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Effects of Atmosphere Dominated Phase Fluctuation and Intensity Scintillation to DPSK System

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***Abstract*—Free-space-optical (FSO) coherent communication is a cost-effective, license-free, high-sensitivity and wide-bandwidth access technique for high date rates applications, whereas its performance strongly suffers from the turbulent atmosphere for the reason of both intensity scintillation and phase fluctuation. In this paper, the intensity scintillation is modeled as a multiplicative random process following the Gamma-Gamma distribution while the phase fluctuation is proved to satisfy Gaussian distribution. The slow varying characteristic of the power spectrum density of the atmosphere is verified. In our research the Mach-Zehnder-interferometer (MZI) based coherent differential-phase-shift-keying (DPSK) transmission system is employed. And its bit-error-rate (BER) performance is studied with BER expression given by Meijer-G function substitution. Besides, a novel simulation model of the atmosphere channel is established based on which effects of the atmosphere to BER performance of the system are simulated. From the simulation, we know over FSO links, to guarantee the performance of the coherent system, measures should be taken to compensate the intensity of the signal when the Rytov variance is over 1.0, and the statistical standard deviation of the received signal frequency should be compensated to less than 100MHz when the transmission rate is 1Gbit/s. By comparing performances of MZI-DPSK and traditional PSK, the advantages of MZI-DPSK are presented.**

I. Introduction

Free-space-optical (FSO) communications have received much attention in recent years as a cost-effective, license-free and wide-bandwidth access technique for high data rate applications [1]. In free space field, up until a few years ago, optical communication systems primarily employed conventional on-off-keyed (OOK) signals in either nonreturn-to-zero (NRZ) or return-to-zero (RZ) format [2]–[4]. Recently, coherent communication have attracted increasing attentions over FSO links. Compared with the traditional intensity modulation/direct detection (IM/DD), coherent detection has higher sensitivity than that of the IM/DD in terms of making full use of phase information [4], [5].

In FSO coherent communication, the most imperative factor related to the performance of communication system is the effect of the atmosphere. The turbulent atmosphere will cause both intensity scintillation and phase fluctuation to the laser beam. There are different models to describe the effect of the atmosphere to the intensity scintillation of the laser beam [1]. Since Gamma-Gamma channel model can cover all turbulence scenarios from weak to strong, we choose Gamma-Gamma model in this paper. But up until now, few paper mentioned the effect of the atmosphere to the phase fluctuation of the laser beam. In this paper, phase fluctuation of the laser beam is proved to satisfy Gaussian distribution.

Moreover, in FSO system, the correlation time of signal variations in turbulent atmosphere is a function of transversal wind velocity, which can be shown to be on the order of millisecond [1]. Therefore, characterized by high speed transmission (Gb/s), one can assume a frozen atmosphere model. There are very few changes of the channel features in the time of millisecond which means channel condition changes not too much for at least two consecutive symbol durations. In this paper, both the intensity scintillation and phase fluctuation are supposed to satisfy this assumption, and the slow varying characteristic of the power spectrum density of the normalized laser intensity is verified by our experiment.

Among the various modulation methods of coherent communication, differential-phase-shift-keying (DPSK) is reviewed in this paper. Many researches on phase modulation including phase-shift-keying (PSK) and DPSK are limited to the study of bit-error-rate (BER) performance considering only the intensity scintillation of the atmosphere without phase fluctuation [6]–[8]. In [9] phase noise caused by the laser linewidth and intensity noise is considered but the phase noise cause by atmosphere is neglected and low-pass characteristic of the atmosphere is not mentioned. In this paper, a MZI-demodulation-based-DPSK (MZI-DPSK) coherent scheme is proposed to apply in FSO links considering both intensity scintillation and phase fluctuation caused by atmosphere and the low-pass characteristic of the atmosphere. In addition, Meijer-G function is employed to simplify the BER expression of our scheme. Finally, a novel simulation model of the atmosphere which can produce a sequence that satisfies both some specific distribution and some specific power spectrum density is discussed. Under this simulation model, effects of intensity scintillation and phase fluctuation to the system performance are simulated and comparisons between the MZI-DPSK scheme and the traditional PSK scheme are carried out.

