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SIMULATION OF EPIDEMIC OUTBREAK USING SIR MODEL FOR SPREAD OF DISEASE

Aim

To simulate the dynamics of the number of susceptible, infected, and recovered population using SIR model with constant immunity loss

Objective

1. To simulate the dynamical system of an epidemic outbreak using the simple mathematical description
2. To assess the impact of the contraction rate, recovery rate, and immunity loss rate on the outbreak of epidemic

Apparatus

MATLAB

Theory

A simple mathematical description of the spread of a disease in a population is the SIR model, which divides the (fixed) population of N individuals into three "compartments" which may vary as a function of time, t:

- S(t) are those susceptible but not yet infected with the disease
- I(t) is the number of infectious individuals
- R(t) are those individuals who have recovered from the disease and now have immunity to it

The SIR model describes the change in the population of each of these compartments in terms of two parameters, β and γ. β describes the effective contact rate of the disease: an infected individual comes into contact with βN other individuals per unit time (of which the fraction that are susceptible to contracting the disease is S/N). γ is the mean recovery rate i.e., $1/\gamma$ is the mean period of time during which an infected individual can pass it on, while Δ is the rate of immunity loss.

The dynamical system of epidemic outbreak in the light of SIR model can be represented in the following block diagram:

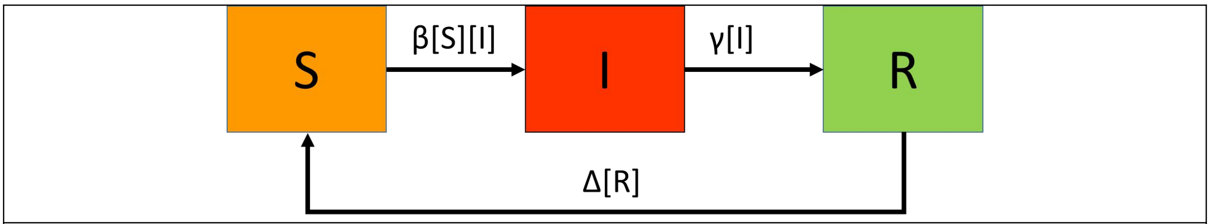

Figure 1: The block diagram of simple SIR model where the blocks represents the states of Susceptible [S], Infected [I], and Recovered [R] and the arrows indicate the flow direction with per capita flow rate

The differential equations describing this model are as follows:

$$\frac{\mathrm{d}S}{\mathrm{d}t} = -\frac{\beta SI}{N} + \Delta R \tag{1}$$

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \frac{\beta SI}{N} - \gamma I \tag{2}$$

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \gamma I - \Delta R \tag{3}$$

Here the characteristic recovery rate (R_0) is $N.\beta/\gamma$ which determines the dynamics of the epidemic outbreak.

Methodology

This problem can be solved as an initial value problem with a finite difference scheme to solve the given ODE system.

A sample code is given below written for MATLAB. The population is considered to be 9×10^7 people with initial number of infected people as 100. The input parameters are chosen as $\beta = 5 \times 10^{-9}$, $\gamma = 0.07$, and $\Delta = 0.01$. The steps are chosen as 1/4th of the days.

MATLAB code

% Model parameters

```
beta = 5*10^-9; % rate of infection
gamma = 0.07; % rate of recovery
delta = 0.01; % rate of immunity loss
N = 9*10^7; % Total population N = S + I + R
I0 = 100; % initial number of infected
T = 300; % period of 300 days
dt = 1/4; % time interval of 6 hours (1/4 of a day)
fprintf('Value of parameter R0 is %.2f',N*beta/gamma)
```

% Calculate the model

```
[S,I,R] = sir_model(beta,gamma,delta,N,I0,T,dt);
% Plots that display the epidemic outbreak
tt = 0:dt:T-dt;
% Curve
figure()
plot(tt,S,'b',tt,I,'r',tt,R,'g','LineWidth',2); grid on;
xlabel('Days'); ylabel('Number of individuals');
legend('S','I','R');
```

