

# **ED5340 - Data Science: Theory and Practise**

## **L20 - Support vector machine (SVM)**

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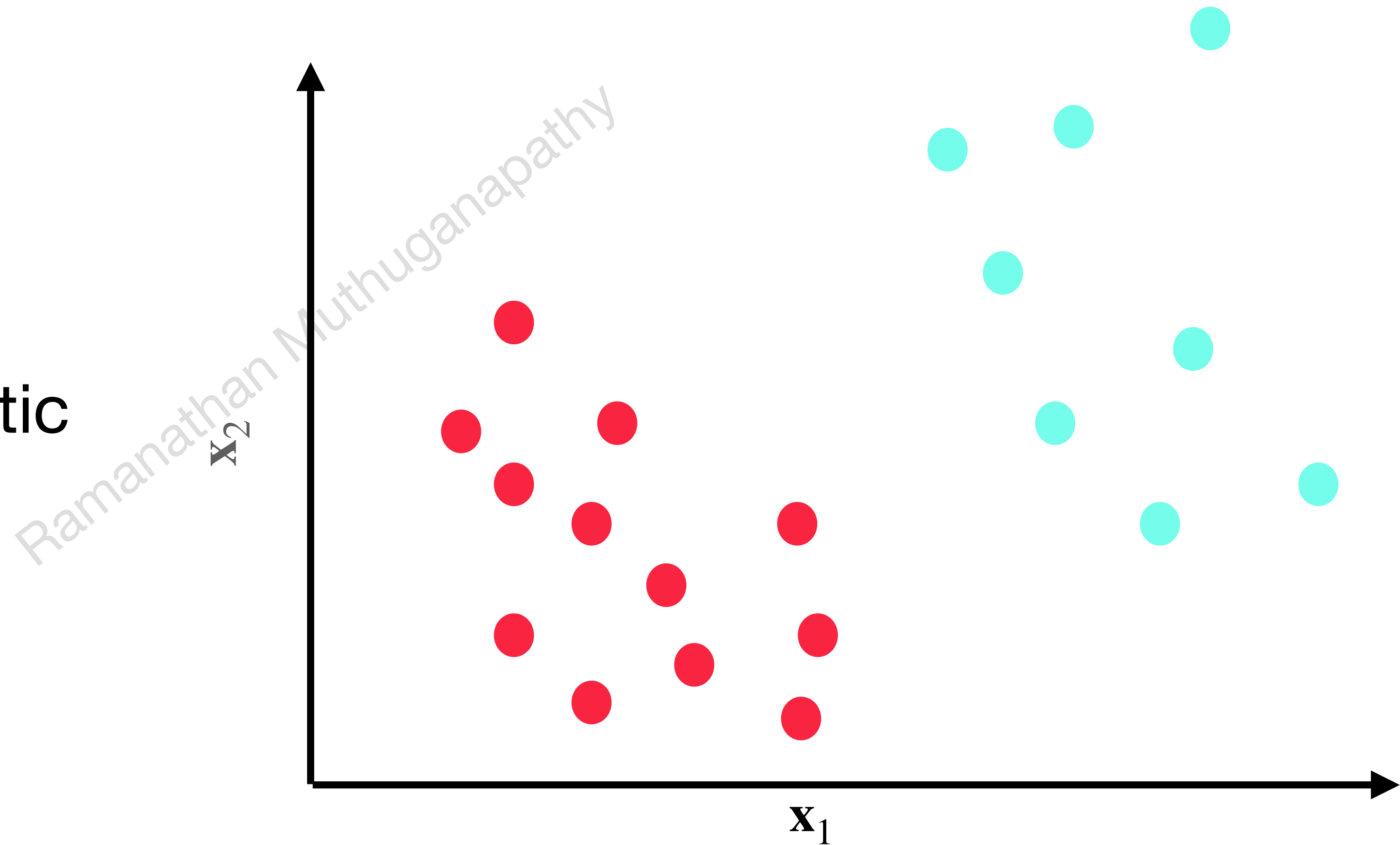
**Course web page: <https://ed.iitm.ac.in/~raman/datascience.html>**

**Moodle page: Available at <https://courses.iitm.ac.in/>**

# Support vector machine

## Decision boundary

- Find a linear separable boundary
- with a margin
- looks very similar to logistic regression.

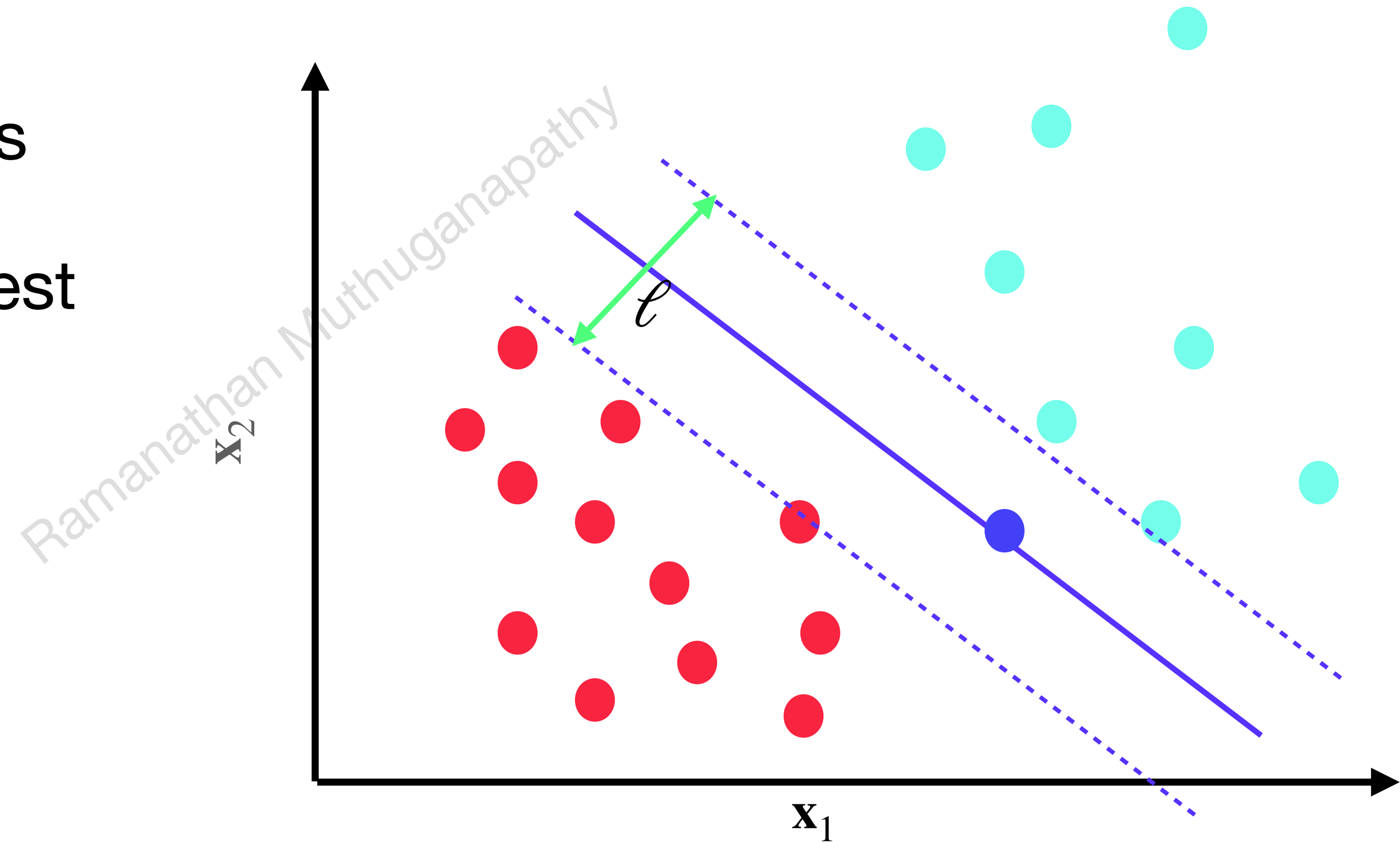


# Support vector machine

## Decision boundary

- Find a hyperplane that separates the two classes
- $\ell$  - the margin is the largest

