A Dynamic Assessment of the Economic Impacts of a Foot-and-Mouth Disease Outbreak on the U.S. Beef Cattle Industry

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Abstract

A dynamic model integrating the optimizing behavior of cattle producers, biology of cattle, stock replacement, age distribution, and marketing processes is conceptualized and calibrated to the U.S. beef cattle industry. The framework is demonstrated by first showing the results from the model capture the recent evolution of the U.S. beef cattle industry and then by estimating the economic impacts of a hypothetical Foot-and-Mouth Disease outbreak. The exogenous shocks from policy changes or disease outbreaks such as international trade, domestic consumption, and supply shocks, are integrated into the data-driven dynamic framework to capture the market response. Scenarios are designed to reflect varying levels of disease outbreak and response, allowing the model to quantify the impact on prices, supplies, and the age distribution of beef cattle. While the specific application in this case is a disease outbreak, the framework can be utilized to capture the market response to a variety of production and policy shifts and quantify the short-run and long-run economic impacts, and the variation of the economic impacts over time. The findings of the study demonstrate the value of the dynamic framework for policy work, including the design, development, and implementation of disease management policies.

1 Introduction

The U.S. beef industry operates in a highly competitive global marketplace. Major competitors include Canada, Australia, New Zealand, Brazil, and Mexico. Historically, the United States has held a comparative advantage in beef production due to a well-developed infrastructure, sound production practices, and a reputation for quality and high food safety (Chen et al. 2020). The United States is the largest producer of beef, producing over 28 billion pounds in 2021, and is the third largest exporter of beef, primarily exporting high-quality grain-fed beef (USDA-FAS 2022). The United States exported 3.4 billion pounds of beef valued at $9.9 billion in 2021 and the domestic cash receipts from cattle and calf production in 2021 were $72.9 billion (USDA-ERS 2021). Due to the importance of the beef industry to the economy and its significance in the international markets, the American beef industry is vital both domestically and internationally. It is crucial that industry leaders and policymakers have access to models that accurately describe the industry and contain the flexibility to investigate both biological and economic factors shaping the industry. This work creates such a model and applies it to explore the potential impacts of an animal disease outbreak.

Foot and Mouth Disease (FMD) is one of the Foreign Animal Diseases (FADs) affecting economies around the world. Although its risk is low, the economic consequences of FMD outbreaks are high. For example, in the United Kingdom, which had been free from FMD since 1967, a major outbreak of FMD occurred in 2001, which lasted 221 days and the disease spread to France, Ireland, and the Netherlands. The outbreak resulted in the depopulation of more than six million animals, with the estimated losses between £2.7 billion to £3.2 billion (Thompson et al. 2002). Since 2000, there have been multiple FMD outbreaks in more than 52 countries worldwide, including the U.K., Argentina, Brazil, Taiwan, and Malaysia.

The United States is considered to have a relatively low risk of FMD occurring; however, the Bovine Spongiform Encephalopathy (BSE) outbreak in the U.S. in 2003 and the immediate loss of the U.S. beef exports to global markets has demonstrated the significant economic implications of animal disease on the U.S. livestock industry.[1] Economic losses from BSE in 2004 due to export restrictions were estimated between $3.2 billion to $4.7 billion (Coffey et al. 2005).[2] FMD is not endemic to the United States. The disease outbreak would be a major economic event and its consequences can be catastrophic for the country because of possible trade bans, depopulation of infected animals, lost productivity distorting the production of animal and animal products, international trade, and threatening the beef supplies, safety and security. Furthermore, recent outbreaks of other diseases, such as highly pathogenic avian influenza in poultry in the United States have sparked interest in the impacts of FADs on the U.S. livestock industry.

Various approaches have been used to investigate the possible economic impacts of a FMD outbreak on the U.S. beef cattle industry. In particular, past studies have heavily relied on static economic models, such as input-output models and partial equilibrium models (Pendell et al. 2015, 2007; Paarlberg and Lee 1998; Garner and Lack 1995). Although these economic models have their own advantages, one limitation of these models is that they do not include the dynamic processes inherent in beef cattle production. Given the nature of biological lags in beef production (one-year birth delay and two-year maturation lags), the decisions made by producers (in regard to retaining animals for breeding versus marketing them for slaughter), and the existence of cattle cycles, it is imperative to include dynamics when analyzing animal disease outbreaks.[3] Seminal works by Jarvis (1974) and Rosen, Murphy, and Scheinkman (1994) emphasized the relevance of the dynamics and biology of cattle. Some studies recognized the dynamic process and incorporated it into the cattle industry analysis (Paarlberg et al. 2008; Zhao, Wahl, and Marsh 2006; Aadland 2004; Aadland and Bailey 2001; Chavas 2000). However, previous economic models analyzing the disease outbreaks

[1]Because of their potential for rapid spread regardless of borders and their ability to cause economic damages and impact trade of animals and animal products, the FADs are monitored by the World Organization for Animal Health (OIE).

[2]Following the discovery of BSE, Japan, South Korea, Canada, and Mexico closed their borders to the U.S. beef imports.

[3]For farmers, cattle are both capital and consumption goods (Rosen, Murphy, and Scheinkman 1994). A calf destined for slaughter as a fed animal is a production good, since it will be sold at the market price. In contrast, a calf that is kept on the farm and added to the breeding stock is a capital good because it will contribute to future production for up to 10 years. In the literature, this process is recognized as a *dynamic* process (Jarvis 1974; Rosen 1987; Rosen, Murphy, and Scheinkman 1994).

lack the dynamic nature of cattle production and optimizing behaviors of producers.

Our study proposes a framework of the U.S. beef cattle industry incorporating the dynamic nature of beef cattle production and producer behavior. In particular, a dynamic model of the U.S. beef cattle industry is conceptualized, calibrated, and employed to accurately replicate recent history for the industry and then utilized to quantify the economic impacts of a hypothetical FMD outbreak on the U.S. beef cattle industry.

Our study complements the existing beef cattle economic models and the economics of animal disease literature as it fully incorporates the optimizing behavior of cattle producers, the biology of beef cattle, stock replacement, the age distribution of the herd, and marketing processes. Since our model incorporates the dynamic processes within the industry, it captures the short-run impacts of production and economic shocks to the industry on the prices, supplies, and inventories as well as long-run impacts and the variability of those impacts over time. The dynamic model proposed in this study allows us to generate more accurate counterfactuals by incorporating the economic assumptions due to supply, trade, and consumer shocks, quantifying the impacts on the prices and stocks, and determining the general trajectories of prices and supplies over time. Furthermore, the evolution of the stock levels determined by the model, along with the economic impacts, will be of interest to policymakers, researchers, and industry stakeholders.

2 The dynamic model framework

The breeding and inventory modeling approach is inspired by Jarvis (1974), Rosen, Murphy, and Scheinkman (1994), Aadland and Bailey (2001), and Chavas (2000). A representative cattle producer is assumed to maximize the discounted future stream of profits for cows of all ages separately. The optimizing behavior of a producer is subject to market and biological constraints. Beef production is modeled to include both fed cattle (steers and heifers) and cull cows (older adult cows). Total cattle numbers and beef supply is determined by producer decisions on slaughtering fed cattle, culling cows, and adding younger breeding stock.[4] Ultimately, the production decisions of a producer depend on market conditions, the age distribution of the herd, biology, and consumer demand for beef from fed cattle and cull cows.

Several assumptions are made in the model framework. They include: (i) a cow can have a calf every year, (ii) once a cow is 10 years old it is culled for the market price $p_{c,t}$, (iii) fed cattle are slaughtered for the market price $p_{s,t}$, (iv) a cow not chosen for slaughter survives to the next period with a probability δ, (v) half of the calves are bulls and castrated and steers are slaughtered when they are two years old, and (vi) heifers are harvested at two years of age or added to the breeding stock as replacements.

Table 1 describes the notation and the fixed parameters (with value) used to calibrate the dynamic model.

Table 1: Parameter description and values

Parameter	Description	Value
$p_{c,t}$	Market price of cows	Data obtained from USDA[1]
$p_{s,t}$	Market price of steers and heifers	Data obtained from USDA[1]
K_t	Total breeding stock	Data obtained from USDA[2]
$k_{j,t}$	Inventory of cows of age j at time t	Constructed within the model
$V_{j,t+1}$	Value of cows of age j	Computed within the model
β	Discount factor	0.98[3]
g	Breeding rate	0.97[3]
δ	Survival rate of cows	0.95[3]
γ_0	Proportion factor for the cost of holding new-born calf	0.90[3]
γ_1	Proportion factor for the cost of holding one-year-old calf	0.95[3]

[1]USDA-NASS (2022a), [2]USDA-NASS (2022b), [3]Aadland (2004), Aadland and Bailey (2001), Baak (1999), Rosen, Murphy, and Scheinkman (1994)

[4]Although low in number, bulls play a role in supplying beef. On average bulls made up about 1.8% of the total federally inspected (FI) cattle slaughter in 2021 (USDA-NASS 2021).

2.1 Holding costs

The discounted cost of holding cows for one more year is given by:

$$z_t = h_t + \beta g \gamma_0 h_{t+1} + \beta^2 g \gamma_1 h_{t+2}. \tag{1}$$

Equation 1 states that if a producer keeps a cow for another year, the producer commits to the cost of feeding that cow for the next year and its progeny for the next two years. Rosen, Murphy, and Scheinkman (1994) specify holding costs in a similar manner.

The unit holding cost, h_t, depends in large part on the price of feed, mainly the prices of corn and forage, in varying proportions depending on the region. Here, we will take holding costs based on the price of corn, assuming that the price of forage and other feeds are correlated with the price of corn and assuming that other costs are fixed.

2.2 Arbitrage conditions by cohort

The temporal arbitrage conditions are specific to the age of cattle. These equations take a similar form but it is important to understand timing, in particular for older and younger cows. As such, we derive all the conditions.

Producers with nine-year-old cows can either breed them for another year or cull them in the current period. If a producer breeds a nine-year-old cow for one more year, the producer will cull the cow, within the context of our model in the next period.

2.2.1 Nine-year old cows

The value of a nine-year old cow is

$$V_{9,t} = \max \left\{ p_{c,t}, \mathrm{E}_t[\beta p_{c,t+1} + g\beta^3 p_{s,t+3} - z_t] \right\}, \tag{2}$$

where $V_{9,t}$ is the value of a cow of age 9, $p_{c,t}$ is the value of culling the cow this year and $\mathrm{E}_t[\beta p_{c,t+1} + g\beta^3 p_{s,t+3} - z_t]$ is the net value of breeding the cow this year, culling the cow next year, and capturing the slaughter value from the calf.

If $\mathrm{E}_t\left[\beta p_{c,t+1} + g\beta^3 p_{s,t+3} - z_t\right] > p_{c,t}$, then a producer keeps the cow for one more year such that $k_{10,t+1} = \delta k_{9,t}$ where δ is the survival rate of nine-year-old cows. That is, if that inequality holds, the producer maximizes profit by breeding all of the nine-year-old cows. Conversely, if $\mathrm{E}_t\left[\beta p_{c,t+1} + g\beta^3 p_{t+3} - z_t\right] < p_{c,t}$, a producer culls all of the nine-year old cows such that $k_{10,t+1} = 0$. Finally, if $\mathrm{E}_t\left[\beta p_{c,t+1} + g\beta^3 p_{s,t+3} - z_t\right] = p_{c,t}$, then the producer will cull only a fraction of the cows such that $k_{10,t+1} \in (0, \delta k_{9,t})$.

2.2.2 Eight-year old cows

The arbitrage condition for an eight-year-old cow is similar. Producers with eight-year-old cows can either breed them or cull them in the current period. A producer would only cull an 8-year cow if the producer had already culled all of the 9-year cows.

If a producer breeds an 8-year cow, they expect to earn $\mathrm{E}_t V_{9,t+1}$ in the next period. Therefore, we write the value of an eight-year-old cow as

$$V_{8,t} = \max\left\{p_{c,t}, \mathrm{E}_t[\beta V_{9,t+1} + g\beta^3 p_{s,t+3} - z_t]\right\}, \tag{3}$$

At equilibrium, $V_{8,t} \geq V_{9,t}$ because the lowest value it can take is $p_{c,t}$ and $\mathrm{E}_t\left[\beta V_{9,t+1} + g\beta^3 p_{s,t+3} - z_t\right] > \mathrm{E}_t\left[\beta p_{c,t+1} + g\beta^3 p_{s,t+3} - z_t\right]$. The value for the 8-year old cow contains the expected value for a 9-year old cow which we write as

$$\mathrm{E}_t V_{9,t+1} = \max\left\{\mathrm{E}_t p_{c,t+1}, \mathrm{E}_t[\beta p_{c,t+2} + g\beta^3 p_{s,t+4} - z_{t+1}]\right\}. \tag{4}$$

Substituting 4 in 3 yields

$$V_{8,t} = \max\left\{p_{c,t}, \mathrm{E}_t\left[\beta \max\{p_{c,t+1}, \beta p_{c,t+2} + g\beta^3 p_{s,t+4} - z_{t+1}\} + g\beta^3 p_{s,t+3} - z_t\right]\right\}. \tag{5}$$

2.2.3 Cows between 3 and 7 years old

For cows between 3 and 7 years old, the arbitrage conditions are analogous to the arbitrage condition for 8-year-old cows. We will not fully expand upon all these conditions as they are repetitive and grow in complexity rapidly.

The value of a seven-year old cow is

$$V_{7,t} = \max\left\{p_{c,t}, \mathrm{E}_t[\beta V_{8,t+1} + g\beta^3 p_{s,t+3} - z_t]\right\}. \tag{6}$$

We can iteratively substitute for $V_{8,t+1}$ and then $V_{9,t+2}$ to find an expression containing the observed and expected prices.

Producers rarely cull younger cows as they will cull older and lower-performing cows first, so culling cows that are six years old or younger is an unlikely event. The main reason is that the conditions that would lead cattle producers to cull younger cows are quite extreme and in practice, we do not expect such conditions to occur regularly. Younger cows have many years of useful life ahead and a producer would need to have dire expectations about the future to cull a young cow.

Based on this observation, we assume that producers never cull cows that are six years old or younger.[5] The assumption has to do with the comparison of the expected future value and their value for culling. For $k \in \{3,4,5,6\}$, we can write

$$V_{k,t} = \mathrm{E}_t[\beta V_{k+1,t+1} + g\beta^3 p_{s,t+3} - z_t] \geq p_{c,t}. \tag{7}$$

[5] In practice, farmers will cull sick or injured cows. We consider these cases as natural mortality using the parameter for the survival rate, δ.

The implication is that for younger cattle the annual transition is determined by the survival rate of each cohort such that we can write $k_{4,t+1} = \delta k_{3,t}$, $k_{5,t+1} = \delta k_{4,t}$, $k_{6,t+1} = \delta k_{5,t}$, and $k_{7,t+1} = \delta k_{6,t}$.

2.2.4 Two-year old heifers

Producers with two-year-old heifers have the choice to either send them to slaughterhouse or add them to the breeding stock. The value of a heifer is

$$V_{2,t} = \max \left\{ p_{s,t}, \mathrm{E}_t[\beta V_{3,t+1} + g\beta^3 p_{s,t+3} - z_{t+1}] \right\}, \tag{8}$$

where $p_{s,t}$ is the slaughter price. For model simplicity and given the very small number of bulls retained for breeding, the model only considers heifers to be kept in the breeding herd, therefore putting an upper limit to the number of heifers bred given by $k_{3,t+1} \leq 0.5gK_{t-1}$. In practice, it is never the case that $k_{3,t+1} = 0$, i.e., no heifers are added to the breeding herd. Thus, we only consider the interior solution where $k_{3,t+1} \in (0, 0.5gK_{t-1})$ such that

$$p_{s,t} = \mathrm{E}_t \left[\beta V_{3,t+1} + g\beta^3 p_{s,t+3} - z_{t+1} \right]. \tag{9}$$

2.3 Demand for beef and cattle

Most meat from fed cattle makes higher-value cuts (e.g., steaks), while meat from cull cows makes lower-value beef (e.g., ground beef). Beef products from fed cattle and from cows are imperfect substitutes, with fed cattle meat being of higher quality. Prices for fed cattle and cull cows reflect the difference in the quality of meat products from cattle of different ages.

Our model recognizes the quality difference between beef products from fed cattle and cull cows. The demand for cattle is derived from the demand for beef. Accordingly, we proceed in two steps. We begin by modeling consumer demand for beef products, assuming that distinct products are made out of fed cattle and cull cows. We then turn to beef packing production technology, which allows us to derive an expression for the demand for fed cattle and cull cows.

2.3.1 Consumer demand for beef products

Beef from fed cattle is considered a higher quality product than beef from cows. In practice, this means that if prices for beef from fed cattle and beef from cows are the same, consumers will choose beef from fed cattle. This is a simplification because an animal yields many cuts with a wide range of values.

The intensity preference for beef will vary across consumers depending on their intrinsic characteristics. We model these preferences using a standard choice model where the diversity of preferences is captured using a distribution function. The utility a consumer derives from one unit of beef is

$$\theta_j - w_j, \tag{10}$$

where w_j is the retail price of beef for $j \in \{s, c\}$. The parameter θ_j is the utility to a consumer of beef of type j, excluding the purchase cost w_j. A consumer purchases beef from fed cattle if

$$\theta \equiv \theta_s - \theta_c > w_s - w_c \equiv w. \tag{11}$$

Equation 11 says that a consumer purchases beef that yields the most utility. We can interpret θ as consumer willingness to pay for beef from fed cattle over beef from cows, and we can interpret w as the premium for fed cattle beef over beef from cows.