% Map

```
figure()
plot(I(1:(T/dt)-1),I(2:T/dt),"LineWidth",1,"Color",'r');
hold on; grid on;
plot(I(2),I(1),'ob','MarkerSize',4);
xlabel('Infected at time t'); ylabel('Infected at time t+1');
hold off;



function [S,I,R] = sir_model(beta,gamma,delta,N,I0,T,dt)
   % if delta = 0 we assume a model without immunity loss
   S = zeros(1,T/dt);
   S(1) = N;
   I = zeros(1,T/dt);
   I(1) = I0;
   R = zeros(1,T/dt);
```

```
    for tt = 1:(T/dt)-1
        % Equations of the model
        dS = (-beta*I(tt)*S(tt) + delta*R(tt)) * dt;
        dI = (beta*I(tt)*S(tt) - gamma*I(tt)) * dt;
        dR = (gamma*I(tt) - delta*R(tt)) * dt;
        S(tt+1) = S(tt) + dS;
        I(tt+1) = I(tt) + dI;
        R(tt+1) = R(tt) + dR;
    end
end
```

Results

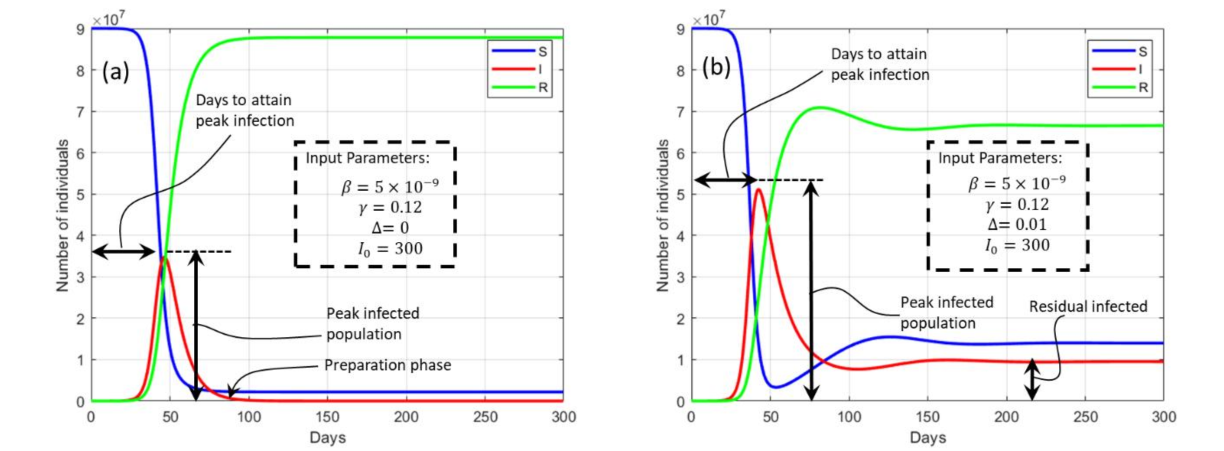

Figure 1: Representative dynamics of susceptible (S), infected (I) and recovered (R) population for the given input parameters (a) without loss of immunity and (b) with loss of immunity

For given N and β, simulate for varied γ such that:

$R_0 = \frac{N.\beta}{\gamma}$	Peak Infected Population	Days to Attain Peak Infection (days)	Initiation of Preparation Phase (days)
$\gamma = 0.07$, R0 = 6.43	51036900	43	115
$\gamma = 0.05$, R0 = 9.00	58472100	42	140
$\gamma = 0.08$, R0 = 5.62	46945400	43.25	112
$\gamma = 0.09$, R0 = 5.00	43590700	44	100
$\gamma = 0.06$, R0 = 7.50	54418900	42.5	127

Vary the immunity loss rate Δ to identify the changes in the SIR dynamics:

Δ	Peak Infected Population	Days to Attain Peak Infection (days)	Residual Infected Population
0.0001	50552500	42.5	1123.45
0.001	50602400	42.5	42529.9
0.01	51089100	42.75	9428740
0.1	55457600	44	44705900

Outcome

- The ratio of the infection and recovery rate influences the dynamics of the epidemic spread
- The immunity loss rate determines the change in the disease spread profile and also the residual number of population infected with the disease

Conclusion

The SIR model can be used to predict the epidemic outbreak trajectory when the appropriate parameters are identified as the inputs.

