List of Errata to

Massive MIMO Networks: Spectral, Energy, and Hardware Efficiency

Updated: February 26, 2020.

This documents lists typos detected in the published manuscript of:

Emil Björnson, Jakob Hoydis and Luca Sanguinetti (2017), "Massive MIMO Networks: Spectral, Energy, and Hardware Efficiency", Foundations and Trends® in Signal Processing: Vol. 11: No. 3-4, pp 154-655. http://dx.doi.org/10.1561/2000000093

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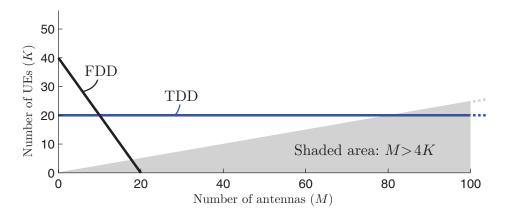
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If you discover a potential typo that is not listed in the latest version of this document, we appreciate if you report it to us. You can send an email to emil.bjornson@liu.se

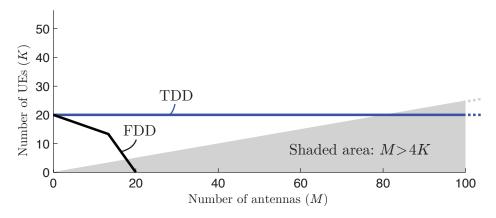
List of Errata

- 1. In the paragraph before Eq. (1.23), "horizontal ULA with antenna spacing $d_{\rm H}$ " should be "horizontal ULA with antenna spacing $d_{\rm H} \in (0, 0.5]$ ".
- 2. In Section 1.3.5, the statement "the same as that of sending M additional UL pilot signals" is only true if $M \geq K$. To make it more accurate, the statement should instead be "the same as that of sending $\max(M,K)$ additional UL pilot signals" and the following sentence should be added to the footnote on the same page: "More precisely, with the multiplexing gain $\min(M,K)$ of SDMA, we need $\max(M,K)$ symbol transmissions to feed back the MK channel coefficients."

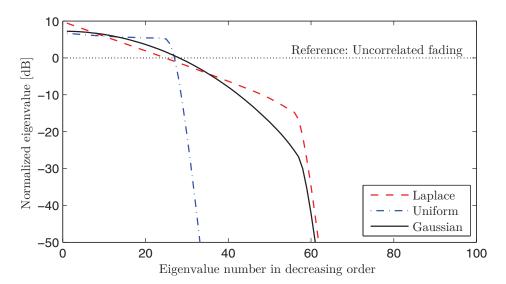
Consequently, the average pilot overhead of the FDD protocol is $\frac{M+K+\max(M,K)}{2}$ and not $M+\frac{K}{2}$ (which is only correct for $M \geq K$). This error is found at several places in this section. Moreover, Figure 1.22 is shown as



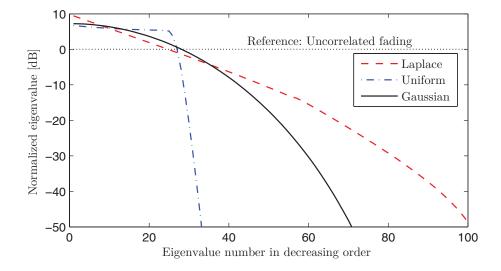
but should be



- 3. In Eq. (2.17), the final expression $\frac{\operatorname{tr}\left(\left(\mathbf{R}_{jk}^{j}\right)^{2}\right)}{(M_{j}\beta_{lk}^{j})^{2}}$ should be $\frac{\operatorname{tr}\left(\left(\mathbf{R}_{jk}^{j}\right)^{2}\right)}{(M_{j}\beta_{jk}^{j})^{2}}$.
- 4. Figure 2.6 is shown as



but should be



This error was a consequence of insufficient accuracy in the computation of the covariance matrices. The related sentence

"In fact, a uniform angular distribution makes 66% of the eigenvalues $50\,\mathrm{dB}$ smaller than in the reference case, while this happens for around 40% of the eigenvalues with Gaussian and Laplace distributions."

