Answer Sheet

1.

Assuming the external load torque is zero.

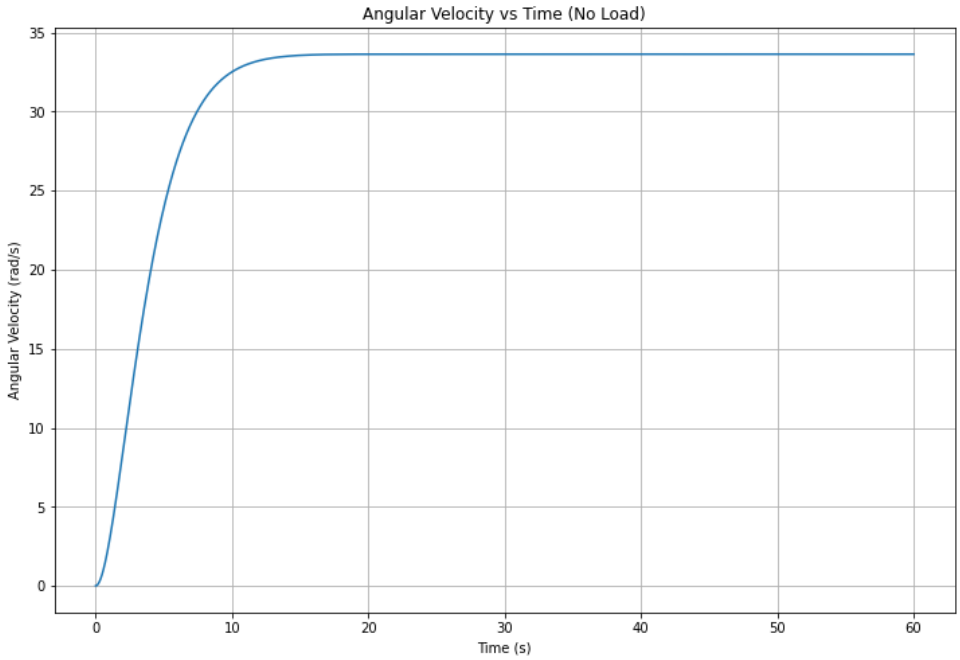


Figure 1: $\dot{\theta}$ **Vs Time** (No Load)

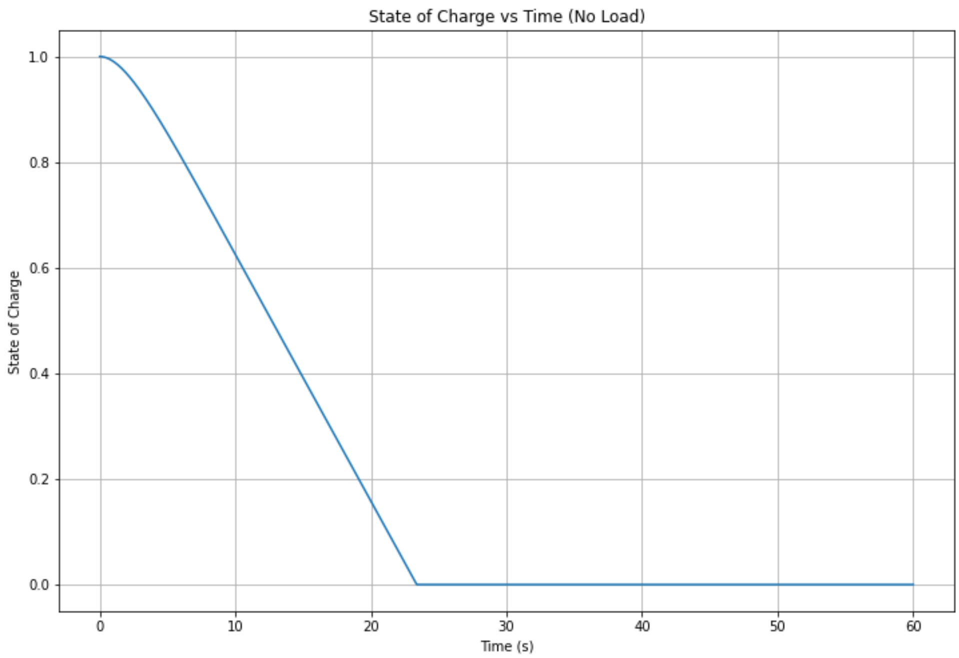


Figure 2: **SoC Vs Time** (No Load)

2.

