Determining Optimal Selling Time of Cattle:
A Stochastic Dynamic Programming Application

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Abstract

World's meat market conditions have forced the beef sector to look for strategies to become competitive and researchers have proved that optimal replacement decisions are one of the most important factors that affect competitiveness. The present paper formulates a model in a discrete stochastic dynamic programming framework which describes manager's decision-making process combining both economic and biological variables and involving uncertainty on price fluctuations: investors can use the model to support their decisions of selling or keeping a fattening animal. The methodology developed is very general and can be used in different regions under similar production conditions by calibrating the parameters and making the required changes according to local regulations. The paper illustrates model's conveniences with an empirical application based on a local Colombian market, proving that researchers were right when ranked the dynamic programming as an excellent modeling tool for evaluating livestock replacement.

Key Words: Cattle, Farm Planning, Replacement Decisions, Stochastic Dynamic Programming.

1. Introduction

Cattle's raising is an old, worldwide disseminated economic activity, which consists on animal handling for productive purposes such as milk and beef production and their derivatives. Since meat has been considered the main source of protein for human nutrition and an important source of nutrients needed for human development (FAO, 2012a), livestock sector plays an important role in many economies by producing food supplies (Randela, 2003) and generating employment and investment in different segments of the beef industry value chain (Randela, 2003).

World’s beef industry has grown at decreasing rates in the last decades (FAO, 2012a; Schroeder and Graff, 2000). Researchers have hypothesized about a restructuring on global meat consumption pattern (Galvis, 2000), in fact, since mid-1970’s net returns for beef cattle feeding have been volatile (Hertzler, 1988) and a significant decay in sales and loss of the meat market share to poultry and pork has been presented (Katz and Boland, 2000). Now at days, world’s meat consumption configuration is 42% pork, 35% poultry and 23% cattle (FAO, 2012b). However, even when beef consumption have decreased on last years, many countries still have a large number of consumers making beef industry an attractive sector for any economy to become a participant in world’s market. In meat producing countries beef industry value-chain is very important, due to the generation of local and foreign businesses, the creation of job in different regions and sectors of the chain, and the fact that meat market share in these countries presents a different configuration to world’s market configuration with beef in the top of the market share, making beef one of the most important items on population diet since it is easily obtained in local markets (Ramírez, 2013)

Beef market is affected by many factors. First, consumer’s preferences have move to other meat types as pork and poultry (Galvis, 2000), this phenomenon is strongly influenced by supply fluctuations, volatility in prices (Kalantari, Mehbarani-Yeganeh, Moradi, Sanders, and De Vries, 2010; Glen, 1987) and foodborne illnesses attributed to red meat (Katz and Boland, 2000). Moreover, asymmetry in chain (Lafaurie, 2011), coordination between production and commercialization (Schroeder and Graff, 2000) and vertical integration (Galvis, 2000) are crucial factors since there is a complete separation between production and processing processes in contrast to substitute industries which are strongly integrated (Katz and Boland, 2000). Moreover, cattlemen avoid the necessary changes to improve the competitive structure due to the rigidity in regulation (Katz and Boland, 2000): input prices, and price and cost structures, added to the few economic incentives perceived by producers (Kalantari *et al*, 2010), reduce their capacity to develop a technical change to increase efficiency (Galvis, 2000). It is clear that dependence on natural conditions as climate changes, interdependence with other human activities and increasing requirements to become a global competitor, as health requirements for exportation of meat (Takahashi, Caldeira, and Peres, 1997) demand a strong reorientation to measure competitiveness on cattle procurements (Crespi and Sexton, 2005), improve information flow (Schroeder and Graff, 2000), valorize taking into account value-generating factors (Scoones, 1992) and increase productivity.

As stated, there are many aspects to be improved in order to increase competitiveness in cattle raising. Although the importance of the beef market in many economies, little effort have been made to use formal methods, instead empirical methods, for decision making: nowadays and for many countries, livestock is still developed through local technologies (Takahashi *et al*, 1997) and decisions are not based on economical or financial data that support them but in cattlemen's intuition and brute force (Glen, 1987; Takahashi *et al,* 1997). It has been proved that optimal replacement decisions are one of the major issues that producers face (Frasier and Pfeiffer, 1994) and one of the most important factors affecting farm profitability (Kalantari *et al,* 2010).

Farm's Strategic Planning concept (Glen, 1987) appears as a mean to improve competitiveness in livestock sector, enhancing information flow and thus, improving farm's profitability by making accurate decisions. Though empirical and intuitively methods have worked well, implementing mathematical, economic, statistical and computational techniques will improve outcomes substantially.

The present research attempts to answer one of the principal questions that any owner of productive assets has: how can the optimal time to sell and replace the asset be determined? Answering this question for the livestock sector involves describing industry's behavior, establishing the problem faced by farm managers and their decision making process, understanding the relation between economic and biological variables and fitting a dynamic model to represent the sequential nature of these decisions. The novelty of our approach lies on modeling the optimization problem faced by the investor as a discrete problem where the cattleman decides whether to sell or not, comparing actual benefits and future expected profits using functional forms that relate economic and biological variables that affect livestock and involving uncertainty considering that optimal decision is founded on the expected value associated with realizations of random variables in future, explicitly specifying the random component of prices as a multiplicative autoregressive process, approximating it by a first order Markov process and calculating the transition probabilities using the method proposed by Tauchen (1986). The multiplicative form represents changes around the expected price that affect manager's perceived utilities and simplifies interpretation and analysis of price's shocks.

The paper is organized as follows: Section 2 presents the theoretical framework, including previous research on cattle's modeling, the model proposed and a description of the methods used, Section 3 exhibits an empirical application with its results, Section 4 discuss the results and show future research paths while Section 5 displays main conclusions.

2. Theoretical Framework and Methodology

2.1. Previous Research on Cattle's modeling

Livestock should be replaced when its performance deteriorates. Performance is affected by age, production, costs, prices and nature conditions, among others. Evaluating the optimal factors setup to replace a productive asset as livestock, involves understanding the sequential nature of replacement decisions (Glen, 1987) and the biological and economic factors that impact these decisions.

Although it has been identified the necessity of involving models in cattle management and the fact that given its sequential features and versatility (Glen, 1987) dynamic programming has been ranked as an excellent modeling tool for evaluating livestock replacement, existing literature on the determination of optimal times in livestock is relatively scarce and, despite their large possibilities of application, stochastic dynamic programming techniques have been slightly used.

