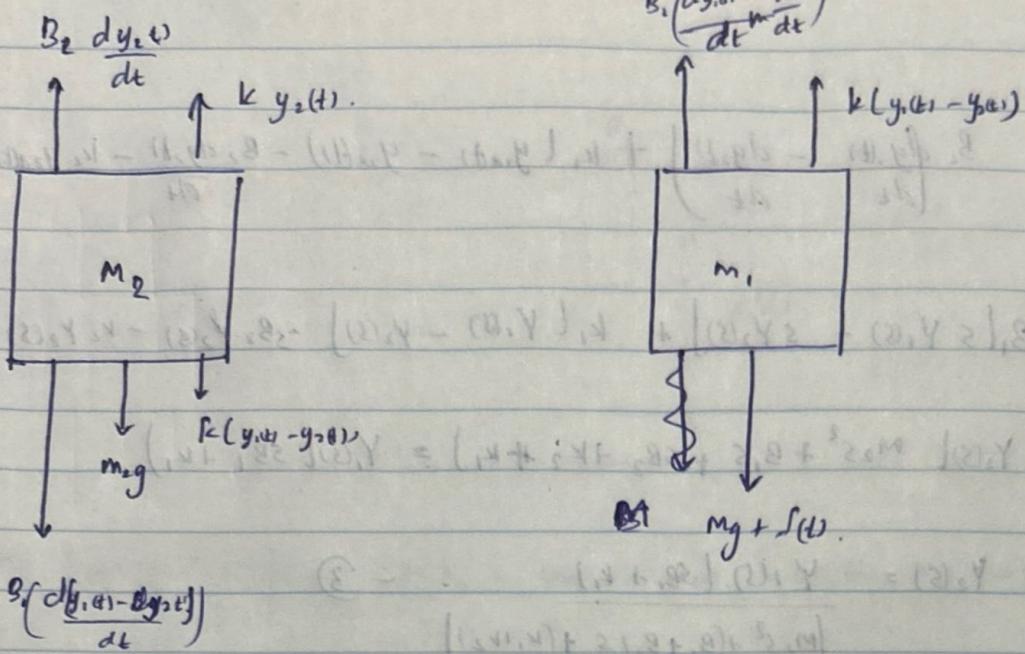


1.



at equilibrium.

$$M_2 g = \cancel{k y_2^0} \quad M_2 g = k y_2^0 - k_1 (y_1^0 - y_2^0)$$

$$M_1 g = k_1 (y_1^0 - y_2^0)$$

$$y_1(t) = y_1^0 + y_1^1(t) \quad \dots \quad (1)$$

$$y_2(t) = y_2^0 + y_2^1(t) \quad \dots \quad (2)$$

$$\frac{dy_1(t)}{dt} = \frac{dy_1^1(t)}{dt}, \quad \frac{dy_2(t)}{dt} = \frac{dy_2^1(t)}{dt} \quad \text{Since } y_1^0 \text{ and } y_2^0 \text{ are constant.}$$

$$f = m_1 a \quad \text{for } M_2, a = \frac{1}{2} M$$

$$M_2 g + B_2 \left(\frac{dy_2(t) - dy_1(t)}{dt} \right) + k(y_{1(t)} - y_{2(t)}) - B_2 \frac{dy_2^1(t)}{dt} - k y_2^1(t) = M_2 \frac{d^2 y_2^1(t)}{dt^2}$$

$$M_2 g + B_2 \left(\frac{dy_1(t)}{dt} - \frac{dy_2(t)}{dt} \right) + k_1 (y_{1(t)} + y_{1(t)}^1 - y_{2(t)}^1 - y_{2(t)}) - B_2 \frac{dy_2^1(t)}{dt} - k y_2^1(t) = M_2 \frac{d^2 y_2^1(t)}{dt^2}$$

$$M_2 g - k_1 (y_{1(t)}^1 - y_{2(t)}^1) +$$

$$M_2 g - [k_2 y_2^1(t) - k_1 (y_{1(t)}^1 - y_{2(t)}^1)] + B_2 \left(\frac{dy_1(t)}{dt} - \frac{dy_2(t)}{dt} \right) + k_1 (y_{1(t)} - y_{2(t)}) - k_2 y_{2(t)} = M \frac{d^2 y_2^1(t)}{dt^2}$$

$$B_1 \left(\frac{dy_1(t)}{dt} - \frac{dy_2(t)}{dt} \right) + k_1 (y_{1d(t)} - y_{2d(t)}) - B_2 \frac{dy_2(t)}{dt} - k_2 y_{2d(t)} = M_2 \frac{d^2 y_2(t)}{dt^2}$$

$$B_1 [s Y_1(s) - y_{1d(s)}] + k_1 [Y_1(s) - y_{2d(s)}] - s B_2 Y_2(s) - k_2 Y_2(s) = M_2 s^2 Y_2(s)$$

$$Y_2(s) [M_2 s^2 + B_1 s + s B_2 + k_2 + k_1] = Y_1(s) [s B_1 + k_1]$$

$$Y_2(s) = \frac{Y_1(s) [s B_1 + k_1]}{[M_2 s^2 + (B_1 + B_2) s + (k_1 + k_2)]} \quad \text{--- (3)}$$

for $M_1 \downarrow f = ma$.