The parameter θ summarizes consumer preferences. Consumers are not identical, and we expect some to have strong preferences for a steak from fed cattle while others are content with ground beef coming mostly from cows. Let $h(\theta)$ represent the marginal distribution of willingness to pay, defined between a lower bound of $\underline{\theta}$ and an upper bound of $\overline{\theta}$. Consumers who purchase beef from fed cattle over beef from cows are those with a willingness to pay a greater price premium as equation 11 shows. The share of consumers' purchases of beef from fed cattle is

$$\int_{w}^{\overline{\theta}} h(\theta)d\theta = 1 - H(w). \tag{12}$$

The share of of consumers' purchases of beef from cows is

$$\int_{\underline{\theta}}^{w} h(\theta)d\theta = H(w). \tag{13}$$

Because the cumulative distribution function $H(w)$ is increasing, the share of beef derived from fed cattle purchased by beef packers decreases as the price premium w increases. Conversely, the share of beef derived from cull cows purchased by packers increases as the price premium w increases.

Equations 12 and 13 give the consumption shares for a given premium w. We multiply these shares by the total consumption of beef. The total consumption of beef is defined as

$$Q_b(w_s, w_c) = Q_s(w_s, w_c) + Q_c(w_s, w_c), \tag{14}$$

where we can write $Q_s(w_s, w_c) = \Big(1 - H(w)\Big)Q_b(w_s, w_c)$ and $Q_c(w_s, w_c) = H(w)Q_b(w_s, w_c)$. The total demand for beef, $Q_b(w_s, w_c)$, depends on prices for the beef categories.

In what follows, it is useful to work with inverse demand equations. We denote the inverse demand functions as $w_s(Q_s, Q_c)$ and $w_c(Q_s, Q_c)$.

2.3.2 Derived demand by packing houses

We assume that packers have Leontief production technology where the total quantity of beef produced by a plant n is given by

$$q_n = q_{ns} + q_{nc} = \min(\phi_s X_{ns} + \phi_c X_{nc}, m), \tag{15}$$

where ϕ_i is meat yield, X_{ni} is the quantity of cattle of category $i \in \{s, c\}$ and m is the quantity of other inputs. The literature provides evidence of increasing returns to scale in meatpacking (Ball and Chambers 1982; Azzam and Anderson 1996; Xia and Steven 2002). However, assuming constant returns to scale simplifies the model and is consistent with fixed processing capacity in the short run.

The profit of a packing plant is denoted as

$$\Pi_n = w_s\phi_s X_{ns} + w_c\phi_c X_{nc} - p_s X_{ns} - p_c X_{nc} - (\phi_s X_{ns} + \phi_c X_{nc})p_m, \tag{16}$$

where p_m is the price of other inputs. Taking the first order conditions and assuming perfect competition yields

$$\phi_s w_s - p_s - \phi_s p_m \leq 0, \tag{17}$$

$$\phi_c w_c - p_c - \phi_c p_m \leq 0. \tag{18}$$

For a competitive packer, the first-order conditions permit three solutions: two corner solutions where the plant processes only either fed cattle or cows and an interior solution where it processes both. Our interest here is in a marginal plant that will process both (cattle) categories, such that the two first-order conditions hold with equality. These equations give us that $p_s = \phi_s(w_s - p_m)$ and $p_c = \phi_c(w_c - p_m)$, such that we can write the inverse demand for fed cattle as

$$p_s(X_s, X_c) = \phi_s(w_s(\phi_s X_s, \phi_c X_c) - p_m), \tag{19}$$

and for cows as

$$p_c(X_s, X_c) = \phi_c(w_c(\phi_s X_s, \phi_c X_c) - p_m). \tag{20}$$

Thus, the demands for fed cattle and cows are proportional to the beef products for the two categories and shifted down reflecting processing costs. Since we are working with beef from fed cattle and cows, and because the beef from these categories has limited substitutability, we are using single-equation demand equations for both types of beef.

For the purpose of calibrating the model to the data, we must specify a functional form for the demands, which requires specifying an expression for $H(w)$. We want a distribution function defined over the positive and negative intervals, capable of capturing a wide range of preferences for beef products and possessing few parameters, so we can more easily calibrate it to the data. One such function is the logistic distribution:

$$H(w) = \frac{1}{1+\exp\left(\frac{\mu-w}{\sigma}\right)}, \tag{21}$$

where μ is the mean and median willingness to pay for beef from fed cattle over beef from cows and σ is a scale parameter.

We can express the distribution of willingness to pay as it applies to the derived demand for cattle by a packer. From the first order conditions for a packer's profit maximization, we write $w_s = \frac{p_s}{\phi_s} + p_m$ and $w_c = \frac{p_c}{\phi_c} + p_m$ such that $w \equiv w_s - w_c = \frac{p_s}{\phi_s} - \frac{p_c}{\phi_c} = \widetilde{p}_s - \widetilde{p}_c \equiv \widetilde{p}$. We adjust the units for μ and σ as they are measured in dollars per pound of beef. To modify these parameters into dollars per pound of live cattle, we multiply them by ϕ which is measured in pounds of beef per pound of live cattle such that $\widetilde{\mu} = \phi\mu$ and $\widetilde{s} = \phi s$. The distribution function becomes

$$H(\widetilde{p}) = \frac{1}{1+\exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)}. \tag{22}$$

We further adjust the parameter for the demand intensity by writing that $\widetilde{A} = \frac{A}{\phi}$ such that the demand for fed cattle is given by $X_s = \widetilde{A}(1-H(\widetilde{p}))$ and the demand for cull cows is given by $X_c = \widetilde{A}H(\widetilde{p})$. Where A and $\tilde{A}$ are estimated total demand for cattle and total demand for beef in quantity respectively.[6]

2.4 Equilibrium

The equilibrium solution system depends on the price expectations and the culling decisions of the producer. We derive the equilibrium system under rational price expectations and for different culling decisions. The equilibrium system under rational price expectations and the numerical methods employed to solve the system and calibrate the model are provided in the appendix. In

[6]These are estimated using the constructed population distribution of cattle.

brief, the equilibrium system includes expressions containing market clearing conditions for supply and demand, price, and expected price conditions. Using the observed and constructed data within the model, the dynamic model is solved and calibrated to fit the observed data until the year 2021. The fitted model is then utilized to project the market several years into the future (2022 – 2031). To test the validity of long-term projections, the dynamic models' projections are compared with USDA and FAPRI projections.

2.5 Historical estimation

Using the constructed and available data along with the fixed parameters, the model is calibrated to capture the dynamics of U.S. beef cattle. The algorithm is written in R programming language and uses a simple sum squared error as the loss function. Since the system of equations is non-linear, a non-linear least squares method is employed in the iterative algorithm. The estimated parameters, fitted prices, and quantities are obtained for the model under rational price expectations. The solution of the model under rational expectations is used to project the prices and quantities into the future.

Figures 1 and 2 illustrate the observed and the model-fitted (the median of the iteration results) prices of fed cattle and cull cows, respectively. Using the fitted quantities, the cattle inventories are replicated and illustrated in Figures 3 and 4.

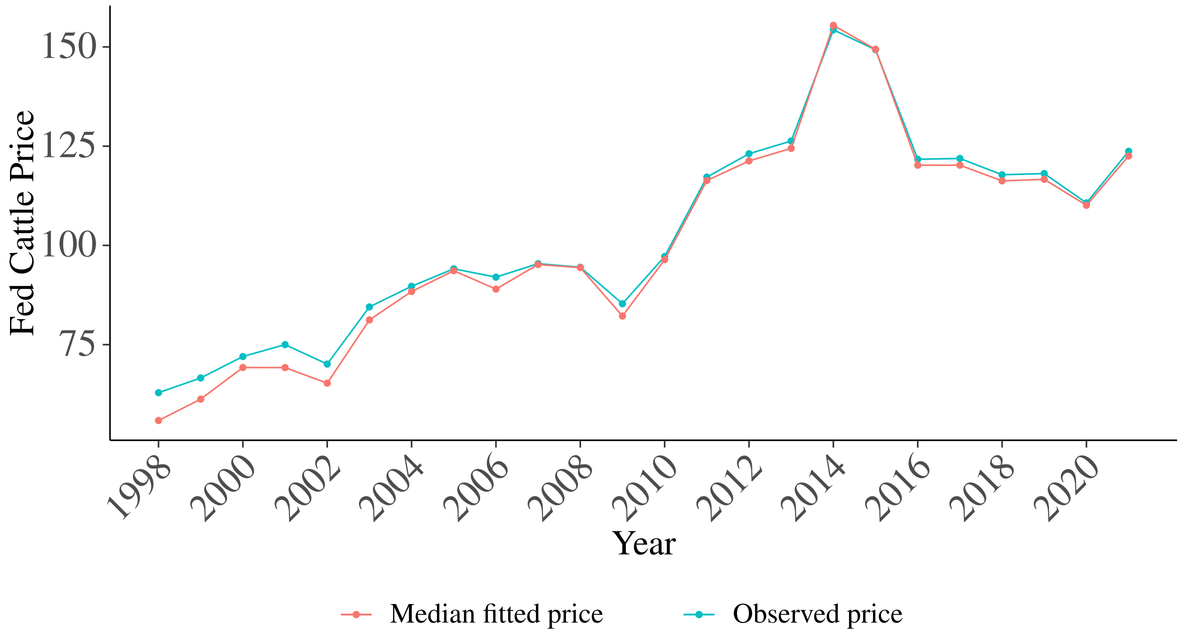

Figure 1: Observed and fitted fed cattle price ($/CWT)

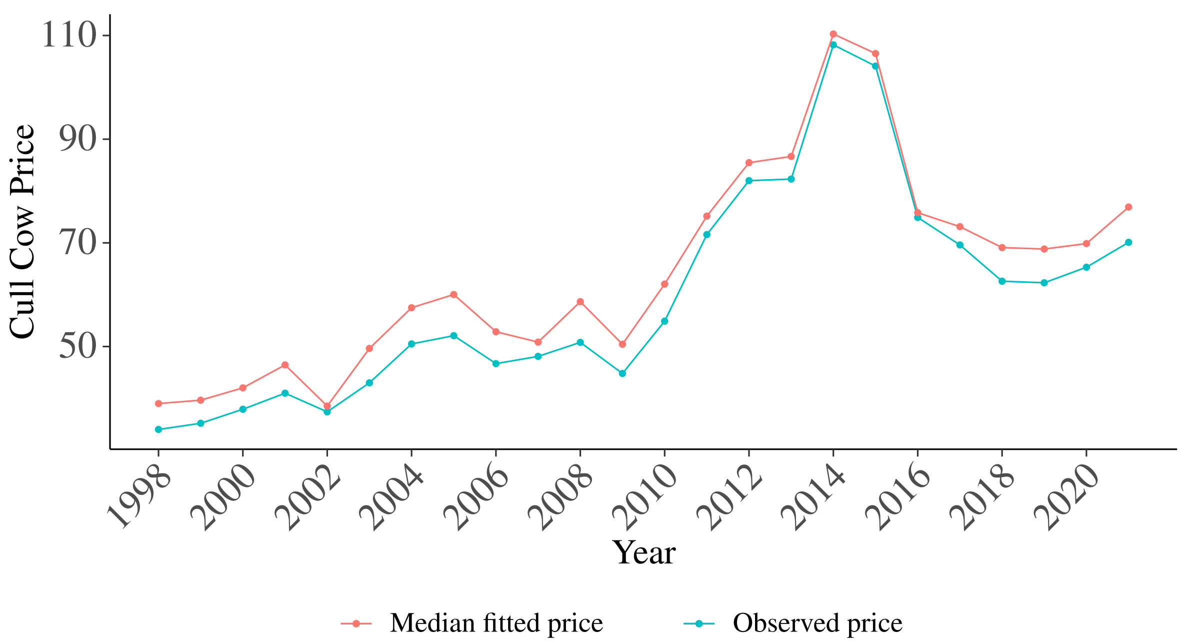

Figure 2: Observed and fitted cull cow price ($/CWT)

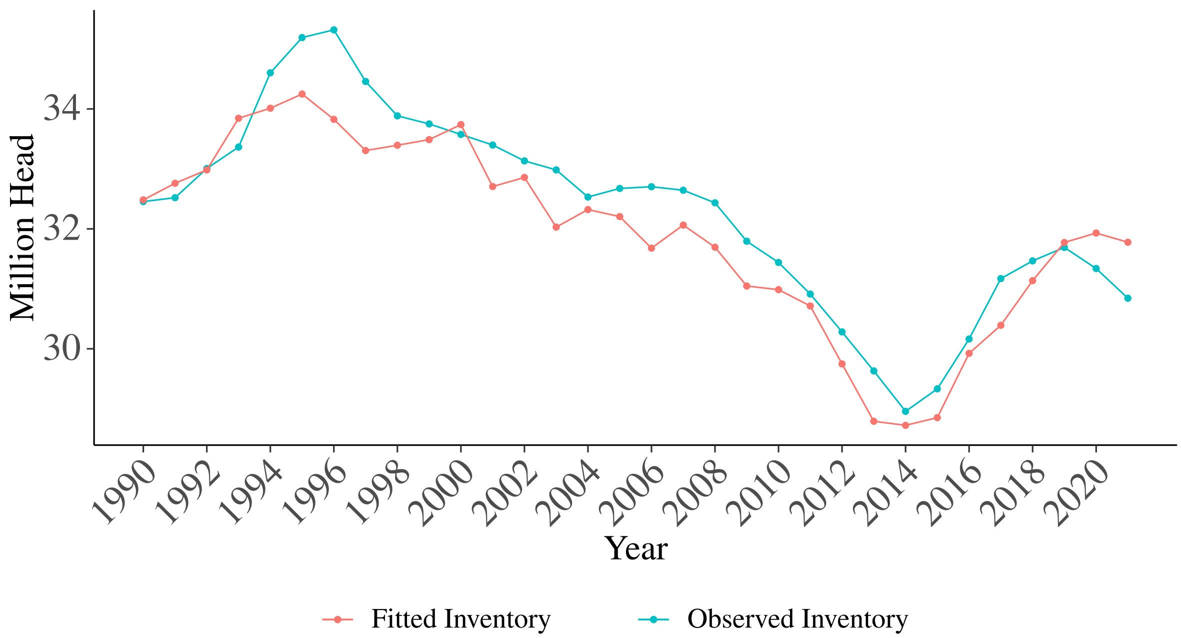

Figure 3: U.S. beef cow inventories and model fitted inventories

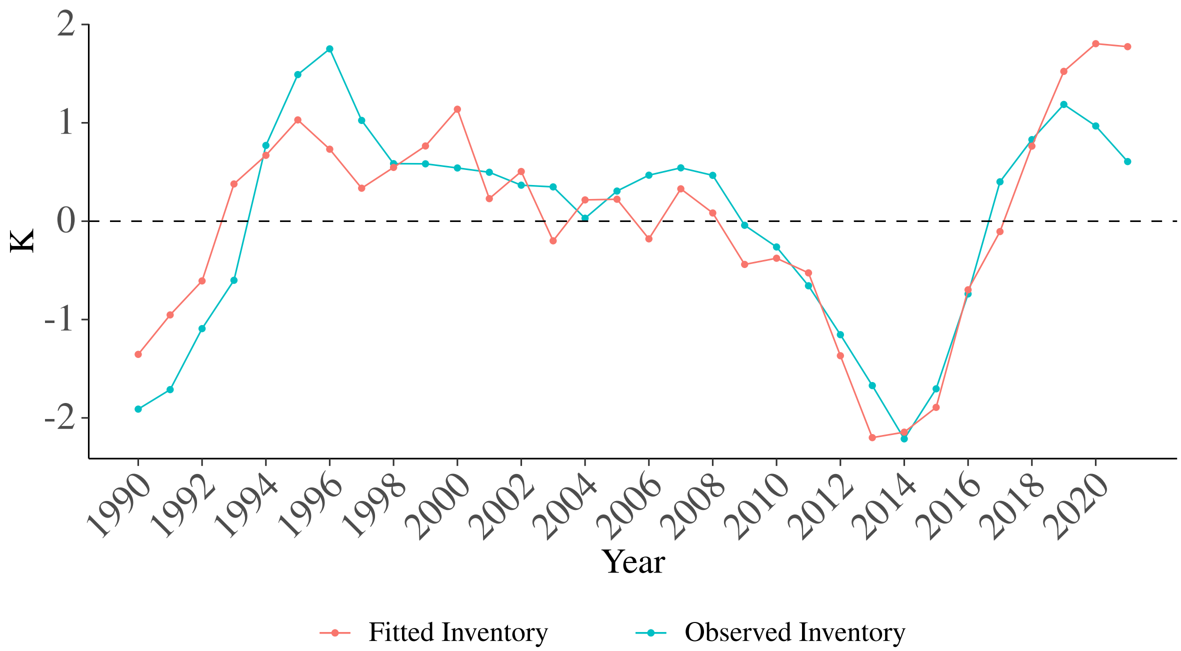

Figure 4: U.S. beef cow inventories and model fitted inventories - Detrended

From the above results, it can be seen that the dynamic model fits the observed data well. In particular, the median of the prices of the iterative algorithm is in the close neighborhood of the observed prices. To support the claim, the model fits the observed data, a percent error (in unit-free form) is computed using $|e| = \left|\frac{(O-\hat{M})}{O}\right|$, where O and $\hat{M}$ are observed and fitted respectively. The model fits the data with a median error of 2.38%, 10.04%, 2.49%, and 6.66% for fed cattle price, cull cow price, fed cattle supply, and cull cow supply respectively, strongly supporting the model's fitness to the observed data. Our replication of the cattle inventories from the fitted results follows the observed inventories and demonstrates the ability of the model to capture the observed dynamics in beef cattle inventories.

2.6 Model baseline projections

The estimated parameters, fitted prices, and fitted quantities of the model are used to project the prices and supply into the future. We use the 2021 estimates from the fitted model to initialize a multi-year cyclical algorithm to project future values, using the values for each year as the starting point for the next year's projections.