Literature can be divided on research focusing on optimizing fattening strategy, research looking for economic basis to determine optimal policies and studies aiming to define the optimal fattening/replacement time.

For optimizing fattening strategy, Meyer and Newett (1970) proposed a deterministic methodology, based on dynamic programming structure, to define the optimal food ration and selling time that would maximize profits for any type of cattle, also Apland (1985) and García, Rodríguez and Ruiz (1998) used linear programming to describe the impact over herd's productivity of the interest rates and diet, respectively.

Looking for economic basis to determine optimal policies, Bentley, Waters and Shumway (1976) used an expression to calculate the net expected revenue for each period of time using prices and costs and including probabilistic uncertainty over asset's productivity due to mortality or infertility; Randela (2003) proposed a method to compute the average total value of an adult cow, that could be understood as the opportunity cost for replacing an animal, and allows the farmer to determine the impact of mortality.

Finally, different methodologies have been used to define optimal times in livestock replacing: Muftuoglu, Escan, and Toprak (1980) and Göncü and Özkütük (2008) employed least squares to find the optimum culling age and weight, Yerturk, Kaplan and Avci (2011) developed an ANOVA to describe fattening's performance, Arnade and Jones (2003) used a SUR regression conjoint with dynamic programming to establish the cattle cycle, Clark and Kumar (1978) proposed a deterministic dynamic programming model to define the optimal time for selling and buying beef cattle using prices and live weight, both variables depending on time and breed, Frasier and Pfeiffer (1994) exploited a Markovian decision analysis with dynamic programming to find the optimal replacement time for cattle breeding according to nutrition path, Takahashi *et al* (1997) presented a new optimization method based on dynamic programming to establish the optimal policy for herd shaping and Kalantari *et al* (2010) used stochastic dynamic programming to define the optimal replacement policy for dairy herds using milk production, parity and pregnancy status as state variables to solve the problem.

2.2. Dynamic Programming

Dynamic programming is a versatile optimization method developed by Bellman (1957) that have begun to be widely used in economics (Lomelí and Rumbos, 2003), which uses the principle of optimality to reduce the number of calculations required to determine the optimal decision path (Kirk, 1970). Bellman's principle of optimality postulates that:

> "An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision" (Bellman, 1957, p. 83)

The principle of optimality applies to problems characterized by having an optimal substructure i.e. when problem's solution can be defined as a function of optimal solutions to minor size subproblems or problems which has overlapping subproblems so the same problem is solved several times when a recursive solution arises.

The idea behind the method is to find a functional form for each problem through the principle of optimality and thereby establishing a recurrence that generates an algorithm which solves the problem. The recursive expression essentially converts a T-period problem into a 2-period problem with the appropriate rewriting of the objective function (Adda and Cooper, 2002). This expression is known as the value function, while the mapping from the state to actions is summarized in the policy function.

For the purposes of the dynamic programming problem, it does not matter how the decision sequence was taken from the initial period, all that is important is that agents are rational and act optimally at each period of time (Guerequeta and Vallecillo, 1998). Indeed, the state variables summarize all the information of the past that is required to make a decision.

Main features of dynamic programming method are the versatility to model both continuous and discrete variables and the capability to introduce uncertainty since is the only general approach for sequential optimization under uncertainty (Bertsekas, 2005) . Since livestock replacement problem can be represented as a multi-stage decision process involving uncertainty (Frasier and Pfeiffer, 1994), dynamic programming is a natural modeling tool for solving it (Glen, 1987)

Because complexities to find a closed form solution are common in dynamic programming problems, numerical methods as the Value Function Iteration procedure, the Policy Function Iteration method or the projection methods are used to solve them. In particular, the Value Function Iteration procedure starts from Bellman's equation and compute the value function by iterations on an initial guess; albeit slower than methods that operate on the policy function rather than the value function, it is trustworthy since it has been proved that "under certain conditions the solution of the Bellman equation can be reached by iterating the value function starting from an arbitrary initial value" (Adda and Cooper, 2002, p. 41). In order to compute the value function using this procedure, functional forms must be defined and state variables must be discretized. In the case of stochastic dynamic programming problems, when a random shock is involved and is assumed to come from a continuous distribution that follows an autoregressive process of order one, the discretization can be done approximating the process by a Markov Chain using the technique presented by Tauchen (1986). This method simplifies the expected values computation in the Value Function Iteration framework (Adda and Cooper, 2002).

2.3. Formulation of the model

The basic problem faced by the farmer is to determine the optimal selling time for cattle, whereas the optimal time is conditioned on a number of productive factors and is defined as the time where the net expected present value of economic returns associated with livestock management is maximized.

Consider an investor who seeks to determine the optimal time for holding a fattening animal, maximizing the expected present value of the investment decision $\Pi(q_t, p_t)$ where q_t is the animal's weight (in kilograms) and p_t is the price per kilogram (in US dollars) and (q_t, p_t) precisely described producer's state at time t. Specifically, at each point of time, the agent must choose whether to sell or not. Since this problem fit on the family of problems called optimal stopping problems (Chow, Robbins and Sigmund, 1971), it can be described as a dynamic stochastic discrete choice problem and can be expressed as a 2-period problem using Bellman's equation, so, at each point of time, the agent chooses whether to sell or to wait another period.

Formally, let $V(q_t, p_t)$ represent the value function of having an animal at state (q_t, p_t) which can be expressed as the maximum value between keeping the animal and selling it, then,

$$V(q_t, p_t) = \max\{V^k(q_t, p_t), V^s(q_t, p_t)\} \tag{1}$$

where, $V^k(q_t, p_t)$ and $V^s(q_t, p_t)$ represent the value functions of *keeping* and *selling* the animal at state (q_t, p_t), respectively.

Define δ as the probability of death, $E[V(.|I_t)]$ as the expected value function conditioned by the information available at period t, I_t and $\Pi(.)$ as the net revenue's present value obtained for selling the animal. Then, the value of keeping the animal is the expected value function of next period conditioned to the available information at time t, multiplied by the survival probability. The value of selling the animal is the net revenue's present value.

$$V^k(q_t, p_t) = (1 - \delta)E[V(q_{t+1}, p_{t+1}|I_t)] \tag{2}$$

$$V^s(q_t, p_t) = \Pi(q_t, p_t) \tag{3}$$

The net present value of the benefits at time t is composed by the present value of income, discounted at rate r minus the initial inversion made when the producer bought the animal at $t = 0$ and the present value of the costs per kilogram earned in each period of time that the investor keep the animal for fattening:

$$\Pi(q_t, p_t) = \beta^t q_t p_t - q_0 p_0 - \sum_{s=1}^{t} \beta^s \tilde{c}(q_s - q_{s-1}) \tag{4}$$

Where, $\beta = (1 + r)^{-1}$, and $\tilde{c}$ is the average cost per kilogram gained.

