

## CALCULATIONS

Calculation of the modulation index ( $\mu$ )

### Time domain representation

When the Time domain representation is used,

$$\mu = \frac{X_{AM/MAX} - X_{AM/MIN}}{X_{AM/MAX} + X_{AM/MIN}}$$

a). Sinusoidal Signal

I. When  $\mu = 0.25$ ,

From figure 01,

$$X_{AM/MAX} = 750 \text{ mV}$$

$$X_{AM/MIN} = 450 \text{ mV}$$

$$\mu = \frac{750 - 450}{750 + 450}$$

$$\underline{\mu = 0.25}$$

II. When  $\mu = 0.5$ ,

From figure 02,

$$X_{AM/MAX} = 900 \text{ mV}$$

$$X_{AM/MIN} = 300 \text{ mV}$$

$$\mu = \frac{900 - 300}{900 + 300}$$

$$\underline{\mu = 0.50}$$

III. When  $\mu = 1.0$ ,

From figure 03,

$$X_{AM/MAX} = 1175 \text{ mV}$$

$$X_{AM/MIN} = 25 \text{ mV}$$

$$\mu = \frac{1175 - 25}{1175 + 25}$$

$$\underline{\mu = 0.96}$$

b) Square Wave Signal

I. When  $\mu = 0.25$ ,

From figure 07,

$$X_{AM/MAX} = 700 \text{ mV}$$

$$X_{AM/MIN} = 425 \text{ mV}$$

$$\mu = \frac{700 - 425}{700 + 425}$$

$$\underline{\mu = 0.24}$$

II. When  $\mu = 0.5$ ,

From figure 09,

$$X_{AM/MAX} = 900 \text{ mV}$$

$$X_{AM/MIN} = 300 \text{ mV}$$

$$\mu = \frac{900 - 300}{900 + 300}$$

$$\underline{\mu = 0.50}$$

III. When  $\mu = 1.0$ ,

From figure 08,

$$X_{AM/MAX} = 1175 \text{ mV}$$

$$X_{AM/MIN} = 25 \text{ mV}$$

$$\mu = \frac{1175 - 25}{1175 + 25}$$

$$\underline{\mu = 0.96}$$

c) Triangular Pulse Signal

I. When  $\mu = 0.25$ ,

From figure 15,

$$X_{AM/MAX} = 750 \text{ mV}$$

$$X_{AM/MIN} = 450 \text{ mV}$$

$$\mu = \frac{750 - 450}{750 + 450}$$

$$\underline{\mu = 0.25}$$

II. When  $\mu = 0.5$ ,

From figure 17,

$$X_{AM/MAX} = 900 \text{ mV}$$

$$X_{AM/MIN} = 300 \text{ mV}$$

$$\mu = \frac{900 - 300}{900 + 300}$$

$$\underline{\mu = 0.50}$$

III. When  $\mu = 1.0$ ,

From figure 16,

$$X_{AM/MAX} = 1175 \text{ mV}$$

$$X_{AM/MIN} = 25 \text{ mV}$$

$$\mu = \frac{1175 - 25}{1175 + 25}$$

$$\underline{\mu = 0.96}$$

## Frequency domain representation

When the frequency domain representation is used,

$$\mu = \frac{2 \times \text{sideband RMS value}}{\text{Carrier RMS voltage}}$$

$P_C$  = Carrier Power

$P_S$  = Sideband Power

$A_{C,RMS}$  = RMS voltage Carrier

$A_{S,RMS}$  = RMS voltage Sideband

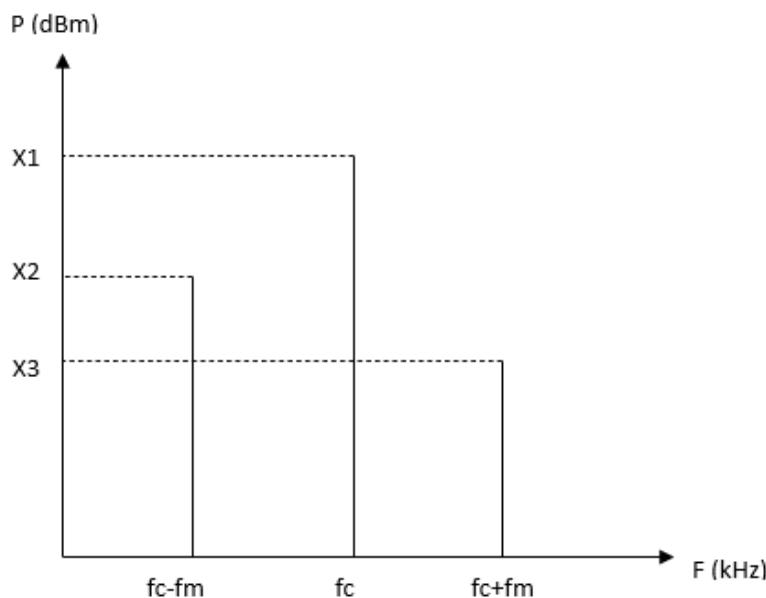
$$P_C = \frac{A_{C,RMS}^2}{2} \quad \dots \dots \dots \quad (1)$$

