DEPARTMENT OF ELECTRONIC AND TELECOMMUNICATION ENGINEERING
UNIVERSITY OF MORATUWA

MUDALIGE DINETH NAVODYA

170401V

BM 2101 - ANALYSIS OF PHYSIOLOGICAL SYSTEMS

Properties of the Hodgkin-Huxley equations

This is submitted as a partial fulfillment for the module
BM 2101 - Analysis of Physiological Systems

—

December 24, 2019

Contents

List of Figures

1 Question 1

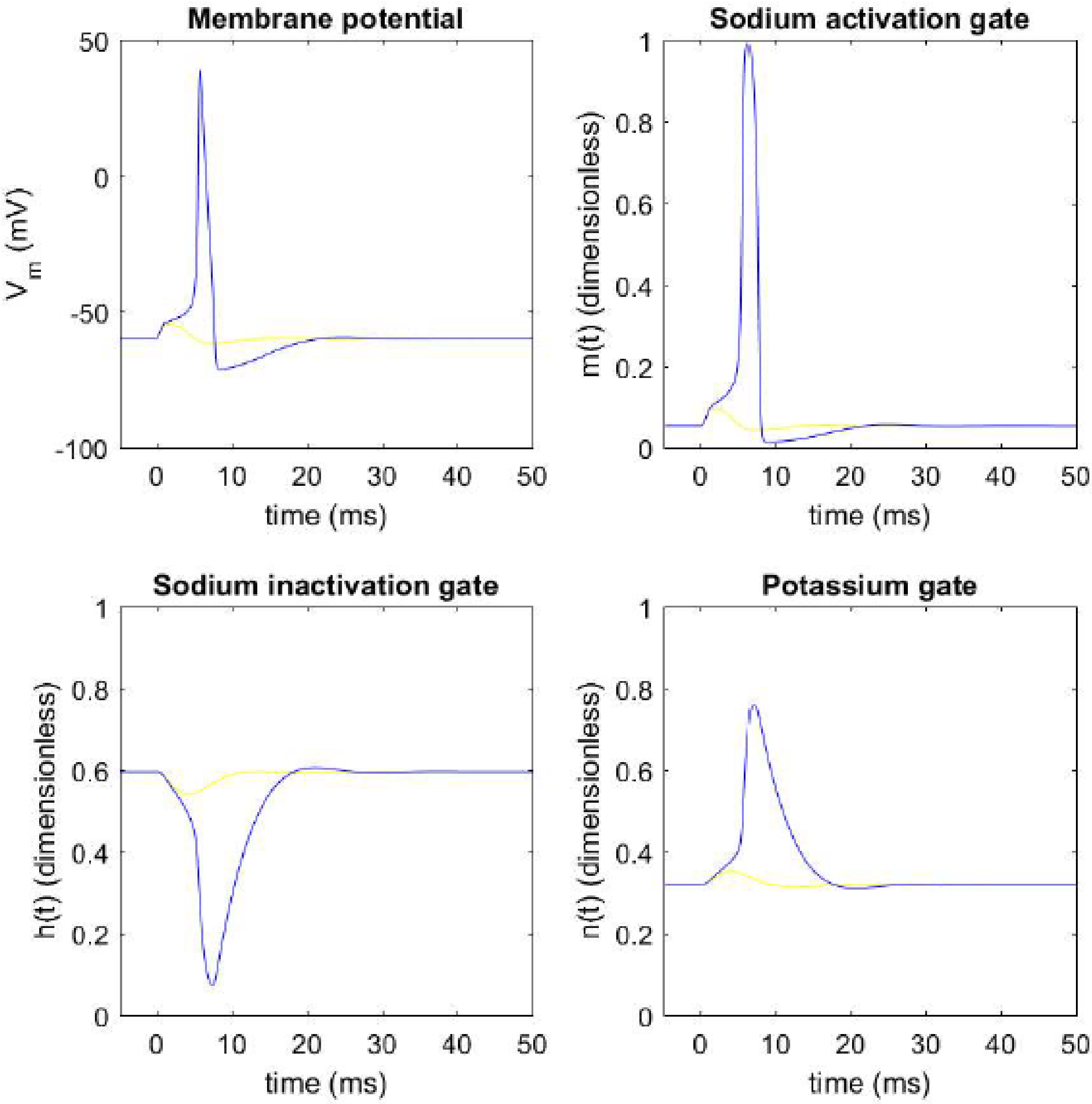


Figure 1: The membrane potential and related plots for initial values assigned in Question 1