Let a_t represents cattle's age; a_t can be implicitly interpreted as a control variable since it keeps a straight relation with the state variables, specifically a_t has a strong relation with the weight q_t, and the real control variable which is the time an investor should keep the animal.

Cattle's weight q_t is a function of the age and a Gaussian stochastic perturbation. Raze is not considered in the formulation of q_t since, cattlemen usually invest in a specific raze according to the environmental characteristics and the pursuing financial objective. Square age is involved in equation to gather the concavity in weight evolution since experience has showed that animals get more weight when they are calves.

Price per kilogram p_t is modeled as the product between two components: first component is the expected price according to the weight q_t, $E[\bar{p}_t|q_t]$ where $\bar{p}_t$ is a stochastic equation relating price and weight. Second component u_t is modeled as an autoregressive gaussian process and represents changes around the expected price that affect manager's perceived utilities. Modeling prices in a multiplicative form, rather than an additive form, simplifies interpretation and analysis of price's shocks since investors are subject to diverse sources of uncertainty both internal (perceived only by the animal's owner) and external (perceived by the entire market) and take decisions according to the price they expect.

The functional forms defined to model the state variables q_t and p_t are:

$$q_t = \eta_1 a_t + \eta_2 a_t^2 + \varepsilon_t \tag{5}$$

$$p_t = E[\bar{p}_t|q_t]u_t \tag{6}$$

$$\bar{p}_t = \gamma_0 + \gamma_1 q_t + \gamma_2 q_t^2 + \epsilon_t \tag{7}$$

$$u_t = \mu(1-\phi) + \phi u_{t-1} + \xi_t \tag{8}$$

Where, $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$, $\epsilon_t \sim N(0, \sigma_\epsilon^2)$ and $\xi_t \sim N(0, \sigma_\xi^2)$.

3. Empirical Application

3.1. The Data

The data used to the empirical application of the model were gathered from a cattle farm located in Puerto Boyacá, Colombia and its market, where cattle production is extensive (or traditional) i.e. animals are fed in large areas.

To estimate weight's coefficients presented in equation (5), 24 representative fattening cattle were followed and weighed at different ages since they were weaned at age of ten months, shaping a database made of 162 observations of age and weight. To estimate price's coefficients, a periodical database which contains information of average weight and average price of lifting and fattening male cattle traded in the market between Oct-2010 and May-2013 is used. Equation (7) is estimated using a total of 180 observations involving both cattle for lifting and fattening purposes, while equation (8) only used 95 observations that correspond to animals reared for fattening purposes to capture the temporal component of the fluctuation on prices.

3.2. Estimation

Table 1 shows the results for the estimation of equation (5). Since the estimation presents heteroskedasticity problems, the standard errors are consistently estimated using the covariance matrix estimator proposed by White (1980) which provides consistent estimates of the coefficient covariances; the estimation passed the Jarque-Bera Normality Test at the 0.05 level and did not present multicolineality problems. Finally, the coefficient signs are the expected by the theory, gathering the concavity in the age, i.e. animals gain more relative weight at first stages of life than at posterior stages. Figure 1 shows the relation between age and weight for a representative animal, the figure shows that weight grows in declining rates: the gaining weight is noticeable at first months of life.

Table 1. Estimated parameters for the weight per age measure in kilograms

Weight $q_t = \eta_1 a_t + \eta_2 a_t^2 + \varepsilon_t$		
Observations	162	
R^2	0.681	
Parameter	Value	Standard error
η_1	26.43***	0.878
η_2	-0.34***	0.046

[a.] ***Significant at the 0.01 level; ** Significant at the 0.05 level

Price’s parameters are estimated in two phases: at the first stage, equation (7) is estimated using information of animals for fattening and lifting purposes, then when the estimation is done, fluctuations, in proportion, between real and estimated expected prices $p_t/E[\bar{p}_t|q_t]$ are calculated and equation (8) is estimated using the periodical data corresponding to fattening cattle.

Table 2 displays the estimation results for both phases of equation (6) estimation. First, equation (7) estimation used 180 observations and presents heteroskedasticity problems that are solved using White’s robust matrix. The estimation does not present multicolineality problems but normality is not achieved, yet this is not a problem given the sample size. Equation (8) estimation used 95 periodical observations of fluctuations calculated from the ratio between real and expected price

$u_t = p_t / E[\bar{p}_t | q_t]$. Coefficients are significant at the 0.05 level and signs corresponds to those expected by theory, moreover, since shocks u_t are expected to be proportions that move around the expected price's mean, so intuitively μ is expected to be 1 given the multiplicative form of prices and the functional form of shocks where μ is the mean[1], a hypothesis test for μ was performed finding that null hypothesis of $\mu = 1$ cannot be rejected as the t-statistic is 0.34 which is not in the rejection region for a confidence level 95%. The t-statistic yielded by the Dickey-Fuller's unit root test was -3.74 rejecting the null hypothesis of an unit root, meaning the multiplicative autoregressive process is stationary. Appendix 1 resumes the results for the statistical tests performed for the econometric estimations.

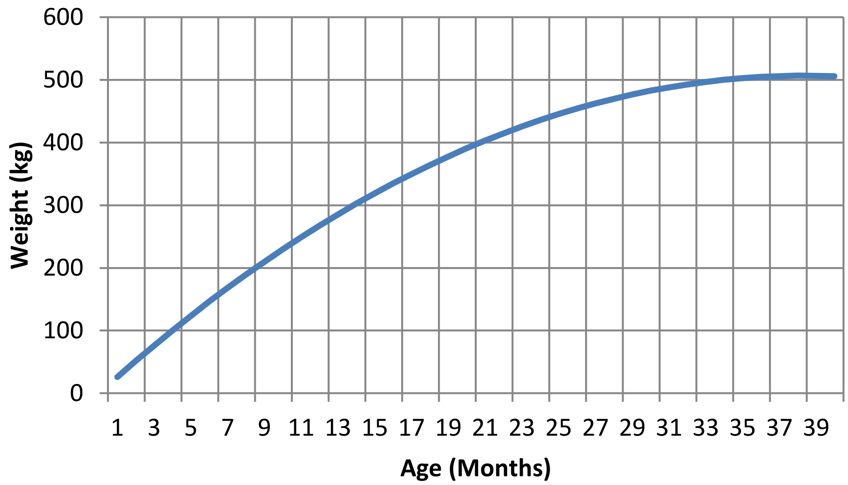


Fig. 1 Relation between age and weight for a representative animal

Figure 2 presents price's first component estimation which is the relation between weight and price per kilogram. Price per kilogram decreases at decreasing rates: as the animal weighs more, the marginal value for gaining a kilogram is lower i.e. the relative price of a kilogram is higher when the animal is younger.

