Q1:

1. $$\mathrm{softmax}(\vec{z} - c1) = \frac{\exp(z_i - c1)}{\sum_{j=1}^{K} \exp(z_j - c1)}$$

$$= \frac{1}{\sum_{j=1}^{K} \exp(z_j - z_i)}$$

$$= \frac{\exp(z_i)}{\sum_{j=1}^{K} \exp(z_j)} = \mathrm{softmax}(\vec{z}).$$

2. ① if $i = j$.

$$\frac{\partial y_i}{\partial z_i} = \exp(z_i) \cdot \frac{1}{\sum \exp(z_j)} + \exp(z_i) \cdot \frac{-\exp(z_i)}{(\sum \exp(z_j))^2}$$

$$= \mathrm{sofmax}(z_i) - \mathrm{sofmax}^2(z_i).$$

② if $i \neq j$.

$$\frac{\partial y_i}{\partial z_j} = -\mathrm{sofmax}(z_i) \cdot \mathrm{softmax}(z_j).$$

3. Chain Rule:

$$\text{softmax}(\vec{z}) = \frac{\exp(z_i)}{\sum_j \exp(z_j)}.$$

①

$$\therefore \frac{\partial y_i}{\partial x} = \sum \frac{\partial y_i}{\partial z_j} \cdot \frac{\partial z_j}{\partial x}$$

$$\therefore \frac{\partial z_i}{\partial x} = w_i \quad , \quad \frac{\partial z_j}{\partial x} = w_j .$$

$$\therefore \frac{\partial y_i}{\partial x} = [s(z_i) - s^2(z_i)] \cdot w_i - \sum_{i \neq j} s(z_i) \cdot s(z_j) \cdot w_j .$$

②

$$\frac{\partial y_i}{\partial w_j} = \sum \frac{\partial y_i}{\partial z_j} \cdot \frac{\partial z_j}{\partial w_j}.$$

$$= [s(z_i) - s^2(z_i)] \cdot x_j - \sum_{i \neq j} s(z_i)(s(z_j)) \cdot w_i$$

③

$$\frac{\partial y_i}{\partial \vec{u}} = \sum \frac{\partial y_i}{\partial z_j} \cdot \frac{\partial z_j}{\partial u} = -\sum \frac{\partial y_i}{\partial z_j} .$$

$$= \left[\sum_{i \neq j} s(z_i) s(z_j)\right] - s(z_i) + s^2(z_i)$$

Q2:

1. $V\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{bmatrix}$ $\quad Vx$: spin.

$V\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}-\frac{\sqrt{2}}{2}\\ \frac{\sqrt{2}}{2}\end{bmatrix}$

2. $V^T x$: spin $\vec{x}$.

3. $\Sigma = \begin{pmatrix}3 & 0\\ 0 & 5\end{pmatrix}$ $\quad V^T = \begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{pmatrix}$

$\therefore x = \begin{pmatrix}\frac{1}{\sqrt{2}}\\ 0\end{pmatrix} = \Sigma V^T x = \begin{pmatrix}\frac{3}{2}\\ -\frac{5}{2}\end{pmatrix}$

$x = \begin{pmatrix}0\\ \frac{1}{\sqrt{2}}\end{pmatrix} = \Sigma V^T x = \begin{pmatrix}\frac{3}{2}\\ \frac{5}{2}\end{pmatrix}$

$x = \begin{pmatrix}-\frac{1}{\sqrt{2}}\\ 0\end{pmatrix} = \Sigma V^T x = \begin{pmatrix}-\frac{3}{2}\\ \frac{5}{2}\end{pmatrix}$

$x = \begin{pmatrix}0\\ -\frac{1}{\sqrt{2}}\end{pmatrix} = \Sigma V^T x = \begin{pmatrix}-\frac{3}{2}\\ -\frac{5}{2}\end{pmatrix}$.

4. calculate SVD of B:

$B = U\Sigma V^T$

$\therefore$ V^T rotate vector

Σ rescale

U rotate again.

5. Σ is singular value.

$$U\Sigma V^T x = 0$$

$$\therefore \Sigma V^T x = 0.$$

$$\therefore \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} x = 0 \quad \therefore \quad x = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$