After the initial values, then bisection method is used to find the supra-threshold and the sub-threshold using a while loop. The action potential can be found through a zero crossing detector in Listing[2] in Appendix A as the peak of depolarisation ends in a positive magnitude. The supra and sub-thresholds are as follows.

$$\text{Supra-threshold} = 6.96\,\mu A cm^{-2}$$
$$\text{Sub-threshold} = 6.95\,\mu A cm^{-2}$$

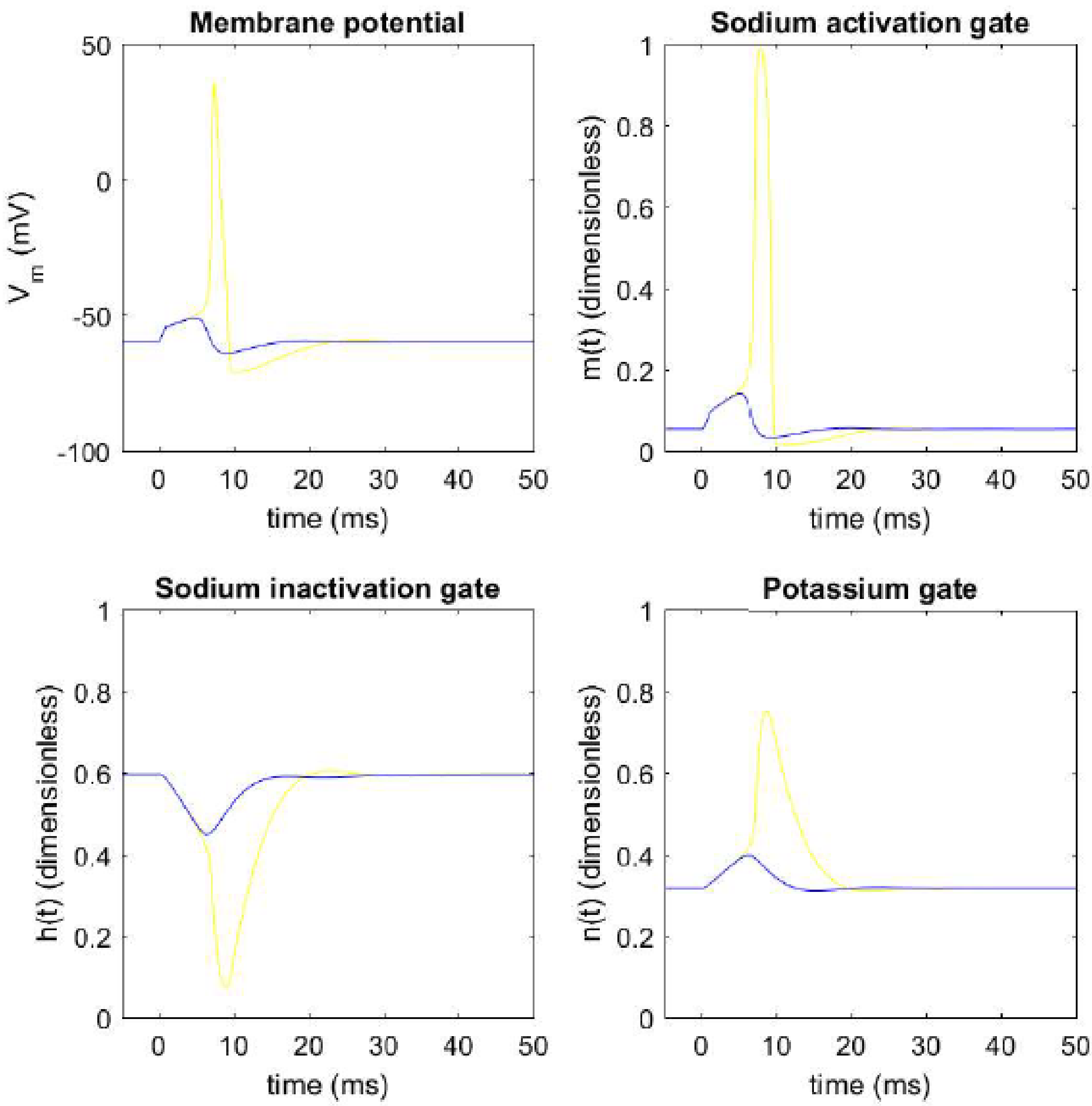


Figure 2: The membrane potential and related plots for sub and supra-thresholds

2 Question 2

Time does not affect the relationship here.
From the results in the Table[1]