Finally, to solve the dynamic programming problem, the average cost per kilogram gained $\tilde{c}$, the monthly interest rate r and the monthly mortality rate δ must be defined. In the cattle farm used for the application, the mortality rate is set about 2%, which is consistent with the Colombian strategic plan for the livestock sector (FEDEGAN, 2006) and the results found in Gómez (1992) who set the

[1] $u_t = \mu(1-\phi) + \phi u_{t-1} + \xi_t$
$\bar{u} = \mu(1-\phi) + \phi\bar{u} \rightarrow \bar{u} = \mu$

mortality rate in Colombia in 2.3%. Also, the average cost for gaining a kilogram, including feeding and medical costs, is set in US$0.5 per gained kilogram. The monthly interest rate is set in 1%, corresponding to an annual interest rate of 12.7% which is consistent with the annual interest rate expected in Colombian Economy.

Table 2. Estimated parameters for the price per kilogram measure in dollars (US$/Kg)

Price $p_t = E[\bar{p}_t \| q_t]u_t$		
First stage $\bar{p}_t = \gamma_0 + \gamma_1 q_t + \gamma_2 q_t^2 + \epsilon_t$		
Observations	180	
R^2	0.250	
Parameter	Value	Standard error
γ_0	1.7799***	0.0514
γ_1	-0.0014***	0.0003
γ_2	1.32×10^{-6}***	4.35×10^{-7}
Second stage $\frac{p_t}{E[\bar{p}_t \| q_t]} = u_t = \mu(1-\phi) + \phi u_{t-1} + \xi_t$		
Observations	95	
R^2	0.122	
Parameter	Value	Standard error
μ	1.002***	0.007
ϕ	0.354***	0.099

[a.] ***Significant at the 0.01 level; **Significant at the 0.05 level

[b.] Do not reject the null hypothesis of $\mu = 1$at the 0.05 level

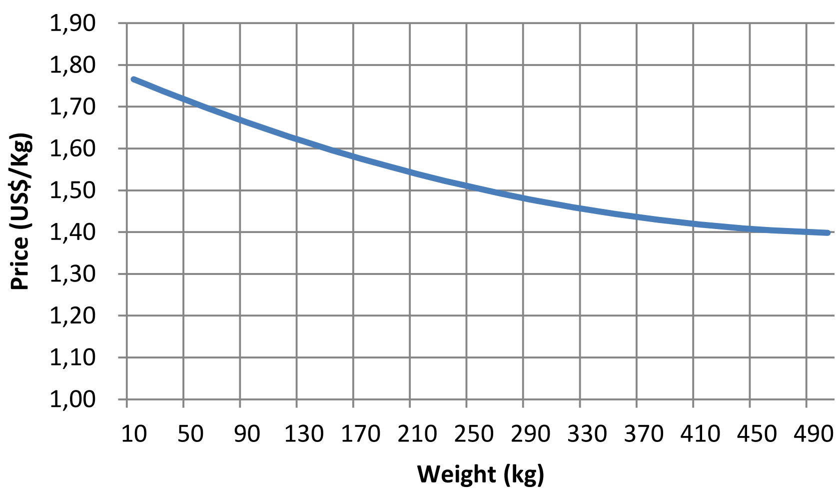


Fig. 2 Relation between weight and price per kilogram in an animal

3.3.Dynamic programming

Since the problem presented in section 2.3 does not possess a closed solution, a numerical technique to approximate the solution should be employed. The Value Function Iteration procedure was implemented to compute the value function by iterations over an initial guess. In order to solve the dynamic problem using the Value Function Iteration method, four steps should be completed: first, specify functional forms, second, discretize both control and state variables, third, perform the iterations calculation and define the tolerance parameters and finally, evaluate the value and the policy function.

First step was performed in Section 2.3 where all the functional forms, including the payoff function for selling the animal, were defined. To complete the second step, the control variable age a_t was discretized in 36 points where each point represents a month, thus time horizon is set in 3 years, which is the maximum time that animals stay at farm. Taking age discretization, weight and expected price can be discretized through equations (5) and (7). Since the multiplicative random shocks of the price are assumed to come from a continuous distribution that follows a Gaussian autoregressive process of order one with parameters (μ, ϕ, σ_ξ), Tauchen's procedure (1986) is performed to avoid the calculation of an integral for the expected value function at each calculation. In few words, the method approximates an autoregressive process of order one by a Markov Chain to create a discrete state space of the shock process, discretizing it in N optimal points and defining the transition matrix $\pi_{ij} = P[u_t = u_i | u_{t-1} = u_j]$ by calculating the transition probabilities between points such that the Markov Chain mimics the autoregressive process (Tauchen, 1986; Tauchen an Hussey, 1991; Adda and Cooper, 2002). To perform the iterations, a Matlab code was developed, Figure 3 shows the pseudo-code used to solve the problem.

The code was run using the parameters found in Section 3.2 and discretizing age a_t in 36 points and price shocks u_t in $N = 500$ points, so the AR process is guaranteed to be well approximated since Tauchen's approximation is only good if N is big enough. The method took 21 iterations to converge to the value function V which is presented in Figure 4.