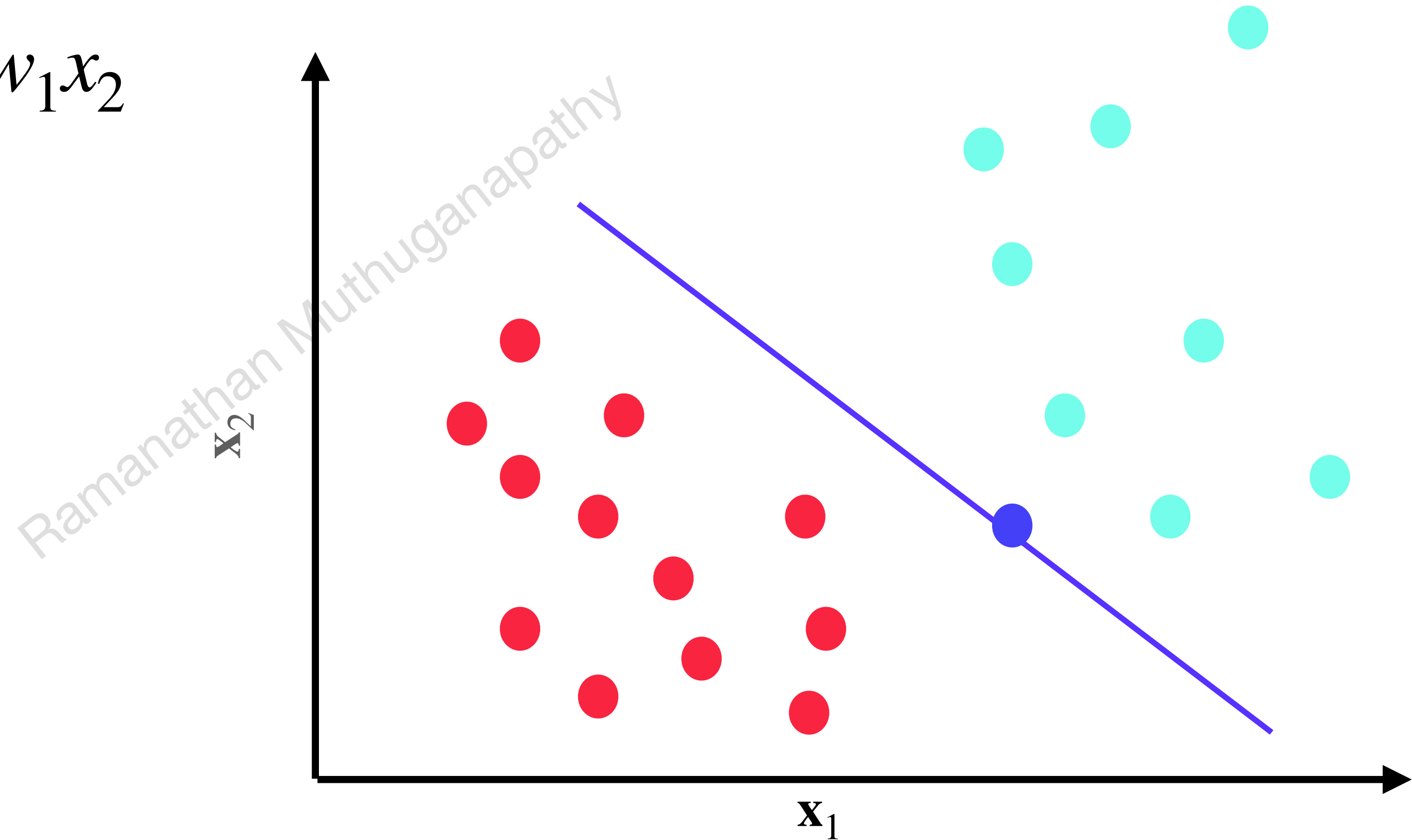
Two problems  
Soft margin problem  
Hard margin problem



# Support vector machine

## Recall vector notation

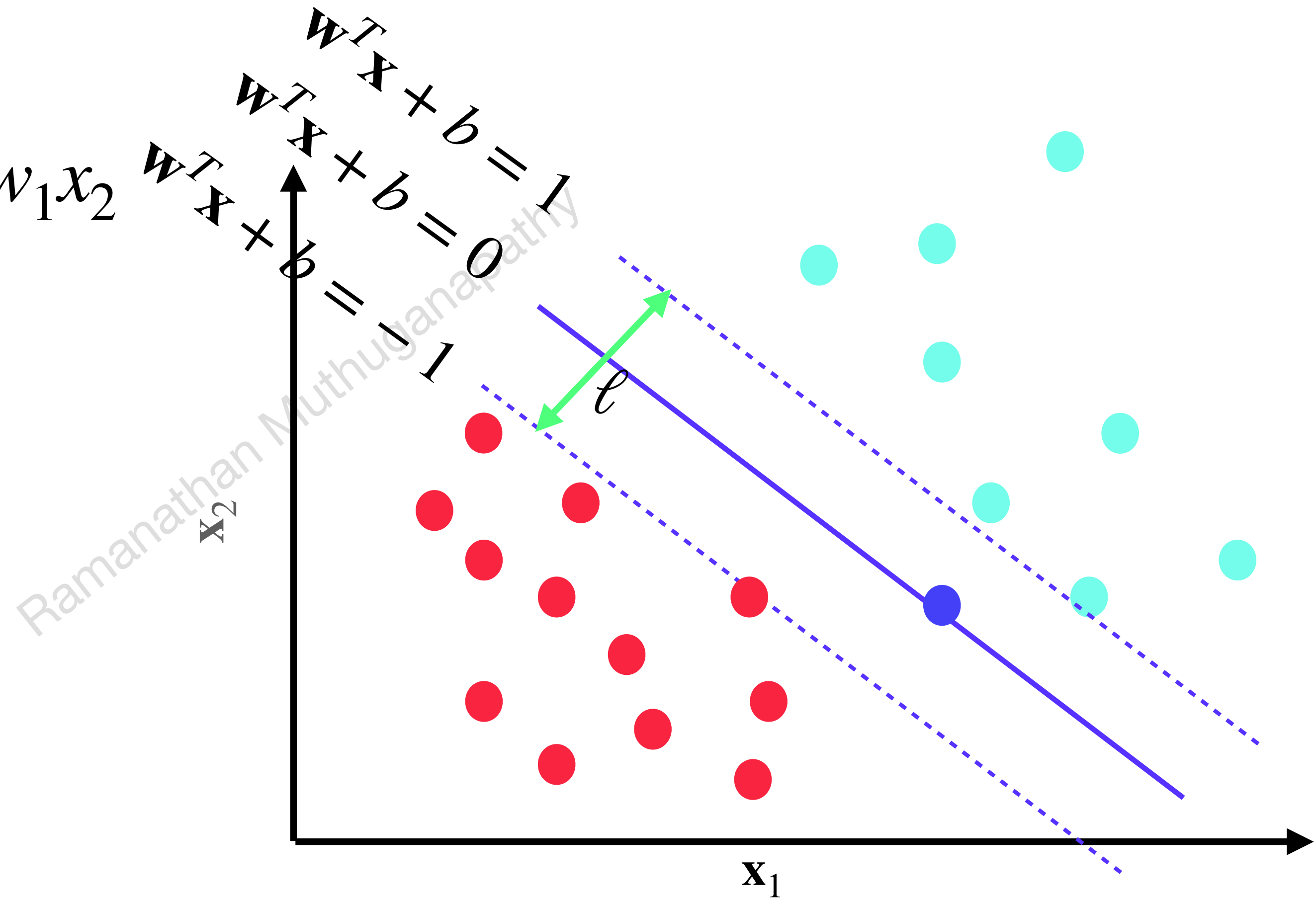
- $h_w(x) = w_0x_0 + w_1x_1 + w_1x_2$
- $\mathbf{w}^T = [w_1 \quad w_2]$
- $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$
- $b = w_0$  (bias)
- $h_w(x) = \mathbf{w}^T \mathbf{x} + b$



# Support vector machine

## Recall vector notation

- $h_w(x) = w_0x_0 + w_1x_1 + w_1x_2$
- $\mathbf{w}^T = [w_1 \quad w_2]$
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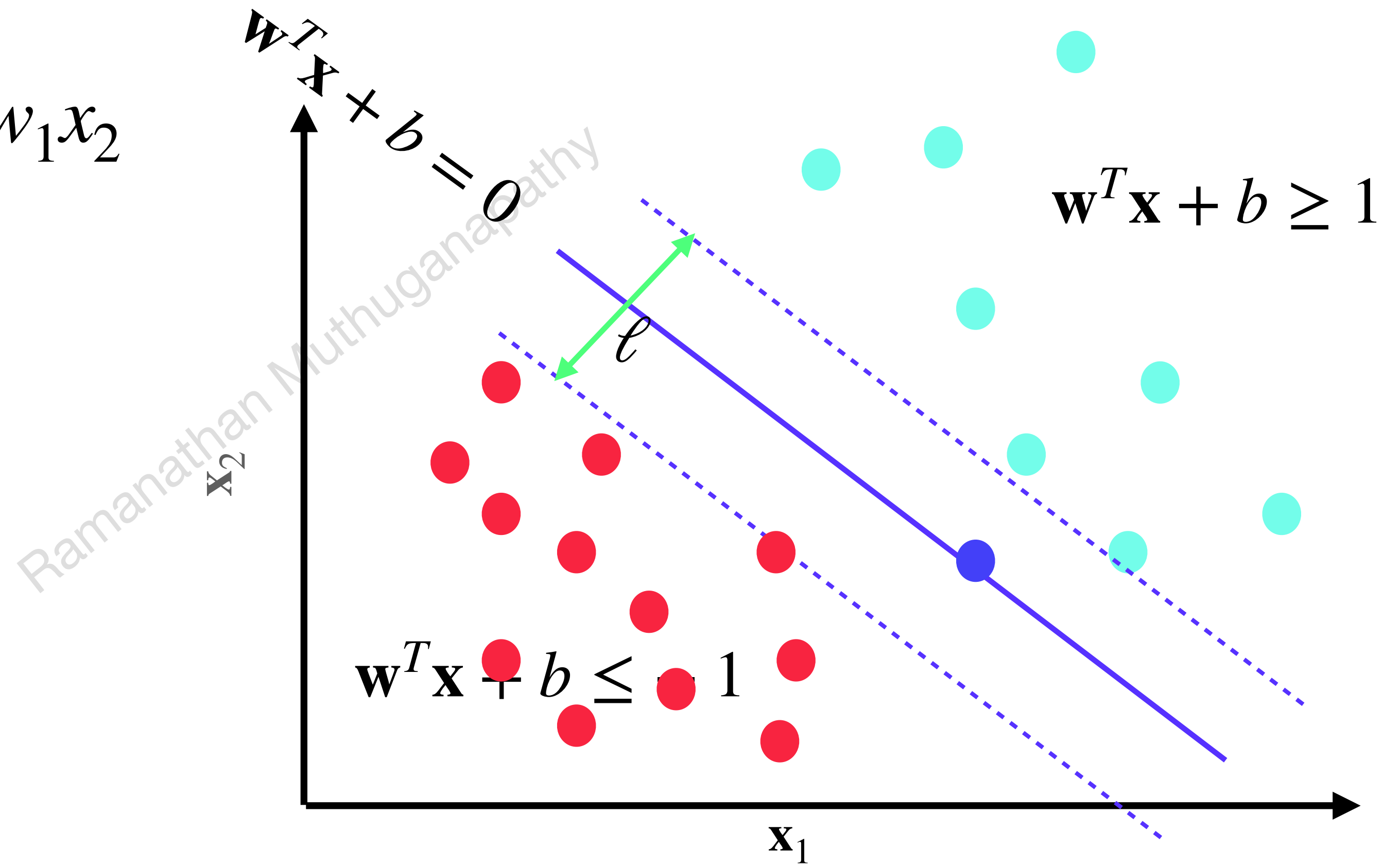