Stimulus current	**Time**	j_k	Σ	j_{ei}
6	5	-71.277+236.818+-159.541	6.000	6.000
6	7	-1415.242+1556.382+-133.141	7.999	8.000
8	5	-1415.242+1556.382+-133.141	7.999	8.000
10	5	-1434.064+1577.969+-133.906	9.998	10.000
15	5	-1447.352+1596.829+-134.478	41.999	15.000

Table 1: The relation between net inward current and ionic current

$$\Sigma\, j_k \;=\; j_{ie}$$

3 Question 3

delay2	I_{2th}
20	11.6
18	11.3
16	12.7
14	17.0
12	25.5
10	40.8
8	70.1
6	145.3

Table 2: The threshold of the second action potential

4 Question 4

From the Table[2], the Figure[3] can be obtained. During the absolute refractory period, the action potential cannot

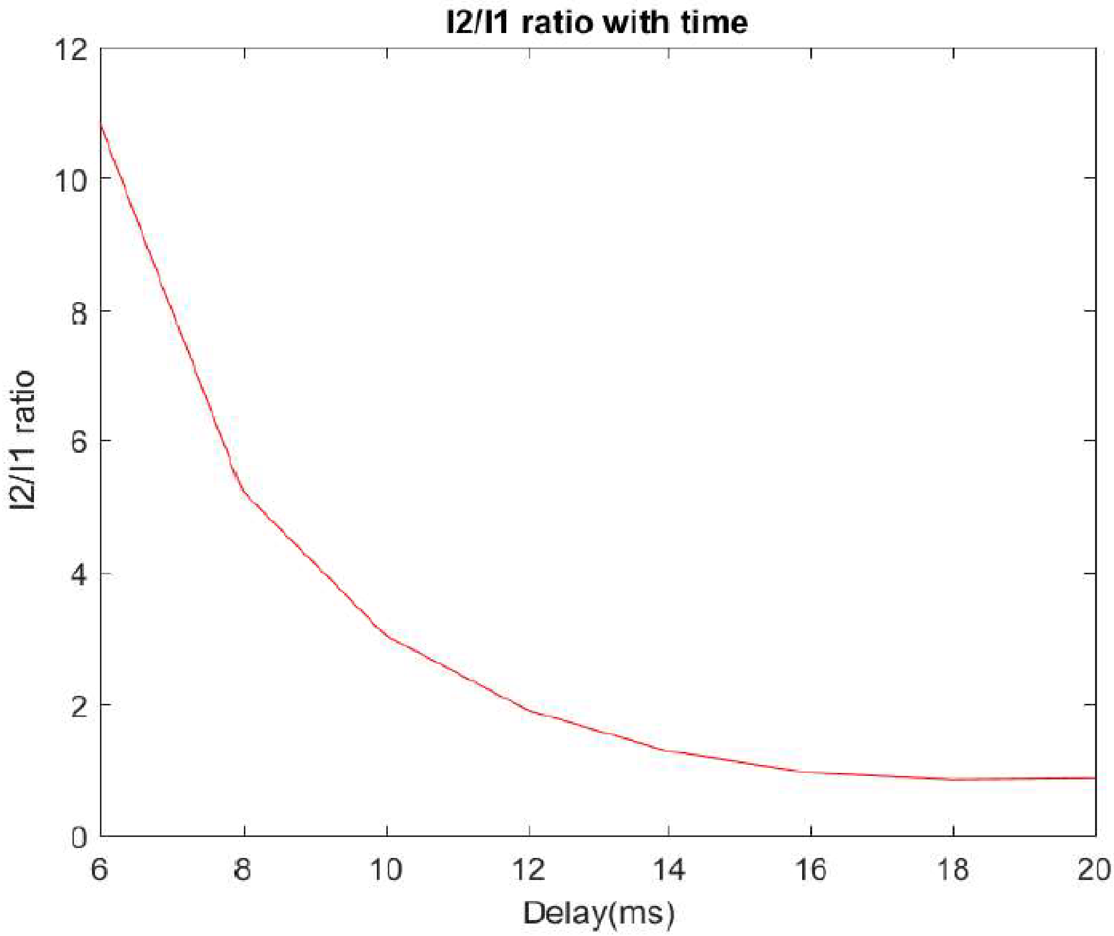


Figure 3: The $I_{2th} : I_{1st}$ with the delay

occur no matter how large the stimulus current is. By the Figure[3], the following approximations can be made.

Absolute refractory period ≈ 6ms
Relative refractory period = (18-6)ms = 12ms

5 Question 5

Stimulus current	Action potential frequency
5	10
10	60
20	70
30	80
50	100
70	110
