Selected Topics in Frontiers of Statistics First Assignment

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Graph Generation

As the question mentioned: First generate a undirected graph with 100 nodes,the smallest degree of each node is 2, the largest is 20. Also, the degree distribution of the graph is: $p(k) = k^{-2.5}$. After that, randomly select edges and turn them into directed edges, making the graph being directed.
As for code implementation, first define a function named *generate_degree_sequence* to generate a sequence of nodes whose degrees according with demand, then also define a function named *generate_directed_graph* for final graph generation.
The network visualization is shown as:

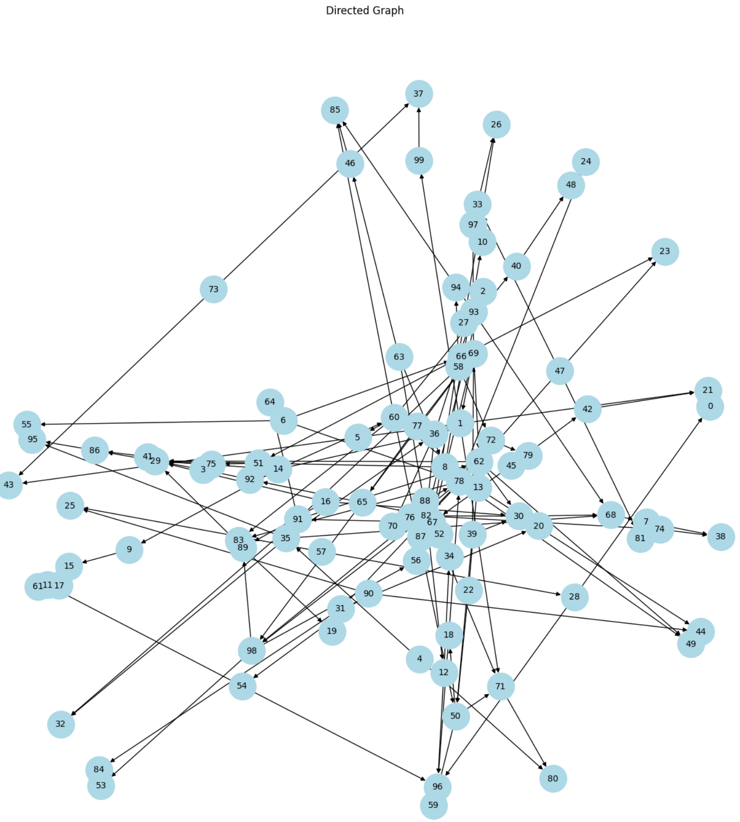


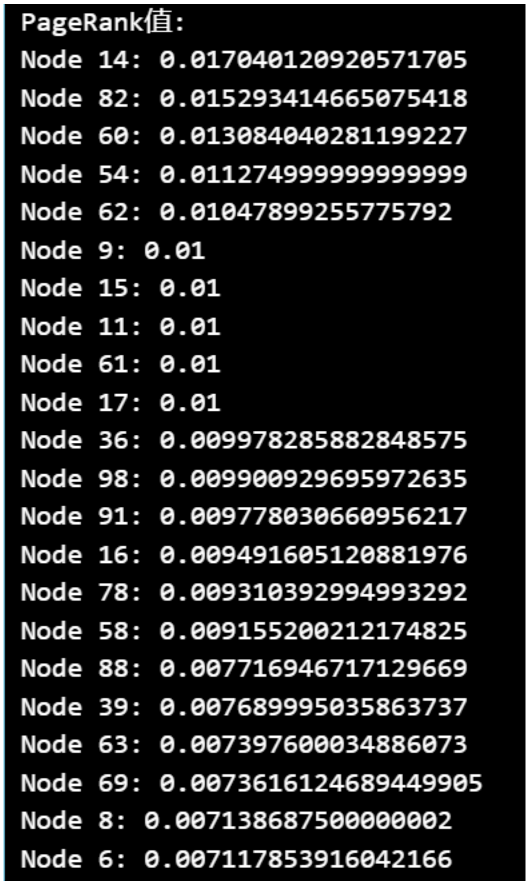
Figure 1: Network Visualization

PageRank Algorithm Implementation

For the first problem, PageRank algorithm is implemented into graph generated before, with $\beta = 0.85$. Below is the implemented detail, which also define in the code script *calculate_pagerank*:

Algorithm 1 PageRank

```
N ← Number of nodes in graph
Initialize the pagerank of each node $\frac{1}{N}$
for _ in range(max_iterations) do
    for node in graph.nodes() do
        $sum\_pr \leftarrow 0$
        if graph has neighbor then
            for node in graph's neighbors do
                if node's neighbor is null then
                    Continue
                end if
                sum_pr += node's pagerank / number of node's neighbors
            end for
        else
            $sum\_pc \leftarrow 0$
        end if
        new pagerank of node ← (1-$\beta$)/N + $\beta$ * sum_pr
    end for
    if Converge then
        break
    end if
    pagerank ← new_pagerank
end for
```



```
PageRank值:
Node 14: 0.017040120920571705
Node 82: 0.015293414665075418
Node 60: 0.013084040281199227
Node 54: 0.011274999999999999
Node 62: 0.01047899255775792
Node 9: 0.01
Node 15: 0.01
Node 11: 0.01
Node 61: 0.01
Node 17: 0.01
Node 36: 0.009978285882848575
Node 98: 0.009900929695972635
Node 91: 0.009778030660956217
Node 16: 0.009491605120881976
Node 78: 0.009310392994993292
Node 58: 0.009155200212174825
Node 88: 0.007716946717129669
Node 39: 0.007689995035863737
Node 63: 0.007397600034886073
Node 69: 0.007361612468944905
Node 8: 0.007138687500000002
Node 6: 0.007117853916042166
```

Figure 2: Part of Pagerank Outcome

Conduct the algorithm, then we can get the pagerank for each node, after that, discover the relationship of pagerank and degrees for nodes, which has a correlation coefficient of 0.503. The relationship is shown by the figure below:

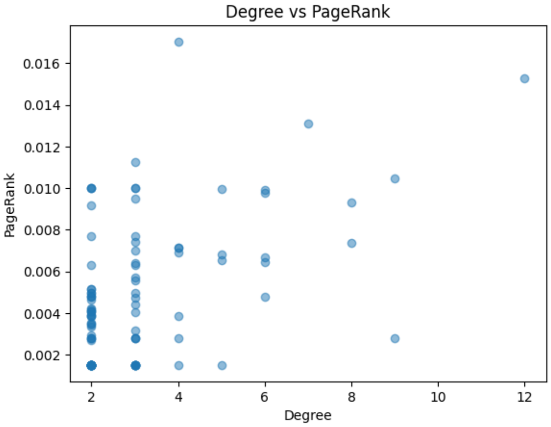


Figure 3: Relationship of Pagerank and Degree of nodes

Spam Farm Design

Based on the graph generated before, design a spam farm to simulate an attack to the origin network to boosting the pagerank of the target page. The structure of spamfarm consists of three aspects:

- Accessible Nodes: Composed by the original node of directed graph, which represents the common webpage(harmless)
- Target Page: Page that we want to boost its pagerank by means of adding spam page
- Spam Pages: Harmful pages in the network, aiming for boosting pagerank of target page; need to be detected and removed.

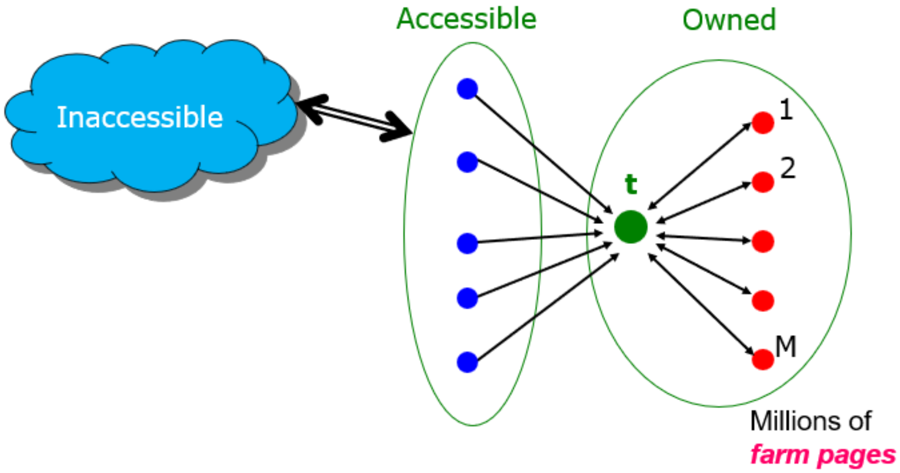


Figure 4: Design of Spamfarm

To discover the relationship of target page's pagerank and the number of spam pages, a figure is shown below:

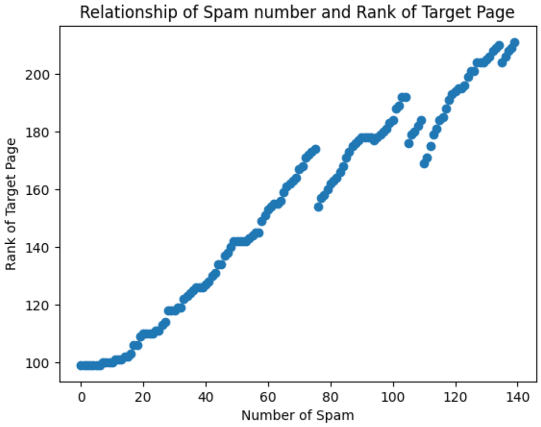


Figure 5: Relationship of Spam Number and Pagerank of Target Page

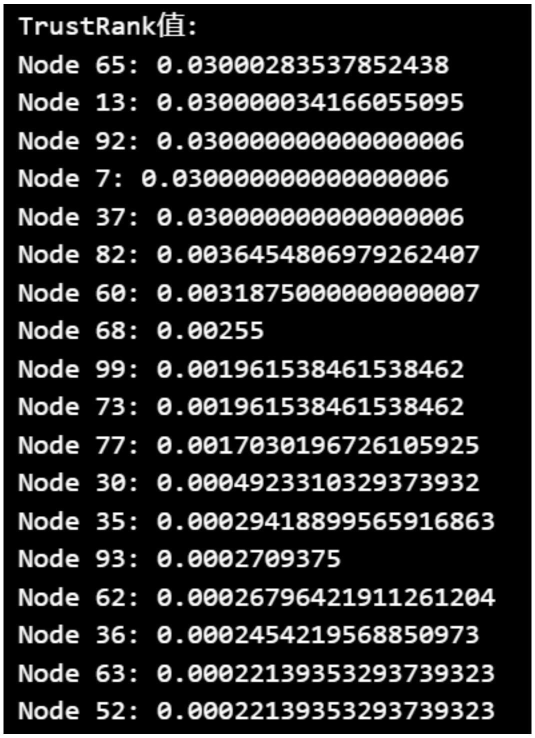
It is easy to find that as the number of spam pages growing, the pagerank of target page will become higher.

TrustRank Algorithm Implementation

In order to solve the spam farm situation, trustrank algorithm is introduced and implemented in the following report. Randomly, select 5% percent of the accessible node to be trust node, according to which trustrank of each node is calculated and analyzed.

Algorithm 2 TrustRank

```
seedsize ← 5% amount of number of accessible nodes
Trustvalue ← $\frac{1}{seedsize}$
Initialize the $trustrank$ of each node        ▷ Randomly Select trust nodes from accessible nodes
for _ in range(max_iterations) do
    for node in graph.nodes() do
        $sum\_pr \leftarrow 0$
        if graph has neighbor then
            for node in graph's neighbors do
                if node's neighbor is null then
                    Continue
                end if
                sum_pr += node's trustrank / number of node's neighbors
            end for
        else
            $sum\_pc \leftarrow 0$
        end if
        if node is trust node then
            new trustrank of node ← $(1 - \beta) * trust\_value + \beta * sum\_pr$
        else
            new trustrank of node ← $\beta *$ sum_pr
        end if
    end for
    if Converge then
        break
    end if
    trustrank ← new_trustrank
end for
```



```
TrustRank值:
Node 65: 0.03000283537852438
Node 13: 0.030000034166055095
Node 92: 0.030000000000000006
Node 7: 0.030000000000000006
Node 37: 0.030000000000000006
Node 82: 0.0036454806979262407
Node 60: 0.0031875000000000007
Node 68: 0.00255
Node 99: 0.001961538461538462
Node 73: 0.001961538461538462
Node 77: 0.0017030196726105925
Node 30: 0.0004923310329373932
Node 35: 0.00029418899565916863
Node 93: 0.0002709375
Node 62: 0.00026796421911261204
Node 36: 0.0002454219568850973
Node 63: 0.00022139353293739323
Node 52: 0.00022139353293739323
```