$$M_1 g + f(t) + k_1 (y_{1d(t)} - y_{2d(t)}) - B_1 \left(\frac{dy_1(t)}{dt} - \frac{dy_2(t)}{dt} \right) = M_1 \frac{d^2 y_1(t)}{dt^2}$$

$$M_1 g + f(t) - M_1 \dot{y}_1 - k_1 (y_{1d(t)} - y_{2d(t)}) - B_1 \left(\frac{dy_1(t)}{dt} - \frac{dy_2(t)}{dt} \right) = M_1 \frac{d^2 y_1(t)}{dt^2}$$

$$f(s) - k_1 [Y_1(s) - Y_2(s)] - B_1 s [Y_1(s) - Y_2(s)] = s^2 M_1 Y_1(s)$$

$$f(s) = [M_1 s^2 + B_1 s + k_1] Y_1(s) + Y_2(s) [k_1 + B_1 s]$$

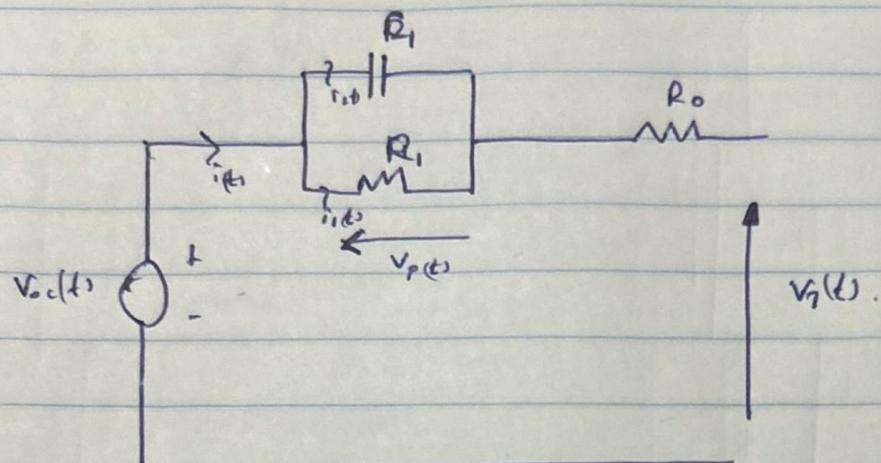
from (3)

$$f(s) = \frac{[M_1 s^2 + B_1 s + k_1] Y_1(s) - (k_1 + B_1 s) [Y_1(s) (s B_1 + k_1)]}{[M_2 s^2 + (B_1 + B_2) s + (k_1 + k_2)]}$$

$$f(s) = Y_1(s) \frac{[M_1 s^2 + B_1 s + k_1] [M_2 s^2 + (B_1 + B_2) s + (k_1 + k_2)] - (s B_1 + k_1)^2}{M_2 s^2 + (B_1 + B_2) s + (k_1 + k_2)}$$

$$\frac{Y_1(s)}{f(s)} = \frac{M_2 s^2 + (B_1 + B_2) s + (k_1 + k_2)}{[M_1 s^2 + B_1 s + k_1] [M_2 s^2 + (B_1 + B_2) s + (k_1 + k_2)] - (s B_1 + k_1)^2}$$

2)



$$V_{oc}(t) = V_p(t) - i(t) R_2 = V_1(t). \Rightarrow k \text{soc}(t) - V_2(t) - i(t) R_2 = V_1(t).$$

$$i(t) = i_1(t) + i_2(t).$$

$$i(t) = \frac{V_p(t)}{R} + C \frac{dV_p(t)}{dt}$$

$$\frac{dV_p(t)}{dt} = -\frac{V_p(t)}{RC} + \frac{i(t)}{C}$$

$$\frac{d\text{soc}(t)}{dt} = \frac{i(t)}{Q}$$

$$\begin{bmatrix} \frac{dV_p(t)}{dt} \\ \frac{d\text{soc}(t)}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_p(t) \\ \text{soc}(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{C} \\ \frac{1}{Q} \end{bmatrix} i(t)$$

$$V_1(t) = [-1 \quad k] \begin{bmatrix} V_p(t) \\ \text{soc}(t) \end{bmatrix} + (-R_2) i(t)$$