Constrained optimization problem

# Support vector machine

## Recall vector notation

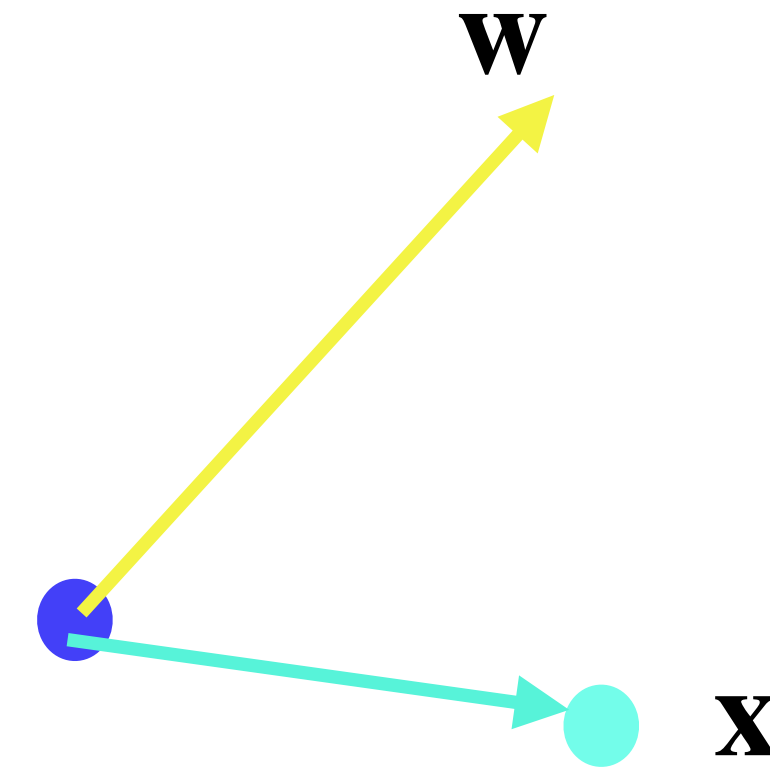
- $h_w(x) = w_0x_0 + w_1x_1 + w_1x_2$
- $\mathbf{w}^T = [w_1 \quad w_2]$
- $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$
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# Support vector machine

## Basic vector calculus

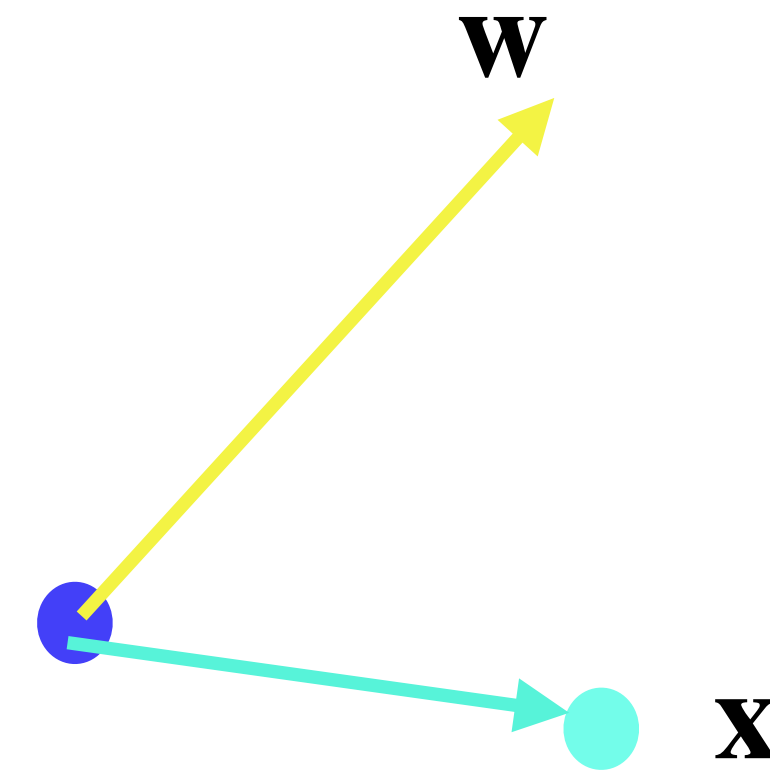
- Two vectors  $\mathbf{w}$  and  $\mathbf{x}$
- What is  $\mathbf{w}^T \mathbf{x}$ ?



# Support vector machine

## Basic vector calculus

- What is  $\mathbf{w}^T \mathbf{x}$ ?
- Dot product between the two vectors

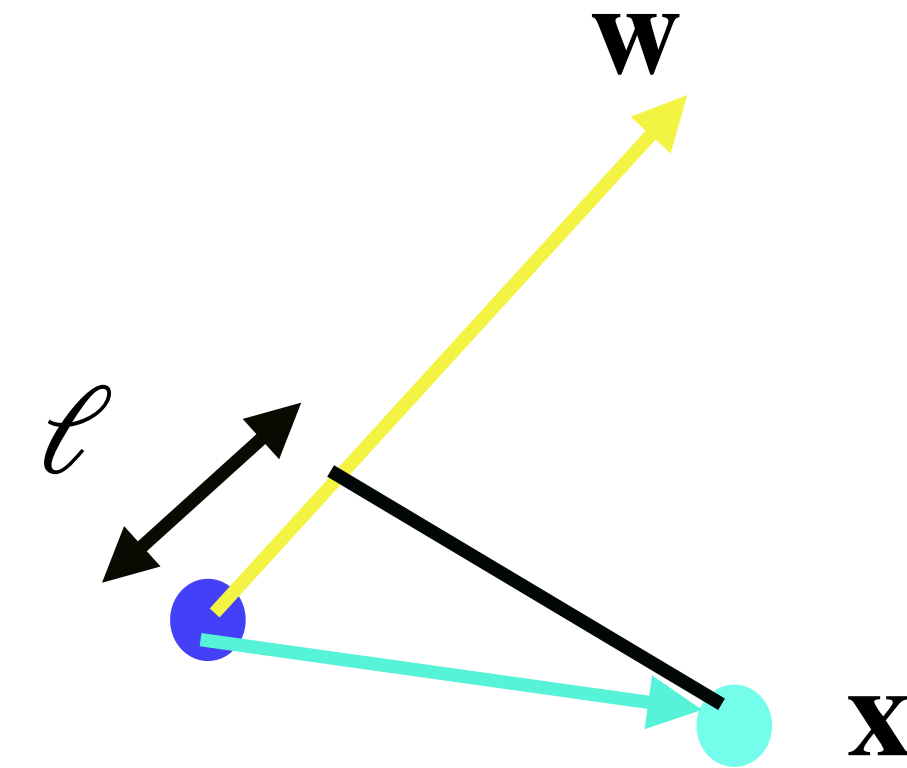




# Support vector machine

## Basic vector calculus

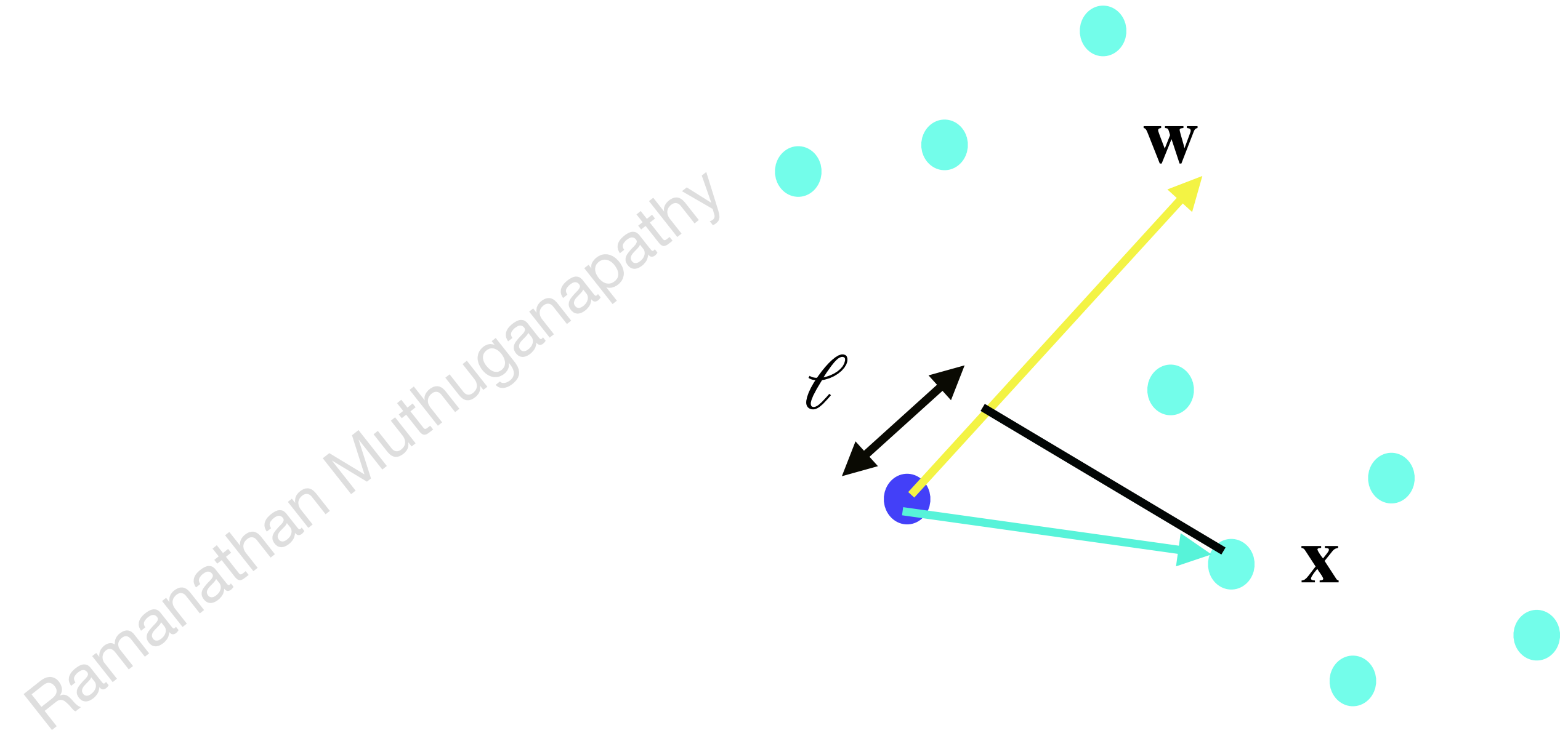
- What is  $\mathbf{w}^T \mathbf{x}$ ?
- Dot product between the two vectors
- $\mathbf{w}^T \mathbf{x} = \ell \|\mathbf{w}\|$
- $\mathbf{w}^T \mathbf{x} \geq 0$



