$$\omega_t + b_{x1}\omega_{t-1} + b_{x2}\omega_{t-1} \equiv (1 - \rho_m L)(1 - \rho_h L)\bar{\epsilon}_t^d \\ + (1 - \rho_d L)(1 - \rho_h L)\bar{\epsilon}_t^m \\ + (1 - \rho_d L)(1 - \rho_m L)\bar{\epsilon}_t^h,$$

with

$$\bar{\epsilon}_t^u = \left(\frac{\rho_u - \lambda_1}{\phi_1 - \rho_u}\right)\epsilon^u \quad \text{for } u = d, m, h.$$

Applying the same procedure to equation (19) results in an ARMA(4, 3) for c_t:

$$(1 - \lambda_1 L)(1 - \rho_d L)(1 - \rho_m L)(1 - \rho_h L)c_t \\ = \tilde{\omega}_t + b_{c1}\tilde{\omega}_{t-1} + b_{c2}\tilde{\omega}_{t-2} + b_{c3}\tilde{\omega}_{t-3}, \tag{25}$$

with i.i.d. random variable $\tilde{\omega}_t$ and moving average constants b_{cj} defined by an expression similar to ω_t. Write equation (24) as $B(L)x_t = $

$A(L)\omega_t$, where $B(L)$ is the sixth-order polynomial in L and $A(L)$ is a second-order polynomial in L. Then equations (20) and (24) imply an ARMA(6, 4) for $\{y_t\}$ of the form $B(L)y_t = G(L)A(L)\omega_t$, with the same autoregressive part as the x process.

Two simplifications are useful.

i) If $\rho_d = \rho_m = \rho_h = \rho$, several terms in $1 - \rho L$ in (24) factor out. There is effectively only one time-independent shock, a linear combination of the ϵ_t^d, ϵ_t^m, and ϵ_t^h shocks. Here equation (24) reduces to an AR(4) and equation (25) to an ARMA(2, 1). The $\{y\}$ process is ARMA(4, 2):

$$(1 - \lambda_1 L)(1 - \phi_2 L)(1 - \phi_3 L)(1 - \rho L)x_t = \omega_t, \tag{26}$$

$$(1 - \lambda_1 L)(1 - \rho L)c_t = \tilde{\omega}_t + b_c \tilde{\omega}_{t-1}, \tag{27}$$

and

$$(1 - \lambda_1 L)(1 - \phi_2 L)(1 - \phi_3 L)(1 - \rho L)y_t = \omega_t + g\omega_{t-1} + g\omega_{t-2}. \tag{28}$$

ii) If $\rho_m = \rho_h$, then $b_{x2} = b_{c3} = 0$. There are two independent shocks, one to "demand" and another to "supply," and x_t becomes an ARMA(5, 1) and c_t an ARMA(3, 2):

$$(1 - \lambda_1 L)(1 - \phi_2 L)(1 - \phi_3 L)(1 - \rho_d L)(1 - \rho_s L)x_t = \omega_t + b_x\omega_{t-1} \tag{29}$$

and

$$(1 - \lambda_1 L)(1 - \rho_d L)(1 - \rho_s L)c_t = \tilde{\omega}_t + b_{c1}\tilde{\omega}_{t-1} + b_{c2}\tilde{\omega}_{t-2}. \tag{30}$$

Here $\{y\}$ follows an ARMA(5, 3) with the same autoregression coefficients as equation (29).

The essential elements of cattle technology are so simple that $(\lambda_1, \phi_2, \phi_3)$ in these formulas are known approximately a priori. They are calculated from equations (11) and (12) for given values of δ, β, and g. In what follows we specify $g = .85$, $\delta = .10$, and $\beta = .909$. These values imply $\lambda_1 = .809$, $\phi_2 = -.2299 + .7565i$, and $\phi_3 = -.2299 - .7565i$. Substituting these theoretical values into the autoregressive parts of the ARMA processes provides benchmarks with which ARMA estimates can be compared.