We compare our model projections to the well-recognized USDA long-term projections (USDA 2022) and FAPRI projections (FAPRI 2022). In particular, we compare the total beef production and the fed cattle price projections. Simple plots of the comparable model projections along with USDA and FAPRI projections are presented. Figures 5 and 6 illustrate the fed cattle price and the total beef supply from the dynamic model and the USDA and FAPRI projections.

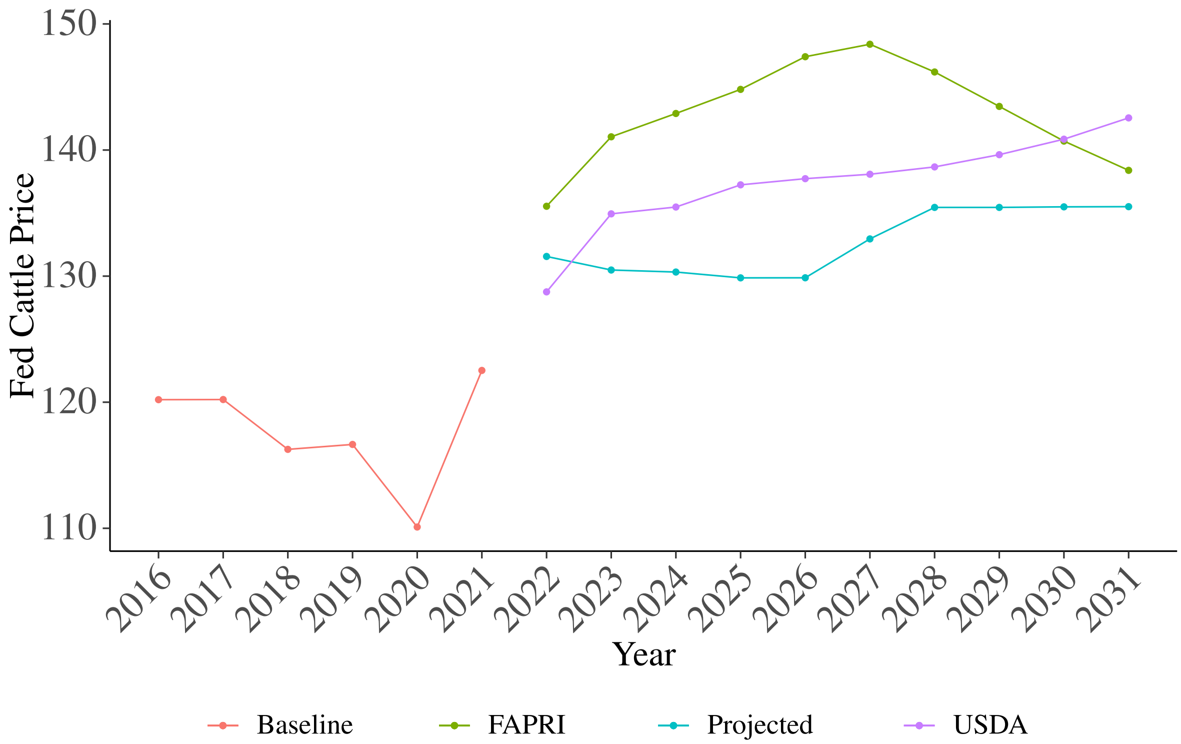

Figure 5: Projected fed cattle price vs USDA, FAPRI counterparts

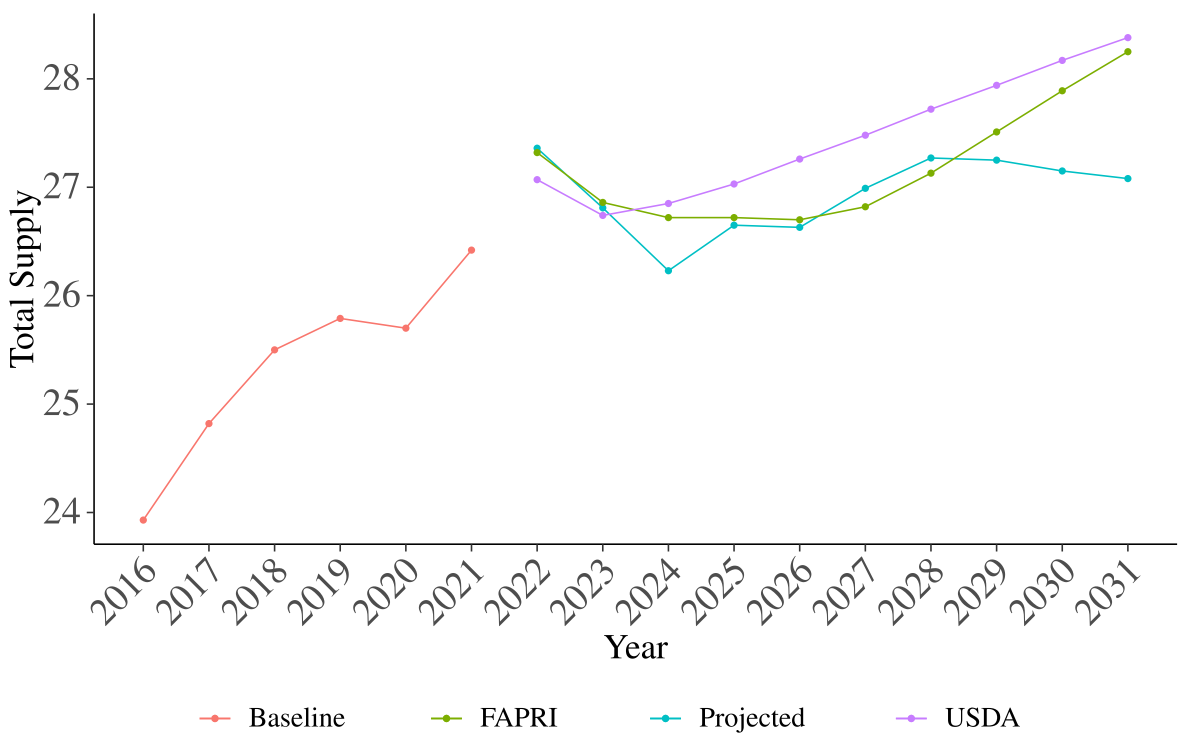

Figure 6: Projected total supply vs USDA, FAPRI counterparts

From the comparable projections above, the projected price and quantities are consistent with the USDA and FAPRI projections in general, providing strong evidence that our model is adequately projecting the market as these other models are considered the "gold standard" for agricultural projections. However, the dynamic processes within our model do provide some differences with the USDA and FAPRI projections, which could be critical in evaluating policy impacts in the future.

3 Foot-and-Mouth Disease (FMD)

FMD is a highly contagious viral disease and poses a threat to all cloven-hoofed domestic and wild animals such as cattle, pigs, sheep, goats, bison, deer, and elk (USDA-APHIS 2021). Common symptoms of FMD include high fever, lesions, and erosions on the lips, tongue, mouth, and feet (USDA-APHIS 2021). FMD is not transmissible to humans and is not considered a public threat. FMD is considered a threat to the agricultural sector, particularly the livestock sector, due to its contagious nature, significant disruptions in the livestock markets, and devastating economic impacts throughout the world.

The transmission of the FMD virus can occur via active subjects (e.g., humans, infected animals) or inactive subjects (e.g., vehicles, clothes, animal byproducts), direct or indirect contact, and air (the virus can transmit up to 100 kilometers by air and can transmit over land and water bodies). An animal that is exposed to the strain can excrete the virus before showing any clinical signs. If undetected, introducing an infected animal carrying the virus into a dense herd can escalate the spread of the disease and infect the entire herd. Although FMD is not fatal to affected adult livestock, it can have negative impacts on the productivity of the animals because the disease makes the animal weak and unable to produce milk.

The FMD virus can survive for extended periods in uncooked processed meats, frozen products, and dairy products over a broad range of climates and regional conditions. The disease is very common and is endemic in parts of Asia, Africa, the Middle East, and South America (USDA-APHIS 2020). FMD was first discovered in the United States in 1870. Since 1870, there have been several outbreaks of FMD in the U.S.; the last known mild outbreak occurred in the state of California in 1929 (Pendell et al. 2007). The United States has been free of FMD (without vaccination) since 1929; however, international travel and trade pose a significant risk of the entry of FMD into the country.

FMD is one of the most difficult animal diseases to control. When detected, aggressive measures must be taken to contain and eradicate the disease. Measures may include restrictions on the movement of animals, the establishment of strict quarantine zones, and the elimination of the infected animals. In addition, a vaccination program may be used in conjunction with cleaning and disinfecting FMD areas. To implement any disease management policies, the USDA-APHIS will coordinate with federal, state, tribal, and local partners to control, contain and eradicate FMD in the event of an outbreak.[7] Potential response strategies are described in the FMD Disease Response Plan or Red Book (USDA-APHIS 2020). The response strategies and disease management policies may further depend on the anticipated economic impacts, as well as cooperation from the states and tribal nations.

Previous estimates of the economic impacts of FMD depend on the economic model used. Using an economic model of the U.S. agricultural sector, Paarlberg et al. (2008) estimated the impacts of a hypothetical outbreak of FMD on different livestock species. Under the assumption that all agricultural sectors recover in 16 quarters, the estimated total losses to the livestock industries by the disease outbreak range between \$2.77 and \$4.06 billion. In another study, a partial equilibrium model for the beef and veal industry was employed to quantify the welfare impacts of FMD (Paarlberg, Lee, and Seitzinger 2003). Paarlberg, Lee, and Seitzinger (2003) concluded the aggregate producer welfare loss due to an outbreak would be between \$138 and \$1,772 million. Using a dynamic modeling approach, Zhao, Wahl, and Marsh (2006) concluded that welfare losses under depopulation rates of 60 to 70 percent of impacted U.S. herds would be between \$30 and \$50 billion. In Pendell et al. (2007), input-output and partial equilibrium economic models were employed to analyze the impacts of a FMD in southwest Kansas. Under different

[7]The USDA-APHIS is a federal agency with primary responsibility and authority for animal disease control.

disease introduction scenarios, the study found the total beef industry producer surplus losses due to a FMD outbreak would range \$43 – 706 million.

Elbakidze et al. (2009) studied the effectiveness of several FMD mitigation strategies and concluded that detecting the virus early can dampen the economic costs of an outbreak. Schoenbaum and Terry Disney (2003) suggested the best mitigation strategy for FMD depends on the speed of the spread of the virus and the demographics of the population.

4 Hypothetical FMD introduction and exogenous shocks

Below is the description of the exogenous shocks caused by a hypothetical FMD disease outbreak, which are introduced into the model.

4.1 Exogenous shock on international trade

Historically, animal diseases have had a significant impact on international trade. Frequent animal disease events over the past three decades have significantly increased the uncertainty in the global meat markets through their impact on animal welfare, consumer preferences, and trade patterns (Morgan and Prakash 2006). In the event of an animal disease outbreak, strict regulations are often placed on livestock and meat trade. These restrictions depend on the disease and the trading partners. In general, countries affected by animal disease outbreaks experienced immediate restrictions by their international trading partners until the affected country showed evidence of disease-free status for a pre-determined period of time.

FMD is one of the animal diseases which is transboundary, that is, it can be spread from one geographical location to another. Hence, in an event of a disease outbreak, the export markets will likely be inaccessible to U.S. beef. Therefore, a trade restriction is imposed on all U.S. beef and live cattle exports (Pendell et al. 2015) as an exogenous shock. Given that the U.S. has not experienced FMD for more than a century, the plausible duration of the trade restrictions was determined by reviewing the previous literature and by studying the reaction of global markets following the BSE events in the U.S. in 2003.

Previous studies analyzing FMD imposed different years of trade restrictions depending on the severity of the outbreak. The European Union imposed a one-year trade ban on the UK following its 2001 FMD outbreak. In analyzing the trade impacts of a hypothetical FMD outbreak in Australia, Tozer and Marsh (2012) applied a one to two-year trade ban. In another study, Nogueira et al. (2011) imposed a one to two-year trade ban in studying a hypothetical FMD outbreak in Mexico. Analyzing the 2000-2001 FMD outbreaks in South America, Rich and Winter-Nelson (2007) concluded that FMD impacts on exports were short-lived. Although previous studies imposed different lengths of trade restrictions, the trade restrictions in reality depend on the product, the disease, the last known active case of the disease, the disease management strategy, trade agreements, and countries involved in the trade. For example, the 2003 BSE outbreak severely impacted U.S. beef exports. The major U.S. beef importers, Japan and South Korea imposed various restrictions on U.S. imports. Japan resumed imports of U.S. beef almost two years after the outbreak, followed by some strict regulations (Kenneth H. Mathews and Gustafson 2006). South Korea, however, took five years to resume imports. In an extreme case, China resumed importing U.S. beef nearly 13 years after the BSE outbreak. This provides evidence that the duration of export restrictions is uncertain.

Based on the literature and the observed trade bans after the 2003 U.S. BSE outbreak, our scenarios cover from a one to a five year trade ban on U.S. beef exports.

4.2 Domestic demand exogenous shock

While the consumption of beef from infected cattle is not considered a health hazard (USDA-ERS 2001), with respect to perceived food safety, food quality, and health concerns, the domestic demand for meat would decline (Schlenker and Villas-Boas 2009; Paarlberg et al. 2008) in the event of a FMD outbreak. Additionally, public food safety information can have a significant impact on meat consumption which can persist for up to two years (Taylor, Klaiber, and Kuchler 2016; Piggott and

Marsh 2004). In the case of Taiwan, consumers have reacted negatively to an FMD outbreak.[8] Thus, an exogenous domestic demand shock is included in the model.

Studies quantifying the impact of animal disease outbreaks on consumer demand in the United States have come to similar conclusions (Schlenker and Villas-Boas 2009; Kuchler and Tegene 2006; Coffey et al. 2005; Piggott and Marsh 2004; Marsh, Schroeder, and Mintert 2004) that the impacts are small and short-lived but can be large in severe outbreaks. In regards to the impact of FMD on consumer demand, the U.K. experienced a FMD outbreak in 2001, which resulted in a 2.7% decline in domestic demand for meat in the U.K. and the market for meat demand recovered to pre-outbreak levels by 5 years (Pendell et al. 2015). Studies on hypothetical FMD outbreaks (Pendell et al. 2015; Schroeder et al. 2015; Zhao, Wahl, and Marsh 2006) and the BSE outbreak (Coffey et al. 2005) in the United States assumed a 5% decline in domestic meat demand.[9] Examining the impact of a hypothetical FMD in the U.S., Paarlberg, Lee, and Seitzinger (2003) assumed a 5% and 10% decline in domestic meat demand. In Mu et al. (2015), by analyzing the observed consumer response in the United States, the impact of BSE and avian influenza on domestic beef demand was found to be less than the impacts established in the literature (less than 5%).

While there are mixed results and views on the extent of the magnitude of the demand shock, following the consensus in the large literature on disease outbreaks and food safety, we use a 5% decline in domestic beef demand in our current analysis. Our choice of demand shock is not intended to imply the immediate demand shock in the event of FMD will be 5%. It is merely to quantify the impacts under that magnitude.[10]

4.3 Production exogenous shock

Despite having painful clinical symptoms, animals infected with FMD can make a full recovery in a relatively short period (two to three weeks) and most adult animals become productive. However,

[8]Livestock diseases such as BSE have been linked to human diseases and some consumers may not distinguish between the human health risks associated with BSE and FMD.

[9]This is consistent with established literature on consumer response to food safety concerns.

[10]The dynamic model can be simulated under varying levels of demand shocks.

in some cases, a permanent reduction in productivity is observed in the infected animals. FMD results in different mortality rates in adult and young animals. The mortality rate in the infected adult animal is between 2% and 5% (Ekboir 1999). In young animals, the mortality rate is much higher. The mortality rates of the infected young animal can range from 20% in smaller herds to 90% in dense herds (Ekboir 1999).

The mitigation strategies of the USDA in the event of a FMD outbreak were reviewed to determine the depopulation levels (reduction of animals). According to the Red Book (USDA-APHIS 2020), the most likely policies for mitigating and eradicating FMD are (1) stamping-out (depopulation) with emergency vaccination to slaughter, (2) stamping-out with emergency vaccination to live, and (3) a combination of stamping-out modified with emergency vaccination to kill, slaughter, and live. The vaccinated animals will live and can be used for their intended purposes (breeding, slaughter, and other purposes). These strategies could depend on the scale of the outbreak and the immediate availability of a vaccine.[11]

In addition to reviewing the USDA mitigation strategies, we examined the 2001 FMD outbreak in the U.K. to determine the depopulation levels. The FMD outbreak in the U.K. resulted in the depopulation of over six million animals (Thompson et al. 2002). In analyzing the impacts of a hypothetical FMD outbreak in the U.S., Paarlberg, Lee, and Seitzinger (2002) assumed a 5% exogenous decline in the beef cattle stocks, which was based on the animal losses in the U.K. In our study, past FMD events and studies examining FMD impacts in the U.S. are used to determine the depopulation levels.

In the current study, assuming the infected animals are depopulated, the hypothetical FMD outbreak is inherently treated as a one-time exogenous shock to the cattle stocks with no recurrent FMD outbreaks. We assume two depopulation levels, in particular, depopulation levels of 5%

[11]When administering a vaccine to infected animals, the USDA must consider the trade repercussions of the vaccination. Vaccination of the infected animal could be one of the mitigation strategies. However, it is recommended in rare cases. This is due to the fact that the FMD vaccine is an inactivated form of the virus and vaccination may prolong the duration of trade bans due to the possibility of vaccinated animals carrying the virus.

and 10%.[12] In the event of a severe outbreak, we expect the USDA may adopt a vaccination approach rather than depopulating all the infected herds. In the event of vaccine unavailability, we expect the USDA may restrict animal depopulation to specific regions, instead of national depopulation. In reality, depopulating all the infected herds (greater than 15% depopulation) might be counter-intuitive, because the infected animals can recover from the disease and lead productive lives.

