AM5510 : BIOMEDICAL SIGNALS & SYSTEMS

Programming Assignment #2: Simulating Action Potential

Please write your own code in Matlab. This assignment can be turned in online in PDF format along with the Matlab code with a filename indicating your unique identity 11/10/2023

Implement the following algorithm to numerically simulate the Hodgkin-Huxley model of the nerve action potential.

Step 0: Initialize values-
E_{rest} = -68mV; E_k = -74.7 mV; E_{Na} = 54.2 mV; C = 1µF/ cm^2; G_K = 12 m℧/cm^2; G_{Na} = 30 m℧/cm^2
n(t=0)=0.3; m(t=0) = 0.065; h(t=0) = 0.6; V_M(t=0) = E_{rest}

Step 1 Calculate ion channel rate constants for the present point in time t.
Calculate the values of α and β at time t using the value of the membrane voltage at the previous point in time, $t - \Delta t$. In the following equations we define the depolarization voltage as the deviation from the resting membrane potential:

$$v(t) = V_M(t - \Delta t) - E_{\text{rest}}$$

$$\alpha_n(t) = \frac{[0.01(10 - v(t))]}{[e^{((10-v(t))/10)} - 1]}, \quad \beta_n(t) = 0.125\,e^{-v(t)/80}$$

$$\alpha_m(t) = \frac{[0.1(25 - v(t))]}{[e^{((25-v(t))/10)} - 1]}, \quad \beta_m(t) = 4\,e^{-v(t)/18}$$

$$\alpha_h(t) = 0.07\,e^{-v(t)/20}, \quad \beta_h(t) = \frac{1}{(e^{(30-v(t))/10} + 1)}$$

Step 2 Using these values of the rate constants calculate the changes in n, m and h:

$$\Delta n(t) = \Delta t\,[\alpha_n(t)\,[1 - n(t - \Delta t)] - \beta_n(t)\,n(t - \Delta t)]$$

$$\Delta m(t) = \Delta t\,[\alpha_m(t)\,[1 - m(t - \Delta t)] - \beta_m(t)\,m(t - \Delta t)]$$

$$\Delta h(t) = \Delta t\,[\alpha_h(t)\,[1 - h(t - \Delta t)] - \beta_h(t)\,h(t - \Delta t)]$$

Now we can calculate the current values of n, m and h:

$$n(t) = n(t - \Delta t) + \Delta n(t)$$

$$m(t) = m(t - \Delta t) + \Delta m(t)$$

$$h(t) = h(t - \Delta t) + \Delta h(t)$$

Step 3 Calculate the ionic conductances and currents.

Calculate the sodium and potassium conductances at time t:

$$g_K(t) = G_K n^4(t)$$

$$g_{Na}(t) = G_{Na} m^3(t)\,h(t)$$

Calculate the corresponding ionic currents:

$$I_K(t) = g_K(t)\,[V_M(t - \Delta t) - E_K]$$

$$I_{Na}(t) = g_{Na}(t)\,[V_M(t - \Delta t) - E_{Na}]$$

Step 4 Calculate the total current.

Add the stimulus current if it is nonzero:

$$I_s(t) = \begin{cases} I_o & \text{if } 0 < t < t_{PW} \\ 0 & \text{if } t_{PW} < t \end{cases}$$

The total capacitive current:

$$I_C(t) = I_s(t) - [I_K(t) + I_{Na}(t)]$$

Step 5 Calculate the membrane voltage.

Calculate the change in membrane voltage effected at time t:

$$\Delta V_M(t) = [I_C(t)/C]\ \Delta t$$

Finally, calculate the membrane voltage at time t:

$$V_M(t) = V_M(t - \Delta t) + \Delta V_M(t)$$

Loop Increment time t and repeat steps 1–5 as many times as required, i.e., while $t < t_{max}$

ASSIGNMENT

- Calculate and plot the discretized functions $V_m(t)$, $g_K(t)$, and $g_{Na}(t)$ on the same timescale. Simulate the membrane behavior with the following stimuli. Use $\Delta t = 0.01$ ms and simulate for a total time $t_{max} = 10$ ms:

  1. $I_o = 5\,\mu A$, $t_{PW} = 0.2$ ms
  2. $I_o = 25\,\mu A$, $t_{PW} = 0.2$ ms
  3. $I_o = 75\,\mu A$, $t_{PW} = 0.2$ ms
  4. $I_o = 5\,\mu A$, $t_{PW} = 0.8$ ms
  5. $I_o = 25\,\mu A$, $t_{PW} = 0.8$ ms
  6. $I_o = 75\,\mu A$, $t_{PW} = 0.8$ ms

- Try one simulation with $\Delta t = 0.1$ ms. How does this change in discretization interval affect the quality of the simulation?

Note: All the units given above are balanced, and the values can be used directly. If the units are converted to SI units, take care that all the quantities are converted correctly.