II. Effect of the Turbulent Atmosphere to the Laser Beam

A. *Effect of Turbulent Atmosphere to Intensity Scintillation*

The most widely accepted theory of the turbulence is attributed to Kolmogorov which accounts for the strength of the turbulence by the unitless Rytov variance

$$\sigma_R^2 = 1.23 C_n^2 k^{7/6} L^{11/6}, \tag{1}$$

where $k = 2\pi/\lambda$ is the wave number, λ is the wavelength, L is the propagation distance, and C_n^2 is the refractive-index structure parameter. According to the Hufnagel-Valley (H-V) turbulence model, the refractive-index structure parameter C_n^2 is determined by wind speed and altitude as discussed in [1], [10].

While studying the effect of turbulent atmosphere, marginal distribution of intensity scintillation is formulated as Gamma-Gamma model [1], [11].

$$f(I) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)} I^{(\alpha+\beta)/2-1} \mathrm{K}_{\alpha-\beta}(2\sqrt{\alpha\beta I}), I > 0, \tag{2}$$

where I is the normalized signal intensity scintillation, Γ is gamma function, and $K_v(\cdot)$ denotes modified Bessel function of the second kind of order v. The positive parameters α and β represent the large-scale and small-scale of the optical wave intensity scintillation, whose expressions can be found in [1], [11].

B. Effect of the Turbulent Atmosphere to Phase Fluctuation

For simplicity, phase fluctuation brought by atmosphere can be equivalently taken as the statistical standard deviation of the received signal frequency Δf_{IF}. In this case, the phase error can be written as [12], [13]

$$\Delta\phi(t) = \int_t^{t+T_b} 2\pi \Delta f_{IF}(t) dt, \tag{3}$$

where T_b is the duration between two consecutive detections, which can also be regarded as the bit duration in our communication system.

A widely accepted theory on phase fluctuation is perturbation approximation theory [1] which is derived by Tatarskii based on the theory of Rytov. By his theory, the distribution of the phase fluctuation can be proved to satisfy Gaussian distribution as discussed in Appednix. The distribution of $\Delta\phi$ can be written as,

$$f_g(\Delta\phi) = \frac{1}{\sqrt{2\pi}\sigma_\phi} e^{\frac{-\Delta\phi^2}{2\sigma_\phi^2}}, \tag{4}$$

where $f_g(\Delta\phi)$ is the probability density function of the $\Delta\phi$, and σ_ϕ^2 is the variance of the phase error, defined as

$$\sigma_\phi^2 = <\Delta\phi^2(t)> = \frac{2\pi\Delta f_{IF}}{f_s}, \tag{5}$$

where $f_s = 1/T_b$ is the signal rate.

C. Power Spectrum Density Model for Refractive-index Fluctuations

Turbulent atmosphere's effect on power spectrum of the intensity scintillation and phase fluctuation has a low-pass spectrum which indicates the slow-varying characteristic of the atmosphere. In [1], there are three kinds of power spectrum

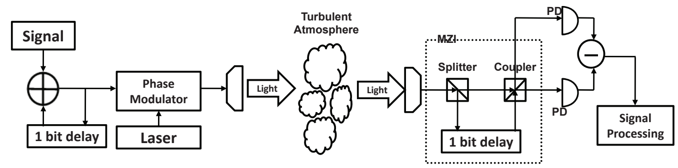

Fig. 1. Schematic of MZI-DPSK transmission system over FSO links

models for refractive-index fluctuation, and in all these three models refractive-index has low-pass characteristic. In this paper, both the intensity scintillation and the phase fluctuation of the laser beam through the atmosphere are supposed to to be low-pass processes.

III. Mathematic Model for MZI-DPSK Transmission System

A. DSPK Transmission Scheme

MZI-DPSK transmission system is shown in Fig. 1. Unlike traditional homodyne PSK or heterodyne PSK detection, in the MZI-DPSK demodulation scheme local oscillation and phase-lock-loop (PLL) are not needed, which makes the design of the system very simple.