# Support vector machine

## Basic vector calculus

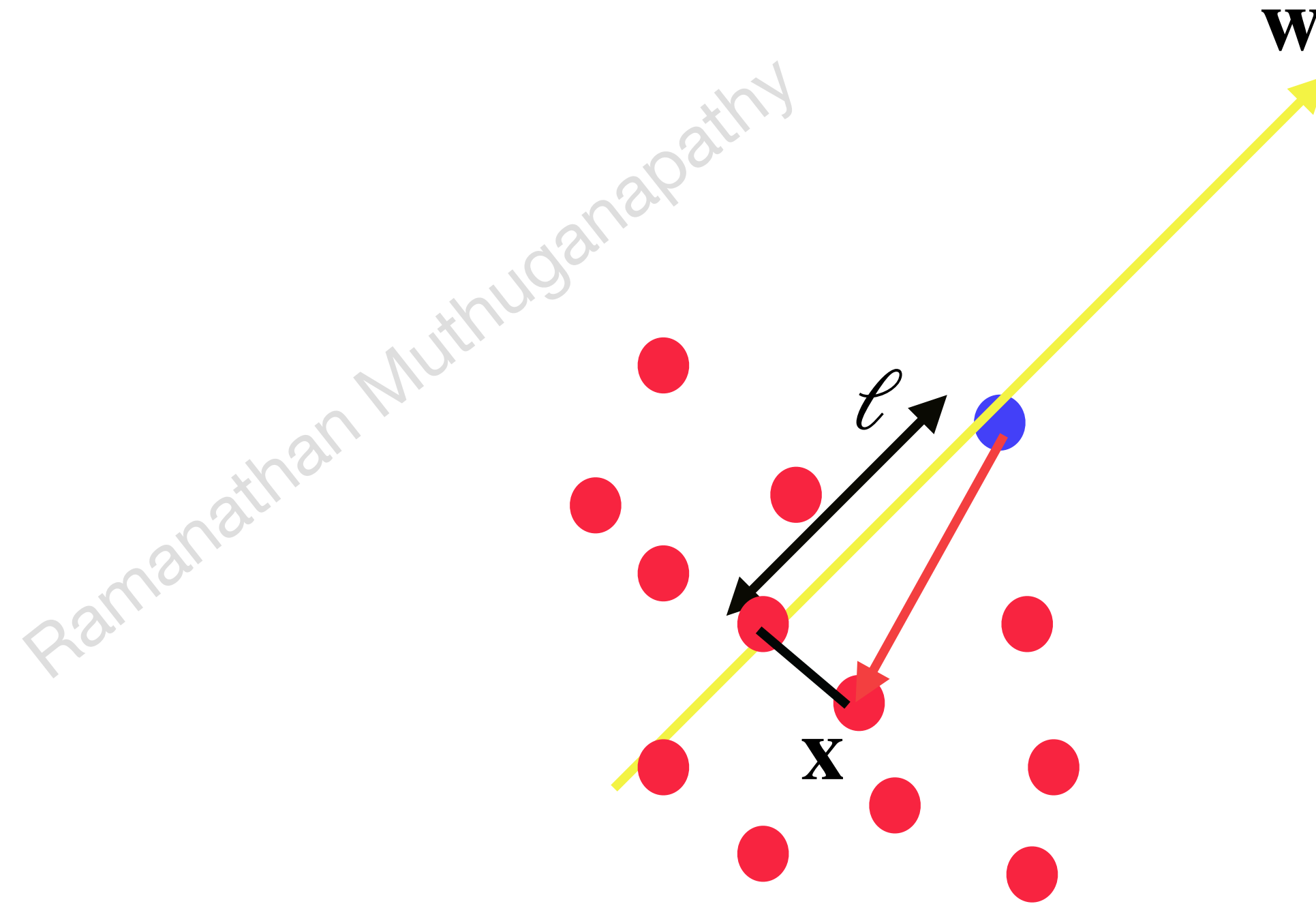
- $\mathbf{w}^T \mathbf{x} \geq 0$



# Support vector machine

## Basic vector calculus

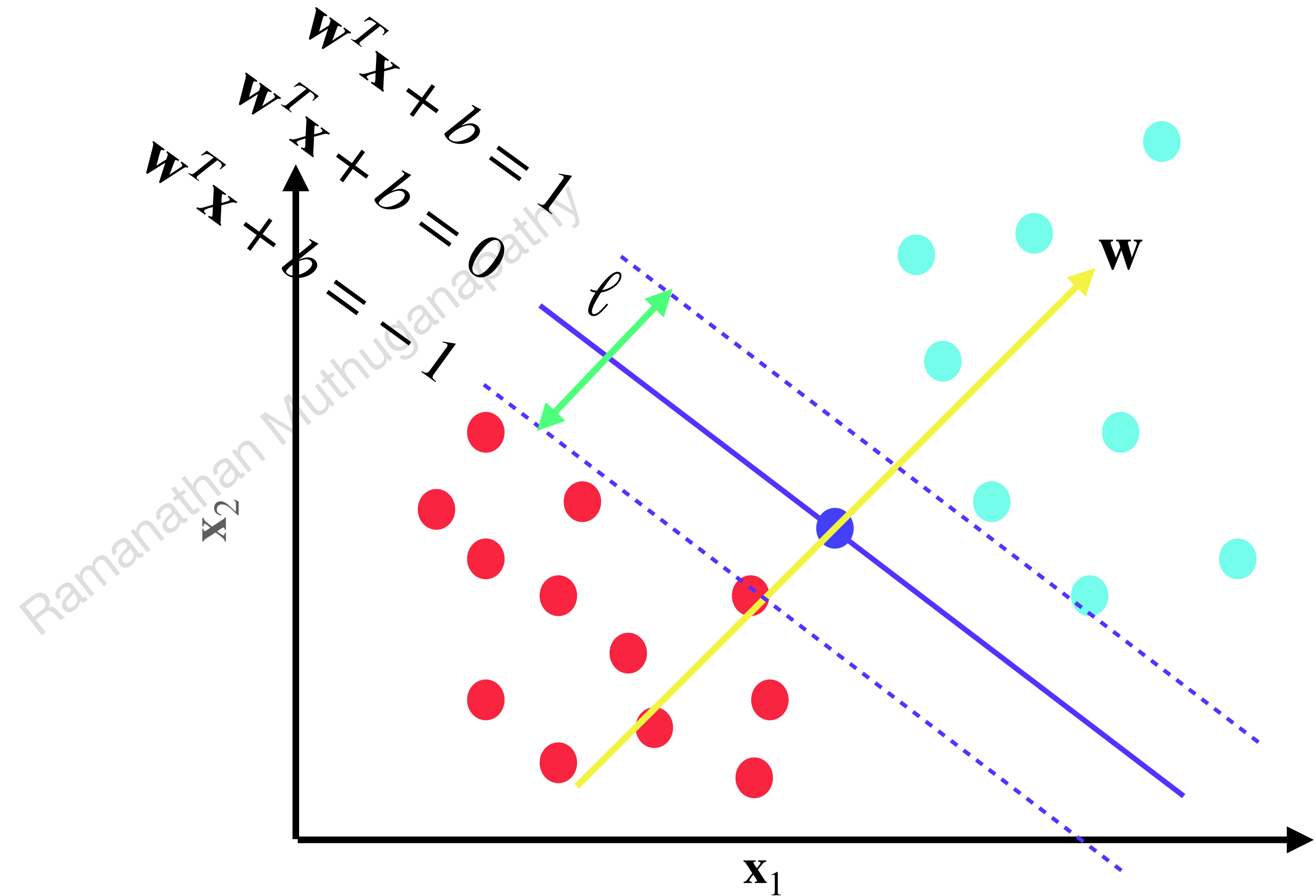
- $\mathbf{w}^T \mathbf{x} \leq 0$



# Support vector machine

## Putting together

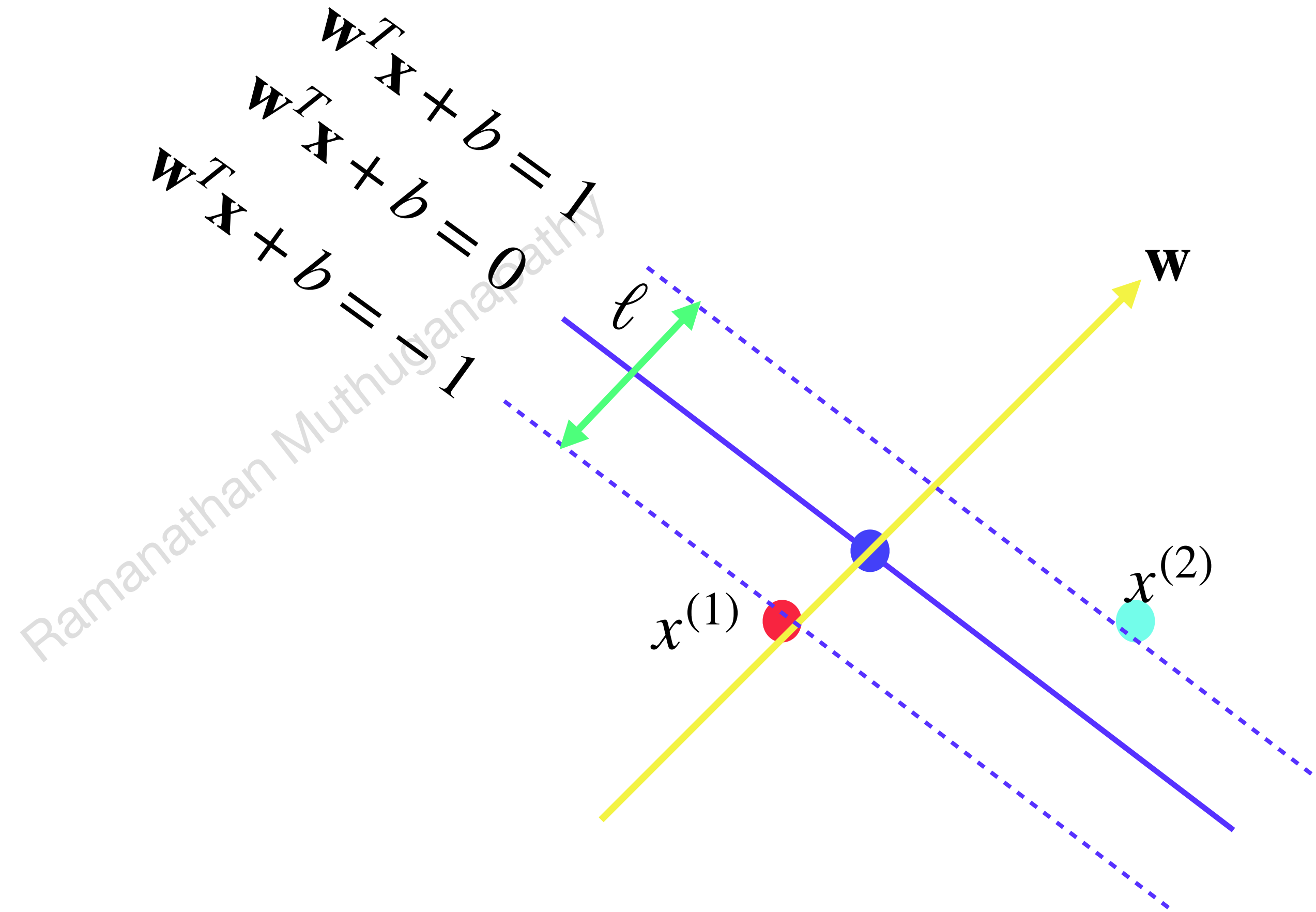
- Hyperplane
