

Assignment 2

1) $x_1(n) = \cos(0.5\pi n)$

$$E = \sum_{n=-\infty}^{\infty} |\cos(0.5\pi n)|^2$$

2)

$$x_1(n) = \cos(2\pi(0.25)n)$$

$$x_1(n+N) = \cos(2\pi(0.25)(n+N))$$

 $\therefore x_1(n)$ is a power signal

$$= \cos(0.5\pi n + \pi n)$$

$$\text{Since } \cos(0.5\pi n) = \cos(0.5\pi n + 2\pi n)$$

$$x_1(n) = x_1(n+N)$$

 $\therefore x_1(n)$ is periodic

b) $x_2(n) = (0.5)^n u(n)$

$$E = \sum_{n=-\infty}^{\infty} |(0.5)^n u(n)|^2$$

$$x_2(n+N) = (0.5)^{n+N} u(n+N)$$

$$= \sum_{n=0}^{\infty} (0.5)^{2n}$$

 $\therefore x_2(n)$ is aperiodic

$$= \frac{1}{1-0.25} = \frac{4}{3}$$

 $\therefore x_2(n)$ is an energy signal

c) $x_3(n) = 1$

this is periodic

$$E = \sum_{n=-\infty}^{\infty} 1^2$$

$$= \infty$$

$$P = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N 1^2$$

$$= \lim_{N \rightarrow \infty} \frac{N+N+1}{2N+1} = \frac{2}{2} = 1$$

Atlas

2). $y(n) = 0.5y(n-1) + x(n)$.

a). impulse response of yw

let $h(n) = 0.5h(n-1) + \delta(n)$.

$n=0$ $h(0) = 0.5h(-1) + 1$ (since $h(-1) = 0$ due to causality)

$h(0) = 1$

$n=1$ $h(1) = 0.5h(0) + 0$ (since $x(1) = 0$)

$h(1) = 0.5$

$n=2$ $h(2) = 0.5h(1) + 0$

$h(2) = 0.5^2$

$n=3$ $h(3) = 0.5h(2) + 0$

$h(3) = 0.5^3$

$h(n) = 0.5^n u(n)$

b). Step response.

let $s(n) = 0.5s(n-1) + x(n)$.

at $n=0$, $s(0) = 0.5s(-1) + \phi$

$s(0) = \phi$

at $n=1$ $s(1) = 0.5s(0) + 1$

$s(1) = 1 + 0.5\phi$

$= \phi + 0.5$

$$\text{at } n=2 \quad S(2) - 0.5 S(1) = 1$$

$$\begin{aligned} S(2) &= 1 + 0.75 \\ &= 1.75 \end{aligned}$$

$$\text{at } n=3 \quad S(3) - 0.5 S(2) = 1$$

$$\begin{aligned} S(3) &= 1 + 0.875 \\ &= 1.875 \end{aligned}$$

$$\underline{\underline{S(n) = (2 - 0.5^n) \text{ uni}}}$$

- c) $h(n)$ is decaying exponentially while $S(n)$ is starting from 1 and converge to 2 when $n \rightarrow \infty$.


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import numpy as np
import matplotlib.pyplot as plt
# Discrete time indices
n = np.arange(0, 21)
# Define input signals
delta_n = np.zeros(21)
delta_n[0] = 1
u_n = np.ones(21)
#  $y[n] = 0.5y[n-1] + x[n]$ 
a1 = 0.5
b0 = 1.0
# Initialize output arrays
h_n = np.zeros(21)
s_n = np.zeros(21)
# Simulate impulse response h[n]
for i in range(21):
    if i == 0:
        h_n[i] = b0 * delta_n[i]
    else:
        h_n[i] = a1 * h_n[i-1] + b0 * delta_n[i]
# Simulate step response s[n]
for i in range(21):
    if i == 0:
        s_n[i] = b0 * u_n[i]
    else:
        s_n[i] = a1 * s_n[i-1] + b0 * u_n[i]
# Plotting the results

#IMPULSE RESPONSE PLOT
plt.figure(figsize=(12, 8))
plt.subplot(2, 1, 1)
plt.stem(n, h_n, linefmt='b-', markerfmt='bo', basefmt=' ', label='Impulse Response h[n]')
plt.title('Impulse Response of the System')
plt.xlabel('n (Discrete Time Index)')
plt.ylabel('h[n]')
plt.legend()
plt.grid()

#STEP RESPONSE PLOT
plt.subplot(2, 1, 2)
plt.stem(n, s_n, linefmt='r-', markerfmt='ro', basefmt=' ', label='Step Response s[n]')
plt.title('Step Response of the System')
plt.xlabel('n (Discrete Time Index)')
plt.ylabel('s[n]')
plt.legend()
plt.grid()
plt.tight_layout()
plt.show()

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