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SIMULATION OF CARDIOVASCULAR SIGNALS – 4-ELEMENT WINDKESSEL MODEL

AIM:

The aim of this experiment is to simulate the blood pressure waveform using the Windkessel model.

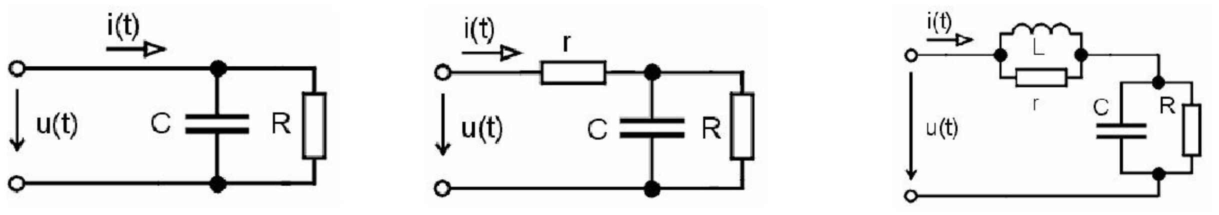
INTRODUCTION:

The largest and most important blood vessel in the human circulatory system is the aortic artery which emerges from the left ventricle of the heart and supplies oxygenated blood to the extremities. The cardiac cycle is best represented as the pulsatile system that operates in a closed loop. The pumping of blood in the systemic circulation phase is in fact best approximated by a pulsed wave pattern.

The cardiac cycle begins with the ventricular diastole which allows for the filling of the atria with fresh oxygenated blood. The ventricles then powerfully contract to pump the oxygenated blood to all parts of the body through the aorta. This phase is called the systole. At this stage i.e. systole, the aortic pressure is at its highest and is called the systolic pressure. The aortic pressure drops to its lowest at the onset of the next phase of atrial filling, and at this point the aortic pressure is called the diastolic pressure. Arterial blood pressure waveform plays the main role to describe the activity of the heart which affects the particular artery in systole and diastole of each cardiac cycle.

The Windkessel Model developed by Otto Frank in the late 1800's describes the circulatory system consisting of the heart and the systemic arterial system as a closed hydraulic system. The Windkessel model takes into account (i) Arterial Compliance i.e. extensibility and elasticity of the aortic artery (ii) Inertia of the blood (iii) Peripheral Resistance i.e. the resistance encountered by the blood in various parts of the arterial system through which it flows.

WINDKESSEL MODELS:



(a) (b) (c)

Figure 1. a) 2-element b) 3-element c) 4-element wind Kessel Model

a. Two-element Wind Kessel model :

Two-element Windkessel model is the simplest mono-compartment model, which is made up of a resistor (R) and a capacitor (C). In this model, the resistor describes the resistance of small peripheral vessels and the capacitor describes the distensibility of large arteries. The two-element Windkessel model simply describes the pressure decay of the aorta in diastole. This model cannot signify the high-frequency effects because there is merely a time constant in the model. Owing to its simplicity, this model can be used in clinical practice readily such as total arterial compliance estimation and blood pressure estimation. The theoretical modeling as seen in the electrical analog is given as:

$$i(t) \ = \ \frac{u(t)}{R} + C\frac{du}{dt} \tag{1}$$

b. Three-element Wind Kessel model:

Adding a characteristic impedance (Z_c) to the two-element Windkessel model, the three-element Windkessel model is formed. The characteristic impedance is equal to oscillatory pressure divided by the oscillatory flow. Although it is found that a resistance numerically equals approximately a characteristic impedance, the characteristic impedance is different from the resistance. The characteristic impedance is merely used to signify oscillatory phenomena. Owing to the inclusion of the characteristic impedance, this model can simulate high-frequency effects. Simultaneously, the introduction of the characteristic impedance also results in some errors at the low frequency. In contrast with the two-element Windkessel model, the three-element Windkessel model can have higher accuracy. The theoretical modeling as seen in the electrical analog is given as:

$$(1+\frac{r}{R})i(t)+CR\frac{di(t)}{dt} \;=\; u(t)+C\frac{du}{dt} \tag{2}$$

c. **Four-element Wind kessel model:**

Taking the inertance of blood flow into consideration on the basis of the three-element Windkessel model. Due to the addition of the inertance, this model can represent middle frequency effects. In other words, the four-element Windkessel model can simulate all frequency effects.