Give the Justification for the differential equations of SIR Model

• S(t) are those susceptible but not yet infected with the disease

• I(t) is the number of infectious individuals

• R(t) are those individuals who have recovered from the disease and now have immunity to it

Total number of individuals, N = S(t) + I(t) + R(t)

- Interpretation of Beta (β): β describes the effective contact rate of the disease: an infected individual comes into contact with βN other individuals per unit time. The parameter β in the equation represents the transmission rate of the disease. A higher value of β indicates a higher transmission rate, leading to a faster spread of the disease within the population. -

Interpretation of Delta (Δ): Δ is the rate of immunity loss. The term ΔR represents a change in the recovered population. This term accounts for factors that influence the recovery rate or the movement of individuals from the infectious compartment to the recovered compartment. It can include factors like treatments, immunity development, or other recovery mechanisms.

Interpretation of Gamma (γ): The parameter γ represents the recovery rate of infectious individuals. It signifies the rate at which infectious individuals recover from the disease or are removed from the infectious compartment. A higher value of γ indicates a faster recovery rate, reducing the number of infectious individuals over time.

In the SIR model first equation

$$\frac{\mathrm{d}S}{\mathrm{d}t} = -\frac{\beta SI}{N} + \Delta R$$

-Multiplication of S and I: The multiplication of S and I in the term -βSI/N represents the interaction between susceptible (S) and infectious (I) individuals. This interaction signifies the rate at which susceptible individuals become infected by coming into contact with infectious individuals, influencing the spread of the disease within the population.

- Negative Sign for the First Term: The negative sign in front of -βSI/N indicates a decrease in the susceptible population. This negative sign signifies that the number of susceptible individuals decreases as they get infected, reflecting the transition of individuals from the susceptible compartment to the infectious compartment.

- Positive Sign for the Second Term (ΔR): The positive sign in front of ΔR indicates an increase in the recovered population. This positive sign signifies the recovery of individuals from the infectious compartment, leading to an increase in the recovered population over time.

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \frac{\beta SI}{N} - \gamma I$$

- Multiplication of S and I: In the equation dI/dt = βSI/N - γI, the multiplication of S and I (βSI/N) represents the rate at which susceptible individuals become infected by coming into contact with infectious individuals. This interaction is fundamental in modeling the transmission of the disease within the population.

-Positive Sign for the First term: The positive sign signifies that as susceptible individuals come into contact with infectious individuals over time due to disease transmission, there is a positive contribution to the rate of new infections, leading to an increase in the number of infectious individuals. This positive term captures the dynamics of disease transmission and the spread of infection within the population.

- Negative Sign for the Second Term (-γI): The negative sign in front of -γI indicates a decrease in the infectious population. It signifies the rate at which infectious individuals recover or are removed from the infectious compartment, leading to a decrease in the number of infectious individuals over time.

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \gamma I - \Delta R$$

- Positive Sign for the First Term (γI): The positive sign in front of γI indicates an increase in the recovered population over time due to individuals recovering from the disease. This term represents the rate at which infectious individuals recover and transition to the recovered compartment, leading to an increase in the number of recovered individuals within the population.

- **Negative Sign for the Second Term ($-\Delta R$):** The negative sign in front of $-\Delta R$ indicates a decrease in the recovered population. This term signifies a reduction in the number of recovered individuals over time, possibly due to factors such as relapse or other processes that decrease the number of individuals in the recovered compartment.