$$P_S = \frac{\mu^2 A_{C,RMS}^2}{4} \quad \dots \dots \dots \quad (2)$$

$$(2)/(1),$$

$$\frac{P_S}{P_C} = \frac{\frac{\mu^2 A_{C,RMS}^2}{4}}{\frac{A_{C,RMS}^2}{2}}$$

$$\frac{P_S}{P_C} = \frac{\mu^2}{2} \quad \dots \dots \dots \quad (A)$$



$$10 \log_{10} P_C \times 10^3 = x_1$$

$$P_C = 10^{\frac{(x_1-30)}{10}} \quad \dots \quad (3)$$

$$P_S = 10^{\frac{(x_2-30)}{10}} + 10^{\frac{(x_3-30)}{10}} \quad \dots \quad (4)$$

(4)/(3),

$$\frac{P_S}{P_C} = \frac{10^{\frac{(x_2-30)}{10}} + 10^{\frac{(x_3-30)}{10}}}{10^{\frac{(x_1-30)}{10}}} \quad \dots \quad (B)$$

(A)=(B),

$$\begin{aligned} \frac{\mu^2}{2} &= \frac{10^{\frac{(x_2-30)}{10}} + 10^{\frac{(x_3-30)}{10}}}{10^{\frac{(x_1-30)}{10}}} \\ \mu &= \sqrt{\frac{2 \left( 10^{\frac{(x_2-30)}{10}} + 10^{\frac{(x_3-30)}{10}} \right)}{10^{\frac{(x_1-30)}{10}}}} \end{aligned}$$

a). Sinusoidal signal

I. When  $\mu = 0.25$ ,

From figure 04,

$$\begin{aligned} \mu &= \sqrt{\frac{2 \left( 10^{\frac{(-52.9-30)}{10}} + 10^{\frac{(-52.9-30)}{10}} \right)}{10^{\frac{(-34.9-30)}{10}}}} \\ \mu &= 0.25 \end{aligned}$$

II. When  $\mu = 0.5$ ,

From figure 05,

$$\begin{aligned} \mu &= \sqrt{\frac{2 \left( 10^{\frac{(-46.8-30)}{10}} + 10^{\frac{(-46.8-30)}{10}} \right)}{10^{\frac{(-34.9-30)}{10}}}} \\ \mu &= 0.51 \end{aligned}$$

III. When  $\mu = 1.0$ ,

From figure 06,

$$\mu = \sqrt{\frac{2 \left( 10^{\frac{(-40.8-30)}{10}} + 10^{\frac{(-40.8-30)}{10}} \right)}{10^{\frac{(-34.9-30)}{10}}}}$$

$$\mu = 1.00$$

b). Square Wave signal

I. When  $\mu = 0.25$ ,

From figure 11,

$$\mu = \sqrt{\frac{2 \left( 10^{\frac{(-60.5-30)}{10}} + 10^{\frac{(-60.5-30)}{10}} \right)}{10^{\frac{(-42.9-30)}{10}}}}$$

$$\mu = 0.\underline{\underline{26}}$$

II. When  $\mu = 0.5$ ,

From figure 13,

$$\mu = \sqrt{\frac{2 \left( 10^{\frac{(-44.5-30)}{10}} + 10^{\frac{(-44.5-30)}{10}} \right)}{10^{\frac{(-34.6-30)}{10}}}}$$

$$\mu = 0.64$$

III. When  $\mu = 1.0$ ,

From figure 12,

$$\mu = \sqrt{\frac{2 \left( 10^{\frac{(-39-30)}{10}} + 10^{\frac{(-39-30)}{10}} \right)}{10^{\frac{(-19.6-30)}{10}}}}$$

$$\mu = 1.21$$

c). Triangular pulse signal

I. When  $\mu = 0.25$ ,

From figure 19,

$$\mu = \sqrt{\frac{2 \left( 10^{\frac{(-53.5-30)}{10}} + 10^{\frac{(-54.5-30)}{10}} \right)}{10^{\frac{(-34.6-30)}{10}}}}$$
$$\mu = \underline{\underline{0.20}}$$