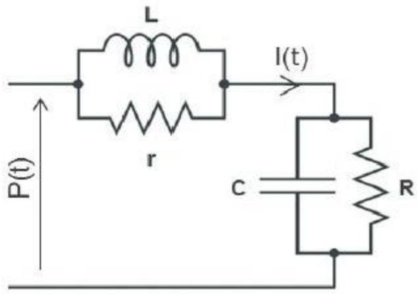


Figure 2. Four-element windkessel model

The model is defined by two differential equations:

$$\frac{du_c(t)}{dt} = -\frac{1}{RC}u_c(t)+\frac{1}{C}i_1(t) \tag{3}$$

$$\frac{di_L(t)}{dt} = -\frac{r}{L}i_L(t)+\frac{r}{L}i_1(t) \tag{4}$$

where,

R = total peripheral resistance = 0.63 ohms

C = Compliance of veins

r = aortic or pulmonary value (resistance to blood flow) = 0.04 ohms

L = inertia of blood flow = 0.03 H

The input current source i(t) is I_0 = 1500 and T_s = 10

$$i(t) = \begin{Bmatrix} I_0 sin^2\left(\pi\frac{t}{T_s}\right) & t\epsilon\{0, T_s\} \\ 0 & t\epsilon\{T_s, T\} \end{Bmatrix} \tag{5}$$

The theoretical modeling as seen in the electrical analog is given as:

$$\frac{d^2u}{dt^2} + (\frac{1}{CR} + \frac{r}{L})\frac{du}{dt} + \frac{r}{LCR}u(t) = r\frac{d^2i}{dt^2} + (\frac{1}{C} + \frac{r}{CR})\frac{di}{dt} + \frac{r}{LC}i(t) \tag{6}$$

The relation between input flow i(t) and the output pressure u(t) in Laplace domain is given by

$$Z(s) = \frac{1.512s^2 + 20.1s + 251}{37.8s^2 + 534s + 400} \tag{7}$$

On taking inverse Laplace transform we get,

$$37.8\frac{d^2u}{dt^2} + 534\frac{du}{dt} + 400u(t) = 1.512\frac{d^2u}{dt^2} + 20.1\frac{du}{dt} + 252i(t) \tag{8}$$

Simulink Model:

Windkessel model is generated in the Simulink using the above transfer function. The input blood flow is given from the MATLAB workspace variable simin. Simin is generated using MATLAB which is i(t). The input is connected to the transfer function. The input and output are scaled and connected to the scope via a MUX. The input and output plots can be obtained from the scope.

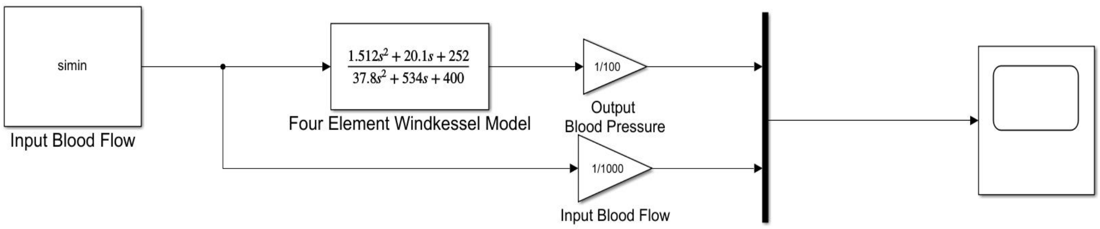


Figure 3. Simulink model for 4-element WindKessel model

RESULTS:

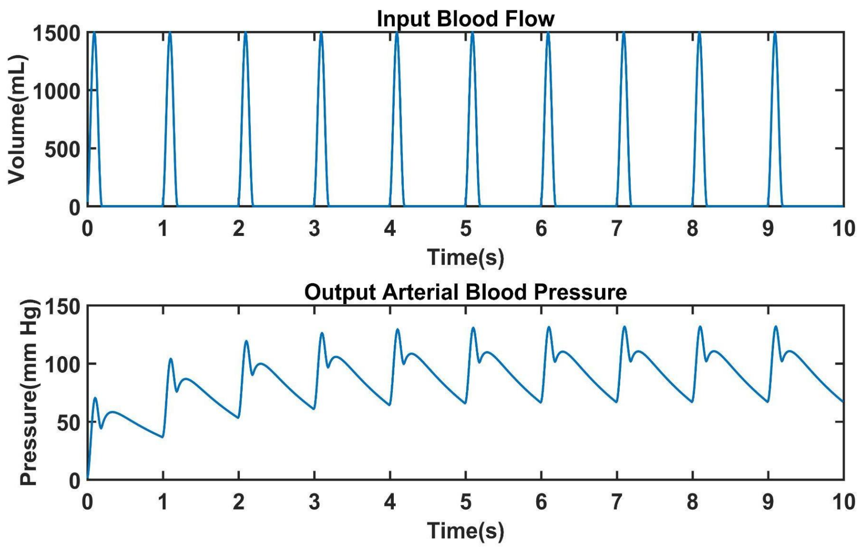


Figure 4. Plot of input blood flow and output arterial blood pressure

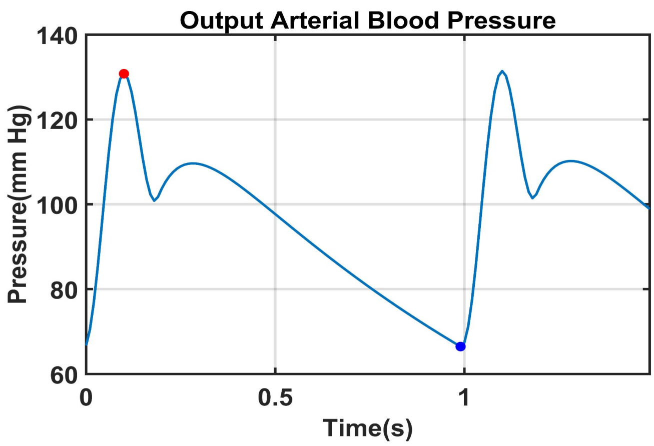


Figure 5. Output arterial blood pressure for a single cycle of input blood flow

MATLAB CODE:

```
Fs = 100; Ts=20; N=10; l = N*Fs;
t=0:1/Fs:(l-1)/Fs;
I=zeros(l,1);
for tx = 1:N
   for ix= 1: Fs
      if(ix<=Ts)
         I((Fs*(tx-1))+ix)=sin(pi*ix/Ts)
      end
   end
end
I=1500*(I.^2)
D=tf([1.512 20.1 252],[37.8 534 400])
t1=1:1:336
response1=lsim(D,I,t)
figure,
subplot(211)
plot(t,I)
title('Input Blood Flow')
xlabel('Time(s)')
ylabel('Volume(mL)')
subplot(212)
plot(t,response1)
title('Output Arterial Blood Pressure')
xlabel('Time(s)')
ylabel('Pressure (mmHg)')
figure
plot(t(5*Fs:6*Fs),response1(5*Fs:6*Fs))
title('Output Arterial Blood Pressure')
xlabel('Time(s)')
```

```
ylabel('Pressure (mmHg)')
grid on
```

CONCLUSION:

Thus, the arterial blood pressure waveform is generated using the Windkessel model from the input blood flow.