V. ARMA Estimates and Comparisons

A. One-Shock Version

The one-shock model in equations (26)–(28) accounts surprisingly well for the major time-series properties of total stock y_t, imputed breeding stock x_t, and slaughter c_t. These series have complicated trends. There was small or no growth in stocks and consumption prior to the 1930s and large growth thereafter (until recently). Per

capita beef consumption fell into the 1920s, took off in the 1930s, and turned down again in the 1970s. This theory has nothing to say about trends, yet trends must be removed for statistical purposes. The ARMAs were fitted to deviations around two piecewise-linear least-squares trends broken at 1930.[11] The maximum likelihood exact method (Hillmer and Tiao 1979) was used for estimation, guaranteeing that the estimated moving average representation has stable roots.

The numbers in the rows labeled $AR(J)$ in table 2 are estimated autoregression coefficients of the Jth lag of the dependent variable (with sign expressed to the right of the equal sign). Those listed in rows labeled $MA(J)$ are coefficient estimates of the Jth moving average term. Theoretical autoregressive benchmarks are listed for each variable, assuming parameter values $g = .85$, $\delta = .10$, and $\beta = .909$ and experimenting with different ρ-values. A value of $\rho = .6$ achieved a reasonably good match between the estimates and theory for breeding stock x, and that same benchmark is applied to the autoregressions for total stock y and consumption c. It is worth noting that the real prices of corn and hay both follow $AR(1)$ processes with estimated coefficient on the order of .6.

The estimated $AR(4)$ coefficients on $\{x\}$ resemble their benchmarks, though better at the first two lags than at the last two. The virtue of $\{x\}$ is that its ARMA has the smallest number of moving average terms, and moving average terms are difficult to estimate. But there is a potential problem: its three-period recursive imputation from equation (20) might induce spurious autocorrelations in the estimates if there are inaccuracies of specification. The higher-order moving average processes for directly observed $\{y\}$ were estimated to check this point. As noted above, the model predicts that $\{y\}$ has the same autoregressive benchmark as $\{x\}$. Its moving average benchmark is $1 + .85L + .85L^2$. The estimates for $\{y\}$ in column 2 confirm that the autoregressive estimates for $\{x\}$ are not spurious consequences of the way $\{x\}$ was constructed. The autoregressions match closely those of $\{x\}$, and the moving average terms are in the right ballpark relative to their benchmarks (though too large: their roots are perilously close to mod 1). The autoregressive estimates for $\{c\}$ compare fairly well to the benchmarks, but the moving average parts do not. Similar results were obtained on series constructed with $g = .8$ and $g = .9$ and are not reported.

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[11] ARMAs based on first differences generally yield similar results. Conceptually, if the forcing variables follow random walks, the order of the differenced ARMA should remain the same as in table 2. If they do not follow random walks, differencing adds an extra moving average term. There are no reasons to expect the shocks to follow random walks in this problem, and we are unable to decide this point. The issue is avoided by using deviations rather than differences. Also, first differences are not quite appropriate because trends are not uniform in these data.