5 Scenarios

We utilize the dynamic model to examine the impact of a disease outbreak by introducing a hypothetical FMD outbreak into the framework in 2021. The resulting exogenous shocks due to the disease outbreak discussed in the previous section are used to simulate the model. In particular, shocks to the cattle stocks, global markets response, and consumer response in the domestic markets are simultaneously included within the model to generate market counterfactuals of an FMD outbreak from 2022 to 2031. In addition, to demonstrate the range of the impacts of an FMD outbreak, we have developed two scenarios: an *optimistic* and a *pessimistic* scenario.

5.1 Optimistic scenario

In the *optimistic* scenario, we assume that the exogenous shocks due to the disease outbreak in the domestic and international markets will be short-lived. In particular, the following shocks are introduced into the model:

- Domestic demand for beef falls by 5% for a year,
- Export markets are inaccessible for two years, and
- Account for either a 5% or 10% depopulation of inventory.

The details of the above are as followed: in year one after the disease outbreak, we assume the

[12] In our study the *depopulation level* is assumed to be a percent decline. For example, a 5% depopulation means a 5% decline in the beef cattle stocks ($K - 0.05K = 0.95K$).

domestic beef demand decline by 5%, and the exports are banned. In the second year, we assume the domestic beef demand recovers and the export restrictions will remain in place. In the third year post-disease outbreak, we assume the export bans have been lifted and the U.S. resumes beef and live animal exports.

5.2 Pessimistic scenario

In the *pessimistic* scenario, we assume that the exogenous shocks in the domestic and international markets are longer-lived. In particular, the following shocks are introduced into the model:

- Domestic demand for beef falls by 5% for three years,
- Export markets are inaccessible for five years, and
- Consider either a 5% or 10% depopulation of inventory.

The further explanation of the *pessimistic* scenario is as followed: for the first three years post-disease outbreak, we assume that the domestic beef demand declines by 5% and exports are banned. In year four, we assume domestic beef demand recovers and export restrictions remain in place and continue through year five. In year six post disease outbreak, we assume the export bans are lifted and the U.S. resumes beef and live animal exports.

5.3 Exports, imports, and culling decisions

In regards to the magnitude of beef exports in the simulation, we take a conservative approach by using the historical (from 2000 to 2021) beef exports relative to the total production before the FMD outbreak to model the beef exports in our simulation period (from 2022 to 2031).[13] To determine the live animal exports in the simulation period (from 2022 to 2031), the historical live animal exports from 2000 to 2021 are utilized to set the relationship between exports and live cattle supplies. The imports of beef and live animals were also determined in a similar fashion. That is,

[13]Instead of keeping the beef exports static, we determine the beef exports dynamically where the beef exports depend on the total beef production in the simulation period.

historical beef and live animal imports are used to determine the beef and live animal imports in our simulation period.[14]

Contrary to the literature, where the culling age of the adult cows is fixed to a specific age, we take current market conditions into consideration to determine the culling age. For example, holding costs and the current market price of a cow are used to determine the culling age of cows. As indicated in the model framework, the holding costs are modeled based on the corn futures (sourced from the Chicago Mercantile Exchange) is used in our simulation period. We model a relationship between the historical holding costs and historical corn prices. The historical relationship is utilized to estimate the holding costs in the future with respect to the corn futures. Consistent with the literature (Hayes et al. 2011), these holding costs are further adjusted (through corn price) to reflect the depopulation levels in the scenarios.

5.4 Algorithm to generate counterfactuals

We simulate the dynamic model to generate price and supply counterfactuals with the introduction of a hypothetical FMD outbreak based on the scenarios described in the previous sub-sections.[15]

1. A one-time exogenous supply shock is introduced by depopulating the existing stocks.
2. Based on the inventories from step 1, the fed cattle and cull cow supplies are determined.
3. The share metric $1 - H(\tilde{p})$ is determined by using the estimated deep parameters.[16].
4. The demand for fed cattle (in quantity) is calculated from the share metric in step 3.
5. The two-year-old heifers, the beef cow inventories of the prior year, and the demand (quantity) for fed cattle from step 4 are used to determine the replacement heifers.[17]
6. The replacement heifers from step 5 are used to determine the total inventories of the present

[14]The historical beef exports and imports relative to the total beef production are used to determine the corresponding beef exports and imports in the simulation period. The historical live animal exports and imports relative to the total inventory are used to determine the corresponding live animal exports and imports in the simulation period.

[15]Due to the biological constraints in the model, the algorithm can be viewed as a *modified evolutionary algorithm* for the beef cattle industry that generates counterfactuals.

[16]These parameters include $\tilde{\mu}$, $\tilde{s}$

[17]The birth rate g and survival rate δ are used to determine the number of mature cattle for the decision making.

period.

7. The replacement heifers and total inventories computed in steps 5 and 6 respectively, are used to generate the number of cows of different age groups (from age 4 to 10).
8. The expected value of a nine-year-old cow is calculated on the basis of the holding costs. The expected value of a nine-year-old cow is then compared to the current market price of the cow and a decision is made on whether the older cows are to be consumed or to keep them in the herd for another year.
9. In this step, the trade and domestic beef demand exogenous shocks are introduced.
10. Once the supplies are determined after the trade shock, the prices of the previous period are used as starting values for solving the equilibrium system. The result of this step contains the equilibrium fed cattle price, cull cow price, corresponding expected prices, and the equilibrium supplies. This is the end of the first iteration.
11. For subsequent years, steps 2 through 10 are repeated with the domestic beef demand and trade shocks described in our scenario.

The above process is performed for both the *optimistic* and *pessimistic* scenarios to generate price and supply counterfactuals and stock evolution. Because our model is calibrated to capture the dynamics at the national level, our simulation assumes the beef industry as one zone. Therefore, the trade impact in the simulation assumes that all international beef trade between the U.S. and trading partners is restricted. In terms of flexibility, the dynamic model can be modified to capture different zones in the United States and appropriate simulations can be run.

6 Scenario results and discussion

Due to the inaccessibility of export markets with a uniform trade ban in our scenarios, the exports are absorbed into the domestic market, increasing domestic supply. Coupled with a decrease in domestic beef demand along with excess domestic supply, we found a significant decrease in domestic prices compared to the baseline (no disease outbreak).[18] Figures 7 and 8 illustrate the price impact on fed cattle for optimistic and pessimistic scenarios, respectively. At higher depopulation levels, the more animals are removed, the more supply is reduced, which drives up the price compared to the price with lower depopulation levels. Over time, however, when domestic beef demand returns to pre-FMD levels and exports resume, domestic prices increase. The largest price increase is found in the scenarios with 10% depopulation levels. Specifically, in the optimistic scenario, on average, the higher prices are sustained for about eight years or approximately one cattle cycle. At the end of our simulation period, in the optimistic scenario, the prices approach baseline (for both 5% and 10% depopulation levels), suggesting that the market will be recovered rapidly if the exogenous shocks are short-lived.

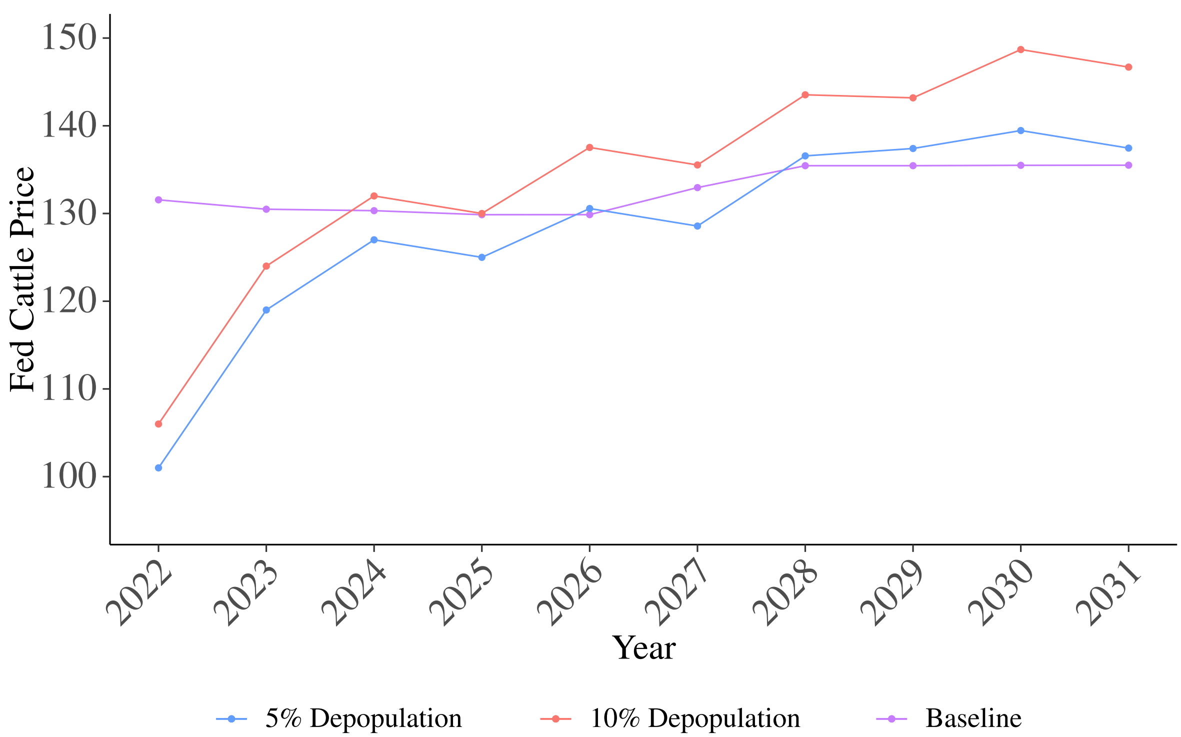

Figure 7: Changes in the fed cattle price ($/CWT) relative to the baseline - **Optimistic Scenario**

[18]The long-run projections of the dynamic model without the disease outbreak are taken as the baseline.

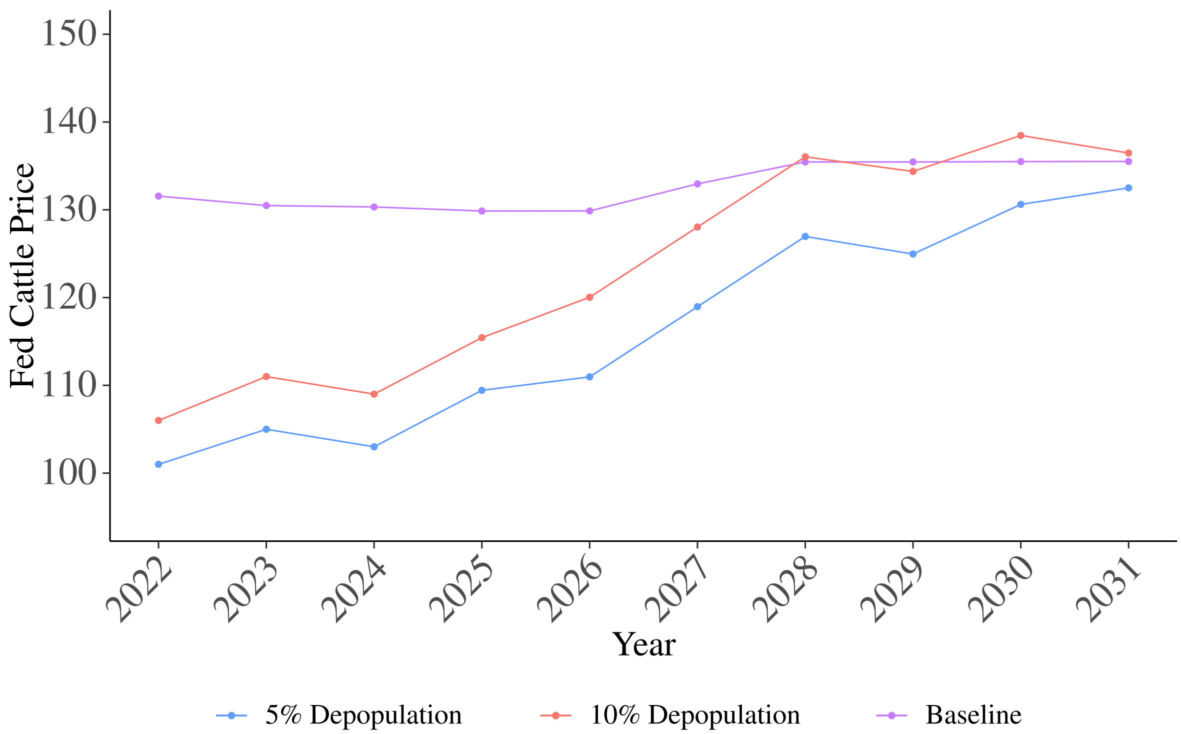

Figure 8: Changes in the fed cattle price ($/CWT) relative to the baseline - **Pessimistic Scenario**

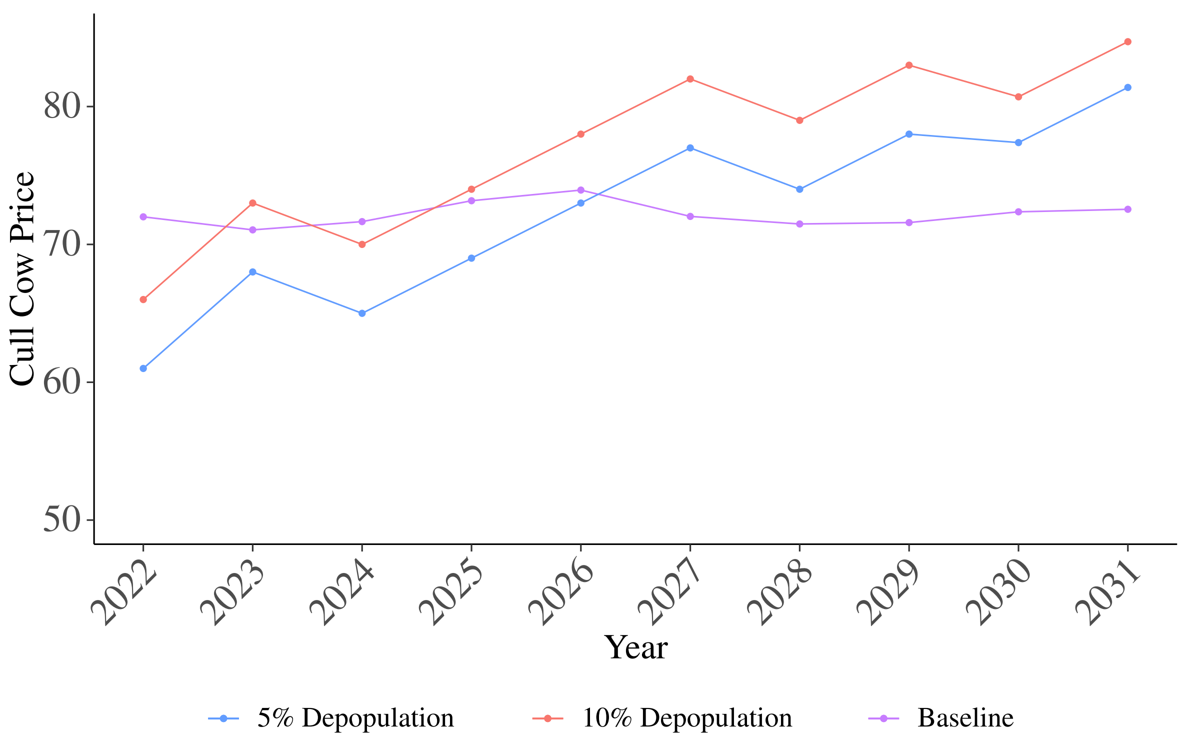

Figure 9: Changes in the cull cow price ($/CWT) relative to the baseline - **Optimistic Scenario**

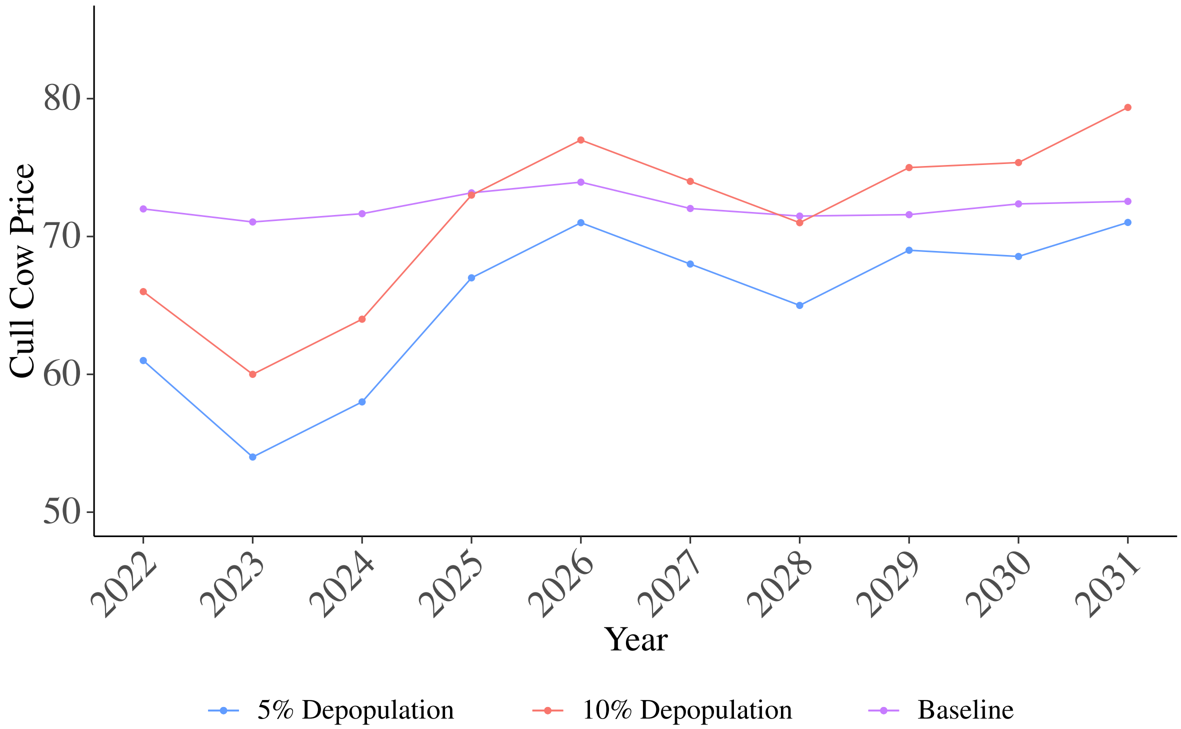

Figure 10: Changes in the cull cow price ($/CWT) relative to the baseline - **Pessimistic Scenario**

The duration of trade bans assumed in our scenarios impacts the prices and supplies in different ways. In the scenarios with shorter trade bans (optimistic scenario), the impact on prices and supplies is short-lived, while in the scenarios with longer trade bans (pessimistic scenario) the impact on the prices and supplies persists for longer periods. However, in both optimistic and pessimistic scenarios, in the long run, the price trajectories approach the baseline. These findings demonstrate that the market recovery after the disease outbreak ultimately depends on the duration of trade restrictions, domestic beef consumption patterns, and other exogenous shocks incorporated into the model.