100	120

Table 3: Stimulus current and the frequency of action potentials

The Figure[4] can be be obtained from the values in Table[3].

With the increase of the stimulus current, the frequency of generating the action potential increases. In the

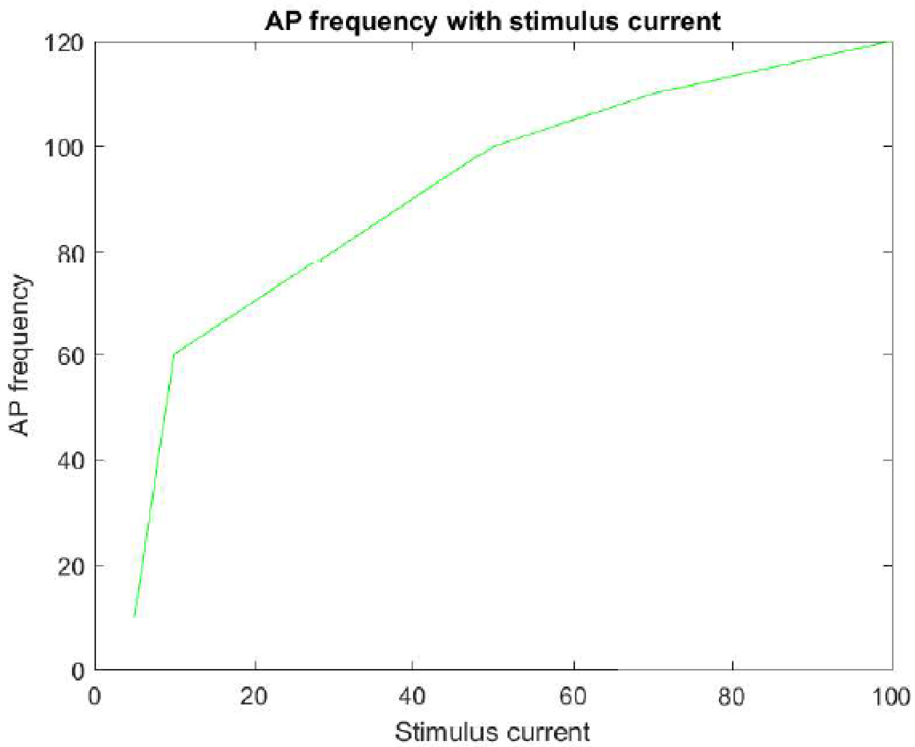


Figure 4: The stimulus current and the action potential frequency

meantime, the amplitude of the action potential decreases. For instance, when the stimulus current is 100 $\mu A cm^{-2}$, only one action potential reaches a positive peak magnitude.

6 Question 6

The number of action potentials that are generated reduces. A plateau appears in the depolarisation region. The parameters n and h are voltage dependent. The Hodgkin-Huxley equation is a second order differential equation and for lower stimulating currents it is in the under-damped state. Thus, it shows a repetitive oscillatory response. When the stimulating current increases, it moves away from the under-damped state and avoids the repetitive action.