Considering the external toque (opposite to the direction of the rotating armature) applied through function 0.05*(1+sin(time))

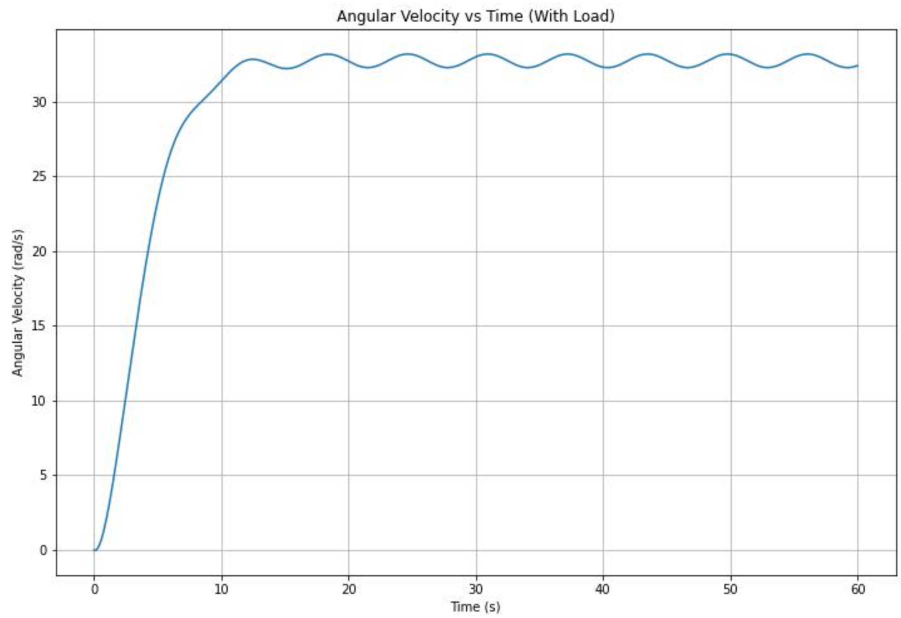


Figure 3: $\dot{\theta}$ **Vs Time** (Loaded)

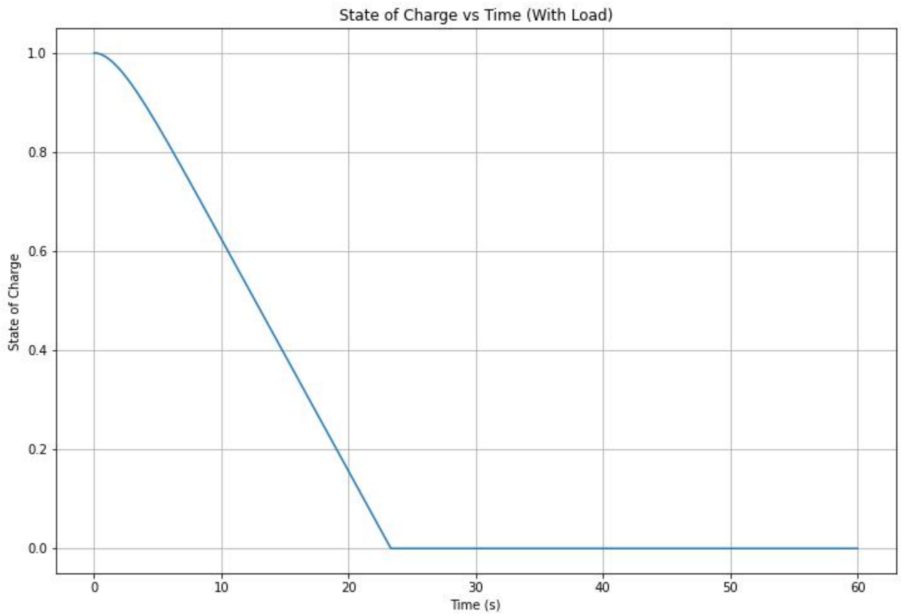


Figure 4: **SoC Vs Time** (Loaded)

3. Bonus Question

Motor System

Table 1: Motor System

State Variables	$x_1 = \dot{\theta}, \quad x_2 = I$
Input Variables	$u_1 = T_{ext}, \quad u_2 = V_t$
State-Space Representation	$\dot{\mathbf{x}}_m = \mathbf{A}_m\mathbf{x}_m + \mathbf{B}_m\mathbf{u}_m$ $\mathbf{A}_m = \begin{bmatrix} -\frac{b}{J} & \frac{K_t}{J} \\ -\frac{K_e}{L_m} & -\frac{R_m}{L_m} \end{bmatrix}$ $\mathbf{B}_m = \begin{bmatrix} -\frac{1}{J} & 0 \\ 0 & \frac{1}{L_m} \end{bmatrix}$