TABLE 2

ARMA MAXIMUM LIKELIHOOD ESTIMATES AND BENCHMARKS

| | ONE-SHOCK MODEL | | | | | TWO-SHOCK MODEL | | | | |
| | Estimates | | | Benchmarks | | Estimates | | | Benchmarks | |
LAG	x (1)	y (2)	c (3)	x, y (4)	c (5)	x (6)	y (7)	c (8)	x, y (9)	c (10)
AR(1)	.96 (.07)	.90 (.07)	1.22 (.31)	.95	1.41	1.48 (.28)	1.51 (.22)	2.38 (.14)	1.90	2.36
AR(2)	−.35 (.08)	−.31 (.06)	−.37 (.28)	−.46	−.49	−.91 (.28)	−.95 (.20)	−2.18 (.23)	−1.36	−1.82
AR(3)	.91 (.08)	1.00 (.07)	...	.66	...	1.12 (.13)	−1.24 (.10)	.78 (.12)	1.10	.46
AR(4)	−.69 (.08)	−.76 (.07)	...	−.30	...	−1.24 (.26)	1.43 (.23)	...	−.93	...
AR(5)	...	...	...	...	...	.49 (.19)	−.58 (.17)	...	.29	...
MA(1)	...	.96 (.05)	−.04 (.32)	0 .85	.74	−.39 (.31)	.46 (.24)	−1.28 (.18)	...	...
MA(2)	...	.99 (.04)	...	0 .85	...	...	.55 (.22)	.59 (.18)	...	...
MA(3)	...	...	...	...	...	...	−.48 (.24)	...	...	...
Root mean square error	1,663	1,575	1,431	...	...	1,589	1,531	1,431	...	...
AR roots	.38 ± .86i .86 ± .21i	−.42 ± .89i .87 ± .21i	.66 .56	−.23 ± .76i .82, .6	.81 .6	−.4 ± .86i .9 .69 ± .35i	−.42 ± .89i .93 .72 ± .35i	.95 .72 ± .56i	−.23 ± .76i .81, .6 .95	.81, .6 .95
Implied cycles	3.16, 13.1	3.12, 13.1	none	3.05	none	3.13, 13.4	3.12, 13.9	9.5	3.4	none

NOTE.—All data are deviations from piecewise-linear trend with 1930 break points. Standard errors are in parentheses.

Table 2 does not conveniently indicate how well this model fits the data in many respects. Graphical evidence indicates that the quality of fit is exceptional.

Sample paths of $\{x\}$, $\{y\}$, and $\{c\}$ predicted by applying the theoretical ARMA benchmark coefficients to residuals calculated from the estimated ARMAs as innovations are compared to actual series in figure 6. In all cases solid lines are data (in the form of deviations from trend) and simulations are marked by circles. The circles apply the theoretical benchmark ARMA coefficients in table 2 directly to the residuals estimated from corresponding table 2 regressions. Let B_i and $\mathbf{B}_i$ represent the theoretical and estimated autoregressive portion for series $i = x, y$ and let G and $\mathbf{G}$ denote theoretical and estimated moving average parts for y. Then the innovations for x are calculated from $\hat{\omega}_t = \mathbf{B}_x(L)x_t$. The circles in the figure impute x from the theoretical benchmark $B_x(L)$, or by $\mathbf{B}_x^{-1}(L)\hat{\omega}_t = B_x^{-1}(L)\mathbf{B}_x(L)x_t$. This would be x_t itself if the estimated autoregression equaled its theoretical benchmark. Applying similar logic to y_t leads to $B_y^{-1}(L)\mathbf{B}_y(L)G(L)\mathbf{G}^{-1}(L)y_t$. A related expression holds for imputations of c_t. The additional moving average terms mean that the simulation of $\{y\}$ has more chances to go wrong than that of $\{x\}$. Figure 6 indicates that differences between simulations and realizations are negligible. We have also examined indirect simulations in which the x's were imputed from estimated y residuals and the y's imputed from estimated x innovations. Those results are very similar to what is shown in figure 6. The fact is that artificially simulated series are very
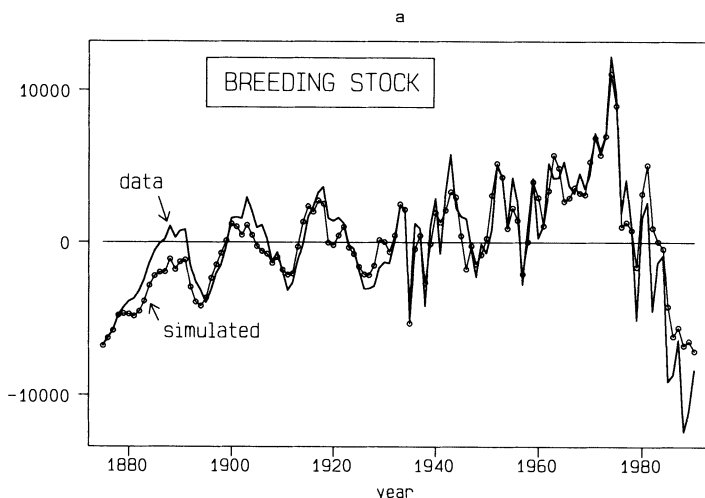