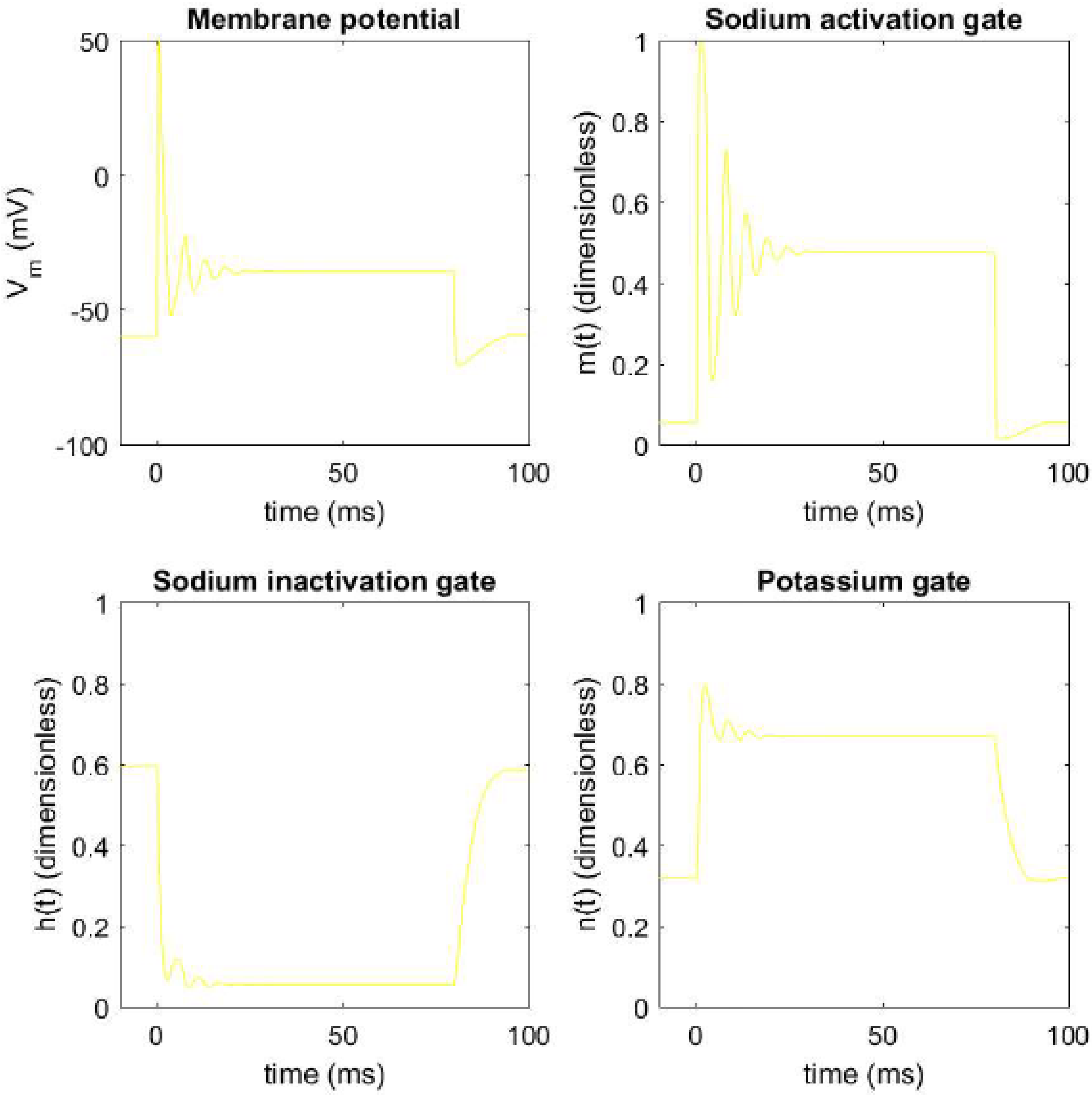
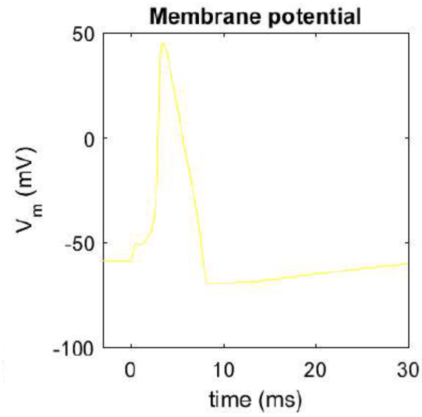