- perpendicular to  $\mathbf{w}$
- with constraints



# Support vector machine

## Putting together

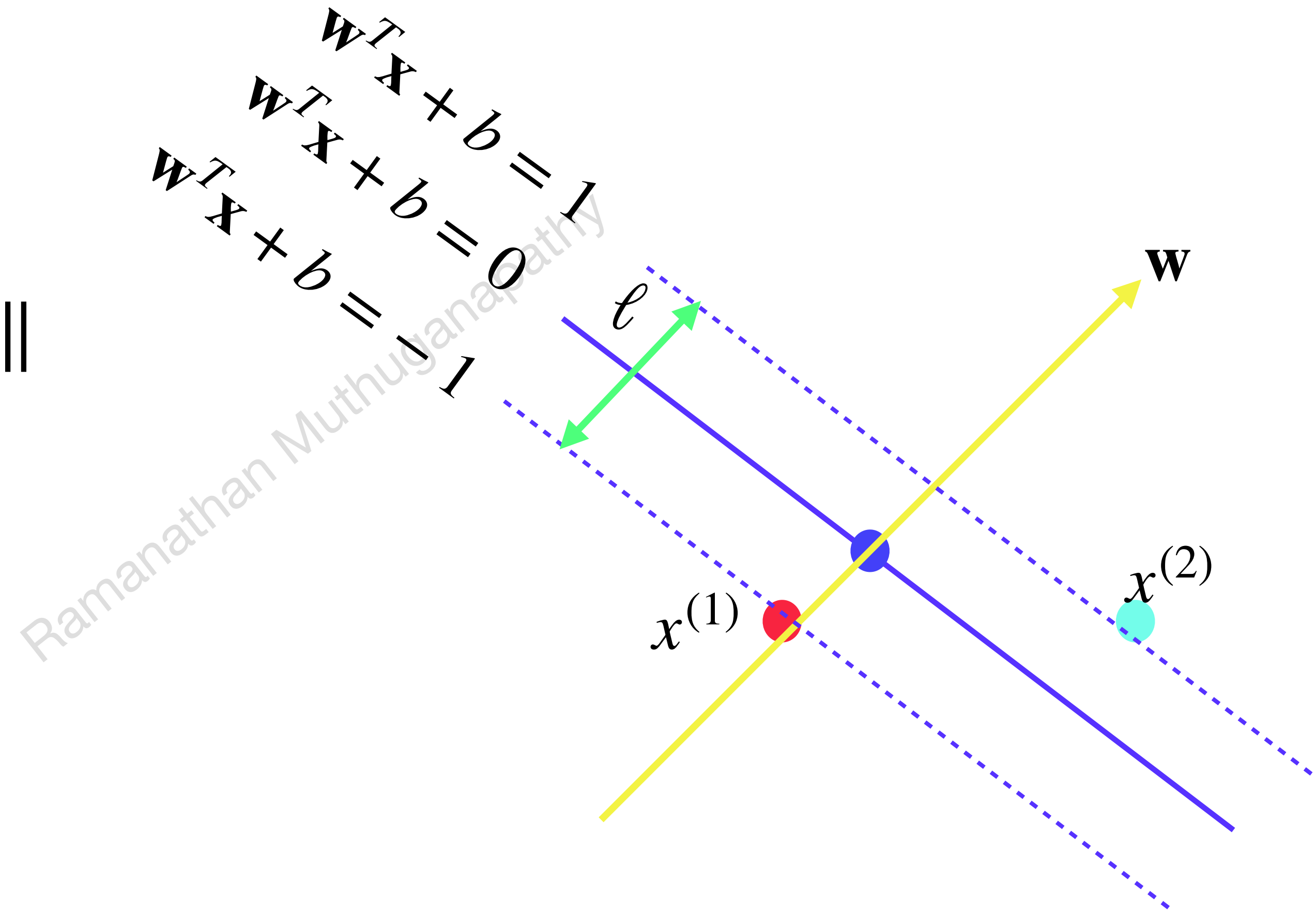
- $\mathbf{w}^T \cdot (x^{(1)} - x^{(2)})$



# Support vector machine

## Putting together

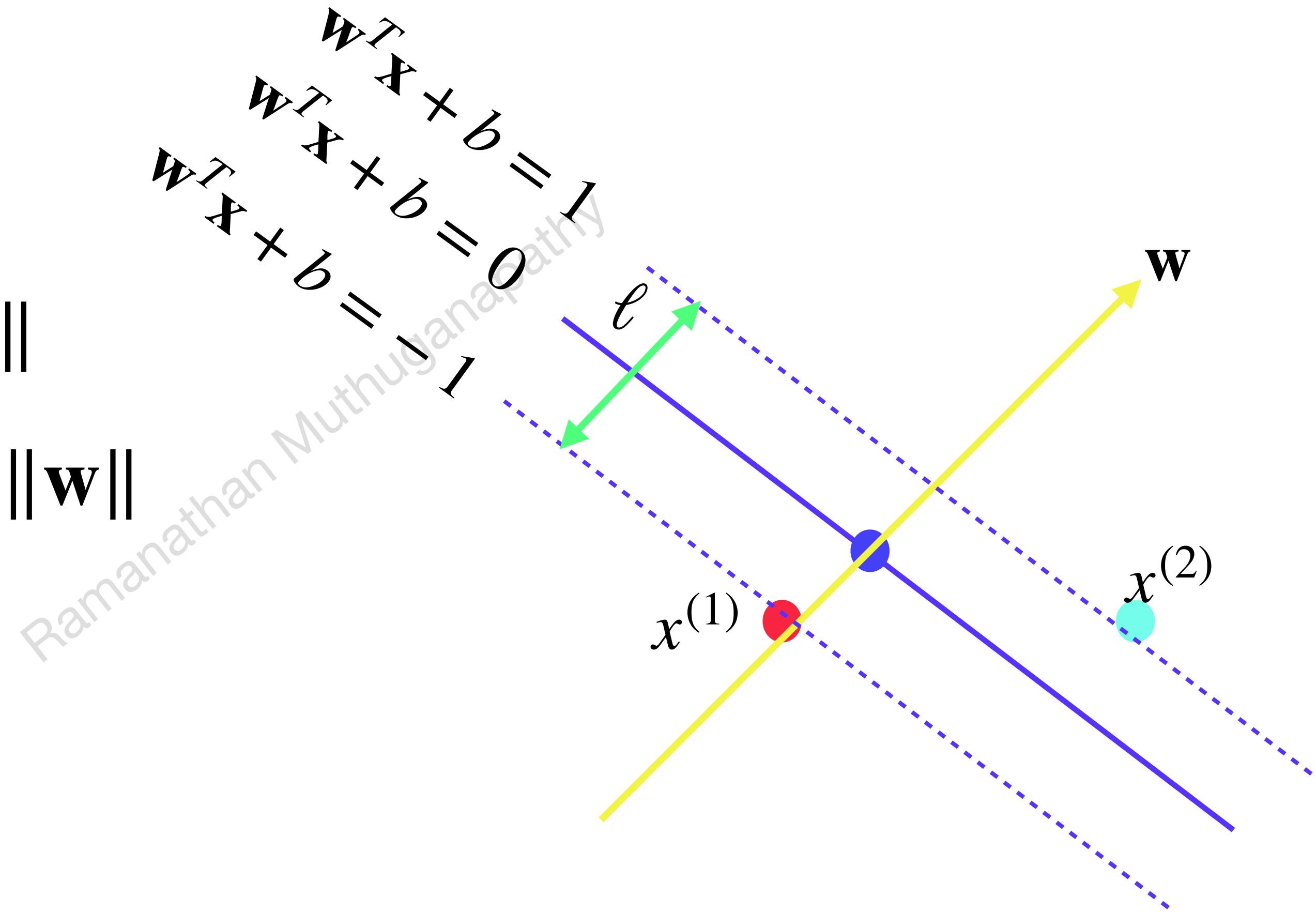
- $\mathbf{w}^T \cdot (x^{(1)} - x^{(2)})$
- $\mathbf{w}^T \cdot (x^{(1)} - x^{(2)}) = \ell \|\mathbf{w}\|$