As shown in Fig. 1, in the transmitting site, signals are firstly precoded by a delay-XOR precoder to generate the differentially encoded data sequences. This process can be seen in the following table

TABLE I
Coding process of DPSK System

Signal		1	0	0	1	0	0	1	1
After XOR	1	0	0	0	1	1	1	0	1
$\phi_s(t)$	0	π	π	π	0	0	0	π	0
$\phi_s(t) - \phi_s(t - T_b)$		π	0	0	$-\pi$	0	0	π	$-\pi$

From table I we can conclude

$$\begin{cases} H_0 : \phi_s(t) - \phi_s(t - T_b) = 0 \\ H_1 : \phi_s(t) - \phi_s(t - T_b) = \pm\pi \end{cases}, \tag{6}$$

where T_b is the bit duration, $\phi_s(t)$ is the phase to be modulated to the carrier wave, H_0 means signal '0' is transmitted, and H_1 means signal '1' is transmitted. Then the data sequences are modulated to phase of the laser carrier wave by phase modulator, which can be expressed as

$$\vec{E}_t(t) = A e^{j\phi_s(t)} e^{j\omega_c t}, \tag{7}$$

where A is the modulation amplitude, ω_c is the frequency of the light carrier wave. At last, signals are sent to the free space by sending telescope.

In the receiving site, signals are received by receiving telescope. Optical amplifier or optical filter can be used here to improve the system performance. But for simplification these two parts are not discussed in this paper. Then an asymmetric MZI splits the signal into two paths, one is directly transmitted to a 180° hybrid, while the other is transmitted to the 180° hybrid after one bit duration delay. After coupled in the 180° hybrid, the electric field of the two light beams can be expressed as

$$\begin{cases} \vec{E}_1(t) = Ae^{j\phi_s(t)}e^{j\omega_c t} \\ \vec{E}_2(t) = Ae^{j\phi_s(t-T_b)}e^{j\omega_c(t-T_b)} \end{cases}. \quad (8)$$

Generally in the MZI-DPSK system, we should guarantee $e^{j\omega_c T_b} = 1$. A balanced receiver follows the interferometer as a multiplier to demodulate the differentially coded signal. The signal from the balanced receiver can be expressed as

$$i(t) = RA^2 \cos(\phi_s(t) - \phi_s(t - T_b)), \quad (9)$$

where R is the photodetector responsivity. By signal decision, the transmitting signals can be recovered.

If taking the effect of intensity scintillation and phase fluctuation described in Section II into consideration, (9) can be written as

$$i(t) = RA^2 I \cos(\Delta\phi)(\cos(\phi_s(t) - \phi_s(t - T_b))). \quad (10)$$

B. Detector Noise of the System

In the transmission system, shot noise and thermal noise of the photodetector are also important factors. These noises can be expressed as [14]

$$\sigma^2 = \sigma_s^2 + \sigma_T^2 \ , \quad (11)$$

where σ^2 denotes the total noise of the detector, $\sigma_s^2 = 2q(i + i_d)\Delta f$ denotes the shot noise, $\sigma_T^2 = (4k_B T/R_L)\Delta f$ denotes the thermal noise, i is the current from the photodetector of the balanced receiver, i_d is the dark current, Δf is the bandwidth of photodetector, k_B is the constant of Boltzmann, q is the electric quantity of elementary charge, and T is absolute temperature. Thus the average signal-to-noise-ratio (SNR) can be written as

$$SNR = \frac{R^2 A^4 I^2 \cos^2(\Delta\phi)}{2\left[2q(RA^2 I|\cos(\Delta\phi)|/2 + i_d)\Delta f + \sigma_T^2\right]}, \quad (12)$$

where $RA^2 I|\cos(\Delta\phi)|/2$ is the current from one branch of the balanced detector. And the detector noise is twice the noise of each branch in intensity. Since the signal current is generally strong enough to ignore the dark current of the photodetector and thermal noise, and therefore (12) can be written as

$$SNR = \frac{RA^2 I|\cos(\Delta\phi)|}{2q}. \quad (13)$$

Because I and $\Delta\phi$ are function of time, so SNR is also function of time. For the purpose of BER calculation, we define equivalent SNR as