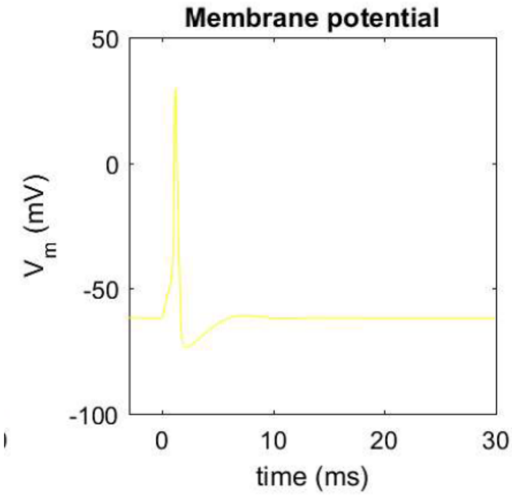
Figure 5: The depolarisation block

7 Question 7

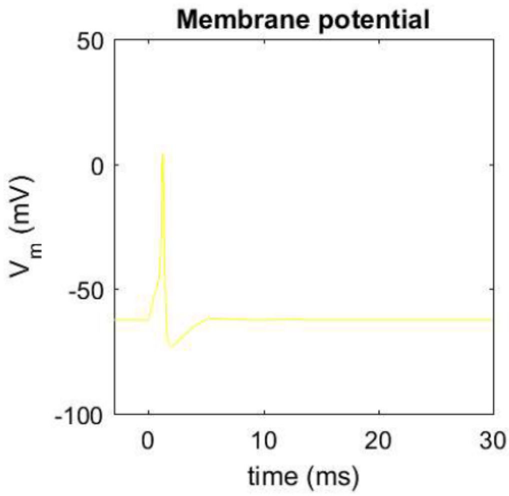
From Figure[6a] to Figure[6d], the amplitude of the membrane potential decreases with the increase of temperature.



