Selected Topics in Frontiers of Statistics Second Assignment

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Problem Setting

This assignment is about to simulate a social network and clarify the "small world phenomenon". The simulated network follows the paradigm of Watts and Strogatz. I only focus on 1-dimension case, and constructed a chain-like model. After constructing the network, I utilized decentralized algorithm to calculate the average path length for single message passing. By changing the clustering exponent r, I can find the best value of r such that the message passing efficiency is the best within the network.

Network Construction

The network is constructed to a chain with a number of nodes which is denoted as message passing candidates. The network is rich in local connection, meaning every node is connected with its neighbors. Also, the nodes have a few long-range connection with some other nodes, simulating long-distance connection for two people. For the probability of connecting to long-distance node, the definition is:
For some value r and two nodes u and v, the probability of the two nodes connected with each other is:

$$P = \frac{1}{d(u,v)^r}$$

Notice that the probability of connected to neighbor is ignored.
For a single node, after I gaining the probability of connecting with other nodes, then after standardization. the probability can be used for sample some long-range connecting nodes.
Besides, a value q will be given, which is the expectation of number of long-range nodes for a single node, determining the sparsity of the network.
A more specific network construction algorithm is showed in Appendix.

Message Passing Path Calculation

This section describe a decentralized algorithm that is utilized for simulating a message passing from one node to another node. There are some basic rule for each message passer u:

- u has the knowledge of the set of its local contact among all nodes
- u knows its own location, and the location of target t.
- u knows the long-range contact nodes' location and their position relationship with the target nodes t.

Based on the setting, we can utilize the algorithm and calculate the average efficiency of message passing given a specific social network. Also, the algorithm implementation detail is shown in Appendix.

Appendix

Algorithm 1 Network Construction

Require: n: number of nodes
Require: q: expectation of number of long-range connection
Require: r: clustering exponent
  Use network x generate a chain graph
  Connect local nodes
  for node1 in G.nodes() **do**
    Continue the loop if the node already have enough long-range connection
    for node2 in G.nodes() **do**
      Calculate distance between node1 and node2
      Calculate connecting probability
    end for
    Normalize the probability
    Append the last value of probability list to former ▷ Gaining a prob list adds up to 1
    Random a number, judge which node to connect
  end for

Algorithm 2 Decentralized Algorithm

Require: G: specific graph
Require: n: number of nodes
  for node1 in G.nodes() **do**
    for node2 in G.nodes() **do**
      Define *now_node* as node1
      while *now_node* is not equal node2 **do**
        Loop the neighbors of *now_node*, find the neighbor that closest to node2
        Accumulate the path length
      end while
    end for
  end for
  Calculate the average message passing length.

More relevant detailed information you can find in: **Github Repository**