The price and supply responses determined by our dynamic model are consistent with previous studies (Tozer and Marsh 2012; Hayes et al. 2011; Paarlberg et al. 2008) and economic theory. Given the differences in the economic models and methodologies, the magnitude of our results cannot be directly compared to those of past studies. However, the direction of our findings is in line with past studies. Additionally, the dynamic model offers value-added features such as stock changes and trade patterns which are discussed below.

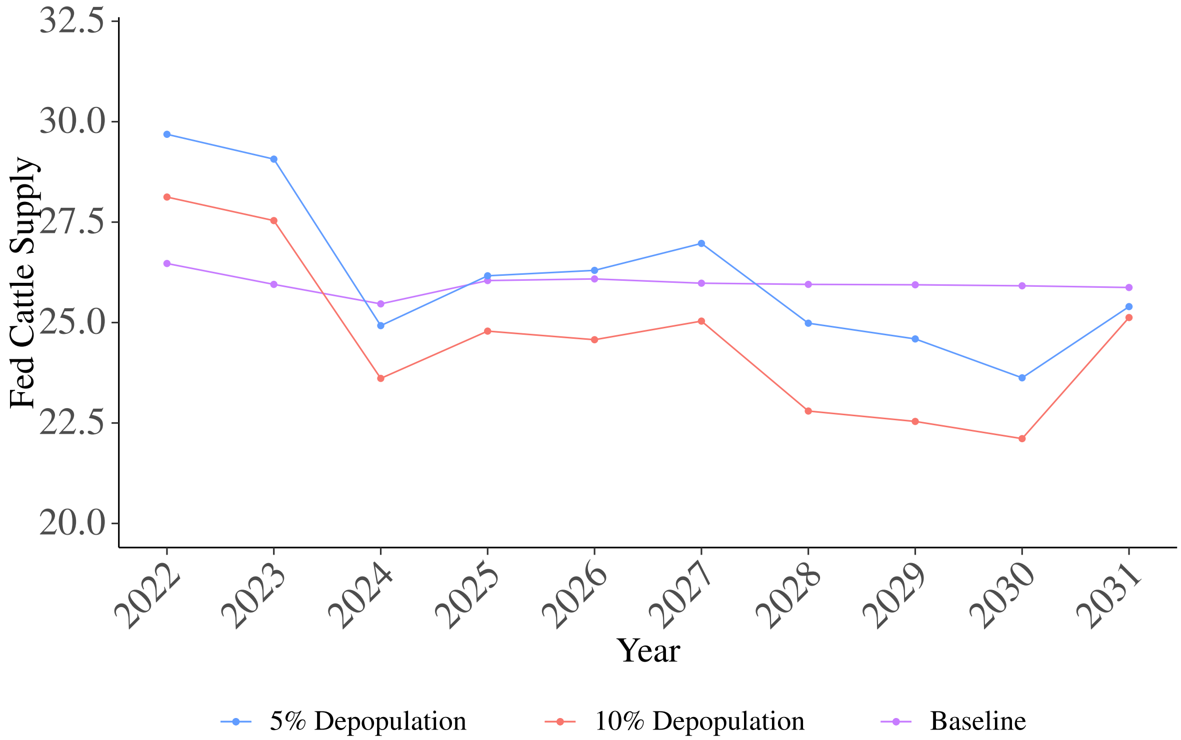

Figure 11: Changes in the fed cattle supply (million head) relative to the baseline - **Optimistic Scenario**

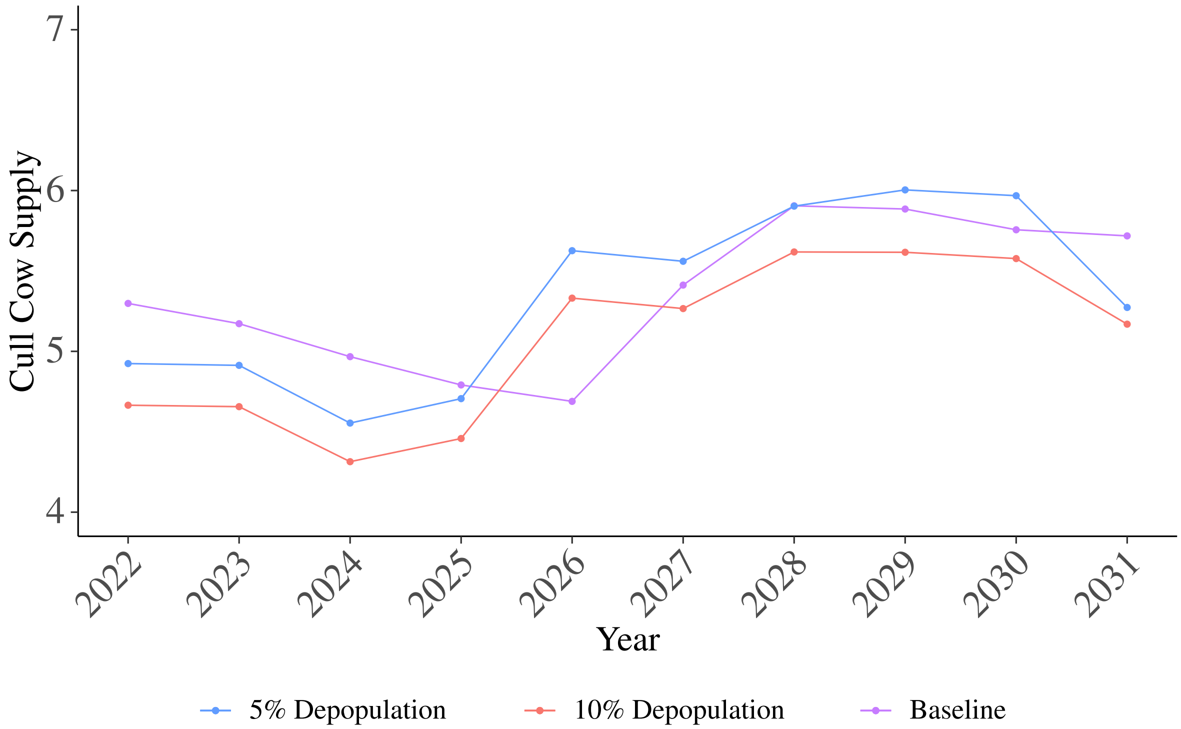

Figure 12: Changes in the cull cow supply (million head) relative to the baseline - **Optimistic Scenario**

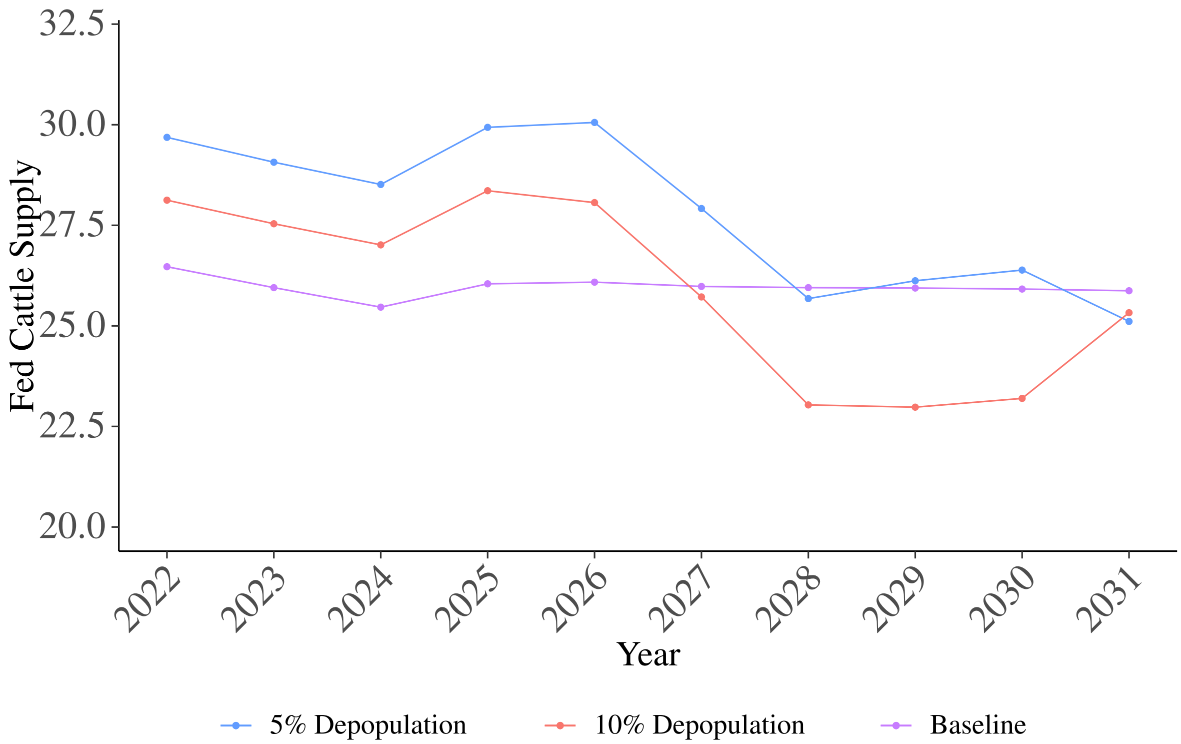

Figure 13: Changes in the fed cattle supply (million head) relative to the baseline - **Pessimistic Scenario**

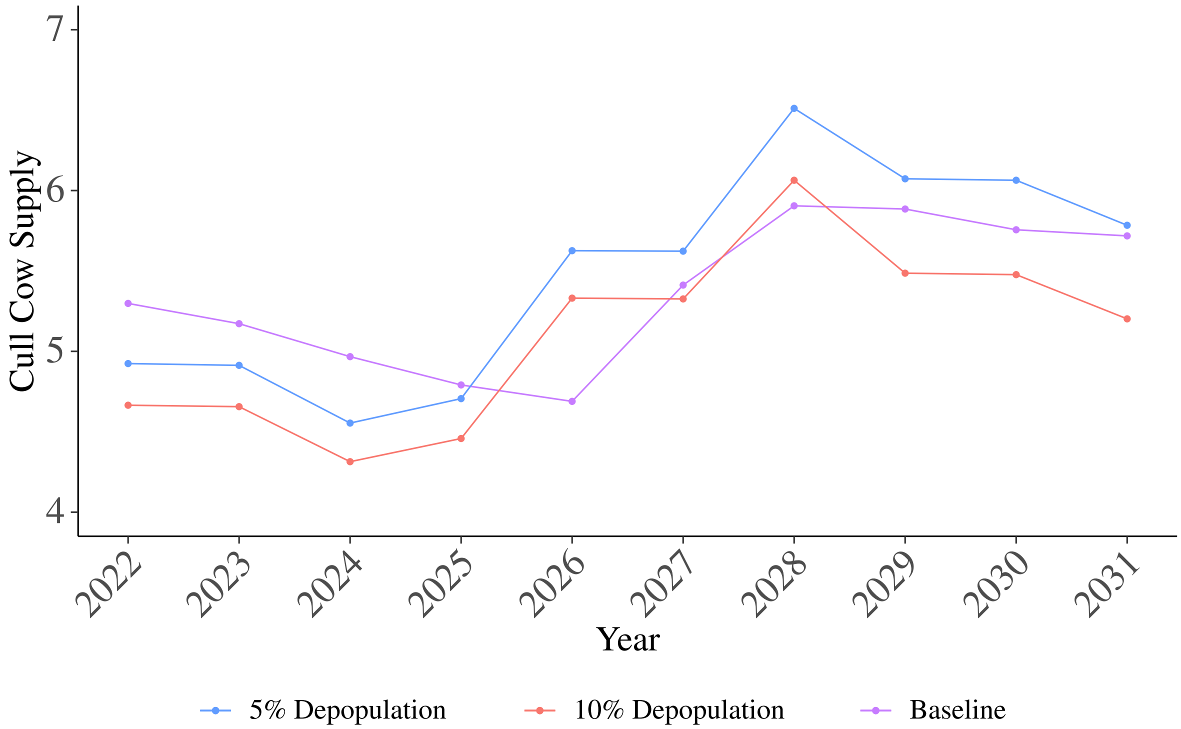

Figure 14: Changes in the cull cow supply (million head) relative to the baseline - **Pessimistic Scenario**

Stock changes (with age groups) after the disease outbreak are shown in Tables 2-5. The stock evolution in Tables 2-5 are consistent with the standard economic theory and intuition. Some years farmers keep both 8 and 9-year-old cows, while in other years they do not keep these older animals. This is due to the holding costs and the market price of the cows. As discussed in the previous subsections, within the model, before deciding to cull the older cows, a representative producer computes the expected value of the older cow which depends on the holding costs, and compares the expected value to the current market price of the cow and makes a decision, resulting in dynamic optimizing behavior.[19] These results show a novelty in this study by determining the stocks dynamically with age distribution in a detailed fashion.

Trade patterns, the magnitude of the imports and exports determined in our simulations, are shown in Tables 6 - 9. In the initial years after the FMD outbreak, as the prices fall, we expect

[19]The conditions that are used for these decisions are specified in the model framework section.

Table 2: Counterfactual inventory (in number of head) distribution under 5% depopulation rate - **Optimistic Scenario**

Year	K	k_3	k_4	k_5	k_6	k_7	k_8	k_9
2022	29995052	6907361	5242532	5045566	4975167	4923723	0	0
2023	30322816	6826104	6561993	4980406	4793288	4726409	0	0
2024	31056546	7248459	6484799	6233893	4731385	4553623	4490088	0
2025	31876000	7370869	6886036	6160559	5922199	4494816	4325942	4265584
2026	31792868	6509256	7002326	6541734	5852531	5626089	4270075	4109645
2027	31016137	5811500	6183793	6652209	6214647	5559905	5344784	4056572
2028	29991855	5525113	5520925	5874604	6319599	5903915	0	0
2029	31244059	7750384	5248857	5244879	5580873	6003619	0	0
2030	31478995	6795726	7362865	4986414	4982635	5301830	0	0
2031	30824267	5917810	6455940	6994721	4737093	4733503	0	0

Note: Here K represents total inventory, k_3 represents the replacement heifers, and k_4 to k_9 represents the cows of ages from 4 to 9 years.

Table 3: Counterfactual inventory (in number of head) distribution under 10% depopulation rate - **Optimistic Scenario**

Year	K	k_3	k_4	k_5	k_6	k_7	k_8	k_9
2022	28416365	6543816	4966610	4780010	4713316	4664580	0	0
2023	28462525	6465566	6216625	4718279	4541010	4477650	0	0
2024	28937730	6896919	6142288	5905794	4482365	4313959	4253768	0
2025	29365580	6873325	6552073	5835174	5610504	4258247	4098261	4041079
2026	29258213	6359499	6529658	6224469	5543415	5329979	4045334	3893348
2027	28890635	6093920	6041524	6203175	5913246	5266244	5063480	3843068
2028	29074059	6626544	5789224	5739448	5893017	5617583	0	0
2029	30159601	7537832	6295217	5499763	5452476	5598366	0	0
2030	32269244	8616211	7160940	5980456	5224775	5179852	0	0
2031	33969432	8312238	8185400	6802893	5681433	4963536	0	0

the beef industry may respond to these low prices by reducing imports. In addition, because of the export bans, additional beef supply is absorbed into the domestic markets, which also reduces imports. This phenomenon is captured in both the optimistic and pessimistic scenarios, with 5% and 10% depopulation levels. The magnitude of beef exports and live animal exports are also determined and presented in Tables 6 - 7. As mentioned in subsection 5.3, the exports in our simulations are determined dynamically.[20]

[20]The trade patterns showcased are consistent with the historical trade data.