$$\gamma = \frac{RA^2}{2q}. \quad (14)$$

C. BER Analysis of DPSK System

Given I and $\Delta\phi$, BER of the system can be expressed as,

$$p(e|I,\Delta\phi) = \begin{cases} \frac{1}{2}\mathrm{erfc}\left(\sqrt{\gamma I|\cos(\Delta\phi)|}\right) \\ \qquad 2n\pi - \frac{\pi}{2} < \Delta\phi < 2n\pi + \frac{\pi}{2} \\ 1 - \frac{1}{2}\mathrm{erfc}\left(\sqrt{\gamma I|\cos(\Delta\phi)|}\right) \\ \qquad 2n\pi - \frac{3\pi}{2} < \Delta\phi < 2n\pi - \frac{\pi}{2} \end{cases} \quad n = 1, 2, 3, 4 \cdots \quad (15)$$

where $\mathrm{erfc}(\cdot)$ denotes the complementary error function. The BER expression is divided into two parts, because when $\cos(\Delta\phi) > 0$, it will decrease the amplitude of the current from the balanced detector, but when $\cos(\Delta\phi) < 0$, it will not only decrease the amplitude of the current from the balanced detector but also change the polarity of the current. When $\cos(\Delta\phi) = 0$, it is a singularity of the expression, which can be ignored.

Because I and $\Delta\phi$ can be considered as ergodic process, so the average BER can be written as

$$p_e = \int_{-\infty}^{+\infty} \int_0^{\infty} p(e|I,\Delta\phi) dI d\Delta\phi. \quad (16)$$

By substitute (4) and (15) into (16), the BER expression can be expressed as,

$$p_e = \sum_{n=-\infty}^{\infty} \int_{2n\pi-\frac{\pi}{2}}^{2n\pi+\frac{\pi}{2}} f_g(\Delta\phi) K d\Delta\phi + \sum_{n=-\infty}^{\infty} \int_{2n\pi-\frac{3\pi}{2}}^{2n\pi-\frac{\pi}{2}} f_g(\Delta\phi)\,(1-K) d\Delta\phi, \quad (17)$$

where

$$K = \int_0^{\infty} f(I) p(e\,|I,\Delta\phi) dI. \quad (18)$$

By expressing the $K_v(\cdot)$ in terms of Meijer-G function and using [15].12, [15].14, [15].21, we simplify K as,

$$K = \frac{(\alpha\beta)^{(\alpha+\beta)/2}(\gamma|\cos(\Delta\phi)|)^{-(\alpha+\beta)/2}}{2\sqrt{\pi}\Gamma(\alpha)\Gamma(\beta)} \times G_{2,3}^{2,2}\left[\frac{\alpha\beta}{\gamma|\cos(\Delta\phi)|}\left| \begin{matrix} 1-\frac{\alpha+\beta}{2}, \frac{1-\alpha-\beta}{2} \\ \frac{\alpha-\beta}{2}, \frac{\beta-\alpha}{2}, -\frac{\alpha+\beta}{2} \end{matrix} \right.\right]. \quad (19)$$

IV. Numerical Results and Discussion

A. Simulation of the Turbulent Atmosphere

In order to identify the low-pass characteristic of the power spectrum density of light beam transmission through the atmosphere, we carried out an experiment by measuring the intensity of the laser beam through the atmosphere. In our experiment, one light beam is sent out from a semiconductor laser at the transmission site while at the 250m away from the transmission site, the laser beam is received by a detector. Fig. 2 shows two examples of our experiment result, (a) is the situation when the transmission power is 12.6mW, while (b) is the situation when the transmission power is 31.5mW. No matter in which situations, the power spectrum density of the normalized intensity has the low-pass characteristic, so we can make a frozen atmosphere assumption. The measurement