II. When  $\mu = 0.5$ ,

From figure 21,

$$\mu = \sqrt{\frac{2 \left( 10^{\frac{(-48.5-30)}{10}} + 10^{\frac{(-48.5-30)}{10}} \right)}{10^{\frac{(-14-30)}{10}}}}$$
$$\mu = \underline{\underline{0.40}}$$

III. When  $\mu = 1.0$ ,

From figure 20,

$$\mu = \sqrt{\frac{2 \left( 10^{\frac{(-42.5-30)}{10}} + 10^{\frac{(-42.5-30)}{10}} \right)}{10^{\frac{(-14-30)}{10}}}}$$
$$\mu = \underline{\underline{0.81}}$$

### Efficiency of the modulated signal

$$Efficiency = \frac{Power\ of\ the\ sidebands}{Total\ power}$$

For a sinusoidal wave,

$$\mu = 0.25,$$

$$\eta = \frac{10^{\frac{(-52.9-30)}{10}} + 10^{\frac{(-52.9-30)}{10}}}{10^{\frac{(-52.9-30)}{10}} + 10^{\frac{(-52.9-30)}{10}} + 10^{\frac{(-34.9-30)}{10}}}$$

$$\underline{\eta = 3.07\%}$$

$$\mu = 0.5,$$

$$\eta = \frac{\left(10^{\frac{(-46.8)}{10}} + 10^{\frac{(-46.8)}{10}}\right)}{10^{\frac{(-46.8)}{10}} + 10^{\frac{(-46.8)}{10}} + 10^{\frac{(-34.9)}{10}}}$$

$$\underline{\eta = 11.44\%}$$

$$\mu = 1.0,$$

$$\eta = \frac{\left(10^{\frac{(-40.8)}{10}} + 10^{\frac{(-40.8)}{10}}\right)}{10^{\frac{(-40.8)}{10}} + 10^{\frac{(-40.8)}{10}} + 10^{\frac{(-34.9)}{10}}}$$

$$\underline{\eta = 33.95\%}$$

### Part 3: Simulation of AM Demodulation using a Synchronous Detector

```
# Lab 2: AM Synchronous Demodulation Simulation
import numpy as np
import matplotlib.pyplot as plt
from scipy.signal import butter, filtfilt

# Parameters
fm = 1000
fc = 20000
fs = 400000
duration = 0.005
t = np.linspace(0, duration, int(fs*duration), endpoint=False)

# 1. Generate Basic Signals (Part 1 of Procedure)
Am1 = 1.0
Ac = 2.0
m_t_basic = Am1 * np.cos(2 * np.pi * fm * t)
c_t = Ac * np.cos(2 * np.pi * fc * t)

# 2. Generate AM Signal (Part 2 of Procedure)
mu = 0.8
Am_mod = mu * Ac
m_t_mod = Am_mod * np.cos(2 * np.pi * fm * t)

# AM Signal
s_t = (Ac + m_t_mod) * np.cos(2 * np.pi * fc * t)

# 3. Synchronous Demodulation
local_carrier = np.cos(2 * np.pi * fc * t)
v_t = s_t * local_carrier

# Step B: Low Pass Filter (Specifics: 5kHz cutoff, 2nd order)
def butter_lowpass_filter(data, cutoff, fs, order=5):
    nyq = 0.5 * fs
    normal_cutoff = cutoff / nyq
    b, a = butter(order, normal_cutoff, btype='low', analog=False)
    y = filtfilt(b, a, data)
    return y

# Cutoff = 5 kHz, Order = 2
demodulated_raw = butter_lowpass_filter(v_t, 5000, fs, order=2)
# Remove DC Component (0.5 * Ac) to recover AC message
demodulated_ac = demodulated_raw - np.mean(demodulated_raw)
# Scale to match original amplitude (Factor of 0.5 introduced by mixing)
demodulated_scaled = demodulated_ac * 2
# Plotting
plt.figure(figsize=(12, 14))

# 1. Message Signal
plt.subplot(5, 1, 1)
plt.plot(t * 1000, m_t_basic)
plt.title('Message Signal $m(t)$')
plt.ylabel('Amplitude (V)')
plt.grid(True)

# 2. Carrier Signal
plt.subplot(5, 1, 2)
plt.plot(t * 1000, c_t)
plt.title('Carrier Signal $c(t)$')
plt.ylabel('Amplitude (V)')
plt.grid(True)

# 3. AM Signal
plt.subplot(5, 1, 3)
plt.plot(t * 1000, s_t)
plt.plot(t * 1000, Ac + m_t_mod, 'g', linewidth=1, label='Upper Envelope')
plt.title('AM Signal $s(t)$ ($\mu=0.8$)')
plt.ylabel('Amplitude (V)')
plt.legend(loc='upper right')
plt.grid(True)

# 4. Product Signal v(t)
plt.subplot(5, 1, 4)
plt.plot(t * 1000, v_t)
plt.title('Product Signal $v(t) = s(t) \cdot c_{\{LO\}}(t)$')
plt.ylabel('Amplitude (V)')
plt.grid(True)
```