Battery System

Table 2: Battery System

State Variables	$x_3 = V_1, \quad x_4 = z$
Input Variable	$u_3 = I_L$
State-Space Representation	$\dot{\mathbf{x}}_b = \mathbf{A}_b\mathbf{x}_b + \mathbf{B}_b\mathbf{u}_b$ $\mathbf{A}_b = \begin{bmatrix} -\frac{1}{C_1 R_1} & 0 \\ 0 & 0 \end{bmatrix}$ $\mathbf{B}_b = \begin{bmatrix} \frac{1}{C_1} \\ -\frac{1}{C_n} \end{bmatrix}$

Clarification for State Variables

In the context of state-space representation, state variables are chosen to capture the system's essential dynamics. They are internal variables that represent the system's current state and whose values evolve over time based on the system's dynamics and inputs.

For the motor system, the state variables are:

- $x_1 = \dot{\theta}$: Angular velocity of the motor.
- $x_2 = I$: Current through the motor.

For the battery system, the state variables are:

- $x_3 = V_1$: Voltage across the RC pair in the battery model.
- $x_4 = z$: State of charge (SoC) of the battery.

These variables are selected because they represent the key dynamic quantities that define the behavior of the motor and battery systems, respectively.

Combined System

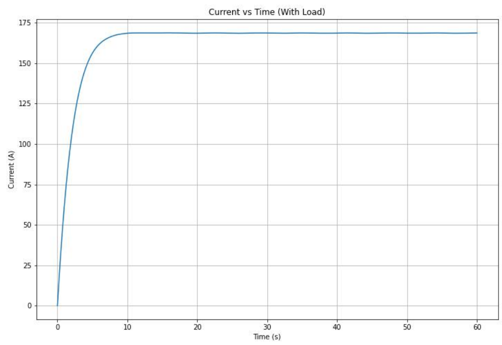
Table 3: Combined System

State Variables	$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ I \\ V_1 \\ z \end{bmatrix}$
Input Variables	$\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} T_{ext} \\ V_t \\ I_L \end{bmatrix}$
State-Space Representation	$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$ $\mathbf{A} = \begin{bmatrix} -\frac{b}{J} & \frac{K_t}{J} & 0 & 0 \\ -\frac{K_e}{L_m} & -\frac{R_m}{L_m} & 0 & 0 \\ 0 & \frac{1}{C_1} & -\frac{1}{C_1 R_1} & 0 \\ 0 & -\frac{1}{C_n} & 0 & 0 \end{bmatrix}$ $\mathbf{B} = \begin{bmatrix} -\frac{1}{J} & 0 & 0 \\ 0 & \frac{1}{L_m} & 0 \\ 0 & 0 & \frac{1}{C_1} \\ 0 & 0 & -\frac{1}{C_n} \end{bmatrix}$
Output Equation	$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$ $\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

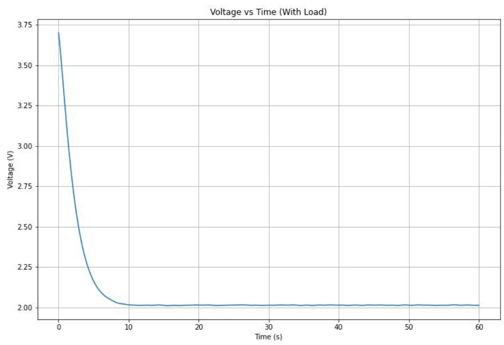
Additional Information

- **Solving Method:** Numerical Integration
  - **Reason:** More familiar with Numerical Integration technique (through coursework) than any other listed methods
- **Numerical method used:** Runge Kutta (RK45) method
  - **Reason:** Adaptive step size, Higher order accuracy, Better error estimation.
- **Software used:** Python
  - **Libraries:** Numpy, Matplotlib, SciPy

Model Verification - Key Points



(a) Current Vs Time



(b) Voltage Vs Time

- A rapid increase (to overcome initial inertia) followed by steady state in angular velocity and current curves.
- A significant voltage drop due to the high current draw.
- A high load current causing rapid depletion of the battery's state of charge (SoC).
- $$P = V_t \times I_{\max} = 2\,\mathrm{V} \times 168.7\,\mathrm{A} = 337.4\,\mathrm{W}$$
- $$T = \frac{P}{\omega} = \frac{337.4\,\mathrm{W}}{33\,\mathrm{rad/s}} \approx 10.23\,\mathrm{Nm}$$
- The motor is designed to deliver significant torque at a relatively low speed (33 rad/s) with a high current draw. This setup is common in applications requiring strong, immediate torque, such as EV drives.
- The resisting external torque with a peak amplitude of 0.1 Nm ends up causing very negligible change in SoC and $\dot{\theta}$ during loaded conditions