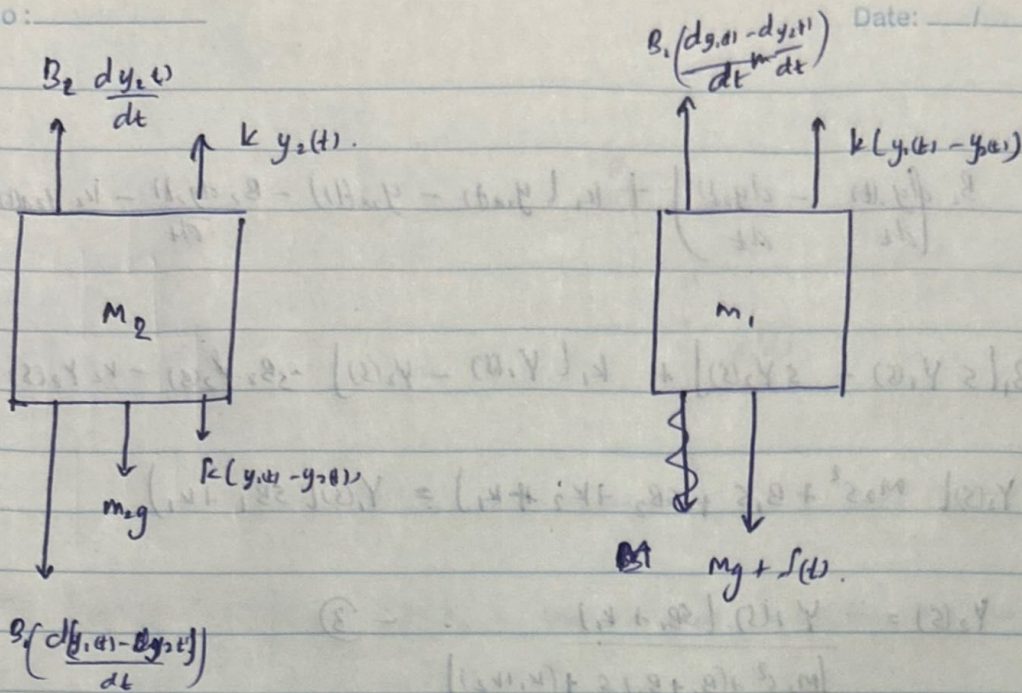


1.



at equilibrium.

$$m_2 g = k y_{20}$$

$$m_2 g = k_2 y_2^0 - k_1 (y_1^0 - y_2^0)$$

$$m_1 g = k_1 (y_1^0 - y_2^0)$$

$$y_1(t) = y_1^0 + q_1(t) \quad \text{--- (1)}$$

$$y_2(t) = y_2^0 + q_2(t) \quad \text{--- (2)}$$

$$\frac{dy_1(t)}{dt} = \frac{dq_1(t)}{dt}, \quad \frac{dy_2(t)}{dt} = \frac{dq_2(t)}{dt} \quad \text{Since } y_1^0 \text{ and } y_2^0 \text{ are constant.}$$

 $\Sigma F = ma$ for m_2

$$m_2 g + B_1 \left(\frac{dy_1(t)}{dt} - \frac{dy_2(t)}{dt} \right) + k(y_2(t) - y_1(t)) - B_2 \frac{dy_2(t)}{dt} - k y_2(t) = m_2 \frac{d^2 y_2(t)}{dt^2}$$

$$m_2 g + B_1 \left(\frac{dy_1(t)}{dt} - \frac{dy_2(t)}{dt} \right) + k(y_1^0 + q_1(t) - y_2^0 - q_2(t)) - B_2 \frac{dq_2(t)}{dt} - k(y_2^0 + q_2(t)) = m_2 \frac{d^2 q_2(t)}{dt^2}$$

$$m_2 g - k_1 (y_1^0 - y_2^0) +$$

$$m_2 g - (k_2 y_2^0 - k_1 (y_1^0 - y_2^0)) + B_1 \left(\frac{dy_1(t)}{dt} - \frac{dy_2(t)}{dt} \right) + k_1 (y_1(t) - y_2(t)) - k_2 y_2(t) = m_2 \frac{d^2 q_2(t)}{dt^2}$$

$$B_1 \left(\frac{dy_1(t)}{dt} - \frac{dy_2(t)}{dt} \right) + k_1 (y_1(t) - y_2(t)) - B_2 \frac{dy_2(t)}{dt} - k_2 y_2(t) = m_2 \frac{d^2 y_2(t)}{dt^2}$$

$$B_1 [s Y_1(s) - s Y_2(s)] + k_1 [Y_1(s) - Y_2(s)] - s B_2 Y_2(s) - k_2 Y_2(s) = m_2 s^2 Y_2(s)$$

$$Y_2(s) [m_2 s^2 + B_1 s + s B_2 + k_2 + k_1] = Y_1(s) [s B_1 + k_1]$$

$$Y_2(s) = \frac{Y_1(s) [s B_1 + k_1]}{[m_2 s^2 + (B_1 + B_2) s + (k_1 + k_2)]} \quad - (3)$$

for $m_1 \downarrow f = ma$.