Table 4: Counterfactual inventory (in number of head) distribution under 5% depopulation rate - **Pessimistic Scenario**

Year	K	k_3	k_4	k_5	k_6	k_7	k_8	k_9
2022	29995052	6907361	5242532	5045566	4975167	4923723	0	0
2023	30300882	6804171	6561993	4980406	4793288	4726409	0	0
2024	31780873	7993623	6463962	6233893	4731385	4553623	4490088	0
2025	32650159	7456917	7593942	6140764	5922199	4494816	4325942	4265584
2026	33229110	7210047	7084071	7214245	5833726	5626089	4270075	4109645
2027	31610106	5041039	6849544	6729868	6853533	5542040	5344784	0
2028	32632151	7601138	4788987	6507067	6393374	6510856	5264938	5077545
2029	32060959	6059003	7221081	4549537	6181714	6073706	6185313	5001691
2030	29950472	4491149	5756053	6860027	4322061	5872628	0	0
2031	31035703	7581342	4266591	5468250	6517026	4105958	0	0

Table 5: Counterfactual inventory (in number of head) distribution under 10% depopulation rate - **Pessimistic Scenario**

Year	K	k_3	k_4	k_5	k_6	k_7	k_8	k_9
2022	28416365	6543816	4966610	4780010	4713316	4664580	0	0
2023	28429279	6432320	6216625	4718279	4541010	4477650	0	0
2024	29454002	7444775	6110704	5905794	4482365	4313959	4253768	0
2025	29718982	6736268	7072536	5805169	5610504	4258247	4098261	4041079
2026	29715092	6480647	6399454	6718909	5514910	5329979	4045334	3893348
2027	27846633	4615883	6156615	6079482	6382964	5239165	0	0
2028	29019848	7564134	4385089	5848784	5775508	6063816	4977207	0
2029	29493092	6922824	7185927	4165834	5556345	5486732	5760625	4728346
2030	29951599	6931750	6576683	6826631	3957543	5278528	0	0
2031	30077238	6621807	6585162	6247849	6485299	3759665	0	0

Using the supply and price counterfactuals, the changes in consumer surplus and producer surplus relative to the base model are computed. The change in consumer surplus is computed by summing the change in consumer expenditure relative to the baseline for both fed cattle beef and cull cow beef markets. The change in producer surplus is computed by summing the changes in producer revenues relative to the baselines for both fed cattle and cull cow markets. Table 10 shows the discounted present value of the change in consumer surplus and producer surplus for optimistic and pessimistic scenarios.

Table 6: Counterfactual trade patterns under 5% depopulation rate - **Optimistic Scenario**

Year	Live Cattle Imports	Live Cattle Exports	Beef Imports	Beef Exports
2022	0	0	0	0
2023	0	0	0	0
2024	0	309619	0	2.118
2025	0	316220	0	2.209
2026	0	306807	0	2.278
2027	0	196852	0	2.325
2028	2442320	192043	0	2.193
2029	0	316009	0	2.169
2030	0	307711	0	2.059
2031	0	194909	0	2.163

Note: Live cattle trade is in million head and beef trade is in billion pounds.

Table 7: Counterfactual trade patterns under 10% depopulation rate - **Optimistic Scenario**

Year	Live Cattle Imports	Live Cattle Exports	Beef Imports	Beef Exports
2022	0	0	0	0
2023	0	0	0	0
2024	0	176232	0	2.006
2025	0	294735	0	2.092
2026	0	181823	0	2.132
2027	0	181158	0	2.164
2028	0	290294	0	2.013
2029	0	306752	0	1.992
2030	0	331605	0	1.938
2031	0	574489	0	2.155

Note: Live cattle trade is in million head and beef trade is in billion pounds

Due to the export restriction, beef prices fall, increasing changes in consumer surplus (Table 10 column 4). In Table 10, under the 5% depopulation level, the present discounted consumer surplus gains are $14.57 billion for optimistic and $19.54 billion for pessimistic scenarios, respectively. The present discounted value of consumer surplus in the pessimistic scenario is the highest at 10% depopulation level in our simulation. The decline in price negatively impacts the producers and the present discounted producer surplus losses due to the disease outbreak are $5.05 billion and $8.24 billion for optimistic and pessimistic scenarios, respectively. The present discounted value of the change in producer surplus is highest at the 10% depopulation level in the pessimistic scenario.

Table 8: Counterfactual trade patterns under 5% depopulation rate - **Pessimistic Scenario**

Year	Live Cattle Imports	Live Cattle Exports	Beef Imports	Beef Exports
2022	0	0	0	0
2023	0	0	0	0
2024	0	0	0	0
2025	0	0	0	0
2026	0	0	0	0
2027	2574099	205745	0	2.395
2028	0	323519	0	2.277
2029	0	202049	0	2.288
2030	2438950	198512	0	2.297
2031	0	312910	0	2.109

Note: Live cattle trade is in million head and beef trade is in billion pounds

Table 9: Counterfactual trade patterns under 10% depopulation rate - **Pessimistic Scenario**

Year	Live Cattle Imports	Live Cattle Exports	Beef Imports	Beef Exports
2022	0	0	0	0
2023	0	0	0	0
2024	0	0	0	0
2025	0	0	0	0
2026	0	0	0	0
2027	2267629	183987	0	2.214
2028	0	299594	0	2.054
2029	0	296077	0	2.020
2030	0	299157	0	2.025
2031	0	296785	0	2.107

Note: Live cattle trade is in million head and beef trade is in billion pounds.

Table 10: Welfare changes associated with a hypothetical FMD outbreak for optimistic and pessimistic scenarios ($ billion)

Scenario	Depopulation level	Change in producer surplus	Change in consumer surplus
Optimistic	5%	-5.05	14.57
Pessimistic	5%	-18.18	32.79
Optimistic	10%	-8.24	19.54
Pessimistic	10%	-21.10	37.32

It should be noted that due to the decline in domestic beef demand and the inaccessibility of the export markets to the U.S., the producer surplus losses and consumer surplus gains are higher in the first few years after the disease outbreak. The domestic beef demand shock and export ban shock amplify the price decline, which reduces the producers' revenues. The same price decline acts in favor of beef consumers and increases the consumer surplus. Over time, as the domestic beef demand increases and the export restrictions on U.S. beef are lifted, the consumer gains and producer losses decrease and converge toward the baseline. All of our welfare results are consistent with the previous literature, in particular, the direction of the changes in the producer surplus and consumer surplus are in line with the past studies (Paarlberg et al. 2008; Pendell et al. 2007; Paarlberg, Lee, and Seitzinger 2002). The welfare changes shown in Table 10 are purely market-based. These changes do not cover the loss of animal value caused by the depopulation of live animals, disinfection and clean-up costs, government support for ranchers, or the implementation of vaccination programs. Incorporating these additional costs would further amplify the losses due to the disease outbreak and would decrease the net welfare impacts.

Our results show that, ignoring the essential dynamic process of beef cattle production can lead to inaccurate long-run estimates and miss the evolution of the industry and the market recovery. By incorporating biological constraints and breeding herd dynamics, we showed not only short-run but also long-run trajectories of market response to the FMD outbreak and the dynamic nature of the market response.

A number of policy recommendations can be made based on our results. First, educating consumers about the health consequences of FMD, in particular, raising awareness that *FMD cannot be transmitted to humans through the consumption of red meat* can reduce the losses from the FMD outbreak. Second, restricting the cattle movement, and convincing trading partners to regionalize the United States can sustain some of the export flow, which can significantly reduce the losses from the FMD outbreak. Our findings about large declines in prices in the initial years post-FMD outbreak also inform that, preventing adverse consumer and trading partners' reactions to the disease outbreak can largely alleviate the negative impacts on prices and revenues.

7 Conclusion

This study develops a dynamic framework for the U.S. beef cattle industry that is consistent with the economic theory, biology, and decision-making about beef production. The framework developed proves to be a valuable tool for analyzing the introduction of exogenous shocks such as a variety of production and policy changes in the beef cattle industry. This conceptual model is demonstrated by first showing the results from the model in capturing the recent evolution of the U.S. beef cattle industry and then by estimating the economic impacts of a hypothetical FMD outbreak. The findings of the study demonstrate how the model can be used to evaluate the potential economic impacts of a disease outbreak and can provide guidance for designing disease control and mitigation policies.

The results of the study, particularly the quantification of short-run and long-run impacts as well as the variation of these impacts over time, demonstrate the ability of the model to capture the evolution of the economic impact of a disease outbreak and response. The detailing of the evolution and the age distribution of cows provide greater information on the physical and economic adjustments within the industry. The dynamic framework developed is not limited to FMD or disease studies, as it can be employed to study various policy proposals affecting the beef cattle industry. As showcased in this study, alternate scenarios of policy proposals can be designed to simulate the model and quantify the economic impacts making the model a valuable tool to design, develop, and deploy policies affecting the beef cattle industry.

References

Aadland, David. 2004. "Cattle Cycles, Heterogeneous Expectations and the Age Distribution of Capital." *Journal of Economic Dynamics and Control* 28 (10): 1977–2002.

Aadland, David, and DeeVon Bailey. 2001. "Short-Run Supply Responses in the U.S. Beef-Cattle Industry." *American Journal of Agricultural Economics* 83 (4): 826–39.

Azzam, Azzeddine M., and Dale G. Anderson. 1996. "Assessing Competition in Meatpacking: Economic History, Theory, and Evidence." *USDA*. https://gipsa.usda.gov/psp/publication/con_ tech%20report/rr96-6.pdf.

Baak, Saang Joon. 1999. "Tests for Bounded Rationality with a Linear Dynamic Model Distorted by Heterogeneous Expectations." *Journal of Economic Dynamics and Control* 23 (9): 1517–43.

Ball, V. Eldon, and Robert G. Chambers. 1982. "An Economic Analysis of Technology in the Meat Products Industry." *American Journal of Agricultural Economics* 64 (4): 699–709.

Chavas, Jean-Paul. 2000. "On Information and Market Dynamics: The Case of the U.S. Beef Market." *Journal of Economic Dynamics and Control* 24 (5): 833–53.

Chen, Chen-Ti, John M. Crespi, William Hahn, Lee L. Schulz, and Fawzi Taha. 2020. "Long-Run Impacts of Trade Shocks and Export Competitiveness: Evidence from the u.s. BSE Event." *Agricultural Economics* 51 (6): 941–58.

Coffey, Brian, James Mintert, John A. Fox, Ted C. Schroeder, and Luc Valentin. 2005. "The Economic Impact of BSE on the u.s. Beef Industry: Product Value Losses, Regulatory Costs, and Consumer Reactions." *Kansas State University Agricultural Experiment Station and Cooperative Extension Service*. https://krex.k-state.edu/dspace/handle/2097/12460.

Ekboir, Javier Mario. 1999. "Potential Impact of Foot-and-Mouth Disease in California: The Role and Contribution of Animal Health Surveillance and Monitoring Services." *Agricultural Issues Center, University of California, Davis*. http://aic.ucdavis.edu/pub/EkboirFMD-part1.pdf.

Elbakidze, Levan, Linda Highfield, Michael Ward, Bruce A McCarl, and Bo Norby. 2009. "Economics Analysis of Mitigation Strategies for FMD Introduction in Highly Concentrated Animal Feeding Regions." *Applied Economic Perspectives and Policy* 31 (4): 931–50.

FAPRI. 2022. "U.S. Agricultural Market Outlook." https://www.fapri.missouri.edu/wp-content/uploads/2022/03/2022-U.S.-Agricultural-Market-Outlook.pdf.

Garner, M. G, and M. B Lack. 1995. "An Evaluation of Alternate Control Strategies for Foot-and-Mouth Disease in Australia: A Regional Approach." *Preventive Veterinary Medicine* 23 (1): 9–32.

Hayes, Dermot J., Jacinto F. Fabiosa, Amani Elobeid, and Miguel Carriquiry. 2011. "Economy Wide Impacts of a Foreign Animal Disease in the United States." *Center for Agricultural and*

Rural Development, Iowa State University 11-WP 525.

Jarvis, Lovell S. 1974. "Cattle as Capital Goods and Ranchers as Portfolio Managers: An Application to the Argentine Cattle Sector." *Journal of Political Economy* 82 (3): 489–520.

Judd, Kenneth L. 1998. *Numerical Methods in Economics*. MIT press.

Kenneth H. Mathews, Monte Vandeveer, Jr., and Ronald A. Gustafson. 2006. "An Economic Chronology of Bovine Spongiform Encephalopathy in North America." *U.S. Department of Agriculture, Livestock, Dairy, and Poultry Outlook No. (LDPM-14301) 18 Pp*. https://www.ers.usda.gov/webdocs/outlooks/37390/29425_ldpm14301_002.pdf?v=9927.2.

Kuchler, Fred, and Abebayehu Tegene. 2006. "Did BSE Announcements Reduce Beef Purchases?" *Washington, DC: U.S. Department of Agriculture, Economic Research Report Number 34*.

Marsh, Thomas L., Ted C. Schroeder, and James Mintert. 2004. "Impacts of Meat Product Recalls on Consumer Demand in the USA." *Applied Economics* 36 (9): 897–909.

Miranda, Mario J. 1997. "Numerical Strategies for Solving the Nonlinear Rational Expectations Commodity Market Model." *Computational Economics* 11 (1): 71–87.

Miranda, Mario J., and Paul L. Fackler. 2002. *Applied Computational Economics and Finance*. MIT press.

Miranda, Mario J., and Joseph W. Glauber. 1995. "Solving Stochastic Models of Competitive Storage and Trade by Chebychev Collocation Methods." *Agricultural and Resource Economics Review* 24 (1): 70–77.

Morgan, N., and A. Prakash. 2006. "International Livestock Markets and the Impact of Animal Disease." *Revue Scientifique Et Technique (International Office of Epizootics)* 25 (2): 517–28.

Mu, Jianhong E., Bruce A. McCarl, Amy Hagerman, and David Bessler. 2015. "Impacts of Bovine Spongiform Encephalopathy and Avian Influenza on U.S. Meat Demand." *Journal of Integrative Agriculture* 14 (6): 1130–41.

Nogueira, Lia, Thomas L. Marsh, Peter R. Tozer, and Derrell Peel. 2011. "Foot-and-Mouth Disease and the Mexican Cattle Industry." *Agricultural Economics* 42 (s1): 33–44.

Paarlberg, Philip L., and John G. Lee. 1998. "Import Restrictions in the Presence of a Health Risk:

An Illustration Using FMD." *American Journal of Agricultural Economics* 80 (1): 175–83.

Paarlberg, Philip L., John G. Lee, and Ann H. Seitzinger. 2002. "Potential Revenue Impact of an Outbreak of Foot-and-Mouth Disease in the United States." *Journal of the American Veterinary Medical Association* 220 (7): 988–91.

———. 2003. "Measuring Welfare Effects of an FMD Outbreak in the United States." *Journal of Agricultural and Applied Economics* 35 (1): 53–65.

Paarlberg, Philip L., Ann H. Seitzinger, John G. Lee, and Kenneth H. Mathews. 2008. "Economic Impacts of Foreign Animal Disease." *Washington, DC: U.S. Department of Agriculture, Economic Research Report No. 57*. https://www.ers.usda.gov/publications/pub-details/?pubid=45991.

Pendell, Dustin L., John Leatherman, Ted C. Schroeder, and Gregory S. Alward. 2007. "The Economic Impacts of a Foot-And-Mouth Disease Outbreak: A Regional Analysis." *Journal of Agricultural and Applied Economics* 39 (s1): 19–33. https://doi.org/10.1017/S1074070800028911.

Pendell, Dustin L., Thomas L. Marsh, Keith H. Coble, Jayson L. Lusk, and Sara C. Szmania. 2015. "Economic Assessment of FMDv Releases from the National Bio and Agro Defense Facility." *PLoS One* 10 (6).

Piggott, Nicholas E., and Thomas L. Marsh. 2004. "Does Food Safety Information Impact U.S. Meat Demand?" *American Journal of Agricultural Economics* 86 (1): 154–74.

Rich, Karl M., and Alex Winter-Nelson. 2007. "An Integrated Epidemiological-Economic Analysis of Foot and Mouth Disease: Applications to the Southern Cone of South America." *American Journal of Agricultural Economics* 89 (3): 682–97.

Rosen, Sherwin. 1987. "Dynamic Animal Economics." *American Journal of Agricultural Economics* 69 (3): 547–57.

Rosen, Sherwin, Kevin M. Murphy, and José A. Scheinkman. 1994. "Cattle Cycles." *Journal of Political Economy* 102 (3): 468–92.

Schlenker, Wolfram, and Sofia B. Villas-Boas. 2009. "Consumer and Market Responses to Mad Cow Disease." *American Journal of Agricultural Economics* 91 (4): 1140–52.

Schoenbaum, Mark A, and W Terry Disney. 2003. "Modeling Alternative Mitigation Strategies for a Hypothetical Outbreak of Foot-and-Mouth Disease in the United States." *Preventive Veterinary Medicine* 58 (1): 25–52.

Schroeder, Ted C., Dustin L. Pendell, Michael W. Sanderson, and Sara Mcreynolds. 2015. "Economic Impact of Alternative FMD Emergency Vaccination Strategies in the Midwestern United States." *Journal of Agricultural and Applied Economics* 47 (1): 47–76. https://doi.org/10.1017/aae.2014.5.

Taylor, Mykel, H. Allen Klaiber, and Fred Kuchler. 2016. "Changes in U.S. Consumer Response to Food Safety Recalls in the Shadow of a BSE Scare." *Food Policy* 62: 56–64.

Thompson, D, P Muriel, D Russell, P Osborne, A Bromley, M Rowland, S Creigh-Tyte, and C Brown. 2002. "Economic Costs of the Foot and Mouth Disease Outbreak in the United Kingdom in 2001." *Rev Sci Tech* 21 (3): 675–87.

Tozer, Peter, and Thomas L. Marsh. 2012. "Domestic and Trade Impacts of Foot-and-Mouth Disease on the Australian Beef Industry." *Australian Journal of Agricultural and Resource Economics* 56 (3): 385–404.

USDA. 2022. "USDA Agricultural Projections to 2031." https://www.ers.usda.gov/webdocs/outlooks/103310/oce-2022-01.pdf?v=7671.2.

USDA-APHIS. 2020. "Foot-and-Mouth Disease Resposne Plan: The Red Book." https://www.aphis.usda.gov/animal_health/emergency_management/downloads/fmd_responseplan.pdf.

———. 2021. "FMD Factsheet." https://www.aphis.usda.gov/publications/animal_health/fs-fmd-general.pdf.