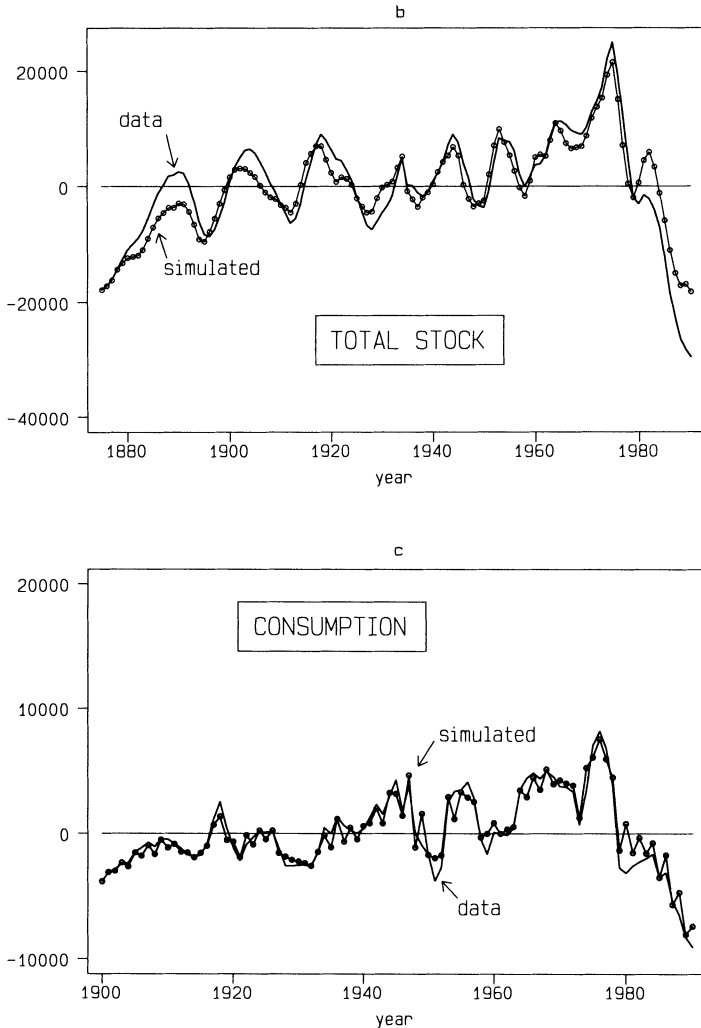FIG. 6.—Comparisons of realizations and benchmark simulations

b

c

FIG. 6.—*Continued*

close to the data, especially if one considers the complexity of the phenomena and the simplicity of the model.