Figure 5 presents the selling and keeping value functions V^s and V^k. In panel (a) it can be seen that when the animal weighs less, i.e. when is younger, the selling function is lower, even negative, meaning that farmer should wait another period to sell, also, when the shock price is higher than one, the farmer should be willing to sell, the opposite behavior is observed in panel (b): when the animal is younger, the keeping value function is higher, so the farmer should wait to sell the animal.

```
optimalSellingTime()
    Define animal information
    Read p_0, a_t, t
    a_0 ← a_t + t
    Define parameters
    Read δ, r, c̃
    β ← (1 + r)^{-1}
    Initialize η_1, η_2, γ_0, γ_1, γ_2, N, μ, ϕ
    Discretize Variables
    Discretize AR u ← Tauchen procedure(N,μ,ϕ)
    Save probability transition matrix π
    Discretize Age a ← a_0: 1: a_0 + 36
    q ← η_1 a + η_2 a^2
    q_0 ← q(1)
    E[p̄|q] ← γ_0 + γ_1 q + γ_2 q^2
    p ← uE[p̄|q]
    Iterate Value Function
    Define maxIter, tol
    for i_q = 1 to size(a) − 1
        for i_p = 1 to size(u)
            t ← i_q
            Initialize V(i_q, i_p) ← β^t q(t) p(i_p, t) − q_0 p_0
        end for
    end for
    for i = 1 to maxIter
        for i_q = 1 to size(a) − 1
            for i_p = 1 to size(u)
                t ← i_q
                δ_q = q(t + 1) − q(t)
                c(t) ← β^t c̃
                sum_c(t) ← Σ_{s=1}^{t} c(s)
                V_s ← β^t q(t) p(i_p, t) − q_0 p_0 − sum_c(t)
                V_k(i_q, i_p) ← (1 − δ) π(i_p, :) V(i_q + 1, :)
                V_aux ← max(V_s, V_k)
            end for
        end for
        error ← max((V_aux − V)/V);
        if error < tol then break else V ← V_aux end if
    end for
    Calculate Policy Function
    Policy function d ← V_s > V_k
end optimalSellingTime
```

Fig. 3 Pseudo-code for the Value Function Iteration Method applied to the Optimal Selling Time problem

The policy function defines the farmer should sell or wait according to animal's weight and price features at time t, specifically, the policy function is one if the selling value function is higher than the keeping value function. Figure 6 shows the policy function. In the figure appears that the investor should wait for a favorable price shock and for the animal to weigh around 300 kilograms, however, if the animal weighs more than 500 kilograms, the investor should not wait for a favorable price shock but must sell at any price.

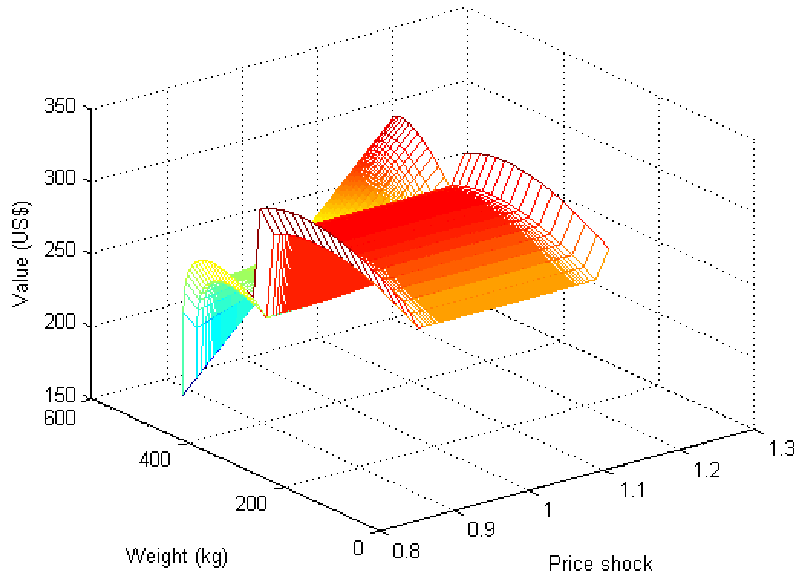
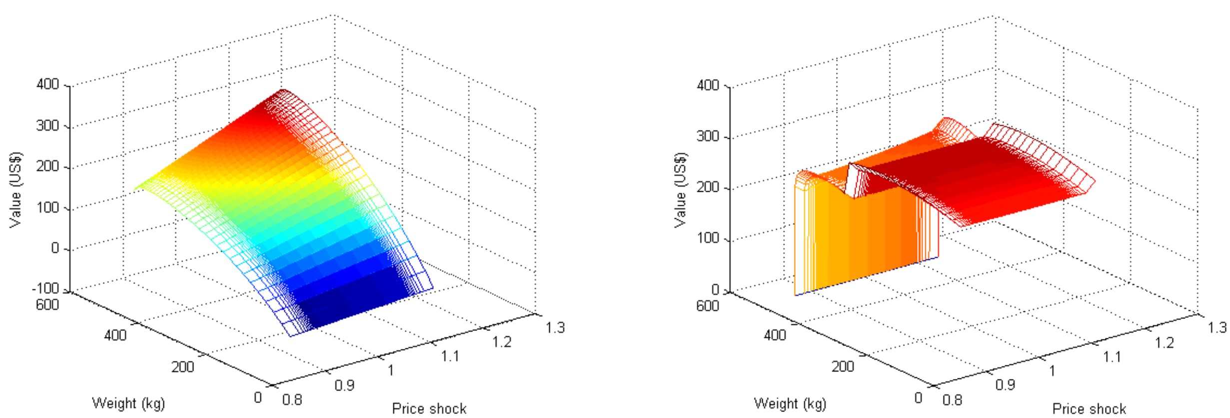


Fig. 4 Value function



(a) Selling Value Function (b) Keeping Value Function

Fig. 5 Selling and Keeping Value Functions

The value function for the model formulated above can be interpreted as the benefits perceived by the farmer at each configuration of the state variables, because the value function is formed by blending both selling and keeping value functions -taking the maximum between them in each point of the grid- and the benefits are measure in money (US$), then the value function represents the amount of cash the state variables denote for the livestock manager. It is important not to interpret the value function as the benefits of selling an animal, since there are some configurations of the state variables where the value function denotes the expected returns of waiting another period. The policy function allows determining where the value function actually displays the benefits for selling the animal. Figure 7 displays the net revenue's present value perceived by the farmer, i.e. the value function given the animal is sold: when the policy function takes the value of one.