# Support vector machine

## Putting together

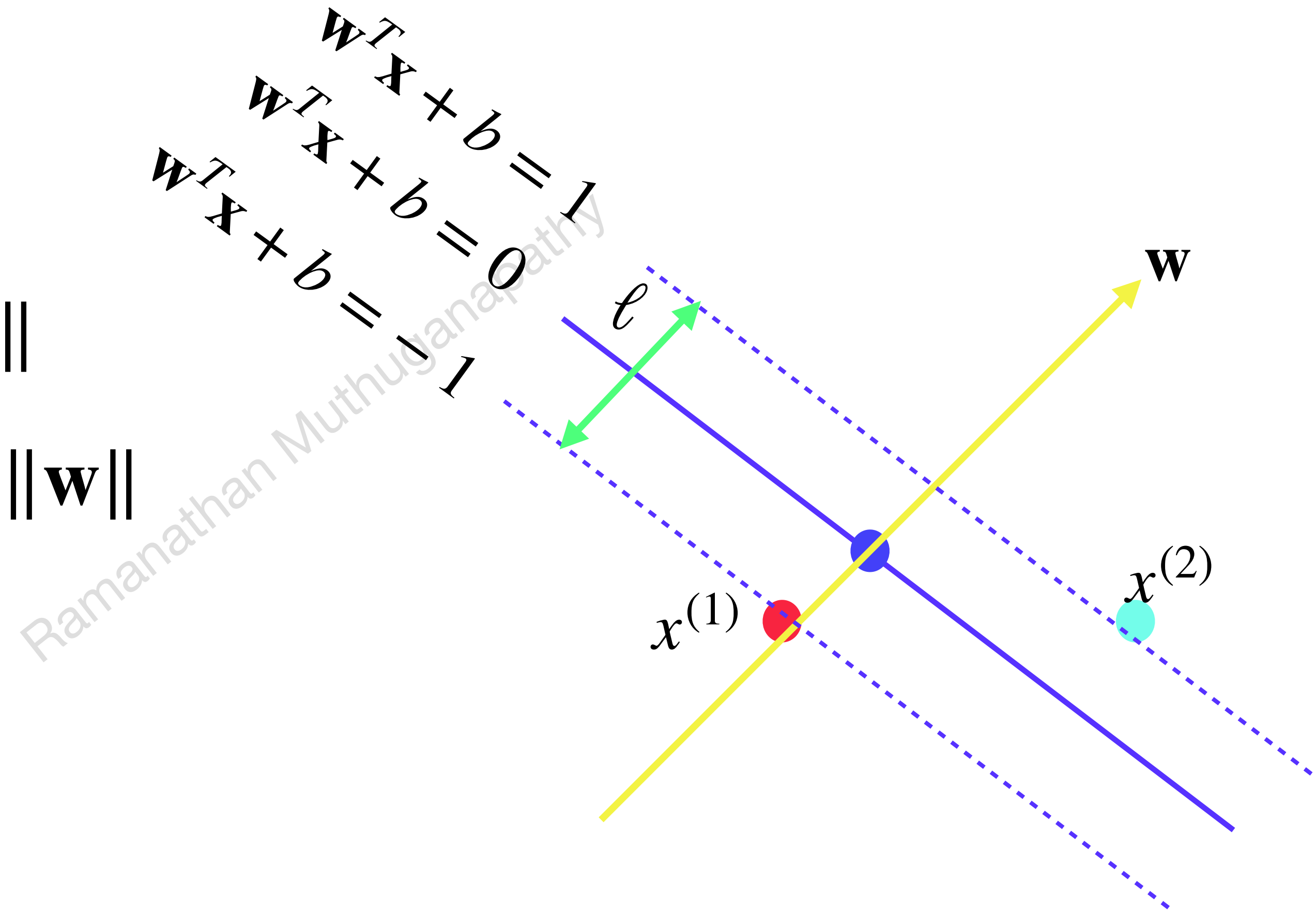
- $\mathbf{w}^T \cdot (x^{(1)} - x^{(2)})$
- $\mathbf{w}^T \cdot (x^{(1)} - x^{(2)}) = \ell \|\mathbf{w}\|$
- $(1 - b) - (-1 - b) = \ell \|\mathbf{w}\|$



# Support vector machine

## Putting together

- $\mathbf{w}^T \cdot (x^{(1)} - x^{(2)})$
- $\mathbf{w}^T \cdot (x^{(1)} - x^{(2)}) = \ell \|\mathbf{w}\|$
- $(1 - b) - (-1 - b) = \ell \|\mathbf{w}\|$
- $2 = \ell \|\mathbf{w}\|$