Another visualization is provided by spectral variance decompositions of estimated and theoretical ARMAs. They are shown (over the entire circle) in figure 7. Spectra implied by the theoretical benchmarks in table 2 are marked by circles. The solid lines are spectra implied by the estimated ARMAs. Again, the comparisons are very close. These calculations are not independent of those underlying figure 6, but they reveal an important point. The accumulation of
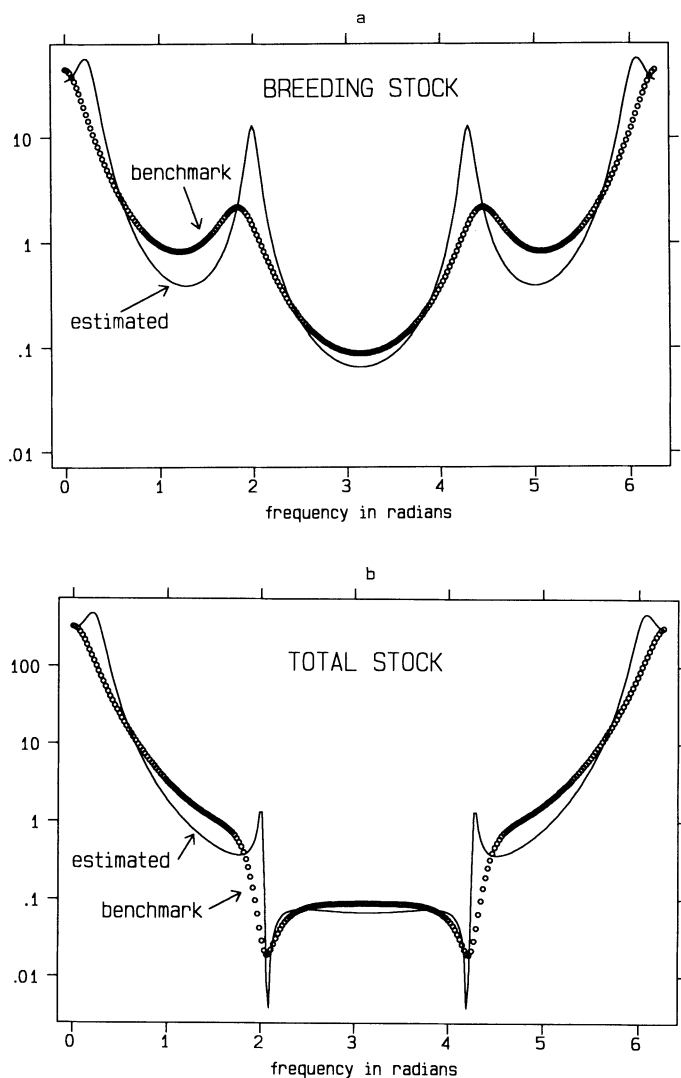
FIG. 7.—Spectral density comparisons

mass near 2 radians in the theoretical spectrum of x implies a cycle in breeding stock of somewhat more than three periods. The mass in the empirical spectrum of x calculated from the ARMA estimates occurs at a slightly higher frequency than predicted, but the difference is minor. We suspect that the three-period recursive construction of $\{x\}$ in equation (20) causes the extra weight at 2 radians in its empirical spectrum in figure 7. However, the spectral comparison for
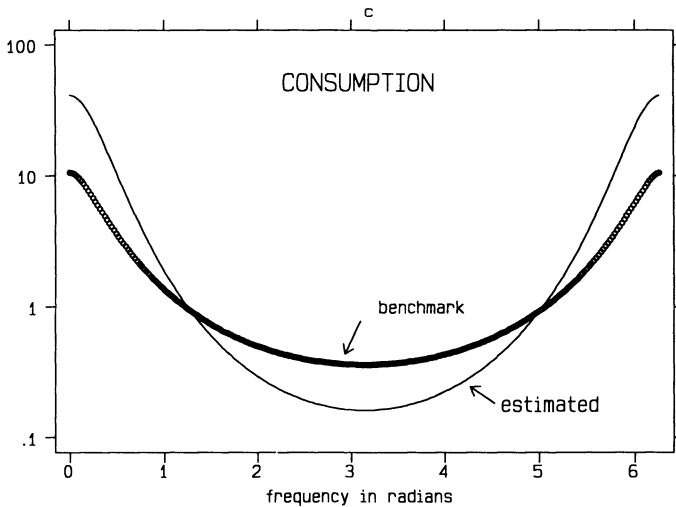
FIG. 7.—*Continued*

y indicates that the empirical mass at 2 for x is not artificial. Theory predicts a "foot" or an absence of mass near 2 radians in the y spectrum, just as the ARMA estimates show. The heavier "shoulder" at the neighboring longer frequency is not implied by theory. Finally, the constant supply price assumption predicts no systematic cycles in $\{c\}$, and none is found in this simple specification.

Similar conclusions follow from the characteristic roots calculated from the estimated autoregressions at the bottom of table 2. The two stable complex roots ϕ_2 and ϕ_3 arising from cattle biotechnology and demographics in equation (14) are what cause the roughly 3-year cycle in breeding stock dynamics. In all cases the estimated and predicted values of these two roots are reasonably close to each other. The comparisons for λ_1 and ρ are less close.