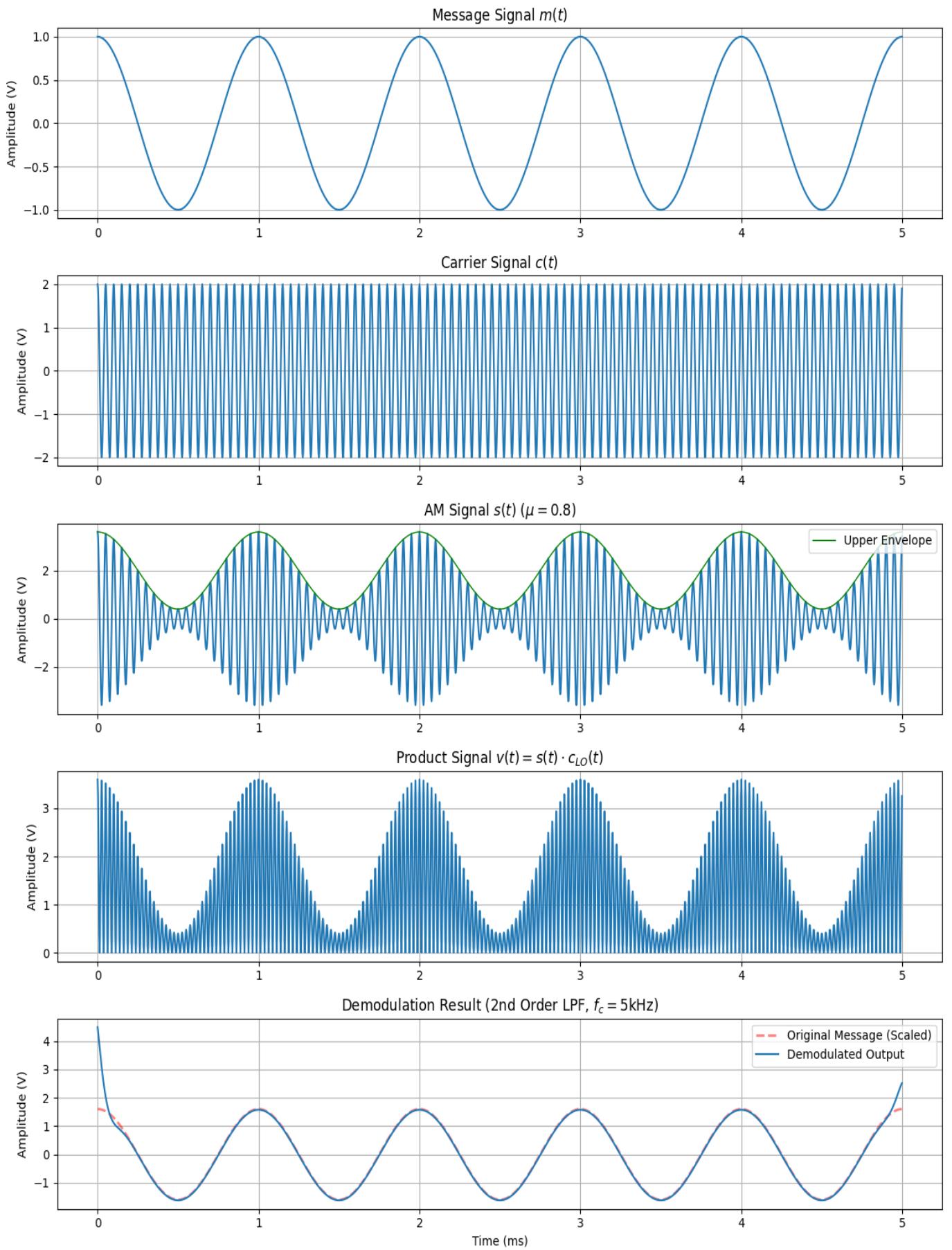


Figure 25: Time domain representation of message signal, carrier signal, amplitude modulated signal, demodulated output signal and filter output signal