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1. MODELLING OF BASIC ECG SIGNAL

Aim: To Simulate a normal lead II ECG waveforms using a simulator based on MATLAB.

Objective: Ideal ECG modelling and simulation using Fourier series approximation based on the quasi-periodic nature of ECG signal.

Apparatus Required: MATLAB Software.

Theory: An electrocardiogram records the electrical signals of the heart. It's a common and painless test used to quickly detect heart problems and monitor heart health. A single period of an ECG signal contains P, Q, R, S, T and U waves**.**

1. The P wave is caused by electrical potentials generated when the atria depolarize before an atrial contraction begins.

2. The QRS complex is caused by potentials generated when the ventricles depolarize before contraction, that is, as the depolarization wave spreads through the ventricles. Therefore, both the P wave and the components of the QRS complex are depolarization waves. The PR interval represents the time taken for an electrical activity to move between the atria and the ventricles.

3. The T wave is caused by potentials generated as the ventricles recover from the state of depolarization. The T wave is known as a repolarization wave.

4. The U wave follows the T wave. The exact cause is uncertain, though it has been suggested that it represents inter-ventricular septal repolarization.

ECG signal is periodic with fundamental frequency determined by the heart beat. It also satisfies the dirichlet's conditions:

• Absolutely integrable over the time period of the signal.

• Finite number of maxima and minima between finite intervals.

• It has finite number of discontinuities over the time period of the signal.

A single period of an ideal ECG signal can be expressed as a combination of sinusoidal/triangular shapes, and they are periodic and also satisfies Dirichlet's conditions. So Fourier series can be used to represent the ECG signal.

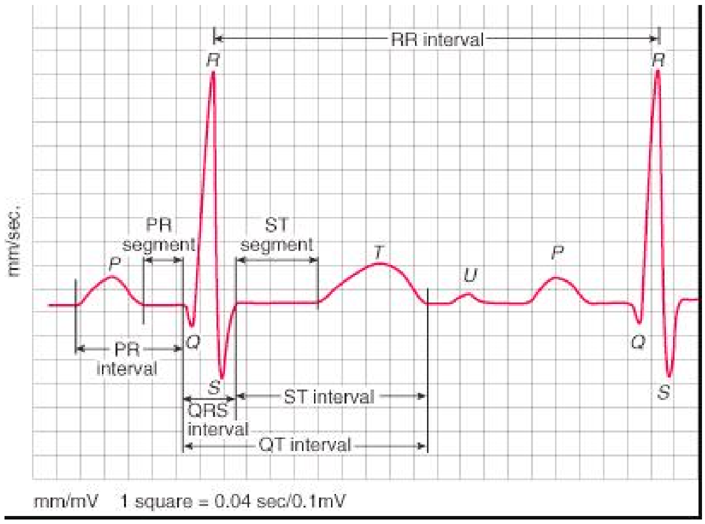


Fig. Ideal ECG

Methodology:

In Fig. 1. Q, QRS, and S waves can be represented by triangular waveform whereas P, T and U can be represented by a sinusoidal waveform.

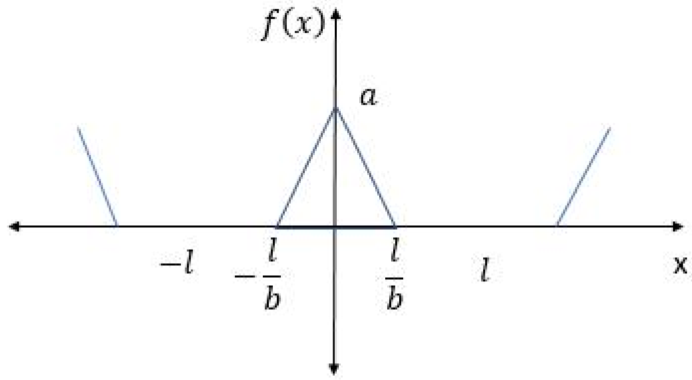


Fig. QRS waveform.

