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week 2 - activity 01

1. $$\frac{d^n y(t)}{dt^n} + a_{n-1}\frac{d^{n-1} y(t)}{dt^{n-1}} + a_{n-2}\frac{d^{n-2} y(t)}{dt^{n-2}} + \cdots + a_1\frac{dy(t)}{dt} + a_0 y(t) = b_m\frac{d^m u(t)}{dt^m} + b_{m-1}\frac{d^{m-1}u}{dt}$$

$$+ \cdots + b_1\frac{du(t)}{dt} + b_0 u(t)$$

$$\cancel{\frac{Y(s)}{U(s)} = \frac{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}}$$

$$\frac{U(s)}{W(s)} \cdot \frac{W(s)}{Y(s)} \Rightarrow \cancel{\frac{U(s)}{W(s)} = s^n + a_{n-1}}$$

$$\frac{Y(s)}{U(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}$$

$$= \frac{1}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0} \times b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0$$

$$= \frac{W(s)}{U(s)} \cdot \frac{Y(s)}{W(s)}$$

$$\frac{W(s)}{U(s)} = \frac{1}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}, \quad \frac{Y(s)}{W(s)} = b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0$$

$$\frac{dw^n}{dt^n} + a_{n-1}\frac{dw^{n-1}}{dt^{n-1}} + \cdots + a_1\frac{dw}{dt} + a_0 w = u(t). \quad \text{— (1)}.$$

$$y(t) = b_m\frac{d^m w}{dt^m} + b_{m-1}\frac{d^{m-1} w}{dt^{m-1}} + \cdots + b_1\frac{dw}{dt} + b_0 w \quad \text{— (2)}.$$

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$x_1(t) = w(t)$

$x_2(t) = \frac{dw(t)}{dt}$

$\vdots$

$x_n(t) = \frac{d^{n-1}w(t)}{dt^{n-1}}$

$\Rightarrow$

$x_2(t) = \frac{dx_1(t)}{dt}$

$x_3(t) = \frac{dx_2(t)}{dt}$

$\vdots$

$\frac{dx_{n-1}}{dt^{n-1}} = x_n(t)$

from ①

$$\frac{dx_n(t)}{dt} = -a_{n-1}x_{n-1}(t) - a_{n-2}x_{n-2}(t) + \cdots + -a_1 x_1(t) + u(t).$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ & & & \vdots & & \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ -a_0 & -a_1 & \cdots & & -a_{n-2} & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} b_0 & b_1 & \cdots & b_m \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}$$

here $n, m \in \mathbb{Z}$ $\quad m \leq n$

et.

2). a). $$G(s) = \frac{s+5}{(s+10)(s^2+s+2)}.$$

order of the system = 3 CE $\Rightarrow$ $(s+10)(s^2+s+2)=0$.

b) from CE

$s+10=0$, $s^2+s+2=0$.

$s=-10$, $s=\dfrac{-1\pm\sqrt{7}j}{2}$

Therefore poles :- $s=-10$, $s=-\frac{1}{2}+\frac{\sqrt{7}}{2}j$, $s=-\frac{1}{2}-\frac{\sqrt{7}}{2}j$

c) zeros :- $s=-5$

d). dominant poles :- $s=-\frac{1}{2}+\frac{\sqrt{7}}{2}j$, $s=-\frac{1}{2}-\frac{\sqrt{7}}{2}j$

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e). for long time behaviour $s << 10$

$\therefore \quad s+10 \approx 10$

$$\therefore \quad G(s) = \frac{s+5}{10(s^2+s+2)}$$

f). $$G(s) = \frac{s+5}{(s+10)(s^2+s+2)}$$

$$= \frac{A}{s+10} + \frac{Bs+C}{(s^2+s+2)}$$

$$s+5 = A(s^2+s+2) + (Bs+C)(s+10)$$

$A+B=0, \quad 2A+10C=5, \quad A = \frac{-5}{92}, \quad B = \frac{5}{92}, \quad C = \frac{47}{92}$

$$G(s) = \frac{-5}{92(s+10)} + \frac{5s+47}{(s^2+s+2)}$$

$$= \frac{-5}{92(s+10)} + \frac{5s+47}{\left[\left(s+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{7}}{4}\right)^2\right]}$$

impulse response

$$h(t) = \frac{-5}{92}e^{-10t} + \frac{5}{92}e^{-t/2}\cos\left(\frac{\sqrt{7}}{2}t\right) + \frac{89}{92}\frac{1}{\sqrt{7}}e^{-t/2}\sin\left(\frac{\sqrt{7}}{2}t\right)$$

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g). $G(s) = \dfrac{s+5}{10(s^2+s+2)}$

$$h(t)_{red} = \frac{1}{10}\left[\frac{s+5}{(s+\frac{1}{2})^2+(\frac{\sqrt{7}}{2})^2}\right]$$

$$= \frac{1}{10}\left[\frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2+(\frac{\sqrt{7}}{2})^2} + \frac{9/2}{(s+\frac{1}{2})^2+(\frac{\sqrt{7}}{2})^2}\right]$$

$$= \frac{1}{10}e^{-\frac{1}{2}t}\cos\left(\frac{\sqrt{7}}{2}t\right) + \frac{9}{10\sqrt{7}}e^{-\frac{1}{2}t}\sin\left(\frac{\sqrt{7}}{2}t\right)$$

$$h(t)_{red} = \frac{1}{10}e^{-\frac{1}{2}t}\cos\left(\frac{\sqrt{7}}{2}t\right) + \frac{9}{10\sqrt{7}}e^{-\frac{1}{2}t}\sin\left(\frac{\sqrt{7}}{2}t\right)$$

h). $h(t)_{red}$ and $h(t)$ have same transient response with different amplitudes however $h(t)_{red}$ model lacks the $\frac{-5}{92}e^{-10t}$ part.