

## Assignment 2

1)  $x_1(n) = \cos(0.5\pi n)$

$E = \sum_{n=-\infty}^{\infty} (\cos(0.5\pi n))^2$

2)

$x_1(n) = \cos(2\pi(0.25)n)$

$x_1(n+N) = \cos[2\pi(0.25)(n+N)]$   $\Rightarrow x_1(n+N)$  is a power signal

$= \cos(0.5\pi n + 0.5\pi N)$

Since  $\cos(0.5\pi n) = \cos(0.5\pi n + 0.5\pi N)$   $\Rightarrow x_1(n+N) = x_1(n)$

$x_1(n) = x_1(n+N)$

 $\therefore x_1(n)$  is periodic

b)  $x_2(n) = (0.5)^n u(n).$

$E = \sum_{n=0}^{\infty} |(0.5)^n|^2$

$x_2(n+N) = (0.5)^{n+N} u(n+N)$

$\Rightarrow \sum_{n=0}^{\infty} |(0.5)^{n+N}|^2 = \sum_{n=0}^{\infty} (0.5)^{2n}$

 $\therefore x_2(n)$  is aperiodic

$= \frac{1}{1-0.25} = \frac{4}{3}$

 $\therefore x_2(n)$  is an energy signal

c)  $x_3(n) = 1$

$E = \sum_{n=-\infty}^{\infty} 1^2$

This is periodic.

$\Rightarrow 1 = (1/2 \cdot 2\pi - 1)2 \Rightarrow 1 = \pi - 1$

$= 100 \approx 100$

$1 = (1/2 \cdot 2\pi - 1)2 \Rightarrow 1 = \pi - 1$

$P = \lim_{N \rightarrow \infty} \frac{1}{2\pi} \sum_{n=-N}^{N+1} 1^2$

$= \lim_{N \rightarrow \infty} \frac{N+1}{2N+1} = \frac{1}{2} = \frac{1}{2}$

2).  $y(n) = 0.5y(n-1) + x(n).$

a). impulse response of  $y(n)$

let  $h(n) = 0.5 h(n-1) = \delta(n).$

$n=0 \quad h(0) = 0.5 h(-1) = 1 \quad$  (since  $h(-1) = 0$  due to cause)

$h(0) = 1$

$n=1 \quad h(1) = 0.5 h(0) = 0$

$h(1) = 0.5$

$n=2 \quad h(2) = 0.5 h(1) = 0$

$h(2) = 0.5^2$

$n=3 \quad h(3) = 0.5 h(2) = 0$

$h(3) = 0.5^3$

$h(n) = 0.5^n u(n)$

b). Step response.

let  $s(n) - 0.5s(n-1) = \delta(n).$

at  $n=0, \delta(0) - 0.5s(-1) = 1$

$s(0) = 1$

at  $n=1, s(1) - 0.5s(0) = 1$

$s(1) = 1 + 0.5$   
 $= 1.5$

$$\text{at } n=2 \quad S(2) - 0.5 S(1) = 1$$

$$S(2) = 1 + 0.75$$

$$= 1.75$$

$$\text{at } n=3 \quad S(3) - 0.5 S(2) = 1$$

$$S(3) = 1 + 0.875$$

$$= 1.875$$

$$\underline{S(n)} = \underline{(2 - 0.5^n) u_n}$$

- c).  $h_{n1}$  is decaying exponentially while  $s_{n1}$  is starting from 1 and converges to 2 when  $n \rightarrow \infty$ .

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import numpy as np
import matplotlib.pyplot as plt
# Discrete time indices
n = np.arange(0, 21)
# Define input signals
delta_n = np.zeros(21)
delta_n[0] = 1
u_n = np.ones(21)
#  $y[n] = 0.5y[n-1] + x[n]$ 
a1 = 0.5
b0 = 1.0
# Initialize output arrays
h_n = np.zeros(21)
s_n = np.zeros(21)
# Simulate impulse response h[n]
for i in range(21):
    if i == 0:
        h_n[i] = b0 * delta_n[i]
    else:
        h_n[i] = a1 * h_n[i-1] + b0 * delta_n[i]
# Simulate step response s[n]
for i in range(21):
    if i == 0:
        s_n[i] = b0 * u_n[i]
    else:
        s_n[i] = a1 * s_n[i-1] + b0 * u_n[i]
# Plotting the results

#IMPULSE RESPONSE PLOT
plt.figure(figsize=(12, 8))
plt.subplot(2, 1, 1)
plt.stem(n, h_n, linefmt='b-', markerfmt='bo', basefmt=' ', label='Impulse Response h[n]')
plt.title('Impulse Response of the System')
plt.xlabel('n (Discrete Time Index)')
plt.ylabel('h[n]')
plt.legend()
plt.grid()

#STEP RESPONSE PLOT
plt.subplot(2, 1, 2)
plt.stem(n, s_n, linefmt='r-', markerfmt='ro', basefmt=' ', label='Step Response s[n]')
plt.title('Step Response of the System')
plt.xlabel('n (Discrete Time Index)')
plt.ylabel('s[n]')
plt.legend()
plt.grid()
plt.tight_layout()
plt.show()

```

Step Response of the System