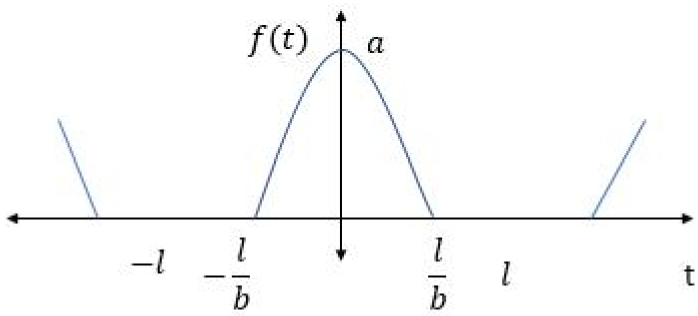


Fig. P,T and U waves

Typical Fourier series is shown in Eq. (1). f (x) represents instantaneous amplitude value of an ECG signal, a_0 is constant representing average amplitude value and ω is a variable representing the angular frequency of ECG signal defined as ω =2 π/T. T stands for the period of ECG signal.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega x + b_n \sin n\omega x)$$

(1)

The constants a_n and b_n are called Fourier coefficient. The calculation of a_0, a_n and b_n are given by

$$a_0 = \frac{2}{T}\int_0^T f(x)dx$$

$$a_n = \frac{2}{T}\int_0^T f(x)\cos(n\omega x)\,dx$$

$$b_n = \frac{2}{T}\int_0^T f(x)\sin(n\omega x)\,dx$$

(2)

Let the period of signal is equal to T =2L and a is assumed the amplitude of signal, f(x) can be calculated by simply following their geometry.

Modelling of Q, QRS and S waves:

$$f(x) = \begin{cases} \frac{-bax}{l} + a, \ if \ 0 < x < \frac{l}{b} \\ \frac{bax}{l} + a, \ if -\frac{l}{b} < x < 0 \end{cases}$$

(3)

$f_{q,qrs,s}(x)$ can be calculated by Eq. (1). It can be seen below no sinusoidal harmonic since $f_{q,qrs,s}(x)$ is symmetric and b_n =0.

$$f_{q,qrs,s}(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\frac{n\pi x}{l}\right)$$

(4)

Let a_0 and a_n in Eq.(4) are solved by the help of Eqs.(2) and (3),we get,

$$a_0 = \frac{a}{b} \quad , \quad a_n = \frac{2ab}{(n\pi)^2}\left(1 - \cos\left(\frac{n\pi}{b}\right)\right)$$

(5)

Then put this values in equ.1.

Modelling of P, T and U waves:

P, T and U waves can be assumed in sinusoidal waveform as shown in Fig. 3, thus, f(x) can be calculated by

$$f(x) = a\cos\left(\frac{\pi bx}{2l}\right), \quad \text{if} \left(-\frac{l}{b} < x < \frac{l}{b}\right) \tag{6}$$

$f_{p.t,u}(x)$ may be written as Eq. (7). It can be seen below no sinusoidal harmonic since $f_{p.t,u}(x)$ is symmetric and b_n =0

$$f_{p.t,u}(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\frac{n\pi x}{l}\right) \tag{7}$$

Let a_0 and a_n in Eq. (7) are solved by the help of Eqs. (2) and (6), we get,

$$a_0 = \frac{4a}{\pi b}, \quad a_n = \frac{4ab\cos\left(\frac{n\pi}{b}\right)}{\pi(b-2n)(b+2n)} \tag{8}$$

Then put these values in equ.1.

Finally, a clear ECG signal consists of the combination of P, Q, R, S, T and U waves. Thus, it can be calculated as

$$f(x) = f_q(x) + f_{qrs}(x) + f_s(x) + f_p(x) + f_t(x) + f_u(x)$$

Matlab codes:

```
%get heart rate
hr = 72;
u = 30/hr;
x = 0.01:0.01:5;
%p wave
p_amp = 0.25;p_dur = 0.09; p_int = 0.16;
p_wave = sinusoid(x,p_amp,p_dur,-p_int,u);
%q wave
q_amp = 0.08; q_dur = 0.066; q_int = 0.166;
q_wave = triangular(x,q_amp,q_dur,-q_int,u);
%qrs wave
qrs_amp = 1.6; qrs_dur = 0.11; qrs_int = 0;
qrs_wave = triangular(x,qrs_amp,qrs_dur,qrs_int,u);
%s wave
s_amp = 0.25; s_dur = 0.066; s_int = 0.09;
s_wave = triangular(x,s_amp,s_dur,s_int,u);
%t_wave
t_amp = 0.35; t_dur = 0.142; t_int = 0.245;
t_wave = sinusoid(x,t_amp,t_dur,t_int,u);
%u wave
u_amp = 0.035; u_dur = 0.0476; u_int = 0.433;
u_wave = sinusoid(x,u_amp,u_dur,u_int,u);
plot(p_wave-q_wave+qrs_wave-s_wave+t_wave+u_wave);
xlabel('Time(s)');ylabel('Amplitude (mV)');
title("Simulated ECG Waveform");
```

```
function y = triangular(x,amp,dur,int,l)
a = amp;
b = (2*l)/dur;
a0 = a/b;
N = 200;
x = x-int;
y = a0/2 + zeros(size(x));
an = zeros(1,N);
for i = 1:N
   y1 = 2*a*b;
   y2 = (i*pi)^2;
   y3 = cos(i*pi/b);
   an(i) = (y1/y2)*(1-y3);
end
for j = 1:N
   y = y + an(j)*cos(j*pi*x/l);
  end
end
```