USDA-ERS. 2001. "From USDA's Economic Research Service : Dissecting the Challenges of Mad Cow & Foot-and-Mouth Disease." *The Food Institute Report*, 9.

———. 2021. "Cattle & Beef: Statistics & Information."

USDA-FAS. 2022. "Livestock and Poultry: World Markets and Trade, July."

USDA-NASS. 2021. "Livestock Slaughter."

———. 2022a. "USDA National Agricultural Statistics Service (NASS) - Agricultural Prices."

https://usda.library.cornell.edu/concern/publications/c821gj76b?locale=en#release-items.

———. 2022b. "USDA National Agricultural Statistics Service (NASS) - January Cattle Inventory." https://usda.library.cornell.edu/concern/publications/h702q636h?locale=en.

Xia, Yin, and Buccola Steven. 2002. "Size, Cost, and Productivity in the Meat Processing Industries." *Agribusiness (New York, N.Y.)* 18 (3): 283–99.

Zhao, Zishun, Thomas I. Wahl, and Thomas L. Marsh. 2006. "Invasive Species Management: Foot-and-Mouth Disease in the U.S. Beef Industry." *Agricultural and Resource Economics Review* 35 (1): 98–115.

Appendix

Equilibrium system and solution to the model

The model equations are constructed to reach market equilibrium and to incorporate the biological and economic considerations on herd size and distribution, trade, and consumer choice. In what follows, we will focus on the equations that describe how producers optimize their profits by choosing to breed, slaughter, or cull cattle. After solving these equations, it will be straightforward to solve for prices and quantities.

The equations of the model are as follows:

$$gK_{t-1} - k_{3,t+1} = \tilde{A} \frac{\exp\left(\frac{\tilde{\mu} - \tilde{p}}{\tilde{s}}\right)}{1 + \exp\left(\frac{\tilde{\mu} - \tilde{p}}{\tilde{s}}\right)}; \tag{23}$$

$$k_{10,t} + \sum_{j=7}^{9} (k_{j,t} - k_{j+1,t+1}) = \tilde{A} \frac{1}{1 + \exp\left(\frac{\tilde{\mu} - \tilde{p}}{\tilde{s}}\right)}; \tag{24}$$

$$p_{s,t} = E_t\left[\beta V_{3,t+1} + g\beta^3 p_{s,t+3} - z_t\right]; \tag{25}$$

$$V_{3,t} = E_t\left[\beta V_{4,t+1} + g\beta^3 p_{s,t+3} - z_t\right] > p_{c,t} \implies k_{4,t+1} = \delta k_{3,t}; \tag{26}$$

$$V_{4,t} = E_t\left[\beta V_{5,t+1} + g\beta^3 p_{s,t+3} - z_t\right] > p_{c,t} \implies k_{5,t+1} = \delta k_{4,t}; \tag{27}$$

$$V_{5,t} = E_t\left[\beta V_{6,t+1} + g\beta^3 p_{s,t+3} - z_t\right] > p_{c,t} \implies k_{6,t+1} = \delta k_{5,t}; \tag{28}$$

$$V_{6,t} = E_t\left[\beta V_{7,t+1} + g\beta^3 p_{s,t+3} - z_t\right] > p_{c,t} \implies k_{7,t+1} = \delta k_{6,t}; \tag{29}$$

$$V_{7,t} = \max\left\{p_{c,t}, E_t\left[\beta V_{8,t+1} + g\beta^3 p_{s,t+3} - z_t\right]\right\}; \tag{30}$$

$$V_{8,t} = \max\left\{p_{c,t}, E_t\left[\beta V_{9,t+1} + g\beta^3 p_{s,t+3} - z_t\right]\right\}; \tag{31}$$

$$V_{9,t} = \max\left\{p_{c,t}, E_t\left[\beta p_{c,t+1} + g\beta^3 p_{s,t+3} - z_t\right]\right\}; \tag{32}$$

$$V_{10,t} = p_{c,t} \implies k_{11,t} = 0. \tag{33}$$

Equation 23 says that the supply of fed cattle equals the demand for fed cattle. Similarly, equation 24 says that the supply of cull cows equals the demand for cull cows. Equation 25 is the arbitrage condition for fed cattle. Equations 26-29 are the arbitrage conditions for cows between 3 and 6 years of age. Note that we assume producers never cull cows 6 years or younger which implies that the number of cows for these younger cohorts carries to the next year, adjusting for natural death. Equations 30-32 determine the choice between keeping cows for one more year or culling cows between 7 and 9 years of age. Finally, equation 33 says that all 10-year-old cows are culled such that there are no 11-year-old cows.

The next step is to specify how producers form their expectations about future prices. In fact, the equations 23-32 imply that producers form price expectations for prices several years into the future. In the following subsections, we present an analytical solution and numerical solution algorithm under rational price expectations.

Rational price expectations

We refer to rational expectations as a situation where the producers use all the information available in the economy to make price expectations. The information may include the present and past prices, production, and disappearance of the fed cattle and cull cows. The system of equations under rational price expectations is as follows:

$$gK_{t-1} - k_{3,t+1} = \tilde{A}\frac{\exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)}{1+\exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)} \tag{34}$$

$$k_{10,t} + \sum_{j=7}^{9}(k_{j,t} - k_{j+1,t+1}) = \tilde{A}\frac{1}{1+\exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)} \tag{35}$$

$$p_{s,t} = \beta \mathrm{E_t} V_{3,t+1} + g\beta^3 \mathrm{E_t} p_{s,t+3} - (1 + g\beta(\gamma_0 + \beta\gamma_1))\mathrm{E_t} h_t \tag{36}$$

$$k_{4,t+1} = \delta k_{3,t} \tag{37}$$

$$k_{5,t+1} = \delta k_{4,t} \tag{38}$$

$$k_{6,t+1} = \delta k_{5,t} \tag{39}$$

$$k_{7,t+1} = \delta k_{6,t} \tag{40}$$

$$V_{7,t} = max\left\{p_{c,t}, \beta \mathrm{E_t} V_{8,t+1} + g\beta^3 \mathrm{E_t} p_{s,t+3} - (1 + g\beta(\gamma_0 + \beta\gamma_1))\mathrm{E_t} h_t\right\} \tag{41}$$

$$V_{8,t} = max\left\{p_{c,t}, \beta \mathrm{E_t} V_{9,t+1} + g\beta^3 \mathrm{E_t} p_{s,t+3} - (1 + g\beta(\gamma_0 + \beta\gamma_1))\mathrm{E_t} h_t\right\} \tag{42}$$

$$V_{9,t} = max\left\{p_{c,t}, \beta \mathrm{E_t} p_{c,t+1} + g\beta^3 \mathrm{E_t} p_{s,t+3} - (1 + g\beta(\gamma_0 + \beta\gamma_1))\mathrm{E_t} h_t\right\} \tag{43}$$

$$V_{10,t} = p_{c,t} \tag{44}$$

Assuming a producer never culls a cow younger than six years of age, we have a variety of conditions that describe the decisions of the producers about culling the older cows. These decisions reduce to four different cases.

Case I: Only 10-year old cows are culled

In this case, producers only cull the 10-year-old cows. Hence, the equations for cows aged between 7 and 10 are as below:

$$k_{j+1,t+1} = \delta k_{j,t} \forall j \in [7,8] \tag{45}$$

$$\beta \mathrm{E_t} p_{c,t+1} + g\beta^3 \mathrm{E_t} p_{s,t+3} - (1 + g\beta(\gamma_0 + \beta\gamma_1))\mathrm{E_t} h_t > p_{c,t} \implies k_{10,t+1} = \delta k_{9,t} \tag{46}$$

$$V_{10,t} = p_{c,t} \implies k_{11,t} = 0 \tag{47}$$

Observe that we do not need to solve for $\mathrm{E}_t V_{10,t+1}$, $\mathrm{E}_t V_{9,t+1}$, or $\mathrm{E}_t V_{8,t+1}$ because we assume that producers cull older cows first. Thus, because producers do not cull 9-year-old cows, they do not cull 8 and 7-year-old cows either.

Case II: Some 9-year old cows are culled

In this case, producers cull some 9-year-old cows in addition to all 10-year-old cows. The set of expressions for cows aged between 7 and 10, in this case, are as below:

$$k_{j+1,t+1} = \delta k_{j,t} \forall j \in [7,8] \tag{48}$$

$$\beta \mathrm{E_t} p_{c,t+1} + g\beta^3 \mathrm{E_t} p_{s,t+3} - (1 + g\beta(\gamma_0 + \beta\gamma_1))\mathrm{E_t} h_t = p_{c,t} \implies k_{10,t+1} \leq \delta k_{9,t} \tag{49}$$

$$k_{11,t} = 0 \tag{50}$$

Again, we do not need to solve for $\mathrm{E}_t V_{9,t+1}$ or $\mathrm{E}_t V_{8,t+1}$ because we assume that producers cull older cows first. We solve for $k_{10,t+1}$ using the arbitrage equality for 10-year-old cows.

Case III: some 8-year old cows are culled

The third case is when producers cull some 8-year-old cows. The equations for cows aged between 7 and 10 years of age for that case are:

$$\begin{aligned} k_{8,t+1} &= \delta k_{7,t}; \\ \beta \mathrm{E_t} p_{c,t+1} + g\beta^3 \mathrm{E_t} p_{s,t+3} - \mathrm{E}_t h_t \left(1 + \beta g(\gamma_0 + \beta\gamma_1)\right) = p_{c,t} \implies k_{9,t+1} &\leq \delta k_{8,t}; \\ k_{j+1,t+1} &= 0 \quad \forall j \in [9,10] \end{aligned} \tag{51}$$

Case IV: some 7-year old cows are culled

Finally, the last case is when producers cull some or all of their 7-year old cows. The equations for cows aged between 7 and 10 years of age below are:

$$\begin{aligned} \beta \mathrm{E_t} p_{c,t+1} + g\beta^3 \mathrm{E_t} p_{s,t+3} - \mathrm{E}_t h_t \left(1 + \beta g(\gamma_0 + \beta\gamma_1)\right) \leq p_{c,t} \implies k_{8,t+1} &\leq \delta k_{7,t}; \\ k_{j+1,t+1} &= 0 \quad \forall j \in [8,9,10] \end{aligned} \tag{52}$$

Expected return for a 2-year old heifer

Equation 36 contains the expected value of the three-year-old cow $\mathrm{E_t} V_{3,t+1}$. We write the expected price as is and by iterative substitution, Equation 36 can be rewritten as

$$\mathrm{E_t} V_{3,t+1} = \beta^4 \mathrm{E_t} V_{7,t+5} + g \sum_{i=3}^{6} \beta^i p_{s,t+1+i} - \sum_{i=0}^{3} \beta^i z_{t+1+i} \tag{53}$$

Solution for Case I: only 10-year old cows are culled

Assuming producers expect to keep their cows until they are 10 years old, after some few manipulations equation 53 can be written as

$$\mathrm{E_t}V_{3,t+1} = \beta^7 p_{c,t+8} + g\sum_{i=3}^{9}\beta^i p_{s,t+1+i} - \sum_{i=0}^{5}\beta^i z_{t+1+i} \tag{54}$$

where z is the discounted holding hosts. By replacing $\mathrm{E_t}V_{3,t+1}$ in equation 36 with equation 54, we get

$$p_{s,t} = \beta^8 p_{c,t+7} + g\sum_{i=4}^{10}\beta^i p_{s,t+i} + g\beta^3 \mathrm{E_t} p_{s,t+3} - \sum_{i=1}^{7}\beta^i z_{t+i} \tag{55}$$

Finally, by replacing equation 36 with equation 55 the final solution system for Case I is as follows

$$gK_{t-1} - k_{3,t+1} = \tilde{A}\frac{\exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)}{1+\exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)} \tag{56}$$

$$k_{10,t} + \sum_{j=7}^{9}(k_{j,t} - k_{j+1,t+1}) = \tilde{A}\frac{1}{1+\exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)} \tag{57}$$

$$p_{s,t} = \beta^8 p_{c,t+7} + g\sum_{i=4}^{10}\beta^i p_{s,t+i} + g\beta^3 \mathrm{E_t} p_{s,t+3} - \sum_{i=1}^{7}\beta^i z_{t+i} \tag{58}$$

$$k_{j+1,t+1} = \delta k_{j,t} \quad \forall j \in [3,4,5,6,7,8] \tag{59}$$

$$p_{c,t} = \beta \mathrm{E_t} p_{c,t+1} + g\beta^3 \mathrm{E_t} p_{s,t+3} - (1+g\beta(\gamma_0+\beta\gamma_1))\mathrm{E_t} h_t \tag{60}$$

Solution for Case II: some 9-year old cows are culled

In this case, producers keep their cows until they are nine years old and then cull them. With some substitutions equation 53 can be written as

$$\mathrm{E_t}V_{3,t+1} = \beta^6 p_{c,t+7} + g\sum_{i=3}^{8}\beta^i p_{s,t+1+i} - \sum_{i=0}^{5}\beta^i z_{t+1+i} \tag{61}$$

where z is the discounted holding hosts. By replacing $\mathrm{E_t}V_{3,t+1}$ in equation 36 with equation 54, we get

$$p_{s,t} = \beta^7 p_{c,t+7} + g\sum_{i=4}^{9}\beta^i p_{s,t+i} + g\beta^3 \mathrm{E_t} p_{s,t+3} - \sum_{i=1}^{6}\beta^i z_{t+i} \tag{62}$$

Finally, by replacing equation 36 with equation 62 the final solution system for Case II becomes

$$gK_{t-1} - k_{3,t+1} = \tilde{A}\frac{\exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)}{1+\exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)} \tag{63}$$

$$k_{10,t} + \sum_{j=7}^{9}(k_{j,t} - k_{j+1,t+1}) = \tilde{A}\frac{1}{1+\exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)} \tag{64}$$

$$p_{s,t} = \beta^7 p_{c,t+7} + g\sum_{i=4}^{9}\beta^i p_{s,t+i} + g\beta^3 \mathrm{E_t} p_{s,t+3} - \sum_{i=1}^{6}\beta^i z_{t+i} \tag{65}$$

$$k_{j+1,t+1} = \delta k_{j,t} \quad \forall j \in [3,4,5,6,7,8] \tag{66}$$

$$p_{c,t} = \beta \mathrm{E_t} p_{c,t+1} + g\beta^3 \mathrm{E_t} p_{s,t+3} - (1+g\beta(\gamma_0+\beta\gamma_1))\mathrm{E_t} h_t \tag{67}$$

The above system is the analytical solution of the rational expectations model (Case I and Case II). The solution system is solved to obtain equilibrium prices and quantities. Note that, the system of equations contains the expected price of the fed cattle three periods ahead, and the expected price of the cull cow one period ahead. We must know these expected prices to find the equilibrium solution.[21]

Under rational expectations, the producers have the ability to make expectations about the price by using all the information available to them. Forming expectations requires knowing the production (in the number of head) of the cattle in future periods. Although it is not possible to know future production with absolute certainty, an approximation of future production is sufficient to make expectations about the price. At time t, a producer knows the total breeding stock $K_t = k_{3,t} + \ldots + k_{10,t}$, the production of fed cattle sl_t, and the production of cull cows cl_t. However, in order

[21]We rely on numerical methods to compute the expected price.

to make expectations about the price, the production (or an approximation of the future production) must be used. We rely on the competitive storage model to construct the production of fed cattle and cull cows into the future. A simple storage type model is specified as $Q_{t+1} = Q_t \varepsilon_t + \text{Storage}_t$, where Q_{t+1} is the production at $t+1$, Q_t, ε_t, Storage_t are the production, production shock, and storage respectively. Note that the solution system includes the expected price in the future, and computing the expected price requires knowledge of random variables, hence the use of a competitive storage model to construct the production.[22]

Using the production shocks and the competitive storage model specification, the production of fed cattle and cull cows can be written as follows.

Fed Cattle Production:

$$
\begin{aligned}
sl_{t+1} &= sl_{t-1}\varepsilon^s_{t-1} + \text{Storage}_{t-1} && (68)\\
&= \Big(gK_{t-2} - k_{3,t}\Big)\varepsilon^s_{t-1} + \text{Storage}_{t-1} && (69)\\
&= \Big(g - gr\Big)K_{t-2}\varepsilon^s_{t-1} + \text{Storage}_{t-1} && (70)\\
&= g\Big(1 - r\Big)K_{t-2}\varepsilon^s_{t-1} + \text{Storage}_{t-1} && (71)
\end{aligned}
$$

where g is the breeding rate, r is the rate of the new born progeny entering the breeding stock. Storage_{t-1} can be further decomposed as:

$$
\begin{aligned}
\text{Storage}_{t-1} &= K_{t-1} - k_{3,t+1} && (72)\\
&= (1-gr)K_{t-1} && (73)\\
&= (1-gr)\delta g\Big[K_{t-2} - sl_{t-2} - cl_{t-2}\Big] && (74)\\
&= (1-gr)\delta g\Big[K_{t-2} - g(1-r)K_{t-3} - (k_{9,t-2} + (1-\delta)k_{8,t-2} + (1-\delta)k_{7,t-2})\Big] && (75)
\end{aligned}
$$

[22]Numerical methods to compute expected price require integration over random variables. The production shocks in the storage model can be used as a random variable to compute the expected price.

Cull Cow Production:

$$cl_{t+1} = cl_t \varepsilon_t^c + \text{Storage}_t \tag{76}$$

$$= \Big[k_{9,t} + (1-\delta)k_{8,t} + (1-\delta)k_{7,t}\Big]\varepsilon_t^c + \Big[(k_{9,t+1} - k_{9,t}) + (k_{8,t+1} - k_{8,t} + (k_{7,t+1} - k_{7,t}))\Big] \tag{77}$$

$$= \Big[k_{9,t} + (1-\delta)k_{8,t} + (1-\delta)k_{7,t}\Big]\varepsilon_t^c + \Big[\delta(k_{8,t} + k_{7,t} + k_{6,t}) - (k_{7,t} + k_{8,t} + k_{9,t})\Big] \tag{78}$$

where the production shock of fed cattle (ε_t^s) and the production shock of cull cows (ε_t^c) are random variables following a Gaussian distribution. The production shocks are constructed by taking the ratio of observed historical production to the constructed production from the model specification. Then, the standard deviation of the constructed production shocks is used to define the Gaussian distribution.