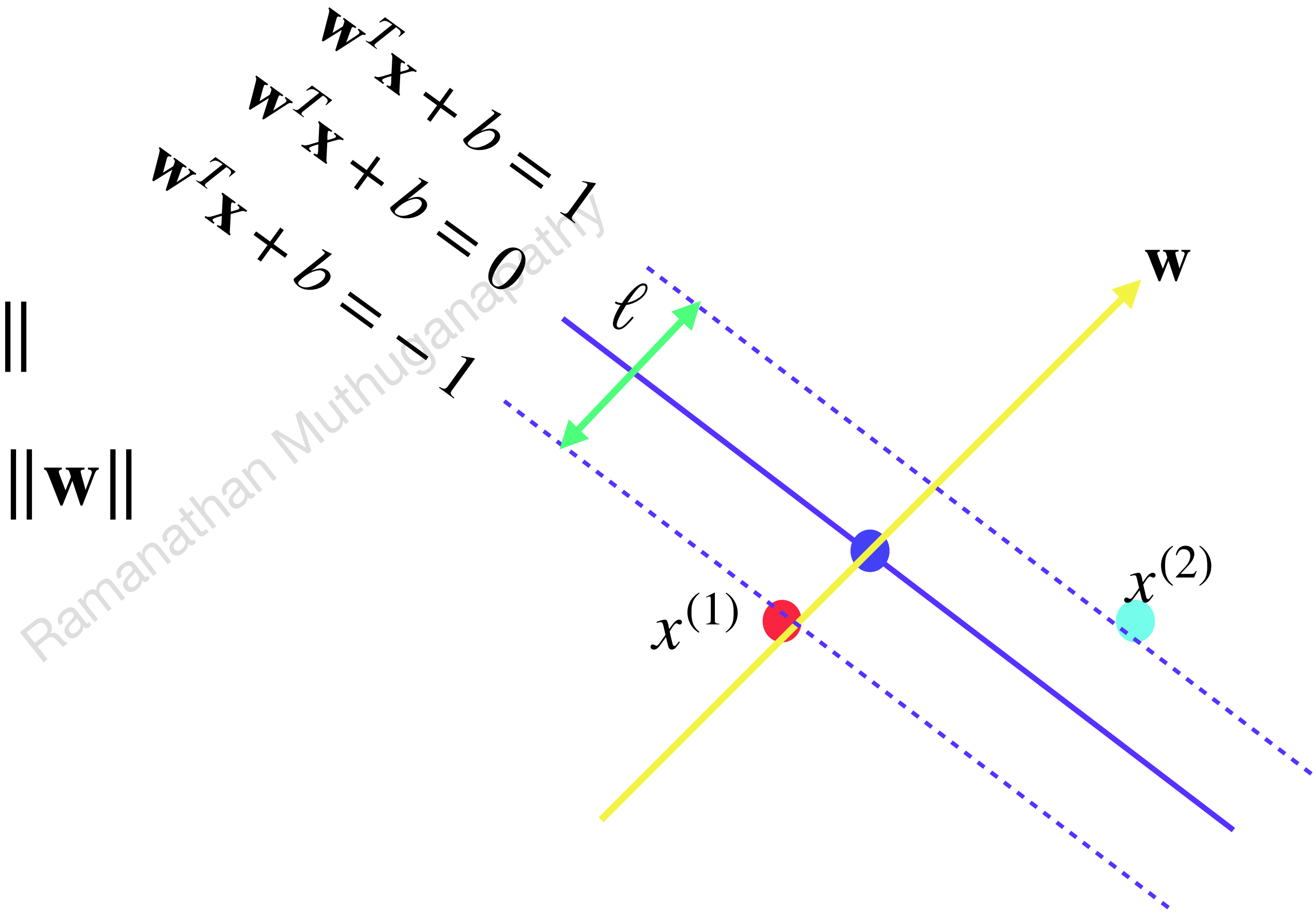




# Support vector machine

## Putting together

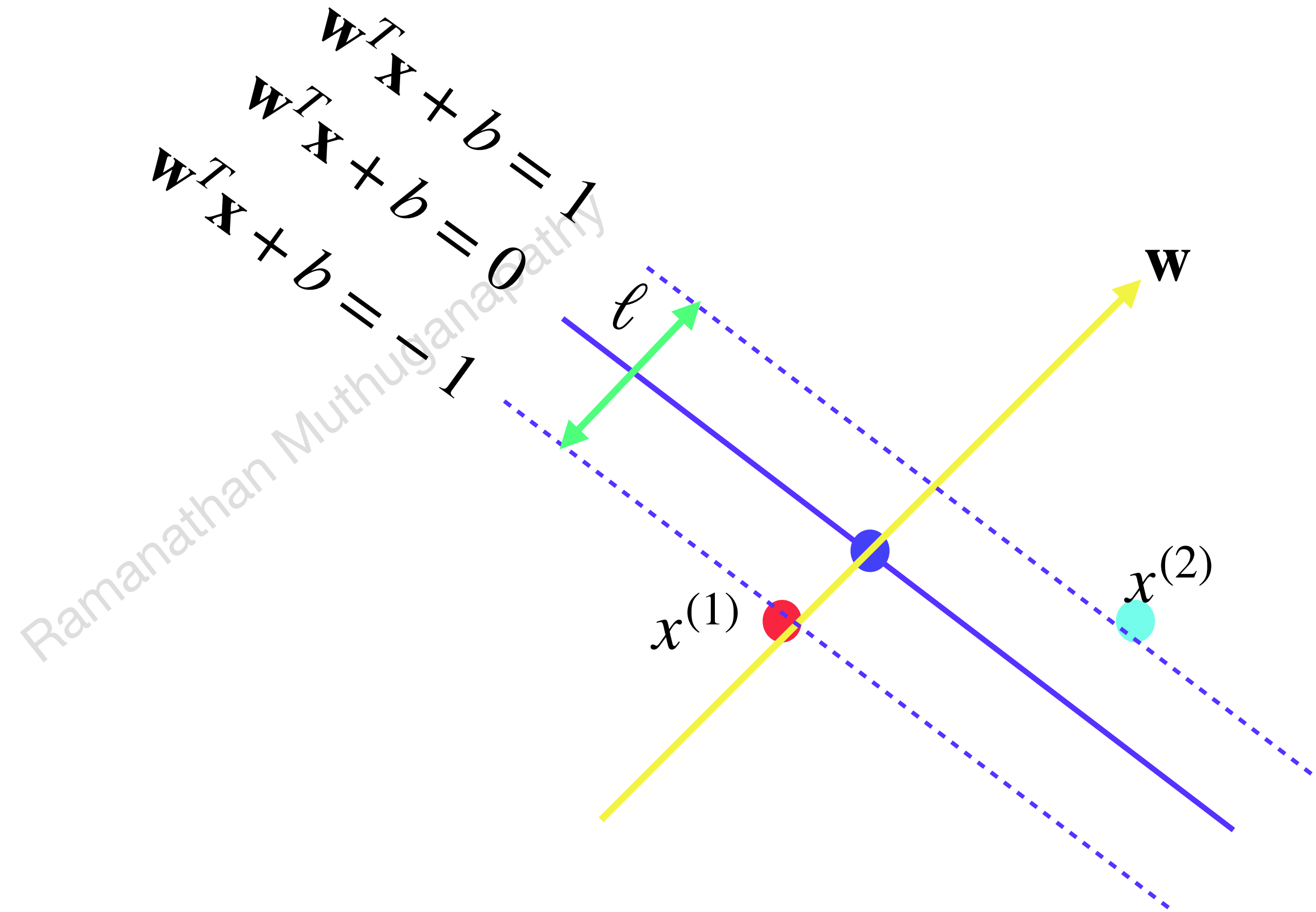
- $\mathbf{w}^T \cdot (x^{(1)} - x^{(2)})$
- $\mathbf{w}^T \cdot (x^{(1)} - x^{(2)}) = \ell \|\mathbf{w}\|$
- $(1 - b) - (-1 - b) = \ell \|\mathbf{w}\|$
- $2 = \ell \|\mathbf{w}\|$
- $\ell = \frac{2}{\|\mathbf{w}\|}$



# Support vector machine

cost function

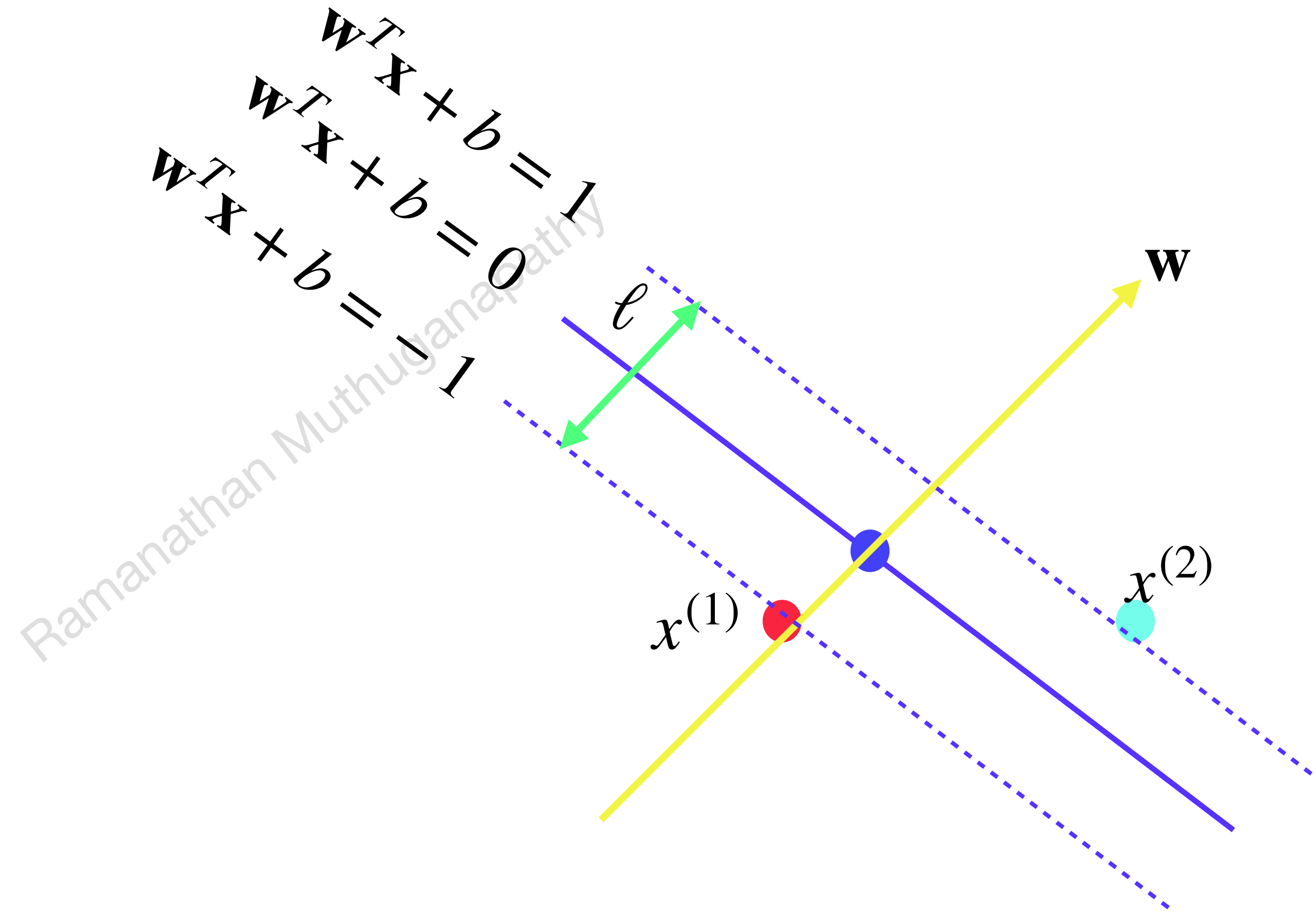
- $\ell = \frac{2}{\|\mathbf{w}\|^2}$



# Support vector machine

## Optimization

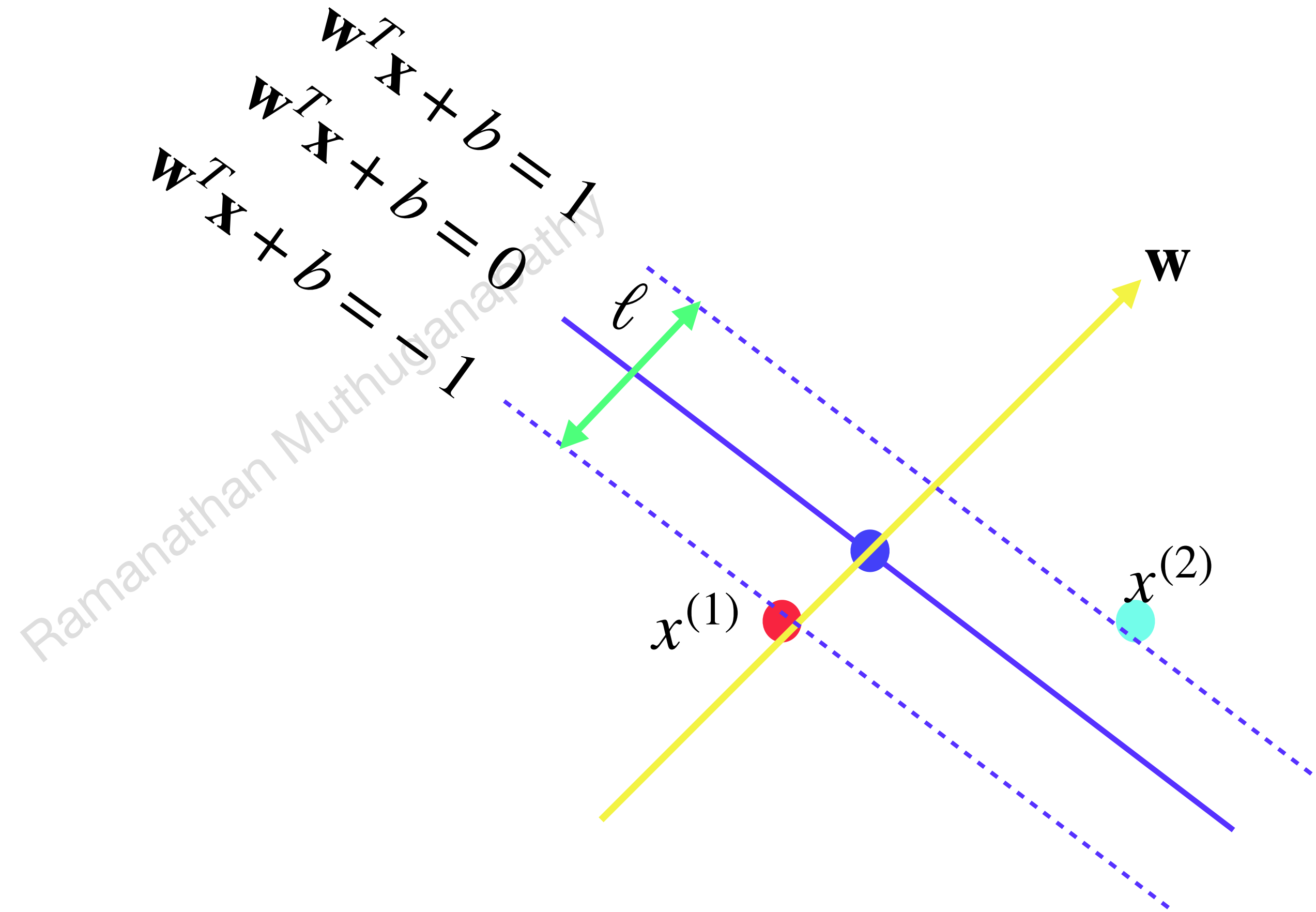
- $\max \ell = \max \frac{2}{\|\mathbf{w}\|^2}$