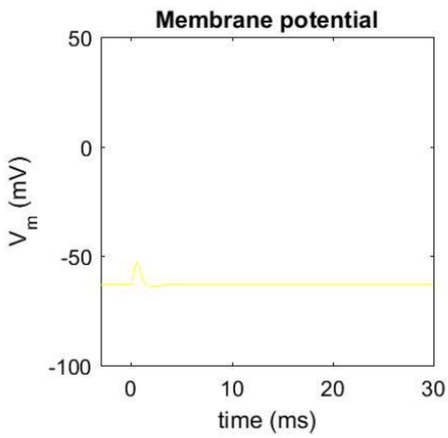
(a) Membrane potential at 0 °C



(b) Membrane potential at 15 °C



(c) Membrane potential at 25 °C



(d) Membrane potential at 30 °C

Figure 6: Variation of membrane potential with temperature

A MATLAB codes

Listing 1: Main code

```
hhconst;
% %Question 1
% Setting the two initial values as defined in the exercise.
%
width1 = 1;
amp1 = 6;
hhmplot(0,50,0);
amp1 = 7;
hhmplot(0,50,1);
% amp1 = 6 is below threshold and amp=7 is above threshold
% detecting the AP generation using zerocrossing as in apdetect.m and giving
% values according to bissection method
amp1 = 6;
b = 7;
while not(apdetect(0,50,amp1,0) & not(apdetect(0,50,amp1-0.01,0)) )
    if apdetect(0,50,amp1,0)
        b = amp1;
    else
        a = amp1;
    end
    amp1 = round(((a+b)/2),2);
end
fprintf('Sub-threshold = %.2f\nSupra-threshold = %.2f\n',amp1-0.01,amp1)
% %Plotting the sub and suprathresholds
% Figure 1 gives the new plots

amp1 = 6.96;
hhmplot(0,50,0);
amp1 = 6.95;
hhmplot(0,50,1);

% Question 2
% To obtain the relationship between the inward current and the sum of ionic
% currents through the gates. Let amp1 be 6,8,10,15 and width be 5,7

for amp1 =[6,8,10,15]
    for width = [5,7]
        [qna,qkl,ql] = hhsplot(0,50);
        jk = qna+qkl+ql;%Total ionic current densities
        jei = amp1;
        fprintf('For amp1 %d and width %d\nThe total current density(jk)=%.3f+%.3f+%.3f
            =%.3f\nThe net inward current(jei) =%.3f\n',amp1,width,qna,qkl,ql,jk,jei)
    end
end
% Total ionic current = amp1 without width consideration

% %Question 3
amp1 = 26.8;
```

```
width1 = 0.5;
width2 = 0.5;
amp2 = 0;
for delay2 = [20,18,16,14,12,10,8,6]
    while (apdetect(0,30,amp1,amp2)<2)
        amp2=amp2+0.1;
    end
    fprintf('For delay of %d, I2th is %.1f\n',delay2,amp2)
    if (apdetect(0,30,amp1,amp2)>=2)
        amp2=0;
    end
end
% Question 4
% Using the values found in Question 3
I2 = [11.6,11.3,12.7,17,25.5,40.8,70.1,145.3];
t = [20,18,16,14,12,10,8,6];
I1 = 13.4;%Predefined first current threshold
figure;
plot(t,I2/I1,'-r');
xlabel('Delay(ms)');
ylabel('I2/I1 ratio');
title('I2/I1 ratio with time');
% Question 5
% By using the apdetect.m derived from hhmplot.m
for amp1=[5,10,20,30,50,70]
    width1=80;
    delay2=0;
    width2=0;
    fprintf('For a current stimulus of %d, there are %d action potentials in 0.1s\n.',
       amp1,apdetect(0,100,amp1,0));
end
width1=80;
delay2=0;
width2=0;
amp1=100;
% Need to plot for 100ms as the AP does not exceed zero
hhmplot(0,100,0);
I = [5,10,20,30,50,70,100];
f = [10,60,70,80,100,110,120];
figure;
plot(I,f,'-g');
xlabel('Stimulus current');
ylabel('AP frequency');
title('AP frequency with stimulus current');
%Question 6
width1=80;
delay2=0;
width2=0;
amp1=200;
hhmplot(0,100,0);
%Question 7
```

```
vclamp = 0;
amp1=20;
width1 = 0.5;
tempc=30;
hhmplot(0,30,0);
hhsplot(0,30);
```

Listing 2: The Matlab code for detecting the number of zerocrossing pulses

```
function n=apdetect(to,tf,amp1,amp2)
%
% Numerical solution of the Hodgkin Huxley equations
% for parameters as set from file hhconst
%
% Gives the number of APs detected in the given time
%

global yo e_vr minfr hinfr ninfr;
global g_na_max g_k_max g_l;
global e_na e_k e_l e_vr;
global g_na_vr g_k_vr;
global delay1  width1;
global width2  delay2 ic vclamp;

% update all neccessary precalculated parameters

hhparams;

[ti,yi] = hode('hh',[to,to+delay1],yo);  % do not really need to integrate here but it
    makes the code more readable
yo = yi';
if vclamp~=0;
        yo = [vclamp; yo(2:4)];
        [t1,y1] = hode('hh',[to+delay1,to+delay1+width1],yo);
        len = length(t1);
        yo = [e_vr; y1(len,2:4)'];
        [t2,y2] = hode('hh',[to+delay1+width1,tf],yo);
        t = [ti;t1;t2];
        y = [yi;y1;y2];
elseif amp1~=0;
        ic = amp1;
        [t1,y1] = hode('hh',[to+delay1,to+delay1+width1],yo);
        len = length(t1);
        yo = y1(len,1:4)';
        ic = 0;
        [t2,y2] = hode('hh',[to+delay1+width1,to+delay1+width1+delay2],yo);
        len = length(t2);
        yo = y2(len,1:4)';
        ic = amp2;
        [t3,y3] = hode('hh',[to+delay1+width1+delay2,to+delay1+width1+delay2+width2],yo);
        len = length(t3);
        yo = y3(len,1:4)';
        ic = 0;
        [t4,y4] = hode('hh',[to+delay1+width1+delay2+width2,tf],yo);
        t = [ti;t1;t2;t3;t4];
        y = [yi;y1;y2;y3;y4];
end

zci = @(v) find(v(:).*circshift(v(:), [-1 0]) <=0);                    % Returns Zero-
```

```
    Crossing Indices Of Argument Vector
zx = zci(y(:,1));                                                          %
    Approximate Zero-Crossing Indices
n = size(zx,1)/2; %Gives the number of APs generated
```