Optimality Criteria:

$J = w^2 + 54/w$

$J' = 2w - \frac{54}{w^2}$

$J'' = w - \frac{(-2(54))}{w^3}$

$= w + \frac{108}{w^3}.$

Critical points

$J' = 0;$

$2w - \frac{54}{w^2} = 0$

$\frac{54}{w^2} = 2w$

$w^3 = 27$

$\boxed{w = \pm 3}$

$J''\Big|_{w=3} = 2 + \frac{108}{27} = 2 + 4$

$= 6$; $\rightarrow$ minimum

$J''\Big|_{w=-3} = 2 + \frac{(108)}{(-3)^3} = -2$

$\rightarrow$ maxima

At $w = 3$, Minimum Occurs//

Lab 8: Multi Variable Optimization

1) $J(w) = w^2 + 54/w$

Optimality criteria:

a) Interval halving:

1st Iteration → Given $J(w) = w^2 + 54/w$.

Step 1:

$a = 0$; $b = 5$, $\epsilon = 10^{-3}$.

$w_m = (0+5)/2 = 2.5$

Initial interval length is $L_0 = L = 5 - 0 = 5$

$f(w_m) = 27.85 \rightarrow *$

Step 2:

$w_1 = 0 + 5/4 = 1.25$

$w_2 = 5 - 5/4 = 3.75$

$f(w_1) = 44.76 \rightarrow ①$

$f(w_2) = 28.46 \rightarrow ②$

Step 3:

From ① and ⊛

$f(w_1) > f(w_m)$

Step 4: $f(w_2) > f(w_m)$,

thus the intervals are,

$(0, 1.25)$ and $(3.75, 5)$.

Now, set $a = 1.25$, $b = 3.75$

Step 5:

$L = 3.75 - 1.25 = 2.5$

$L_0 = 5$,

$|L|$ is not small, so we continue the same algorithm again.

2nd iteration →

Step 2:

$w_1 = 1.25 + 2.5/4 = 1.875$,

$w_2 = 3.75 - 2.5/4 = 3.125$

$f(w_1) = 32.32$

$f(w_2) = 27.05$

Step 3:

$f(xw_1) > f(xwm) = 27.85$

4:

$f(w_2) < f(xwm)$.

Thus, we eliminate interval $(1.25, 2.5)$

Set $a = 2.5$, $wm = 3.125$

5: $L = 3.75 - 2.5 = 1.25$ which is again half of the previous iteration. But this interval is not smaller than ϵ.

3^{rd} iteration

2: $w_1 = 2.8125$

$w_2 = 3.4375$

$f(w_1) = 27.11$

$f(w_2) = 27.53$.

3: $f(w_1) > f(w_m)$.

4: $f(w_2) > f(w_m)$, we eliminate the

boundary intervals.

$a = 2.8125$, $b = 3.4375$

5: $L = 0.625$.

The iterations will be carried out till

we attain $|L| < \epsilon$.

b) Newton-Raphson :-

Given $J(w) = w^2 + 54/w$.

We know

$$x^{t+1} = x^t - \frac{f'(x^{(t)})}{f''(x^{(t)})}.$$

$$f'(w) = 2w - 54/w^2.$$

$$f''(w) = w + \frac{2(54)}{w^3}$$

Step 1

1st Iteration $\rightarrow w^1 = 1, \quad \epsilon = 10^{-3}, k=1$ (initial iteration).

2:

exact derivative at w^1;

$$f'(w^1) = -52.$$

$$f''(w^1) = 110.001.$$

3: $$w^2 = w^1 - \frac{f'(w^1)}{f''(w^1)}$$

$$= 1 - (-52.005)/110.011$$

$$= 1.473.$$

$$f'(w^2) = -21.944.$$

4: $$|f'(a^2)| \nless \epsilon,$$

we increment $K \longrightarrow 2$,

2nd iteration $\rightarrow$

2: $$f''(w^2) = 35.796$$

3: Next guess,

$$w^3 = w^2 - \frac{f'(w^2)}{f''(w^2)}$$

$$= 1.473 - \frac{-21.944}{35.796}$$

$w^3 = 2.086$

$f'(x^3) = -8.239$

4: $|f'(w^3)| \not< \varepsilon \quad k \to 3,$

3rd iteration

2: $f''(w^3) = 13.897$

3: $w_4 = w^3 - \dfrac{f'(w^3)}{f''(w^4)}$

$w^4 = 2.679$

$f'(w^4) = -2.167$

$|f'(w^4)| \not< \varepsilon, \quad k \to 4$

4th iteration

$f''(w^4) = 8.2960$

At 7th iteration

$w^7 = 3.001,$

$f'(w^7) = -4(10)^{-8}$

$f'(w^7) < \varepsilon,$

terminate //