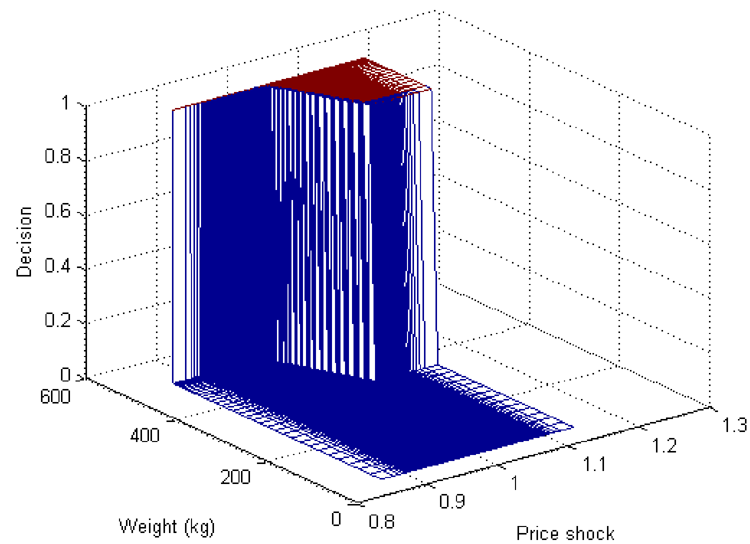


Fig. 6 Policy function

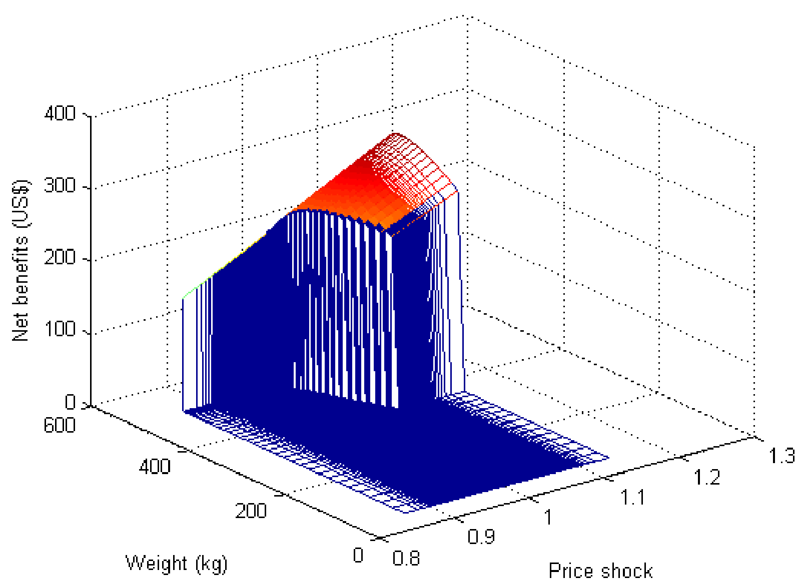


Fig. 7 Value function given the animal is sold

Variable u_t is an unknown shock that investors cannot predict so for decision-making process, managers will always expected that shocks take the value of one which is the mean, since hypothesis testing in Section 3.2 does not reject the null hypothesis for $\mu = 1$ i.e. investors compare their conditions to the expected price assuming $u_t = 1$. Table 3 summarizes the maximum value for each function and age a_t, weight q_t and price p_t configuration given $u_t = 1$, it is marked that the maximum found in the value function equals the maximum of the keeping value function though the maximum in the selling function is lower, this is explained by the fact that prices have a stochastic component, and calculation, when the animal is younger, generates expected values which are slightly higher than the real values once the animal gains weight. Also, it can be seen that the maximum found in the value function once the animal is sold, i.e. the real net revenue's present value, is lower than the maximum found in the other functions, this happens because the configuration of variables that generates the highest value in the selling value function produces a higher value in the keeping value function indicating that is better for the owner to wait another period for a higher price shock in future which will represent higher utilities but risking for the price shock to be one or lower which denotes inferior profits. In sum, given the expected shock price $u_t = 1$, the manager would receive a net present revenue of US$ 238.98 if the animal is sold when it weighs 497.6 kilograms, furthermore, if the farmer gets lucky so the shock price is higher, e.g. 14% when the animal weighs 480.53 kilograms, the net present revenue value would be US$ 313.79 which is the maximum point in the entire value function given the animal is sold.

Table 3. Maximum Values and Variables Configuration given Price Shock $u_t = 1$

Function	Maximum Value (US$)	Variables Configuration		
		Age a_t (Months)	Weight q_t (kg)	Price p_t (US$)
Selling - V_s	241.64	29	480.53	1.44
Keeping - V_k	295.29	12	268.20	1.51
Value – V	295.29	12	268.20	1.51
Value*	238.98	32	497.60	1.44

*Value Function given the animal is sold

Stochastic discrete problems, like the one presented above, have the feature that a threshold function, representing the point where is indifferent to stop or not can be computed. In the model, the threshold p^* is interpreted as the price where is indifferent to sell or keep the animal such that if $p > p^*$, the policy function d takes the value of one, i.e. the investor should decide to sell.

The threshold is calculated by equating V^s and V^k and solving for p^*:

$$V^s(q_t, p_t) = V^k(q_t, p_t)$$

$$\Pi(q_t, p_t) = (1-\delta)E[V(q_{t+1}, p_{t+1}|I_t]$$

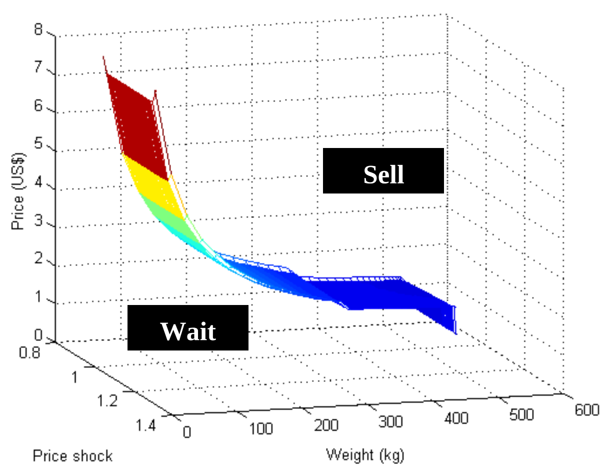
$$\beta^t q_t p_t - q_0 p_0 - \sum_{s=1}^{t} \beta^s \tilde{c}(q_s - q_{s-1}) = (1-\delta)E[V(q_{t+1}, p_{t+1}|I_t]$$

$$p^* = \frac{(1-\delta)E[V(q_{t+1}, p_{t+1}|I_t] + q_0 p_0 + \sum_{s=1}^{t} \beta^s \tilde{c}(q_s - q_{s-1})}{\beta^t q_t} \qquad (9)$$

