1. Show that TSP is NP-complete

**Answer:**

1. **Show that TSP belongs to NP**

We need to check that all the vertices are covered once except the beginning and ending vertex and the cost is minimal. This can be done at polynomial time. Hence, TSP is NP.

1. **Showing that TSP is NP-hard**

We can use that Hamiltonian Cycle can be reduced to TSP, since Hamiltonian cycle is NP-complete. Assume ***G = (V, E)*** be an instance of a Hamiltonian cycle. Now we want to construct a TSP from the given instance of the Hamiltonian cycle.

Let’s assume,

***G’* = *(V, E’)*** where, E***’ = {(i, j) : i, j are elements of v } and i is different from j***

We construct the edges by adding the edges missing in the Hamiltonian cycle Graph with a cost of 1 and cost of 0 for the edges that are not in the Hamiltonian cycle.

Now, suppose that a Hamiltonian cycle *H* exists in *G*. The cost of each edge in *H* is 0 in *G'* as each edge belongs to *E*. Therefore, *H* has a cost of 0 in *G'*. Thus, if graph *G* has a Hamiltonian cycle, then graph *G'* has a path of 0 cost.

Conversely, we assume that *G'* has a tour *H'* of cost at most 0. The cost of edges in *E'* are 0 and 1. Hence, each edge must have a cost of 0 as the cost of *H'* is 0. We therefore conclude that *H'* contains only edges in *E*.

We have thus proven that *G* has a Hamiltonian cycle, if and only if *G'* has a tour of cost at most 0. TSP is NP-complete.

1. True / False. Explain
2. **False** - The fact that A can be polynomial reducible to B doesn’t tell anything, if any NP problem can be reduced using polynomial time transformation algorithm to B.
3. **False** - B is not necessary NP problem
4. **True** - Yes using the transitive property of polynomial reducibility.
5. **False** - We can’t conclude about B as it might not necessarily NP problem.

Actual Vertex Cover: C = {A, B}

Vertex Cover from Approximation

Ca = {A, C, B, D}

The approximation algorithm creates 2\*S

­­­­4. The decision problem formulation of the Vertex Cover problem is this: Given a positive integer k, and a graph G, is there a vertex cover for G having size <= k? Show that this decision problem belongs to NP.

**Answer:**

Given a solution U as a vertex cover, we need to verify that

* U is subset of V takes O(size(U)) = O(n)
* Every e of E has an end point on the vertex cover U – O(m) ~ O(n) for complete graph
* |U| <= |V| - O(1)

Overall , the running time of the above algorithm to verify the solution takes linear time which is in polynomial.