Assignment 3: Convolution and System Output

1. **Convolution by Hand**
   Compute the convolution $y[n] = x[n] * h[n]$ where

$$x[n] = \{1, 2, 1\}, \quad n = 0, 1, 2, \qquad h[n] = \{1, -1, 2\}, \quad n = 0, 1, 2,$$

   and $x[n] = h[n] = 0$ outside these ranges.

   (a) Compute $y[n]$ step-by-step using the convolution sum.

   (b) Tabulate $y[n]$ for all n where it is nonzero.

2. **Cascade of LTI Systems**
   Two LTI systems are cascaded. Their impulse responses are

$$h_1[n] = \{1, 1\}, \ n = 0, 1, \qquad h_2[n] = \{1, -1, 1\}, \ n = 0, 1, 2.$$

   (a) Determine the overall impulse response $h_{\text{eq}}[n] = h_1[n] * h_2[n]$.

   (b) Comment on how the overall duration and shape of $h_{\text{eq}}[n]$ relate to those of $h_1[n]$ and $h_2[n]$.

3. **Python – Verifying Convolution**

   (a) Use Python to compute the convolutions in Q1 and Q2 (e.g. using `numpy.convolve` or a custom loop).

   (b) Print or plot the resulting sequences and verify they match your manual calculations.

   (c) Briefly explain the importance of index alignment and zero-padding when comparing manual and numerical convolution.