Tutoral 02

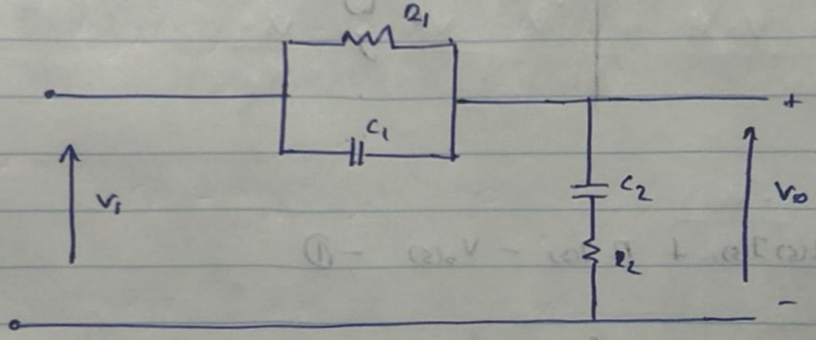
1. $$\frac{d^2y}{dt^2} + \frac{3dy}{dt} + 2y = x + \frac{dx}{dt}$$

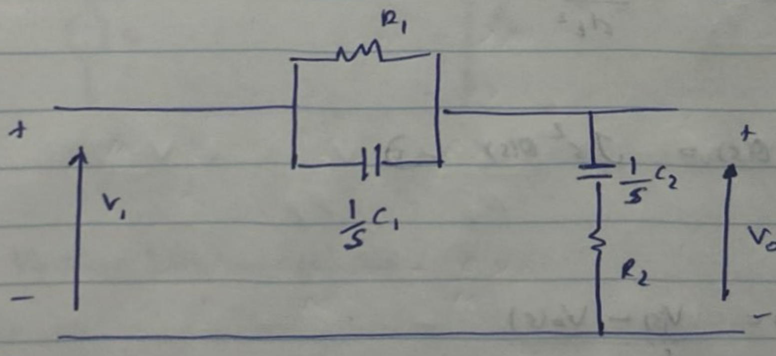
$$s^2 Y(s) + 3s Y(s) + 2Y(s) = X(s) + sX(s).$$

$$(s^2 + 3s + 2)\,Y(s) = X(s)(1+s)$$

$$\frac{Y(s)}{X(s)} = \frac{(s+1)}{(s+1)(s+2)}$$

2.





$$Z_1 = \left[\frac{1}{R_1} + \frac{s}{C_1}\right]^{-1} \qquad Z_2 = \frac{1}{s}C_2 + R_2$$

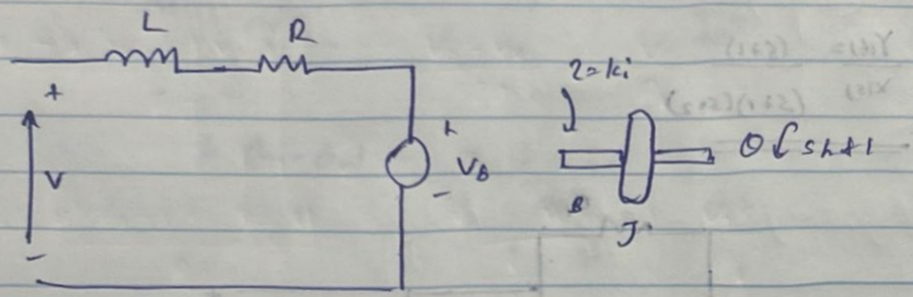
$$Z_1 = \frac{C_1R_1}{C_1 + R_1 s}$$

$$V_o(s) = \frac{V_1(s)Z_2}{(Z_1+Z_2)} = V_1(s) \times \frac{\frac{C_2+R_2s}{s}}{\frac{(C_2+R_2s)}{s} + \frac{C_1R_1}{(C_1+R_1s)}}$$

$$\cancel{\frac{V_o(s)}{V_i(s)}} \quad \& \quad \frac{V_o(s)}{V_i(s)} = \frac{\left[\frac{C_2 + R_2 s}{s}\right] s(C_1 + R_1 s)}{(C_2 + R_2 s)(C_1 + R_1 s) + sC_1R_1}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{C_1C_2 + s(C_1R_2 + C_2R_1) + s^2R_1R_2}{C_1C_2 + s(C_1R_1 + C_1R_2 + C_2R_1) + s^2R_1R_2}$$

3.



$$V(s) = sL(s)I(s) + RI(s) - V_b(s) \quad -①$$

$$T - B\frac{d\theta(t)}{dt} = J\frac{d^2\theta(t)}{dt^2}$$

$$kI(s) - sB\theta(s) = Js^2\theta(s) \quad -②$$

from ①

$$I(s) = \frac{V(s) - V_b(s)}{(R + sL(s))}$$

$$k\left[\frac{V_s - V_b(s)}{R + sL(s)}\right] = \theta(s)\left(s^2J + sB\right)$$

$\cancel{K_b}$ V.