The equilibrium system of equations can be solved when the price expectations of fed cattle and cull cows are known. The constructed production from the competitive storage type model is utilized to determine the price and the expected price of the fed cattle and cull cows. The rational expectations model is a functional equation problem. The solution takes the form of a function rather than a finite-sized vector of prices and quantities. Therefore, the model cannot be solved analytically and requires numerical methods to solve it. The competitive storage model with rational expectations is also a functional equation problem and the solution takes a functional form instead of a finite-sized vector. Therefore, numerical methods from the competitive storage literature are borrowed to find a solution to the rational expectations model. In particular, a collocation method is applied to find an equilibrium solution.

Using the collocation method, the following system of equations is solved to determine the equilibrium prices and quantities.

$$gK_{t-1}-k_{3,t+1}=\tilde{A}\frac{\exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)}{1+\exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)} \tag{79}$$

$$k_{10,t}+\sum_{j=7}^{9}(k_{j,t}-k_{j+1,t+1})=\tilde{A}\frac{1}{1+\exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)} \tag{80}$$

$$p_{s,t}=\beta^{7}p_{c,t+7}+g\sum_{i=4}^{9}\beta^{i}p_{s,t+i}+g\beta^{3}\mathrm{E_t}p_{s,t+3}-\sum_{i=0}^{6}\beta^{i}z_{t+i} \tag{81}$$

$$p_{c,t}=\beta\mathrm{E_t}p_{c,t+1}+g\beta^{3}\mathrm{E_t}p_{s,t+3}-(1+g\beta(\gamma_0+\beta\gamma_1))\mathrm{E_t}h_t \tag{82}$$

The above system of equations is non-linear, so to solve the system of equations we need to provide the initial values for the price and the expected price. First, we determine the cattle price and use it to compute the expected price. We posit that the price of fed cattle (cull cows) depends on the total supply of fed cattle (cull cows), demand shock, and corn price. Without assuming a functional form of the relationship, we approximate the price function by a linear combination of independent basis functions $\phi_1,\phi_2,\ldots,\phi_m$ of the supply of fed cattle (sl_t) for the fed cattle price, supply of cull cows (cl_t) for cull cow price, demand shock (ε_t^D), and corn price (c_t^p).

Specifically, the price of fed cattle is expressed as

$$p_{s,t}=p_{s,t}\left(sl_t,\varepsilon_t^D,c_t^p\right) \tag{83}$$

$$p_{s,t}=p_{s,t}\left(sl_t,\varepsilon_t^D,c_t^p\right)\approx\tilde{p}_{s,t}\left(sl_t,\varepsilon_t^D,c_t^p\right) \tag{84}$$

$$=\sum_{l_1=1}^{m_1}\sum_{l_2=1}^{m_2}\sum_{l_3=1}^{m_3}c_{l_1l_2l_3}\phi_{s1}^{(l_1)}\left(sl_t\right)\phi_2^{(l_2)}\left(\varepsilon_t^D\right)\phi_3^{(l_3)}\left(c_t^p\right) \tag{85}$$

and the price of cull cows is expressed as

$$p_{c,t} = p_{c,t}\left(cl_t, \varepsilon_t^D, c_t^p\right) \tag{86}$$

$$p_{c,t} = p_{c,t}\left(cl_t, \varepsilon_t^D, c_t^p\right) \approx \tilde{p}_{c,t}\left(cl_t, \varepsilon_t^D, c_t^p\right) \tag{87}$$

$$= \sum_{l_1=1}^{m_1} \sum_{l_2=1}^{m_2} \sum_{l_3=1}^{m_3} c_{l_1 l_2 l_3} \phi_{c1}^{(l_1)}\left(cl_t\right) \phi_2^{(l_2)}\left(\varepsilon_t^D\right) \phi_3^{(l_3)}\left(c_t^p\right) \tag{88}$$

where $\left\{\phi_{s1}^{(1)}, \phi_{s1}^{(2)}, \phi_{s1}^{(3)}, \ldots, \phi_{s1}^{(m_1)}\right\}$ and $\left\{\phi_{c1}^{(1)}, \phi_{c1}^{(2)}, \phi_{c1}^{(3)}, \ldots, \phi_{c1}^{(m_1)}\right\}$ are univariate basis functions of sl_t and cl_t, $\left\{\phi_2^{(1)}, \phi_2^{(2)}, \phi_2^{(3)}, \ldots, \phi_2^{(m_2)}\right\}$ are basis functions of the demand shock ε_t^D, and $\left\{\phi_3^{(1)}, \phi_3^{(2)}, \phi_3^{(3)}, \ldots, \phi_3^{(m_3)}\right\}$ are basis functions of corn price c_t^p. And m_1, m_2, m_3 are the number of univariate basis functions for sl_t, cl_t, ε_t^D, and c_t^p respectively. The coefficient vector $c_{l_1 l_2 l_3}$ contains $M = m_1 \times m_2 \times m_3$ elements. These elements must be solved to determine the price.

With the above specification, the equations 63 - 67 are required to hold at a selected number of collocation nodes $x_0, x_1, \ldots, x_m$. The nodes must cover the range of possible values of the variables included in the approximation and don't necessarily need to be equidistant. The polynomial specification is determined by studying the functional properties of the price. The price must be non-negative and must be greater than zero indicating that there will be no corner solution and the range will be positive. Therefore a Chebyshev polynomial interpolation is used (Miranda and Fackler 2002; Judd 1998; Miranda 1997; Miranda and Glauber 1995). The nodes that are used are Chebyshev nodes (Chebyshev nodes are not evenly spaced and are more concentrated on the boundaries of the interval), over a bounded interval $[a, b]$ and takes the following form

$$x_i = \frac{a+b}{2} + \frac{b-a}{2}\cos\left(\frac{m-i+0.5}{m}\pi\right), \forall i = 1, 2, \ldots, m \tag{89}$$

Additionally, the Chebyshev polynomial basis are defined recursively as

$$T_0(z) = 1 \tag{90}$$

$$T_1(z) = z \tag{91}$$

$$T_2(z) = 2z^2 - 1 \tag{92}$$

$$T_{m+1}(z) = 2zT_{m-1}(z) - T_{m-2}(z) \tag{93}$$

where $z_i = \frac{2(x_i - a)}{b-a} - 1, \forall i = 1, 2, \ldots, m$ is the normalized node such that the polynomials are defined on the domain $[-1, 1]$. Alternatively, the Chebyshev polynomials can also be formulated using a trigonometric definition $T_m(z) = \cos(\arccos(\mathrm{z})\mathrm{m})$.

The system of equations that needs to be solved contains the expected price of fed cattle and cull cows. Determining the expected price requires numerical integration. A Gaussian Quadrature for integration is used to compute the expected price (Miranda and Fackler 2002; Judd 1998). In the Gaussian quadrature for the integration method, the continuous distribution is approximated by a finite number of discrete points. The expected price is then computed by assigning weights to the Gaussian nodes and then by taking a weighted average of all the nodes. The expected price for fed cattle and cull cows are specified below:

$$\mathrm{E}_t\left[p_{s,t+3}\right] = \frac{1}{n}\sum_{i=1}^{n}\int\int \tilde{p}_{s,t+3}\left(sl\left(sl_{t+1}, \mathrm{E}_t\left(p_{s,t+3}\right)\right)\varepsilon^s_{t+1,j} + \mathrm{storage}_{t+1}, \varepsilon^D_{t+3,l}, c^p_t\right)\mathrm{d}\varepsilon^s\mathrm{d}\varepsilon^D \tag{94}$$

$$= \frac{1}{n}\sum_{i=1}^{n}\sum_{j,l=1}^{M} w_{j,l}\tilde{p}_{s,t+3}\left(sl\left(sl_{t+1}, \mathrm{E}_t\left(p_{s,t+3}\right)\right)\varepsilon^s_{t+1,j} + \mathrm{storage}_{t+1}, \varepsilon^D_{t+1,l}, c^p_t\right) \tag{95}$$

$$\mathrm{E}_t\left[p_{c,t+1}\right] = \frac{1}{n}\sum_{i=1}^{n}\int\int \tilde{p}_{c,t+1}\left(cl\left(cl_t, \mathrm{E}_t\left(p_{c,t+1}\right)\right)\varepsilon^c_{t,j} + \text{storage}_t, \varepsilon^D_{t+1,l}, c^p_t\right)\mathrm{d}\varepsilon^c \mathrm{d}\varepsilon^D \tag{96}$$

$$= \frac{1}{n}\sum_{i=1}^{n}\sum_{j,l=1}^{M} w_{j,l}\tilde{p}_{c,t+1}\left(cl\left(cl_t, \mathrm{E}_t\left(p_{c,t+1}\right)\right)\varepsilon^s_{t,j} + \text{storage}_t, \varepsilon^D_{t+1,l}, c^p_t\right) \tag{97}$$

The price of fed cattle and cull cows must satisfy the equilibrium conditions. In equilibrium, the system of equations is non-linear. Therefore, we provide the estimated price and expected price from the above price approximation using the Gaussian quadrature method as starting points to solve the system of equations. Additionally, we provide bounds for the price so the price satisfies the equilibrium conditions.

Equations 71 and 78 are used to compute fed cattle and cull cow production and corresponding Chebyshev nodes. In order to attain the equilibrium price, a multi-year iterative method is performed using the Chebyshev nodes of fed cattle production, cull cow production, production shocks, and demand shocks. Using the iterative algorithm, the prices are approximated, expected prices are computed, and the results are used as initial values to solve the system of equations 63 - 67. In particular, we solve equations 63, 64, 65, and 67. Where equations 63 and 64 are the equilibrium conditions (supply equal demand) for the fed cattle and cull cows respectively. Equations 65 and 67 contain both price and expected price conditions.

An initial guess of the price is made and the price coefficients are determined (from the linear price function relationship) as a beginning step before the following multi-year iteration begins. The initial guess of the price (both fed cattle and cull cows) along with the computed expected price are used to solve the system of equations simultaneously. A non-linear least squares estimation method is used to solve the system. The prices that solve the system are then used to update the coefficients and are compared with the previous iteration coefficients. A simple *Euclidean* distance is measured between the updated and previous iteration coefficients. The Euclidean distance is then compared with a predetermined tolerance level. If the Euclidean distance is below the predetermined tolerance level, the iteration stops, and the updated coefficient vector is part of the solution. If the Euclidean distance is above the predetermined tolerance level, the guessed coefficient vector is replaced with the updated coefficient vector, and the iteration continues until the tolerance level is met.

The iterative algorithm

1. Specification of the Chebyshev nodes and polynomials

Using equations 89 and 93, Chebyshev nodes and Chebyshev polynomials are defined. The selection criteria for the number of polynomials depend on the execution time and precision. A higher number of polynomials means more precision, but the execution time is also increased. In this work, for both fed cattle and cull cows, $m_1 = m_2 = m_3 = m$ number of independent Chebyshev polynomials for each variable is used to approximate the price function. The variables for fed cattle and cull cows are $(sl_t, \varepsilon_t^D, c_t^p)$ and $(cl_t, \varepsilon_t^D, c_t^p)$ respectively.[23] Therefore, a total number of $M = m_1 \times m_2 \times m_3 = m^3$ independent Chebyshev polynomials are constructed separately for fed cattle and cull cow price approximation.

The Chebyshev nodes are selected over the domains $[sl_{min}, sl_{max}]$, $[cl_{min}, cl_{max}]$, $[\varepsilon^D_{min}, \varepsilon^D_{max}]$, and $[c^p_{min}, c^p_{max}]$. Historical data are used to define the domains of each variable that go into the price approximation. Following the Chebyshev polynomial structure, m Chebyshev nodes for each variable are determined. For both fed cattle and cull cow price approximation, a grid of $M = m^3$ interpolation nodes is constructed by using a cartesian product of each univariate interpolation node. The number of nodes is chosen to optimize the accuracy and execution time:

$$\left\{ sl_{l_1}, \varepsilon^D_{l_2}, c^p_{l_3} \mid l_1, l_2, l_3 = 1, 2, \ldots, m \right\} \tag{98}$$

$$\left\{ cl_{l_1}, \varepsilon^D_{l_2}, c^p_{l_3} \mid l_1, l_2, l_3 = 1, 2, \ldots, m \right\} \tag{99}$$

2. Start with initial guess for the coefficients

[23] Note that the state variables for both fed cattle and cull cows price approximation are the same except for the production and the corresponding production shock.

Using the past price, the initial guess for the coefficients is determined. The initial guess of the coefficients are then applied to the price approximation functions

$$\tilde{p}_{s,t} = \sum_{l_1=1}^{m_1} \sum_{l_2=1}^{m_2} \sum_{l_3=1}^{m_3} c_{l_1 l_2 l_3,t} \phi_{i1}^{(l_1)} \phi_2^{(l_2)} \phi_3^{(l_3)} \tag{100}$$

$$\tilde{p}_{c,t} = \sum_{l_1=1}^{m_1} \sum_{l_2=1}^{m_2} \sum_{l_3=1}^{m_3} c_{l_1 l_2 l_3,t} \phi_{j1}^{(l_1)} \phi_2^{(l_2)} \phi_3^{(l_3)} \tag{101}$$

Once the price is approximated, using equations 95 and 97 the expected price of fed cattle and cull cow is computed, the Chebyshev node $(sl_t^1, \varepsilon_t^{D1}, c_t^{p1})$, $(cl_t^1, \varepsilon_t^{D1}, c_t^{p1})$ is applied to the system of equations and solved with approximated price and the expected price as initial values. This is repeated for all the m^3 Chebyshev nodes to get a complete set of fed cattle prices, expected fed cattle prices, cull cow prices, and expected cull cow prices:

$$\mathbf{p_{s,t}} = \begin{pmatrix} p_{s,t}^1 \\ p_{s,t}^2 \\ \vdots \\ p_{s,t}^M \end{pmatrix}; \mathbf{p_{c,t}} = \begin{pmatrix} p_{c,t}^1 \\ p_{c,t}^2 \\ \vdots \\ p_{c,t}^M \end{pmatrix}; \mathrm{E}_\mathbf{t}[\mathbf{p_{s,t+3}}] = \begin{pmatrix} \mathrm{E}_t[p_{s,t+3}]^1 \\ \mathrm{E}_t[p_{s,t+3}]^2 \\ \vdots \\ \mathrm{E}_t[p_{s,t+3}]^M \end{pmatrix}; \mathrm{E}_\mathbf{t}[\mathbf{p_{c,t+1}}] = \begin{pmatrix} \mathrm{E}_t[p_{c,t+1}]^1 \\ \mathrm{E}_t[p_{c,t+1}]^2 \\ \vdots \\ \mathrm{E}_t[p_{c,t+1}]^M \end{pmatrix} \tag{102}$$

3. Update the coefficients

An $M \times M$ interpolation matrix Φ (for both fed cattle and cull cows) is determined. Each element in the interpolation matrix is defined by evaluating each Chebyshev polynomial at each interpolation node and the matrix is specified as below:

$$\Phi_{M\times M} = \begin{bmatrix} \phi_1^{(1)}\phi_2^{(1)}\phi_3^{(1)}\,|_1 & \cdots & \phi_1^{(m_1)}\phi_2^{(m_2)}\phi_3^{(m_3)}\,|_1 \\ \phi_1^{(1)}\phi_2^{(1)}\phi_3^{(1)}\,|_2 & \cdots & \phi_1^{(m_1)}\phi_2^{(m_2)}\phi_3^{(m_3)}\,|_2 \\ \vdots & \ddots & \vdots \\ \phi_1^{(1)}\phi_2^{(1)}\phi_3^{(1)}\,|_M & \cdots & \phi_1^{(m_1)}\phi_2^{(m_2)}\phi_3^{(m_3)}\,|_M \end{bmatrix} \tag{103}$$

In a simpler matrix notation, the above $M \times M$ interpolation matrix can be specified by a tensor product of univariate interpolation matrices as $\Phi_{M\times M} = \Phi_1 \otimes \Phi_2 \otimes \Phi_3$.

Using the relationship of the price, the approximation function is:

$$\mathbf{p}_{h,t} = \begin{pmatrix} p^1_{h,t} \\ p^2_{h,t} \\ \vdots \\ p^M_{h,t} \end{pmatrix} = \Phi \times c_1 \quad \text{for} \quad h = \{s,c\} \tag{104}$$

The coefficients are computed and updated by simply solving the linear interpolation equation 104, as $c_1 = (\Phi_{M\times M})^{-1}\mathbf{p}_{h,t} \quad \text{for} \quad h = \{s,c\}$. Note that, we don't have to invert the $M \times M$ interpolation matrix as this will increase the complexity with each additional node. Instead, we can invert the individual univariate interpolation matrices and multiply them together.

4. Equilibrium conditions

After the first iteration, using the updated price, for each node, the differences between the estimated supply and demand are calculated for both fed cattle and cull cows. The computed differences are then used to give prices a specific direction to hold in equilibrium (to avoid local optimum solutions and to increase the precision). At a given node, if the difference between the supply and demand is positive, then the price is reduced and vice versa. The updated prices, along with the expected prices, are used again to solve the model. This is repeated until the price reaches equilibrium and the difference between supply and demand reaches a predetermined tolerance level. The fed cattle production of fed cattle and the production of cull cows at the equilibrium price are also determined for each node.

Finally, the equilibrium prices are used to update the coefficients and the iteration is continued with the equilibrium prices and quantities until the coefficients converge.