B.   Two-Shock Version

Estimates of the two-shock model also appear in table 2 and reinforce these conclusions. Theoretical benchmarks are harder to establish here. We have little prior knowledge of the roots of the shock processes for the autoregressive part. The .6 value of ρ used in the one-shock model was arbitrarily retained, leaving one free autoregression parameter for the second shock. A theoretical value of .95 matches up reasonably well, but by no means perfectly. The three theoretical autoregressive roots λ_1, ϕ_2, and ϕ_3 remain unchanged. The moving average benchmarks are very complicated now (see eqq. [29] and

[30]) and require much more detailed technical knowledge of the costs, production structure, and shock processes than is available. These benchmark entries are left blank for this reason.

The estimated autoregression coefficients approximate theoretical benchmarks in each case, and the implied empirical spectra (including the moving average part) are similar to those in figure 7. The two-shock model confirms the approximate 3-year cycle in $\{x\}$ and the absence of 3-year cycles in $\{c\}$. However, the empirical spectrum of $\{c\}$ shows a small mass point at about .66 radians, or a tendency toward a 9.5-year consumption cycle, which is not predicted by the model. Finally, the implied roots for ϕ_2 and ϕ_3 are close to their benchmarks. However, other complex roots appear here (and also in the one-shock model), suggesting that the shock structure of the processes is more complicated than AR(1). Finally, the three-shock model did not yield reliable estimates. The estimates we managed to produce are very imprecise, but their implied spectral decompositions are much like those reported here. We also estimated table 2 on differenced data and obtained results that are too similar to be presented here.

VI. Conclusion

The evidence suggests that rational demographic models of this kind contribute toward understanding cattle cycles. Shocks to demand and supply have persistent long-term effects on future stocks by changing farmers' incentives to carry breeding stock and altering the age composition and reproductive capacity of herds. The large size of breeding stock in the total cattle population and the lengthy gestation and maturation periods of cattle probably make them a pathological case compared to other managed animal populations. Still, pathologies are useful because their properties are exaggerated and easier to observe.

In spite of its extremely simplified specification, the model fits the 115-year period of the data extremely well. This is surprising in some ways. For example, the period of production was basically cut in half by corn feeding and finishing when it was introduced in the 1930s and 1940s. This would surely change the dynamic structure of a fully specified age distribution model, yet it does not appear to affect our simple three-stock model over the entire period. We do not exactly know why this is the case, but we think it has to do with the fact that there are basically three kinds of animals in beef cattle production, independent of technology. There are cows, calves, and those in between, which naturally correspond to the breeding stock, babies, and

pipeline animals in our model. No doubt further work will clarify this and other interesting issues.

The findings suggest that a recursive structural model with long-run constant returns to scale is a good working approximation. In a fully simultaneous model with rising supply price, the reduced-form autoregressive parts of equations (16) and (19) for $\{c\}$ and $\{x\}$ would be exactly the same. Both would exhibit period 3 cycles. A three-cycle is observed in stocks (fig. 7), but not in consumption. Empirical experimentation revealed that the autoregressive part of the consumption ARMA undoubtedly is of lower order than the autoregressions for stocks, and this is what a constant supply price, recursive structure requires. In this respect, also, the empirical estimates for slaughter are inconsistent with cobweb expectation models that have been commonly suggested as causing cattle cycles. We do not pursue this point here because a constant supply price model cannot nest static (cobweb) price expectations into breeding inventory decisions; that is, static expectations and a constant supply price imply corner solutions in which all adult stocks are either held or slaughtered, contrary to what the data show. Nonetheless, a little reflection reveals that static cobweb price expectations wedded to this technology and a rising supply price generate 3-year cycles in consumption (and prices) not observed in the data. Some longer cycles in consumption and stocks not explained by this model appear to be present in the data. Perhaps some of these cycles can be better understood with improved specifications of cattle dynamics and deeper investigation of the shock processes that drive this market.

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