algo 2 : $O(n^2)$

```
for (int i=0; i<2n; i++) {
    for (int j=0; j<n; j++) {
        System.out.println(i+" "+j);
    }
}
```

$\downarrow$

$C \cdot n^2$

algo 1

$n \neq n^2$ $\quad$ $n \neq n^2$ $\downarrow$

1 loop : n

```
for i in range(0,n):
    print i
```

$\leftarrow$? n

$O(n)$

how many time is the statement printed? $2n^2$ $\quad$ $O(n^2)$

2 loops :

i=0	0	1	i=1	1	1	i = n-1	n-1 1
	0	2		1	2	, , .	n-1 2
	0	3		1	3		⋮
	⋮			⋮			
	0	n-1		1	n-1		2n-1 n

algo 3:

```
for i=0..2n       <-
  for j=0..4n     <-
    for k=0..6n   <-
      print (i j k)
```

$O(n^3)$

print $= 48n^3$

algo 4:

```
def factorial(n):
    if n==1:
        return 1
    else:
        n* factorial(n-1)
```

$n=3$

$n! = 1 \cdot 2 \cdots n.$

$O(\log_2 n)$

CS : $\log_2$

math: $\log_e x = \ln(x)$

f(x) $\qquad$ g(x)

some positive number $\downarrow$ $\qquad$ some real # $\swarrow$

$f(x) = O(g(x))$ if there exists $C > 0$ and $x_0 \in \mathbb{R}$ st

$5x^2 - 2x + 1$ $\qquad$ x^2

$(\, |f(x)| \leq C \cdot |g(x)|$ for all $x \geq x_0 .)$

f(x) is bounded by g(x)

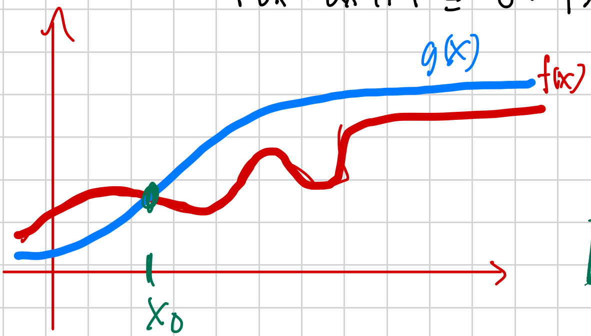
$x^2 = O(x^2)$ $\qquad$ $3x^2 = O(x^2)$ $\qquad$ $5x^2 - 2x + 1 = O(x^2)$

$2x^2 = O(x^2)$ $\qquad$ $3x^2 - 1 = O(x^2)$ $\qquad$ $3x^3 + 6x - 2 = O(x^3)$

$|f(x)| \le C \cdot |g(x)|$ for all $x \geq 0$

$f(x) = 5x^2 - 2x + 1 \qquad = O(x^2) \qquad g(x) = x^2$

$|5x^2 - 2x + 1| \le C \cdot |x^2|$ for all $x \geq X_0$



$|5x^2 - 2x + 1| \le C \cdot |x^2|$ for all $x \geq X_0$

$5x^2 - 2x + 1 \le 6x^2 \qquad X_0 ?$

$\boxed{5x^2 - 2x + 1 \le 6x^2}$ $\qquad X_0 ?$ $\quad x \geq X_0$

$x^2 + 2x - 1 \geq 0$

$x \geq -1 + \sqrt{2} \approx 0.414 = X_0$

$x \le -1 - \sqrt{2} \approx -2.414$

$5x^2 - 2x + 1 = O(x^2) \qquad \underbrace{C = 6 \quad X_0 = -1 + \sqrt{2}}$

not unique

$C = 5 \qquad X_0 = 0.5$

$C = 4$

C	X_0
4	1
5	0.5
6	$-1+\sqrt{2}$

$|5x^2 - 2x + 1| \le 4 |x^2| \qquad X_0 = 1 \quad C = 4$

$4, 5, 6, 7, 8 \ldots$

$5x^2 - 2x + 1 \le 4x^2$

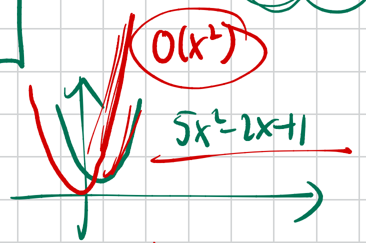
$x^2 - 2x + 1 \le 0$

$\rightarrow (x-1)^2 \le 0 \qquad \boxed{\rightarrow (x-1)^2 = 0}$

$\rightarrow \underline{(x-1)^2 \geq 0}$

$5x^2 - 2x + 1 \le C x^2$

$O(x^2)$



$5x^2 - 2x + 1 = O(x^2)$

" $5x^2 - 2x + 1$ belongs to the $O(x^2)$ class "

polynomial time

$5x^2 - 2x + 1 = O(x^3)$

$|5x^2 - 2x + 1| \le C \cdot |x^3|$ for some C

for all $x \geq X_0$

$5x^2 - 2x + 1 = O(x^{500})$

$O(1)$ constant time ✓
$O(n)$ linear time ✓
$O(n^a)$ poly time ✓
$O(2^n)$ exponential time ☆
$O(\log n)$ logarithmic time ✓
$O(n\log n)$ ✓

$n=6 \rightarrow n^2 = 36$
$2^n = 64 > n^2 = 36$

$\log_2 6 = 2.5 < 6=6$

$n(\log_2 n) < (n)n$

$n \cdot \log_2 n < n^2$

$1 < \log_2 n$ after awhile
$n < n\log_2 n$
$n < n\log_2 n$
constant time — better than $n\log n$

fast efficient → slow inefficient

< linear time <

bad !

constant < log time < nlog n time < poly time < exponential

cryptography : polynomial time