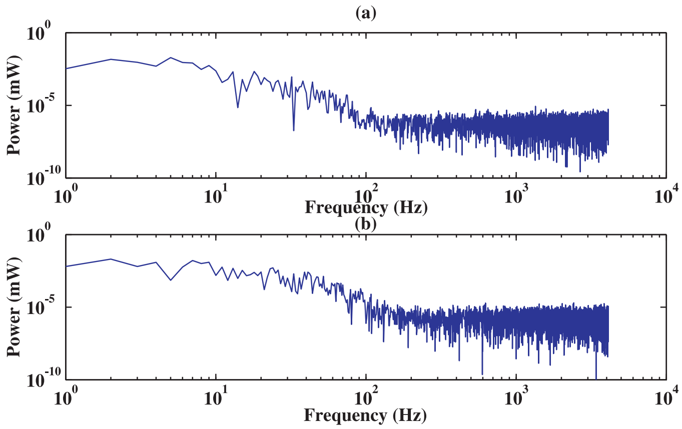

Fig. 2. Experiment result of the power spectrum density of the refractive-index

of the phase fluctuation is not carried out for the reason that it is hard to measure the phase of a light beam.

In the simulation field, it is complex to produce a sequence that satisfies both some specific distribution and some specific power spectrum density. In [16], [17], the exact way to produce such a sequence is introduced but it is very complex. In this section, we provide a novel method to approximate this sequence, although our method is not as accurate as the method in [16], [17], but it is very simple and is enough for our system.

To simulate a sequence with Gaussian distribution and low-pass spectrum density, the first step is to produce a sequence which satisfies Gaussian distribution with white spectrum density, then let the sequence go through a low-pass filter, the output sequence satisfies both Gaussian distribution and low-pass characteristic. By using this method, we can simulate the phase fluctuation of the atmosphere.

To simulate a sequence with Gamma-Gamma distribution and low-pass spectrum density, we propose the following scheme. First of all, we produce a sequence which satisfies Gamma-Gamma distribution with white spectrum density, which can be written as $A(n), n = 1, 2, 3, 4....$, then we produce another sequence which satisfies Gaussian distribution and low-pass power spectrum density, which can be written as $B(n), n = 1, 2, 3, 4...$, and then we make the third sequence $C(n)$, where $C(km+1) = C(km+2) = ... = C(km+m) = B(k), k = 0, 1, 2, 3...$, m is the cutoff frequency of the power spectrum of the refractive-index fluctuation. In our simulation we assume that $m = 100$. And $A(n) * C(n), n = 1, 2, 3, 4....$ is the sequences we need which both has Gamma-Gamma distribution and low pass characteristic. In Fig. 3, (a) and (b) are distribution and pow spectrum density of the normalized intensity of the light through the atmosphere which satisfies both Gamma-Gamma distribution and low-pass characteristic respectively. (c) and (d) are distribution and pow spectrum density of the phase fluctuation of the light through the atmosphere which satisfies both Gaussian distribution and low-pass characteristic.

From (a) and (c), we can see the distributions of the two sequences agree with our requirement, and hypothesis

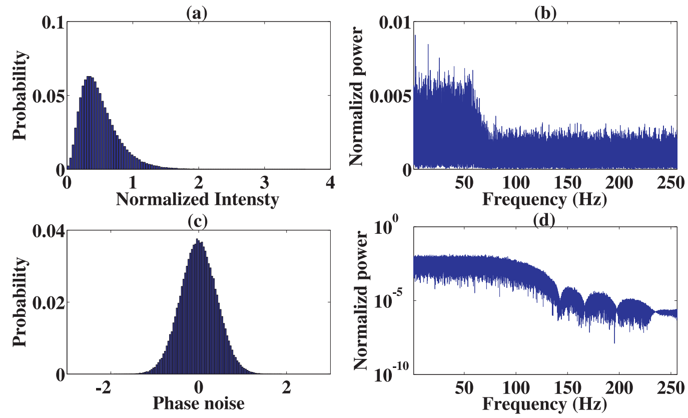

Fig. 3. Effect simulation of turbulent atmosphere to the intensity scintillation and phase fluctuation, (a) and (b) are the distribution and power spectrum density of the simulated effect of the atmosphere to the intensity of the laser beam, (c) and (d) are the distribution and power spectrum density of the simulated effect of the atmosphere to the phase of the laser beam