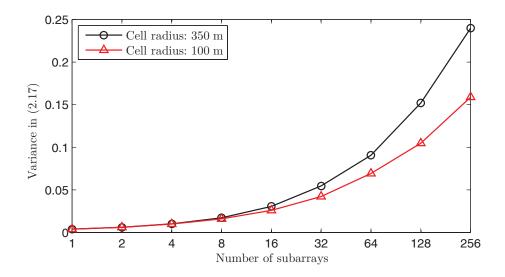
should read as

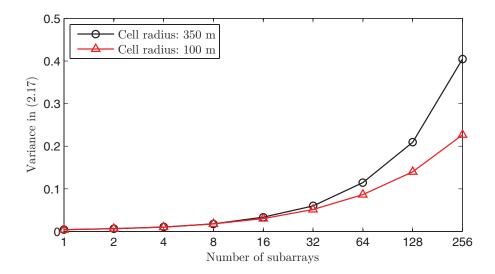
"In fact, a uniform angular distribution makes 68% of the eigenvalues $30\,\mathrm{dB}$ smaller than in the reference case, while this happens for 40% of the eigenvalues with Gaussian distribution and 19% with Laplace distribution."

- 5. In the first paragraph of Section 3.3.3, $\mathbf{h}_{li}^j \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}_M, \mathbf{R}_{li}^j)$ should be $\mathbf{h}_{li}^j \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}_{M_j}, \mathbf{R}_{li}^j)$.
- 6. In Eq. (3.36), I_M should be I_{M_i} . This typo also appears on the row right above Eq. (3.37).

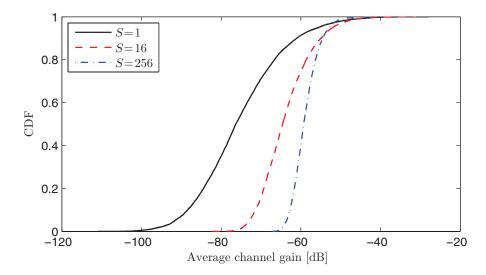
7. In Eq. (3.38),
$$\frac{1}{p_{li}} \left(\mathbf{\Psi}_{li}^j \right)^{-1}$$
 should be $\frac{1}{p_{li}\tau_p} \left(\mathbf{\Psi}_{li}^j \right)^{-1}$.

- 8. In the paragraph after Theorem 4.6, the statement "reduce the number of samples τ_d used for DL data" should be "reduce the number of samples τ_u used for UL data".
- 9. In Eq. (4.29), $\sigma_{\rm UL}^2$ should be $\sigma_{\rm DL}^2$.
- 10. In Section 5.3, the statement "The efficiency of a PA is defined as the ratio of input power to output power" should be "The efficiency of a PA is defined as the ratio of output power to input power".
- 11. In Section 6.1.2, after Eq. (6.7), the statement "LTE only requires EVM ≤ 0.0175 " should be "LTE only requires EVM ≤ 0.175 "
- 12. The subsection title "7.4.1 Physical Array Size and Antenna Spacing" should be "7.4.1 Preliminaries on Physical Array Size".
- 13. Figure 7.26 is shown as

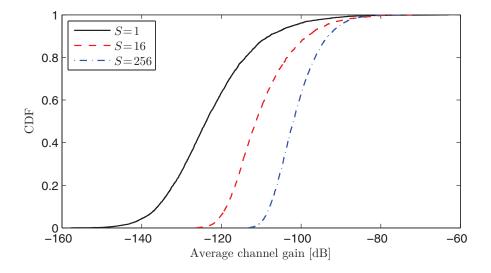




14. Figure 7.27 is shown as



but should be



In the paragraph that describes this figure, the sentence "However, going from S=1 to S=16 improves the median of β by around 9 dB; increasing this number to S=256 adds another 5 dB." should read as: "However, going from S=1 to S=16 improves the median of β by around 12.5 dB; increasing this number to S=256 adds another 9.5 dB."

15. The proof of Lemma B.14 only holds when $\bf B$ is a Hermitian matrix. A correct proof that applies to any deterministic $\bf B$ is:

"Note that $\mathbf{a} = \mathbf{A}^{\frac{1}{2}}\mathbf{w}$ for $\mathbf{w} = [w_1 \dots w_N]^{\mathrm{T}} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}_N, \mathbf{I}_N)$, thus we can write

$$\mathbb{E}\{|\mathbf{a}^{\mathsf{H}}\mathbf{B}\mathbf{a}|^{2}\} = \mathbb{E}\{|\mathbf{w}^{\mathsf{H}}(\mathbf{A}^{\mathsf{H}})^{\frac{1}{2}}\mathbf{B}\mathbf{A}^{\frac{1}{2}}\mathbf{w}|^{2}\} = \mathbb{E}\{|\mathbf{w}^{\mathsf{H}}\mathbf{C}\mathbf{w}|^{2}\}$$
(B.23)