$$m_1 g + f(t) + k_1 (y_1(t) - y_2(t)) - B_1 \left(\frac{dy_1(t)}{dt} - \frac{dy_2(t)}{dt} \right) = m_1 \frac{d^2 y_1(t)}{dt^2}$$

$$m_1 g + f(t) - m_1 g - k_1 (y_1(t) - y_2(t)) - B_1 \left(\frac{dy_1(t)}{dt} - \frac{dy_2(t)}{dt} \right) = m_1 \frac{d^2 y_1(t)}{dt^2}$$

$$f(s) - k_1 [Y_1(s) - Y_2(s)] - B_1 s [Y_1(s) - Y_2(s)] = s^2 m_1 Y_1(s)$$

$$f(s) = [m_1 s^2 + k_1 + B_1 s] Y_1(s) + Y_2(s) [k_1 + B_1 s]$$

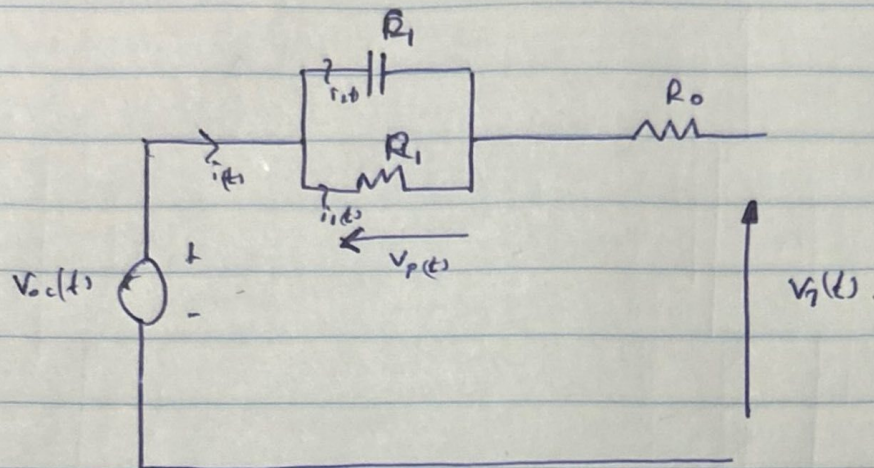
from (3)

$$f(s) = \frac{[m_1 s^2 + B_1 s + k_1] Y_1(s) - (k_1 + B_1 s) [Y_1(s) (s B_1 + k_1)]}{[m_2 s^2 + (B_1 + B_2) s + (k_1 + k_2)]}$$

$$f(s) = Y_1(s) \frac{[m_1 s^2 + B_1 s + k_1] [m_2 s^2 + (B_1 + B_2) s + (k_1 + k_2)] - (s B_1 + k_1)^2}{m_2 s^2 + (B_1 + B_2) s + (k_1 + k_2)}$$

$$\frac{Y_1(s)}{f(s)} = \frac{m_2 s^2 + (B_1 + B_2) s + (k_1 + k_2)}{[m_1 s^2 + B_1 s + k_1] [m_2 s^2 + (B_1 + B_2) s + (k_1 + k_2)] - (s B_1 + k_1)^2}$$

2).



$$V_{oc}(t) = V_p(t) - i(t) R_o = V_7(t) \Rightarrow K_{SOC}(t) - V_p(t) - i(t) R_o = V_7(t)$$

$$i(t) = i_1(t) + i_2(t)$$

$$i(t) = \frac{V_p(t)}{R} + C \frac{dV_p(t)}{dt}$$

$$\frac{dV_p(t)}{dt} = -\frac{V_p(t)}{RC} + \frac{i(t)}{C}$$

$$\frac{dSOC(t)}{dt} = \frac{i(t)}{Q}$$

$$\begin{bmatrix} \frac{dV_p(t)}{dt} \\ \frac{dSOC(t)}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_p(t) \\ SOC(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{C} \\ \frac{1}{Q} \end{bmatrix} i(t)$$

$$V_7(t) = \begin{bmatrix} -1 & R \end{bmatrix} \begin{bmatrix} V_p(t) \\ SOC(t) \end{bmatrix} + \begin{bmatrix} -R_0 \end{bmatrix} i(t)$$