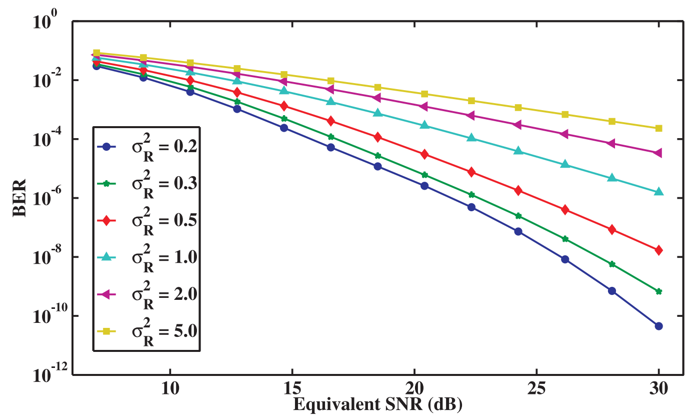

Fig. 4. BER performance of MZI-DPSK in different intensity scintillation situations when $\Delta f_{IF} = 200M$ at the transmission rate of 1GHz

testing of the two sequence also indicates that sequence in (a) satisfies Gamma-Gamma distribution and sequence in (c) satisfies Gaussian distribution. But (b) and (d) are not totally agree with Fig. 2 which is plotted by our experiment data. The problem with (b) is that the high-frequency part of the power spectrum does not descend quickly, and the problem with (d) is that the high-frequency part shows the oscillation characteristic. The reason is that we use Chebeshave low-pass filter to simulate the low-pass characteristic of the atmosphere, and there are some differences between the simulation model and the real system. But their low-pass characteristic are the similar, so it is proper to use this method to simulation the effect of the atmosphere to the intensity scintillation and phase fluctuation of the laser beam.

B. Simulation Result

In this part, numerical results are given with the transmission rate of the system fixed to 1Gbit/s. The BER performances of MZI-DPSK in different intensity scintillation situations are given in Fig. 4. The statistical standard deviation of the received signal frequency Δf_{IF} is fixed to $200MHz$. When σ_R^2 is less than 1.0, which means the turbulence of

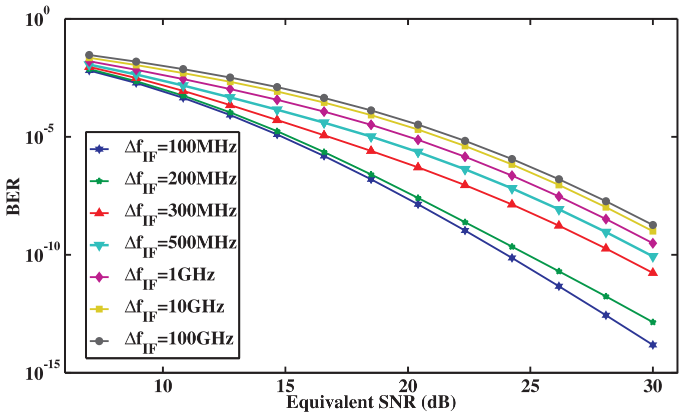

Fig. 5. BER performance of MZI-DPSK in different phase fluctuation situations when $\sigma_R^2 = 0.3$ at the transmission rate of 1GHz

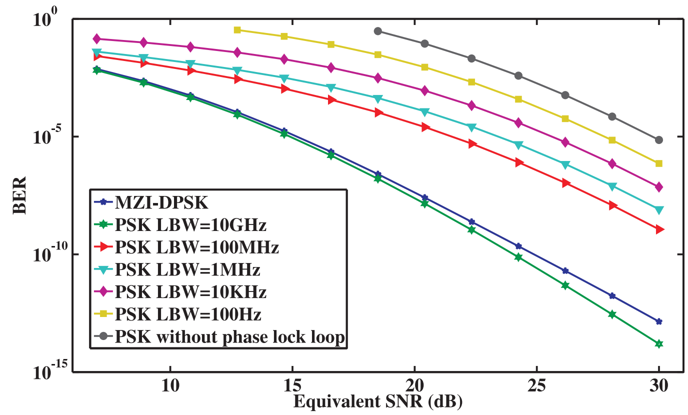

Fig. 6. BER comparison of MZI-DPSK and PSK when $\sigma_R^2 = 0.3$, $\Delta f_{IF} = 200M$ at the transmission rate of 1GHz, LBW in the legend means lock bandwidth of the phase lock loop