but $v_b(t) = \cancel{k\phi}\; k_b \dfrac{d\theta}{dt}$

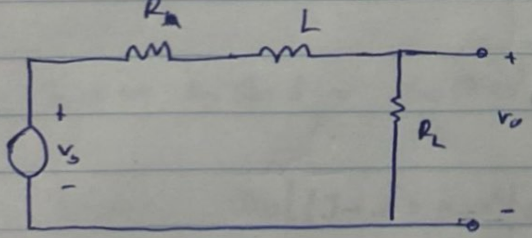
$$V_b(s) = s k_b \theta(s)$$

$$k_t\left[\frac{V_s - sk_b\theta(s)}{R+sL}\right] = \theta(s)(s^2J+sB)$$

$$k_t V(s) = \theta(s)\left\{(s^2J+sB)(R+sL) + sk_bk_t\right\}$$

$$\frac{\theta(s)}{V(s)} = \frac{k_t}{(s^2J+sB)(R+sL) + sk_bk_t}$$

4.



$$V_b = k_f \frac{d\theta}{dt}$$

$$V_b(s) = k_f s\theta(s)$$

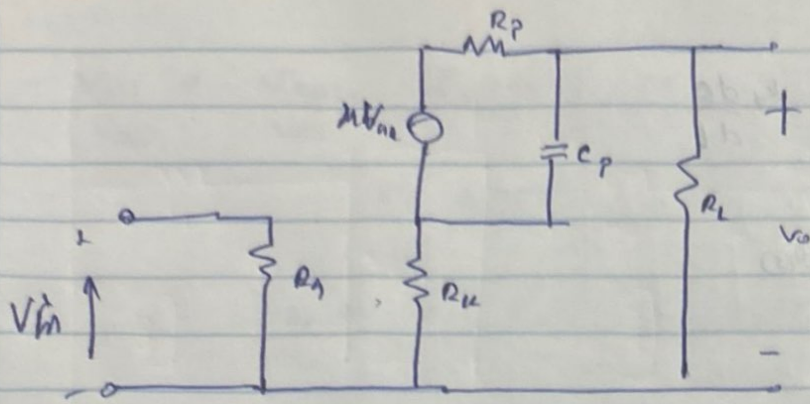
$$\cancel{V_b(s) - [R_b \quad sL]\, I(s)}$$

$$V_o(s) = V_b(s) \cdot \frac{R_L}{(R_L+R_m+sL)}$$

$$V_o(s) = sk_f\theta(s) \cdot \frac{R_L}{R+sL}$$

$$\frac{V_o(s)}{\theta(s)} = \frac{sR_Lk_f}{R+sL}$$

Q5)



$V_{GK} = V_{in} - V_K$

$$\frac{(1-\mu)V_K + V_{in}\mu}{R_P} = \frac{V_o - V_K}{2P} + \frac{V_o}{R_L}$$

$$\frac{(1-\mu)V_K(s) + V_{in}(s)\mu}{R_P} = \frac{V_o(s) - V_K(s)}{\frac{1}{C_o s}} + \frac{V_o(s)}{R_L}$$

$$\frac{V_o(s)}{R_L} = \frac{-V_K(s)}{R_K}$$

$$V_{Ks} = -\left(\frac{R_K}{R_L}\right)V_o(s)$$

$$\frac{(1-\mu)V_K(s)}{R_P} + C_{P}s\,V_K(s) = \frac{C_P V_o(s)}{R_L} - \frac{V_{in}(s)}{R_P} + V_o(s)Cs$$

$$-\left(\frac{R_K}{R_L}\right) V_o(s)\left[\frac{1-\mu) + C_P R_P s}{R_P}\right] = \frac{V_o(s)}{R_L} - \frac{V_{in}(s)\mu}{R_P} + V_o(s)C_P s$$

$$\frac{\mu V_{in}(s)}{R_P} = V_o(s)\left[\frac{1}{R_L} + \frac{R_K}{R_L}\left[\frac{1-\mu + C_P R_P s}{R_{in}}\right] + C_P s\right]$$

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$$\frac{V_o(s)}{V_{in}(s)} = \frac{\mu R}{R_p[R_p + R_k(1-\mu) + C_p R_p s)] + R_L R_p C_p s)}$$

$$= \frac{\mu R_L}{R_k(1-\mu) + R_o[1 + C_p s(R_k + R_L)]}$$

6) a)

$$\frac{1}{2} m_p v^2 = \frac{1}{2} J\omega^2$$

$$J = \frac{M_p v^2}{\omega^2}$$

$$J = M_p R^2$$

b)

$$T - B_m \frac{d\theta_o}{dt} = J_m \frac{d^2\theta}{dt^2} + M_p R^2 \frac{d^2\theta_o}{dt^2}$$

c

$$T_o(s) - B_m \theta(s) s = J_m \theta(s) s^2 + M_p R^2 s^2 \theta(s)$$

$$T_o(s) = \theta(s)[(J_m + m_p R^2) s^2 + B_m s)$$

$$\frac{\theta_o(s)}{T_o(s)} = \frac{1}{s^2(J_m + m_p R^2) + B_m s}$$

d)

