Assignment 5: System Characterization via z-Transform

1. **Transfer Function from Difference Equation**
   Consider the LTI system:

$$y[n] - 1.1\,y[n-1] + 0.3\,y[n-2] = x[n] + 0.5\,x[n-1].$$

   (a) Derive the system transfer function $H(z) = \dfrac{Y(z)}{X(z)}$.

   (b) Express $H(z)$ as

$$H(z) = \frac{b_0 + b_1 z^{-1} + \cdots}{1 + a_1 z^{-1} + a_2 z^{-2}},$$

   and list the coefficients $\{b_k\}$ and $\{a_k\}$ explicitly.

2. **Pole-Zero Analysis and Stability**

   (a) Find the poles and zeros of $H(z)$.

   (b) Sketch (or describe) the pole-zero plot in the complex z-plane.

   (c) Based on the pole locations, determine if the system is BIBO stable. Justify your answer.

   (d) State whether the system is FIR or IIR and explain why.

3. **Python – Frequency Response Plot**

   (a) Using Python (e.g. `scipy.signal.freqz`), compute and plot the magnitude and phase of $H(e^{j\omega})$ for $0 \le \omega \le \pi$.

   (b) Mark the DC gain and the magnitude at $\omega = \pi$. Identify any resonant peaks or notches.

   (c) Explain how the pole-zero locations from Q2 are reflected in the frequency response characteristics observed in the plot.