Assignment 1: Introduction to Sampling and Discrete-Time Signals

1. **Sampling and Periodicity**
   A continuous-time sinusoid
   $$x_c(t) = \cos(50\pi t)$$
   is sampled at rate $F_s = 30$ Hz.

   (a) Derive the discrete-time signal $x[n]$ obtained after sampling. Give a general formula in terms of n.

   (b) Determine the discrete angular frequency ω_0 (in rad/sample) of $x[n]$. Is $x[n]$ periodic? If so, find its fundamental period N.

   (c) The analog frequency is 25 Hz. Comment on whether aliasing occurs at the given sampling rate. Identify the aliased discrete-time frequency and explain your reasoning.

2. **Python – Aliasing Demonstration**
   Use Python to illustrate aliasing.

   (a) Generate a continuous-time signal
   $$x_c(t) = \sin(2\pi \cdot 1000\, t).$$
   Simulate sampling it at two rates: (i) $F_s = 600$ Hz and (ii) $F_s = 1500$ Hz. For each case, generate 1 second of samples $x[n] = x_c(nT)$.

   (b) Plot the discrete-time waveforms for both sampling rates (stem plots or line plots).

   (c) Compare the sampled signals. Explain how the 1000 Hz sinusoid appears when sampled *below* and *above* the Nyquist rate, and identify the apparent (aliased) frequency in the undersampled case.

3. **Sampling Theorem Application**

   (a) A continuous-time signal contains frequency components up to 5 kHz. What is the minimum sampling rate required to avoid aliasing?

   (b) If the signal contains components at 5 kHz and 15 kHz, what sampling frequency is required to avoid aliasing of both components?

   (c) Briefly explain the role of a zero-order hold (ZOH) in reconstructing a band-limited signal from its samples when the sampling rate satisfies the Nyquist criterion.