07/03/2024.

Gradient - Multiple Variable

⇒ $\min J(w_1, w_2) = (w_1 - 2)^2 + (w_2 - 2)^2$ - Partial derivatives

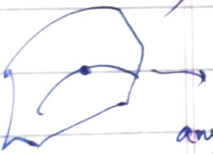
$CW: w_1^2 - w_2^2$

$$\frac{\partial J}{\partial w_1} = 2(w_1 - 2) = 2w_1 - 4 \qquad 2w_1$$

$$\frac{\partial J_2}{\partial w_2} = 2(w_2 - 2) = 2w_2 - 4. \qquad -2w_2$$

$$\underbrace{\nabla J(w_1, w_2)}_{\text{Grad } J \text{ (vector)}} = \left(\frac{\partial J}{\partial w_1}, \frac{\partial J}{\partial w_2}\right)$$

⇒ $f(w_1^{(t)}, w_2^{(t)})$



$$\alpha. = \frac{\partial f}{\partial w_1}\frac{dw_1}{dt} + \frac{\partial f}{\partial w_2}\frac{dw_2}{dt}.$$

any curve,

⇒ $f(x(t), y(t), z(t))$

$$\frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt}$$

$$\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) \cdot \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right) = 0$$

tangent on the surface.

product = 0.

. Taking grad ∇f, will have ⊥r. to the tangent

Hessian vs Jacobian
Symmetric → Skew.

$\frac{\partial^2 J}{\partial w_1^2} = \frac{\partial}{\partial w_1}\left[\frac{\partial J}{\partial w_1}\right] = 2 \quad 2$

$\frac{\partial^2 J}{\partial w_2^2} = \frac{\partial}{\partial w_2}\left[\frac{\partial J}{\partial w_2}\right] = 2 \quad -2$

$\frac{\partial^2 J}{\partial w_1 \partial w_2} = \frac{\partial}{\partial w_1}\left[\frac{\partial J}{\partial w_2}\right] = 0 \quad 0$

$0.$

$\frac{\partial^2 J}{\partial w_2 \partial w_1} = \frac{\partial}{\partial w_2}\left[\frac{\partial J}{\partial w_1}\right] = 0.$

$$H = \begin{bmatrix} \frac{\partial^2 J}{\partial w_1^2} & \frac{\partial^2 J}{\partial w_1 \partial w_2} \\ \frac{\partial^2 J}{\partial w_1 \partial_1} & \frac{\partial^2 J}{\partial w^2} \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Positive definite: "all eigen values → +ve"

H is +ve definite.

diagonal matrix → E. Values → 2, 2

H is +ve def, then minima. → E. Values -ve's

H: -ve → maxima

H: indefinite → Saddle pt → H tends to

(EV → +ve, -ve)

$\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$

Ow: $w_1^2 - w_2^2$

$H = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$

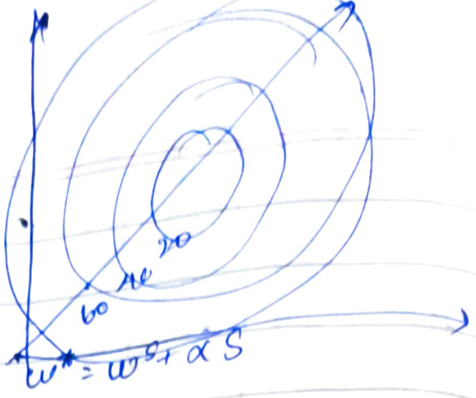
E.V → 2, -2

H → indefinite

(..) → saddle pts.

$\frac{\partial J}{\partial w} = 0$ → we get critical pts

Unidirectional Search



w^0 = st. point.

⇒ What is the α that the line reaches the minimum value y?

↳ boils down to single variable opt.