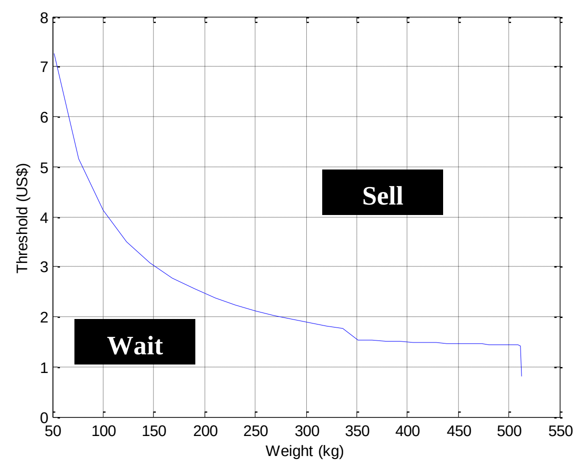
Since the price was formulated in a multiplicative form, the threshold function can be divided by the conditional expected price $E[\bar{p}_t|q_t]$ to find the shock threshold then,

$$u^* = \frac{p^*}{E[\bar{p}|q]} \qquad (10)$$

Figure 8 presents the price threshold calculated, panel (a) shows the function i.e. it shows the price at which the cattleman is indifferent to sell or keep the animal in each point of the grid and panel (b) shows the threshold given the expected shock $u_t = 1$: whereas the price is higher than the threshold at weight q_t then the investor must decide to sell.



(a) Threshold function



(b) Price threshold given $u_t = 1$

Fig. 9 Price threshold

Finally, simulations can be performed to describe multiple agents' behavior and market's configuration pattern through time using the policy function to find out the optimal choice for each period, since the threshold function is a mapping of the policy function, a stochastic discrete problem can be simulated comparing to the threshold values.

In order to perform model simulations representing S agents, a price shock for each agent at each point of time should be defined by simulating S autoregressive processes with the parameters found in Section 3.2, then the price perceived at each time can be calculated by multiplying the shock and the expected price at that point of the grid, finally, if the price is higher than the threshold, then the agent should sell an animal, this can be used to find the percentage of cattle at age a_t in the herd that are traded in market.

Notice that when comparing the threshold and the expected price at each point of the grid, it is implicitly assumed that the owner has an animal at each point of the grid, so it is allowed to happen that the investor sells more than one animal, however, it might be useful to define the evolution of sales in a herd through time i.e. the portion of animals that have been sold up to a specific weight, by including both, animals that was already decided to sell in a previous period and animals which are sold at the current time. Figure 10 shows the pseudo-code to perform the simulations.

```
Simulations()
        Define information
        Define number of periods a
        Read threshold function given u = 1 T_{a×1}
        Read expected price Ep_{a×1}
        Define parameters
        Initialize number of simulations S
        Initialize AR Parameters μ, ϕ, σ_u
        Simulate AR
        Define Burn-in iterations B
        e_{(B+a)xS} ← generate shocks ~ N(0, σ_u^2)
        Initialize u(1,:) ← μ(1 − ϕ) + e(1,:)
        for t = 2:(B + a)
                for s = 1 to S
                        u(t,s) ← μ(1 − ϕ) + ϕu(t − 1,s) + e(t,s)
                end for
        end for
        Drop B first simulations of u
        Simulate agent's behavior
        for t = 1:a
                for s = 1 to S
                        p(t,s) ← u(t,s)E_p(t)
                        if p(t,s) ≥ T(t) → sell(t,s) = 1 else sell(t,s) = 0  end if
                        if sell(t,s) = 1 → Csell(t,s) = 1 else Csell(t,s) = 0  end if
                        if t > 1
                           if Csell(t − 1,s) = 1 → Csell(t,s) = 1 end if
                        end if
                end for
        end for
end Simulations
```

Fig. 10 Pseudo-code for simulating sales behavior applied to the Optimal Selling Time Problem

Figure 11 illustrates the simulated sales according to weight for S agents whom operate under the same conditions that the ones presented above, for example, 12% of the animals that weigh 351 kg or 30% of the animals that weigh 417 kg are sold in market. It can be seen that every owner that waits for a cattle to weigh more than 510 kg should sold it immediately finally, an interesting result is that 50% is crossed exactly when the animal weighs 497.6 kg that is the maximum value found in Table 3 for the Value Function given the animal is sold, this means that on average, the agents will act rational assuming the price shock is the expected one, i.e. $u_t = 1$. Figure 12 shows herd sales evolution, it can be seen that the entire herd is sold when the animal weighs 497.6 kg which is the maximum found in Table 3.

