

# Quiz 15

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$$(a) A = \begin{pmatrix} 2 & 0 & 0 & 4 & 4 & 4 \\ 0 & 2 & 0 & 0 & 4 & 4 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

$$\lambda = 2$$

$$a_1 = 6$$

$$(A - 2I)w_1 = v_1$$

$$\begin{pmatrix} 0 & 0 & 0 & 4 & 4 & 4 \\ 0 & 0 & 0 & 0 & 4 & 4 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$4w_5 + 4w_6 + 4w_4 = 1 \Rightarrow 4w_4 = \frac{1}{4}$$

$$4w_5 + 4w_6 = 0$$

$$w_5 = -w_6$$

$$w_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1/4 \\ 0 \\ 0 \end{pmatrix}$$

$$J = \begin{pmatrix} 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

$$P = \begin{pmatrix} v_1 & w_1 & v_2 & w_2 & v_3 & v_4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1/4 & 0 & -1/4 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

(b)

$$(A - 2I)v = \begin{pmatrix} 0 & 0 & 0 & 4 & 4 & 4 \\ 0 & 0 & 0 & 0 & 4 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} v = 0$$

$$\sim \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = 0$$

$$x_4 = 0$$

$$x_5 = -x_6$$

$$V = \text{span} \left\{ \begin{pmatrix} v_1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} v_2 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} v_3 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} v_4 \\ 0 \\ 0 \\ 0 \\ 1 \\ -1 \end{pmatrix} \right\}$$

$$g_1 = 4$$

$$\# \text{ of Jordan blocks} = 4$$

$$(A - 2I)v_2 = v_2$$

$$\begin{pmatrix} 0 & 0 & 0 & 4 & 4 & 4 \\ 0 & 0 & 0 & 0 & 4 & 4 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$4w_4 + 4w_5 + 4w_6 = 0 \Rightarrow 4w_4 = -1$$

$$4w_5 + 4w_6 = 1$$

$$w_5 = \frac{1}{4} - w_6$$

$$\text{let } w_6 = 0$$

$$w_5 = \frac{1}{4}$$

$$w_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1/4 \\ 1/4 \\ 0 \end{pmatrix}$$

$$AP = PJ \Rightarrow \begin{pmatrix} 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & -2 \end{pmatrix}$$

②

$$A^3 = PJ^3P^{-1}$$

$$\left. \begin{array}{l} A^3 = PJ^3P^{-1} \\ A^3 = PKIP^{-1} \end{array} \right\} \Rightarrow K = J^3$$

$$K = \begin{pmatrix} 8 & 12 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 8 & 12 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8 \end{pmatrix}$$