# Support vector machine

## Optimization Objective

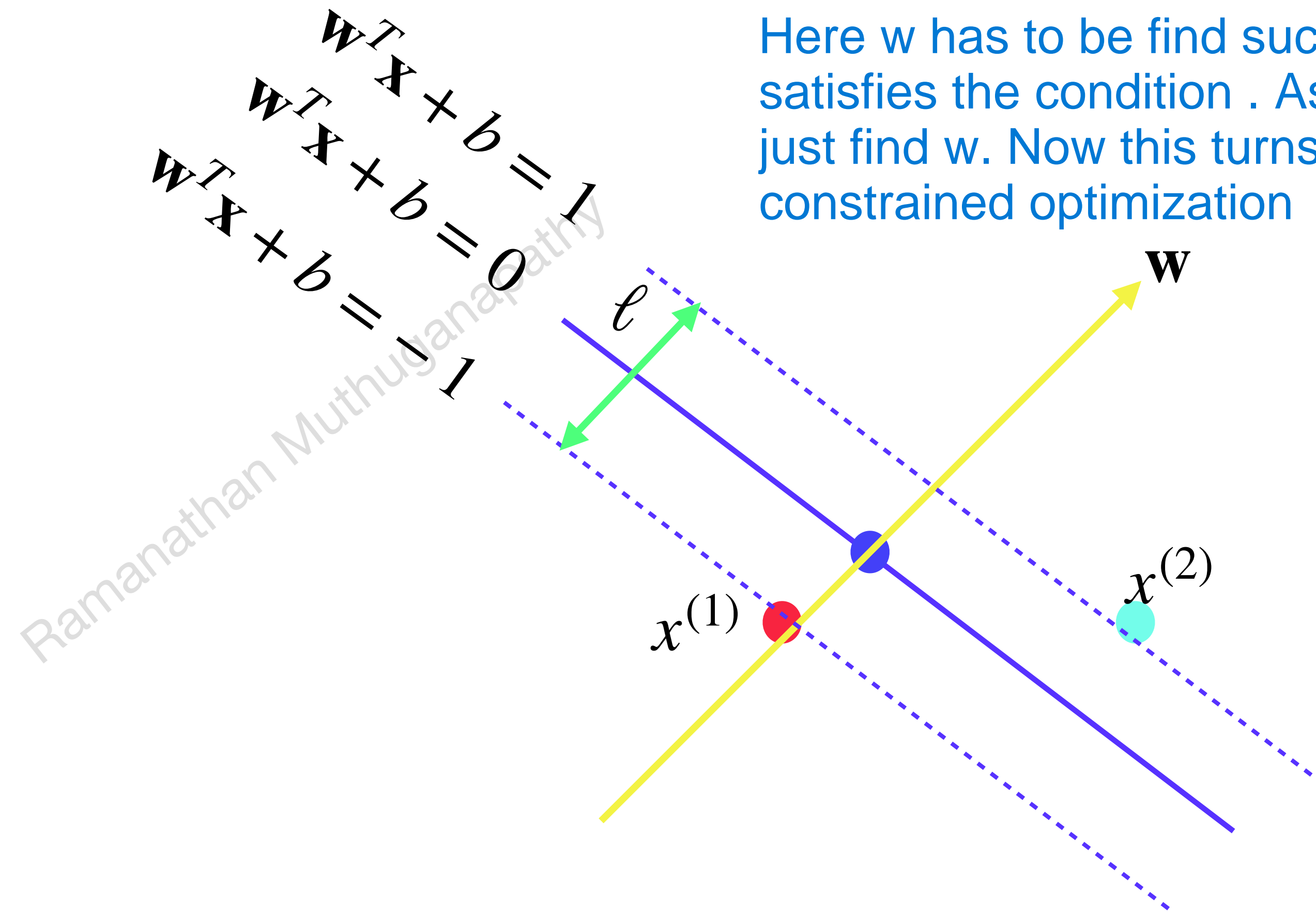
- $$\min \frac{\|w\|^2}{2}$$



# Support vector machine

## Optimization Objective

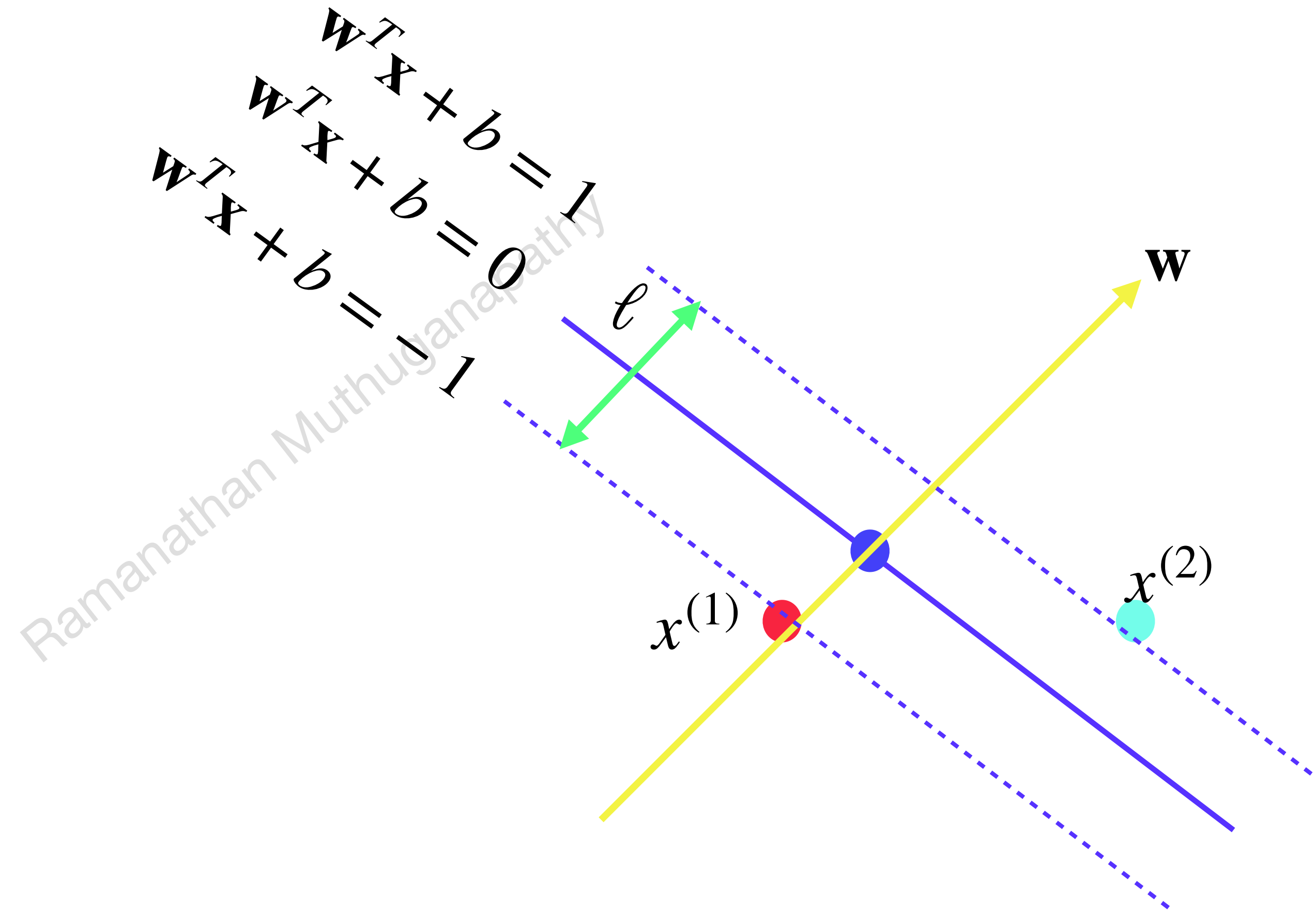
- $\min \frac{\|\mathbf{w}\|^2}{2}$
- $\mathbf{w}^T \mathbf{x} + b \leq -1$
- $\mathbf{w}^T \mathbf{x} + b \geq 1$



# Support vector machine

## Optimization Objective

- $\min \frac{\|\mathbf{w}\|^2}{2}$
- $1 - y^{(i)}(\mathbf{w}^T \mathbf{x} + b) \leq 0$



# Support vector machine

## Constrained Optimization Problem

- $\min \frac{\|\mathbf{w}\|^2}{2}$
- $y^{(i)}(\mathbf{w}^T \mathbf{x} + b) - 1 \geq 0$