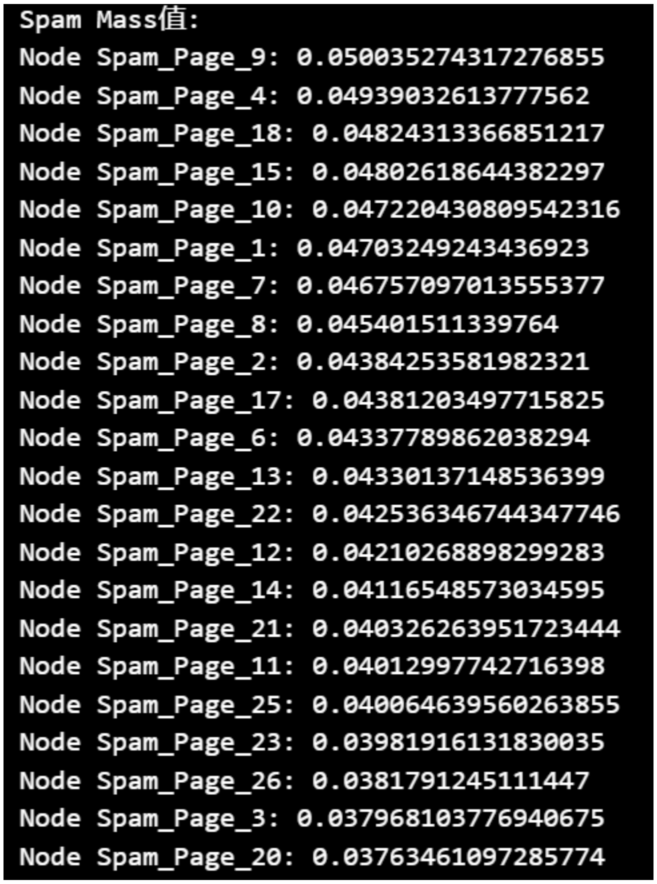
Figure 6: Part of Trustrank Outcome

Spam Mass Calculation

By utilizing the trust page, pagerank of the spammed graph is calculated, combined with the trustrank, spammass can be calculated:

$$SpamMass = \frac{P - T}{P}$$

where P denotes pagerank, and T denotes trustrank.
Part of Spam Mass outcome is shown below: It is obvious that spam page has heavier spam mass, which means they are more likely to be spam pages.



```
Spam Mass值:
Node Spam_Page_9: 0.050035274317276855
Node Spam_Page_4: 0.04939032613777562
Node Spam_Page_18: 0.04824313366851217
Node Spam_Page_15: 0.04802618644382297
Node Spam_Page_10: 0.047220430809542316
Node Spam_Page_1: 0.04703249243436923
Node Spam_Page_7: 0.046757097013555377
Node Spam_Page_8: 0.045401511339764
Node Spam_Page_2: 0.04384253581982321
Node Spam_Page_17: 0.04381203497715825
Node Spam_Page_6: 0.04337789862038294
Node Spam_Page_13: 0.04330137148536399
Node Spam_Page_22: 0.042536346744347746
Node Spam_Page_12: 0.04210268898299283
Node Spam_Page_14: 0.04116548573034595
Node Spam_Page_21: 0.040326263951723444
Node Spam_Page_11: 0.04012997742716398
Node Spam_Page_25: 0.040064639560263855
Node Spam_Page_23: 0.03981916131830035
Node Spam_Page_26: 0.0381791245111447
Node Spam_Page_3: 0.037968103776940675
Node Spam_Page_20: 0.03763461097285774
```

Figure 7: Part of Spam Mass Outcome