



## DT0060 Design tip

### Exploiting the gyroscope to update tilt measure and e-compass

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Main components	
LSM6DS3	iNEMO inertial module: 3D accelerometer and 3D gyroscope
LSM6DS3H	iNEMO inertial module: 3D accelerometer and 3D gyroscope
LSM6DS33	iNEMO inertial module: 3D accelerometer and 3D gyroscope
LSM6DS0	iNEMO inertial module: 3D accelerometer and 3D gyroscope
LSM330	iNEMO inertial module: 3D accelerometer and 3D gyroscope

#### Purpose and benefits

This design tip explains how to exploit gyroscope data to update tilt measurements (Roll and Pitch angles) and e-compass (Yaw angle). A quaternion implementation is also shown, which does not suffer from singularity problem, also known as gymbal-lock.

Benefits:

- Enhanced functionality with respect to simple 6-axis Acc+Mag data fusion which cannot be performed when high-g motion or magnetic anomalies are present.
- Reduction of firmware footprint with respect to using the full-blown data fusion provided by osxMotionFX library, see Open.MEMS in design Support Material paragraph.
- Short essential implementation, which enables easy customization and enhancement by end-user (osxMotionFX is available only in binary format, not as source code)
- Easy to use on every microcontroller (osxMotionFX can only be run on STM32 and only when the proper license has been issued by Open.MEMS license server).

#### Description of Euler angle implementation

Step 1: Compute angle derivatives  $\Phi'$  /  $\Theta'$  /  $\Psi'$  based on current angles  $\Phi$  /  $\Theta$  /  $\Psi$  and on gyroscope data  $W_x$  /  $W_y$  /  $W_z$  (see figure 1 for reference):

**Roll derivative:**  $\Phi' = W_x + W_y \cdot \sin(\Phi) \cdot \tan(\Theta) + W_z \cdot \cos(\Phi) \cdot \tan(\Theta)$

**Pitch derivative:**  $\Theta' = W_y \cdot \cos(\Phi) - W_z \cdot \sin(\Phi)$

**Yaw derivative:**  $\Psi' = W_y \cdot \sin(\Phi) / \cos(\Theta) + W_z \cdot \cos(\Phi) / \cos(\Theta)$



Note: if  $\Theta = \pm 90$  deg, then  $\cos(\Theta)$  is zero and  $\tan(\Theta) = \sin(\Theta) / \cos(\Theta)$  is  $\pm \infty$ . These singularities make it impossible to compute derivatives for Roll and Yaw. This is also known as gymbal-lock: when  $\Theta = \pm 90$  deg,  $\Phi$  and  $\Psi$  will describe a rotation around the same vertical axis and one degree of freedom is lost. Because of these singularities, the quaternion implementation should be preferred.

Step 2: Compute updated angles  $\Phi$  /  $\Theta$  /  $\Psi$  based on angles derivatives:

$$\text{Roll: } \Phi(t+T_s) = \Phi(t) + \Phi' \cdot T_s$$

$$\text{Pitch: } \Theta(t+T_s) = \Theta(t) + \Theta' \cdot T_s$$

$$\text{Yaw: } \Psi(t+T_s) = \Psi(t) + \Psi' \cdot T_s$$

Step 3: Mix with other angles, e.g. computed by Acc+Mag data fusion (optional step)

The Roll and Yaw range may have spurious discontinuities: e.g. 0 and 360 deg represent the same angle, when averaged the output is 180 which is clearly wrong (should be 0 or 360); as another example, -180 and +180 deg represent the same angle, when averaged the output is 0 which is clearly wrong (should be -180 or +180). Correct weighted average is computed as follows:

$$\text{While } (\text{Abs}(\text{Angle1} - \text{Angle2}) > 180) \text{ Angle1} = \text{Angle1} - 360 \cdot \text{Sign}(\text{Angle1} - \text{Angle2})$$

$$\text{MixedAngle} = \text{Angle1} \cdot \alpha + \text{Angle2} \cdot (1 - \alpha), \text{ where } 0 < \alpha < 1$$

Pitch cannot have spurious discontinuities as it goes from -90 to +90 deg.