where we defined $\mathbf{C} = (\mathbf{A}^{\mathrm{H}})^{\frac{1}{2}} \mathbf{B} \mathbf{A}^{\frac{1}{2}}$. Let c_{n_1,n_2} denote the element in \mathbf{C} on row n_1 and column n_2 . Using this notation, we can expand (B.23) as

$$\mathbb{E}\{|\mathbf{w}^{\mathsf{H}}\mathbf{C}\mathbf{w}|^{2}\} = \sum_{n_{1}=1}^{N}\sum_{n_{2}=1}^{N}\sum_{m_{1}=1}^{N}\sum_{m_{2}=1}^{N}\mathbb{E}\{w_{n_{1}}^{\star}c_{n_{1},n_{2}}w_{n_{2}}w_{m_{1}}c_{m_{1},m_{2}}^{\star}w_{m_{2}}^{\star}\}$$

$$\stackrel{(a)}{=} \sum_{n=1}^{N} \mathbb{E}\{|w_{n}|^{4}\}|c_{n,n}|^{2} + \sum_{n=1}^{N} \sum_{\substack{m=1\\m\neq n}}^{N} \mathbb{E}\{|w_{n}|^{2}\}\mathbb{E}\{|w_{m}|^{2}\}c_{n,n}c_{m,m}^{\star}$$

$$+ \sum_{n_{1}=1}^{N} \sum_{\substack{n_{2}=1\\n_{2}\neq n_{1}}}^{N} \mathbb{E}\{|w_{n_{1}}|^{2}\}\mathbb{E}\{|w_{n_{2}}|^{2}\}|c_{n_{1},n_{2}}|^{2}$$

$$\stackrel{(b)}{=} \sum_{n=1}^{N} 2|c_{n,n}|^{2} + \sum_{n=1}^{N} \sum_{\substack{m=1\\m\neq n}}^{N} c_{n,n}c_{m,m}^{\star} + \sum_{n_{1}=1}^{N} \sum_{\substack{n_{2}=1\\n_{2}\neq n_{1}}}^{N} |c_{n_{1},n_{2}}|^{2}$$

$$= \sum_{n=1}^{N} \sum_{m=1}^{N} c_{n,n}c_{m,m}^{\star} + \sum_{n_{1}=1}^{N} \sum_{n_{2}=1}^{N} |c_{n_{1},n_{2}}|^{2}$$

$$\stackrel{(c)}{=} |\operatorname{tr}(\mathbf{C})|^{2} + \operatorname{tr}(\mathbf{CC}^{H})$$

$$(B.24)$$

where (a) utilizes that circular symmetry implies that $\mathbb{E}\{w_{n_1}^*w_{n_2}w_{m_1}w_{m_2}^*\}$ is only non-zero when the terms with conjugates have matching indices to the terms without conjugates. The first expression is given by $n_1 = n_2 = m_1 = m_2$, the second term is given by $n_1 = n_2$ and $m_1 = m_2$ with $n_1 \neq m_1$, and the third term is given by $n_1 = m_1$ and $n_2 = m_2$ with $n_1 \neq n_2$. In (b), we utilize that $\mathbb{E}\{|w_n|^2\} = 1$ and $\mathbb{E}\{|w_n|^4\} = 2$. In (c), we write the sums of elements in \mathbf{C} using the trace. The resulting expression is equivalent to (??), which is shown by replacing \mathbf{C} with \mathbf{A} and \mathbf{B} and utilizing the fact that $\mathrm{tr}(\mathbf{C}_1\mathbf{C}_2) = \mathrm{tr}(\mathbf{C}_2\mathbf{C}_1)$ for any matrices \mathbf{C}_1 , \mathbf{C}_2 such that \mathbf{C}_1 and $\mathbf{C}_2^{\mathrm{T}}$ have the same dimensions."

16. The second paragraph of Definition B.8 should read as follows:

"If the random variable x, with support in \mathcal{X} and PDF f(x), is given and the conditional PDF is f(y|x), then the conditional differential entropy is

$$\mathcal{H}(y|x) = -\int_{\mathcal{V}} \int_{\mathcal{X}} \log_2 \left(f(y|x) \right) f(y|x) f(x) dx dy.$$
 (B.57)

17. In the paragraph after (C.63), $\mathbf{A} = \tau_p \Psi_{jk}^j$ should be $\mathbf{A} = \tau_p (\Psi_{jk}^j)^{-1}$.