$$x(t) = R\,\theta_o(t)$$

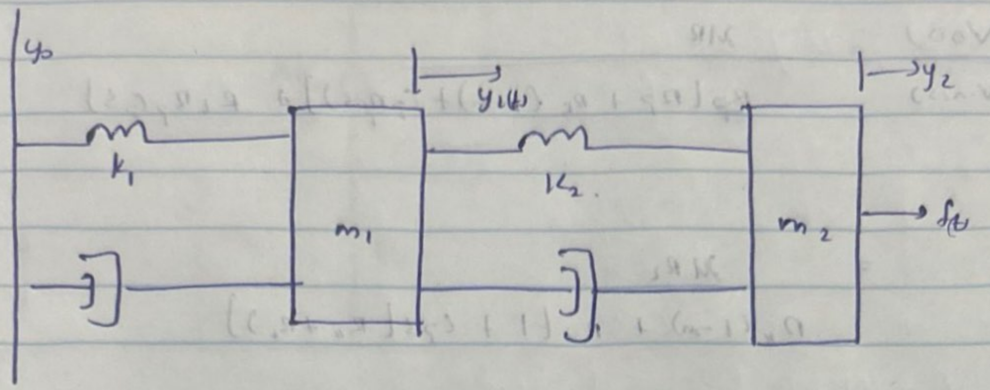
e)

$$\frac{X(s)}{\theta_o(s)} = R$$

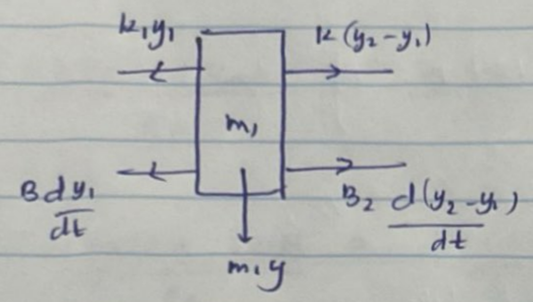
f)

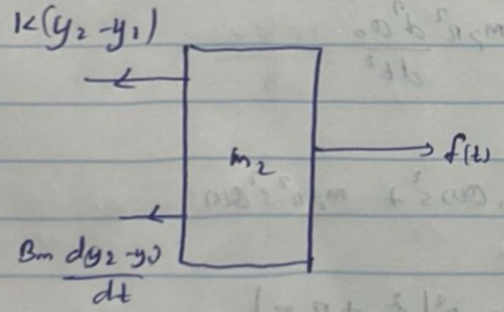
$$\frac{X(s)}{T_o(s)} = \frac{X(s)}{\theta_o(s)} \cdot \frac{\theta_o(s)}{T_o(s)} = \frac{R}{s^2(J_m + M_p R^2) + B_m s}$$

7)



b)





2)

For m_1

$\rightarrow$

$$k(y_2(t) - y_1(t)) + B_2 \frac{d(y_2(t) - y_1(t))}{dt} - B\frac{dy_1}{dt} - k_1 y_1 = M_1 \frac{d^2 y_1(t)}{dt^2}$$

for m_2

$\rightarrow$

$$f(t) - k(y_2(t) - y_1(t)) - B_m \frac{d(y_2(t) - y_1(t))}{dt} = M_2 \frac{d^2 y_2(t)}{dt^2}$$

3). $y_1(t) = x_1(t)$. $\frac{dy_1(t)}{dt} = x_2(t)$, $y_2(t) = x_3(t)$, $\frac{dy_2(t)}{dt} = x_4(t)$

$\frac{dx_1(t)}{dt} = x_2(t)$, $\frac{dx_3(t)}{dt} = x_4(t)$.

$$K_2[x_3(t) - x_1(t)] + B_2[x_4(t) - x_2(t)] - B_1 x_2(t) - K_1 x_1(t) = m_1 \frac{dx_2(t)}{dt}$$

$$\frac{dx_2(t)}{dt} = x_1(t)\left[\frac{-K_2 - K_1}{m_1}\right] + x_2(t)\left[\frac{-B_2 - B_1}{m_1}\right] + \frac{K_2 x_3(t)}{m} + \frac{B_2 x_4(t)}{m}.$$

$$f(t) - K[x_3(t) - x_1(t)] - B_m \cancel{d x_3(t)} [x_4 t - x_2(t)] = m_2 \frac{dx_4(t)}{dt}.$$

$$\frac{dx_4(t)}{dt} = x_3(t)\frac{B_m}{m_2} + x_2(t)\frac{B_m}{m_2} - \frac{K}{m_2}x_3(t) - \frac{B_m x_4(t)}{m_2} + \frac{f(t)}{m_2}$$

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \\ \frac{dx_3}{dt} \\ \frac{dx_4}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-(K_1+K_2)}{m_1} & \frac{-(B_2+B_1)}{m_1} & \frac{K_2}{m_1} & \frac{B_2}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{K}{m_2} & \frac{B_m}{m_2} & -\frac{K}{m_2} & -\frac{B_m}{m_2} \end{bmatrix} \begin{bmatrix} x \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{M_2} \end{bmatrix} f(t)$$

$\dot{x}(t) = Ax + Bf(t)$, $y = Cx(t) + DV$