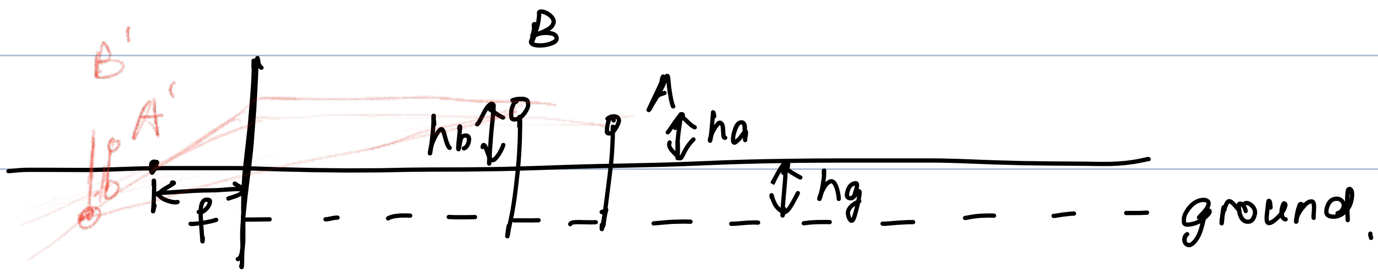
Problem 1

1. A is taller

2. According to question, we get:



$$\begin{cases} \dfrac{h_g}{b_1} = \dfrac{h_b}{b_2} \\ \dfrac{h_g}{a_1} = \dfrac{h_a}{a_2} \\ h_a + h_g = h. \end{cases}$$

$\therefore h_g = \dfrac{a_1}{a_1+a_2} \cdot h.$

$h_b = \dfrac{b_2}{b_1} \cdot h_g.$

$\therefore$ height of b

$= h_b + h_g = \dfrac{b_1+b_2}{b_1} \cdot \dfrac{a_1}{a_1+a_2} \cdot h$

$$\begin{cases} \dfrac{1}{D_b} + \dfrac{1}{D_b'} = \dfrac{1}{f} \\ \dfrac{D_b}{D_b'} = \dfrac{h_b}{b_2} \end{cases} \Rightarrow D_b = f \cdot (1 + \dfrac{h_b}{b_2})$$

$= [1 + \dfrac{a_1 h}{b_1(a_1+a_2)}] \cdot f$

Problem 2

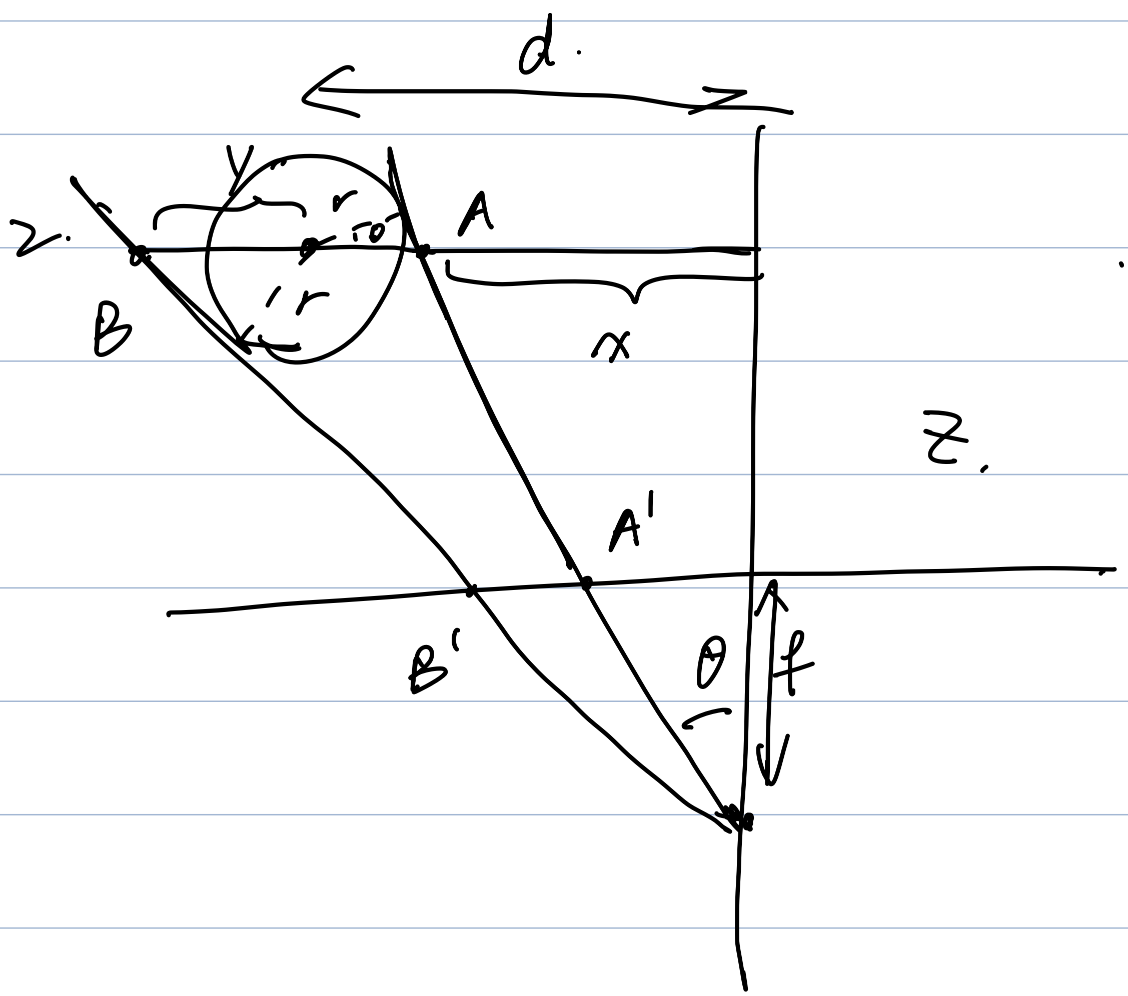
1. start point A (d, Y, Z)

ending point B $(d+L, Y, Z)$

$\Rightarrow A' (f \cdot \frac{d}{Z}, \frac{fY}{Z})$

$B' (f \cdot \frac{d+L}{Z}, \frac{fY}{Z})$.

$\therefore l = L \cdot \frac{f}{Z}$, l is a constant.



$$\therefore \begin{cases} \dfrac{z}{\sqrt{z^2+x^2}} = \dfrac{r}{d-x} \\ \dfrac{r}{y} = \dfrac{z}{\sqrt{(y+d)^2+z^2}} \end{cases} \Rightarrow$$

$$\therefore \begin{cases} L = y+d-x \\ l = \dfrac{f}{z} \cdot L \end{cases}$$

$$\therefore \begin{cases} x = \dfrac{dz^2 - \sqrt{(d^2+z^2-r^2)\cdot r^2 z^2}}{z^2-r^2} \\ y = \dfrac{dr^2 + \sqrt{(d^2+z^2-r^2)\cdot r^2 z^2}}{z^2-r^2} \end{cases}$$

$$\therefore y-x = \frac{d(r^2-z^2) + 2rz\sqrt{d^2+z^2-r^2}}{z^2-r^2}$$

$$\therefore L = \frac{2rz\cdot\sqrt{d^2+z^2-r^2}}{z^2-r^2}$$

$$\therefore l = \frac{2rf\cdot\sqrt{d^2+z^2-r^2}}{z^2-r^2}.$$