After mixing angles, angles can be reduced to their target range. As an example, this is the processing needed to reduce angles to range from -180 to +180 deg (Roll, Pitch and Yaw):

$$\text{MixedAngle} = \text{mod}(\text{MixedAngle}, 360 \text{ deg})$$

$$\text{If } (\text{MixedAngle} > 180) \text{ MixedAngle} = \text{MixedAngle} - 360$$

$$\text{If } (\text{MixedAngle} < -180) \text{ MixedAngle} = \text{MixedAngle} + 360$$

And this is the additional processing needed to reduce range to -90 to +90 (Pitch):

$$\text{If } (\text{MixedAngle} > 90) \text{ MixedAngle} = 180 - \text{MixedAngle}$$

$$\text{If } (\text{MixedAngle} < -90) \text{ MixedAngle} = -180 - \text{MixedAngle}$$

## Description of Quaternion implementation

Step 1: Compute quaternion derivative based on current quaternion and on gyroscope data  $W_x$  /  $W_y$  /  $W_z$  (see figure 1 for reference):

$$Q_w' = -Q_x \cdot W_x - Q_y \cdot W_y - Q_z \cdot W_z$$

$$Q_x' = +Q_w \cdot W_x - Q_z \cdot W_y + Q_y \cdot W_z$$

$$Q_y' = +Q_z \cdot W_x + Q_w \cdot W_y - Q_x \cdot W_z$$



$$Qz' = -Qy*Wx + Qx*Wy + Qw*Wz$$

Step 2: computation of updated quaternion based on quaternion derivatives:

$$Q(t+Ts) = Q(t) + Q' * Ts$$

Step 3: mixing with other quaternion, e.g. computed by Acc+Mag data fusion (optional)

Quaternions have no spurious discontinuities but they have redundant representations: +Q and -Q do represent the same orientation, the same set of angles. Simply averaging would give the incorrect result. Correct weighted average is computed as follows:

$$\text{If } (Q1w*Q2w+Q1x*Q2x+Q1y*Q2y+Q1z*Q2z)<0, Q2=-Q2$$

$$\text{MixedQ} = Q1 * \alpha + Q2 * (1-\alpha), \text{ where } 0<\alpha<1$$

Step 4: conversion from Quaternion to Euler angles (optional step)

$$Q_{\text{mod}} = Qw^2 + Qx^2 + Qy^2 + Qz^2$$

$$Qt = Qw*Qy - Qx*Qz, \text{ to check for singularities}$$

$$\text{If } (Qt>Q_{\text{mod}}/2), \text{ Roll } \Phi = 0, \text{ Pitch } \Theta = +90, \text{ Yaw } \Psi = 2*\text{Atan2}(Qx, Qw)$$

$$\text{If } (Qt<-Q_{\text{mod}}/2), \text{ Roll } \Phi = 0, \text{ Pitch } \Theta = -90, \text{ Yaw } \Psi = -2*\text{Atan2}(Qx, Qw)$$

$$\text{Roll: } \Phi = \text{Atan2}(2*(Qw*Qx+Qy*Qz), Qw^2 - Qx^2 - Qy^2 + Qz^2)$$

$$\text{Pitch: } \Theta = \text{Asin}(2*Qt / Q_{\text{mod}}), \text{ when argument is between -1 and +1}$$

$$\text{Yaw: } \Psi = \text{Atan2}(2*(Qw*Qz+Qx*Qy), Qw^2 + Qx^2 - Qy^2 - Qz^2)$$

## Notes

Gyroscope data usually has a non-zero output even if the angular rate is zero. This is known as gyroscope bias and must be subtracted before the data is used. As an example, the bias can be estimated by averaging the gyroscope output when the system is standing still. System is standing still when the data from the accelerometer and magnetometer is constant and their respective modulus is near 1g and local earth magnetic field.

