

Assignment 4.1

1. $x_c(t) = \cos(502t)$ $f_s = 30 \text{ Hz}$

a) $x_d(t)$ after sampled at $f_s = 30 \text{ Hz}$

$$x(n) = \cos(502 \times n)$$

$$= \cos\left(2\pi n \left[\frac{5}{6}\right]\right) = \cos\left[2\pi n - \frac{\pi n}{3}\right] = \cos\left[\frac{\pi n}{3}\right]$$

Therefore

$$x(n) = \cos\left[2\pi n \left[\frac{5}{6}\right]\right] = \cos\left[\frac{\pi n}{3}\right]$$

b) Since $x(n)$ can be written as

$$x(n) = \cos(\omega_0 n)$$

$$\omega_0 = \frac{\pi n}{3} \times (n-1) \text{ rad}$$

$$\omega_0 = \frac{\pi}{3} \text{ rad/sample}$$

For fundamental frequency,

$$\omega_0 n = 2\pi$$

$$\frac{\pi n}{3} = 2\pi$$

$$n = 6$$

fundamental period $n = 6$

c) according to the Nyquist theorem $f_s \geq 2f_1$ to be not aliasing

Therefore since $30 < 2 \times 25 \text{ kHz}$ the sampling will cause aliasing.

if we reconstruct the signal back

$$x(t) = \cos\left(\frac{2\pi f_1}{3} \times 30\right)$$

$$= \cos(102\pi t)$$

$$= \cos(2\pi 5.1 t)$$

Therefore aliased frequency = 5 Hz

3.

a) according to the Nyquist theorem if a signal has frequency components upto 5 kHz minimal sampling frequency should be 10 kHz to stop aliasing

b) if 15 kHz and 5 kHz components are available $f_s = 30 \text{ kHz}$

c) Zero order hold is a reconstruction method where the value of the sample is kept constant when reconstructing until the next sample which will last for T_s time.