Assignment 4: z-Transform and Inverse z-Transform

1. **Forward z-Transform**
   Find the z-transform $X(z)$ and region of convergence (ROC) of

   $$x[n] = 0.5^n u[n].$$

   Start from the definition and express $X(z)$ as a rational function. Clearly state the ROC.

2. **Inverse z-Transform by Partial Fractions**
   Given

   $$X(z) = \frac{2z}{(z-1)(z+0.5)},$$

   and assuming a causal $x[n]$:

   (a) Perform a partial fraction expansion to express $X(z)$ as a sum of simpler terms.

   (b) Find the corresponding time-domain sequence $x[n]$.

   (c) Sketch $x[n]$ and comment on its decay pattern.

3. **Python – Verifying the Inverse Transform**

   (a) Use Python (e.g. `sympy`) to carry out the partial fraction expansion of $X(z)$ and derive $x[n]$ symbolically.

   (b) Compare the symbolic expression for $x[n]$ with your analytical result in Q2.

   (c) Discuss how symbolic tools can aid in z-transform manipulations for more complicated systems.