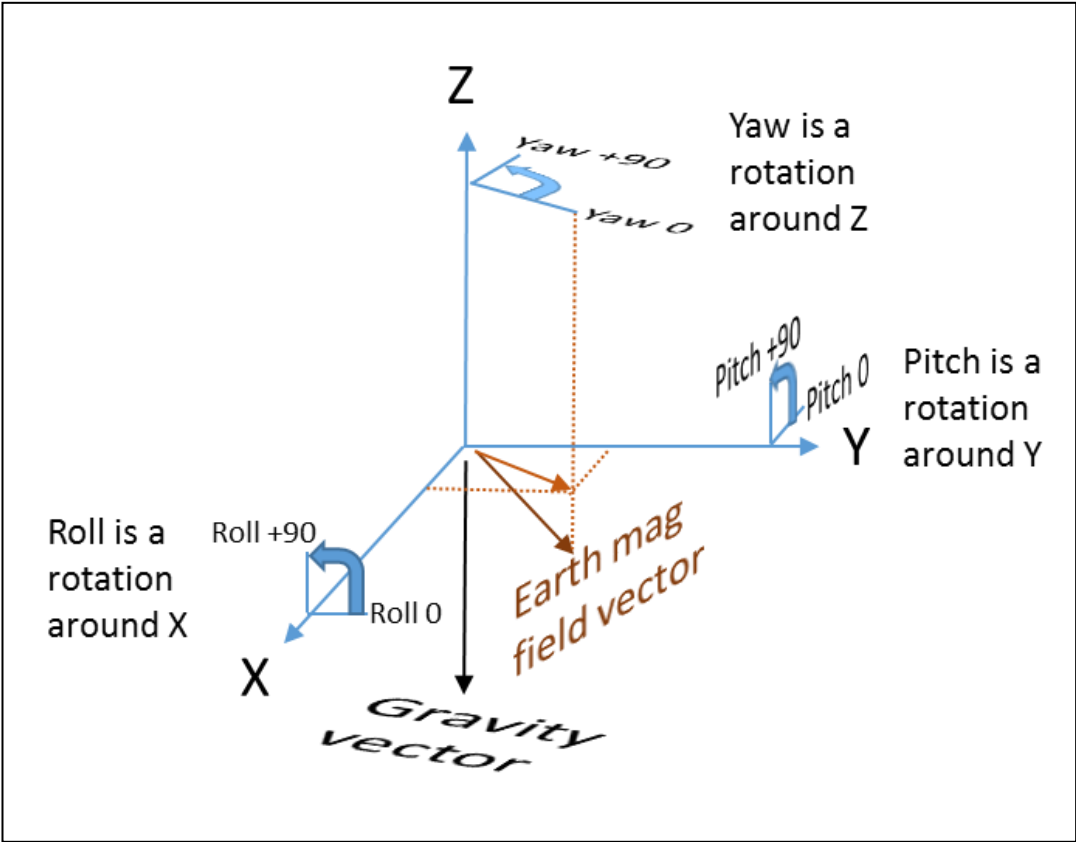
Gyroscope sensitivity may be non-unity, i.e. there can be a 3% tolerance. Calibration may improve the output. Calibration can be done by performing a full rotation around a given axis and comparing the final angle estimated by exploiting the gyroscope with the angle measured by other sensors such as an accelerometer and magnetometer.

Time interval  $T_s$  is critical to get accurate results. The actual value should match as closely as possible to the target value; any discrepancy will cause errors similar to non-unity gyroscope sensitivity.

The smaller the time interval  $T_s$  is, the more accurate the output will be, so it is better to use the faster output data rate which is available from the gyroscope (e.g. LSM6DS3 can reach 1.6kHz) and/or use interpolation.



Figure 1. Reference orientation for input data from gyroscope, and reference orientation for output data: roll, pitch and yaw angles.



Support material

Related design support material	
BlueMicrosystem1, Bluetooth low energy and sensors software expansion for STM32Cube	
Open.MEMS, MotionFX, Real-time motion-sensor data fusion software expansion for STM32Cube	
Documentation	
Design Tip, DT0058, Computing tilt measurement and tilt-compensated e-compass	

Revision history

Date	Version	Changes
11-Apr-2016	1	Initial release