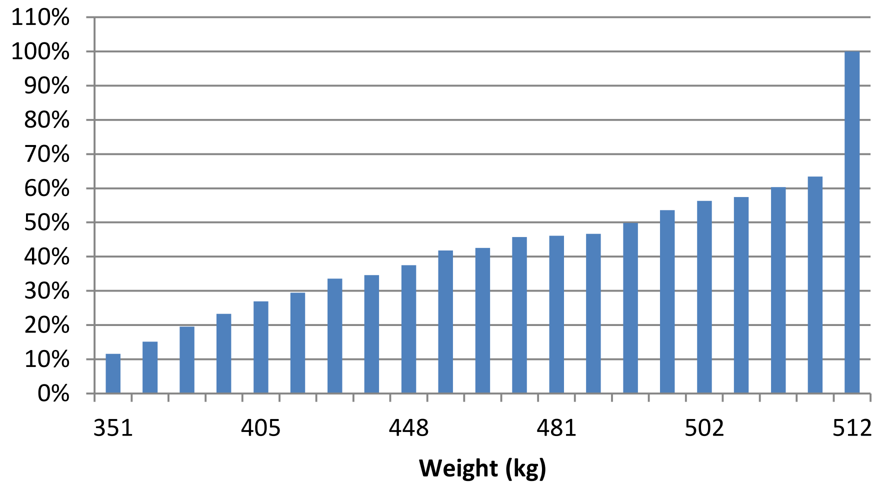


Fig. 11 Simulated sales according to age

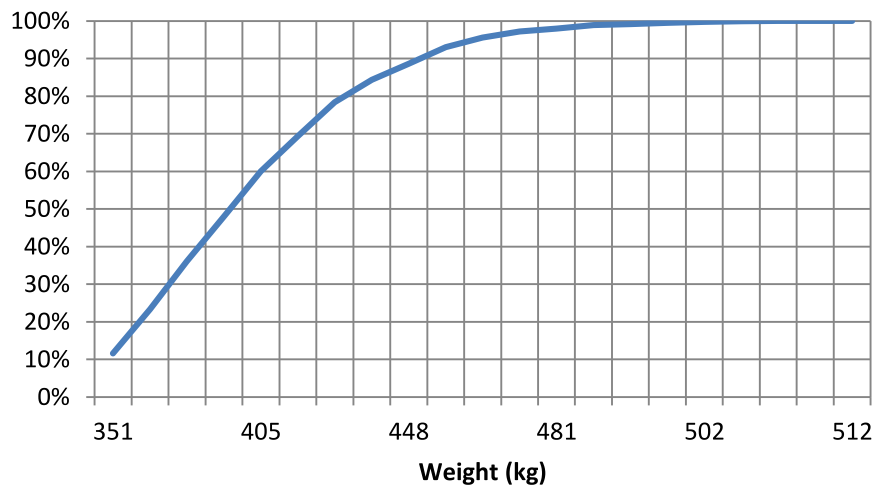


Fig. 12 Herd sales behavior

4. Discussion

Current competitive environment added to livestock sector complexities, vantage position of substitute industries and changes on global meat consumption setup have forced the cattle sector to look for strategies to improve competitiveness, as livestock sector plays an important role in many economies by producing food supplies and activating local economies: generating employment and increasing investment.

Strategies to improve competitiveness go from developing a technical change and changing price and cost structures to achieve vertical integration on beef industry value chain, also, it has been proved that optimal replacement decisions are crucial factors affecting farm profitability: nowadays, reality shows that decisions are strongly based on farmer empirical knowledge; but, even when intuition is very important in beef industry and the decision-making process faced by investors requires all their knowledge, management techniques, that are widely used in other kind of investments, should be implemented to improve accurateness. Empirical methods have worked well from some countries but implementing formal methods will shed accurate decisions.

The model presented in this paper allows the investor to gather more information about the best time for selling a fattening cattle: contains both economic and biological variables, involves uncertainty derived from future price realizations and describes formally the manager decision making process including the discrete component by which the investor decides to sell or wait at each point of time according to the information collected. Also, the model permits to find the optimum by comparing financial outcomes rather than other measurements, this makes easier to interpret results and to explain managers the benefits of implementing this kind of models.

The methodological framework developed is very general, so it can be used in different regions under similar production conditions by calibrating the parameters and making the needed changes according to local regulation, moreover, it can be used as basis to model other economic sectors. The stochastic discrete dynamic programming approach allows combining statistics, mathematics and economy in a computational framework, and given the nature of the problem and the features involved in the formulation, the present paper proved that the dynamic programming methodology is an appropriate tool for solving it.

Although dynamic programming sequential features and its versatility are widely recognized and livestock researchers have ranked it as the modeling tool for evaluating replacement decisions, literature is very scarce, opening a wide range of study in livestock economy, not only for beef cattle, but for dairy herds and other livestock industries. Plainly, the stochastic dynamic programming methodology carries many application options.

Though the value function iteration method is slower than others, it was appropriate to solve the problem formulated in Section 2.3 since it did not take a lot of iterations to compute the value function. Also, as it was said above, the formulation of the problem simplify interpretation of the value function since it can be understood as financial benefits which are the classical measures used by investors. Yet, it is the policy function that indicates when to sell and is the blending of both, value and policy functions, which indicates the real benefits for selling. In particular, for the policy function, the investor could seek the exact point on the grid where the current biological and economical features of the animal and market are and, with this information, take the decision of selling or waiting another period.

As the formulation in the problem includes expected values for the future, using Tauchen's procedure (1986) to discretize and approximate the shock is a good choice because it simplifies the expected values computation, and has the advantage that the points can be discretized prior the numerical method is implemented avoiding the calculation of a cumbersome integral at each iteration, however, it is important to choose a number of discretization points N large enough to ensure a correct approximation.

An important feature of dynamic programming framework is its facility to simulate models using the policy function and the ease to interpret results, furthermore, when the problem can be described as a stochastic discrete model, simulations are simplified since the policy function is mapped in the threshold function. Simulations can be used to describe market and agents behavior and, if there is availability of an extent data base which gathers market's features, it can be used for estimating model's parameters.

The empirical application yielded results according to theory which was endorsed by experts, yet it is imperative to highlight the importance of implementing good information systems and to create a collecting data culture since it is a fundamental input to implement formal methods. Commonly, in countries where livestock exploitation is extensive, data is infrequent, inaccurate and difficult to access.

Future work lies on improving the formulated model involving the causality relation between costs and prices, and enhancing the estimation results using Bayesian Statistics to include expertise or estimating the parameters simulating the dynamic model with the policy function, to achieve these improvements a larger and accurate data base and macro data about country production should be collected.

5. Conclusions

Modelling investor's decision making process using the dynamic programing approach allows the user to capture the sequential nature of the decision process and to introduce economic and biological variables in a stochastic environment.

The model presented could be used in different regions under similar conditions after adjusting it to local regulations, allowing the investor to gather more information than empirical knowledge to support selling decisions.

In order to improve competitiveness by making accurate decisions, livestock sector should enforce farm's strategic planning concept using formal methods, like the presented in this paper, since farm's profitability can be highly improved. This paper confirmed that dynamic programming is an excellent modeling tool for evaluating livestock replacement and provides an original strategy to solve investor's problem in a financial language common to asset managers.

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Appendix 1

Table A summarizes the statistical tests performed for the econometric estimations presented in Section 3.2. The p-value appears in parenthesis beneath the statistic used at each test. For reminder, null hypothesis for Jarque-Bera's test is normality and White's test checks homoscedasticity

Table A. Statistical Tests

	Equation	Normality Jarque-Bera	Heterocedasticity White
Weight	$q_t = \eta_1 a_t + \eta_2 a_t^2 + \varepsilon_t$	1.1 (0.578)*	3.77 (0.012)
Price	First component: $\bar{p}_t = \gamma_0 + \gamma_1 q_t + \gamma_2 q_t^2 + \epsilon_t$	320.74 (0.00)	3.51 (0.0319)
	Stochastic component: $u_t = \frac{p_t}{E[\bar{p}_t \mid q_t]} = \mu(1-\phi) + \phi u_{t-1} + \xi_t$	17.20 (0.00)	0.69 (0.504)*

[a.] * Do not reject null hypothesis