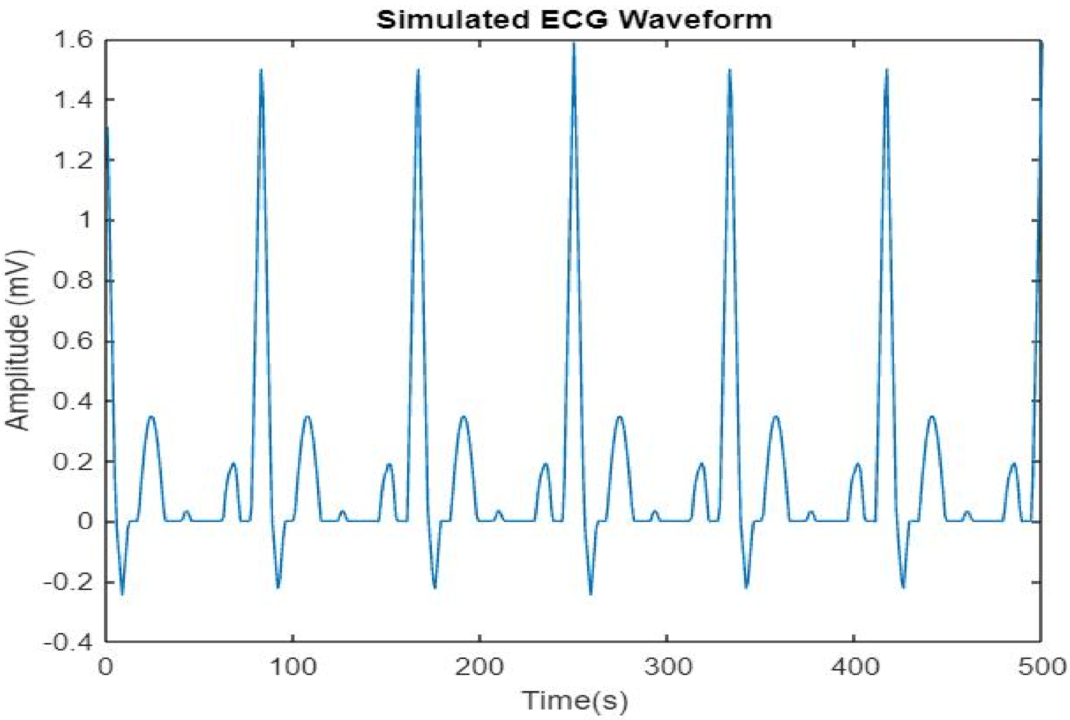
```
function y = sinusoid(x,amp,dur,int,l)
a = amp;
b = (2*l)/dur;
a0 = 4*a/(pi*b);
N = 200;
x = x - int;
an = zeros(1,N);
for k = 1:N
    x1 = 4*a*b*cos(k*pi/b);
    x2 = pi*(b-2*k)*(b+2*k);
    an(k) = x1/x2;
end

y = a0/2 + zeros(size(x));
for w = 1:N
    y = y + an(w)*cos(w*pi*x/l);
end
end
```

Results:

Default specifications:

MORPHOLOGY	AMPLITUDE(mV)	DURATION	INTERVAL(s)
P wave	0.25	0.09	0.16
Q wave	0.025	0.066	0.166
QRS complex	1.6	0.11	-
S wave	0.25	0.066	0.09
T wave	0.35	0.142	0.245
U wave	0.035	0.0476	0.433

A typical output for the above specification will be like this:



Outcomes: This simulation enables anyone to model an ideal ECG pattern by selecting the pre-set ideal values on the system. Any value of heart rate, intervals, and amplitude can be set for each of the peaks using this modelling.

Conclusion: This model is effective to simulate an ideal ECG which can be validated by inspection.