the atmosphere is weak, the transmission has low BER. But when σ_R^2 is bigger than 1.0, which means the turbulence of the atmosphere is very high, the BER is very high. Because our system has no forward-error-correction (FEC) coding and no interleave, the result of the simulation can totally reflect the effect of the intensity scintillation to the performance of the communication system. When $\sigma_R^2 = 0.2, \;\; 0.3, \;\; 0.5$, we need about 24dB, 25.2dB, 27.5dB to get a BER of 10^{-6}, respectively, but when $\sigma_R^2 = 1.0, \;\; 2.0, \;\; 5.0$, we need much higher SNR to get such a performance. And if we want to decrease the effect of the intensity fluctuation, we should take measure to compensate the power of the signal through to turbulent atmosphere especially when σ_R^2 is over 1.0.

The BER performance of MZI-DPSK in different phase fluctuation situations is given in Fig. 5. The Rytov variance σ_R^2 is fixed to 0.3. When $\Delta f_{IF} < 100MHz$, the performance of the system is almost the same as when $\Delta f_{IF} = 100MHz$ which is not shown in the figure. As the phase noise increases, the performance of the system decreases. When Δf_{IF} is at the level of $100MHz$, the performance of the system decreases very quickly. When Δf_{IF} is $1GHz$, $10GHz$, $100GHz$, the differences of the performance of the system are not very obvious. We need about $22.5dB$, $23dB$, $27dB$, $28dB$, $29dB$, to guarantee the performance of the system to achieve the BER of 10^{-6} when $\Delta f_{IF} = 100MHz$ is $200MHz$, $300MHz$, $500MHz$, $1GHz$. So under this situation, phase noise should be controlled by some self-adapt optical technologies to make sure that Δf_{IF} is about $100MHz$. The reason is that when $\Delta f_{IF} < 100M$, the performance of the system will not be improved obviously, but when $\Delta f_{IF} > 100M$, the performance of the system will declines heavily as Δf_{IF} increases. In [18], [19], effect of phase noise to performance of the optical DPSK is given, but their system is the fiber optical system. In our study of FSO system, the phase noise is much heavier than theirs and our standpoint is to give the bottom limit of the phase noise we can accept, so that we can know to what extent the phase noise should be controlled. Despite in different systems, our research results have the similar trend with theirs because we are facing the similar problems in optical DPSK system.

MZI-DPSK system does not need local oscillation and phase lock loop, which is the advantage of MZI-DPSK in system design. When considering the BER performance MZI-DPSK has an obvious advantage in overcoming the intensity scintillation and phase fluctuation. Fig. 6 gives the comparisons of MZI-DPSK with traditional PSK. The curve which is titled “PSK without phase lock loop” represents the BER performance of a PSK system without any phase lock loop, from which we know without phase lock loop, PSK is unusable in FSO link. The performance of the PSK is related to the bandwidth of the phase lock loop which represents the performance of the phase lock loop. When the lock bandwidth is lower than $100M$, the performance of PSK is much worse than MZI-DPSK. Generally it is hard for PSK to achieve the same performance as MZI-DPSK except that the bandwidth of phase lock loop is extremely large. If the bandwidth of phase lock loop achieve $10GHz$, the performance of PSK can be better than MZI-DPSK. But it is hard to design such a phase lock loop in practice. Another advantage of the MZI-DPSK is that we do not need to consider cycle slips, which should not be neglected in PSK system. All in all, MZI-DPSK system is more suitable for FSO link than traditional PSK system.

V. Conclusions

A MZI-DPSK scheme is reviewed in this paper in which Gamma-Gamma model is employed to describe the model of intensity scintillation, and the phase fluctuation is proved to satisfy the Gaussian distribution. Besides we do experiment to verify the low-pass characteristic of the turbulent atmosphere. And then considering the detector noise and the turbulence channel, the BER of the MZI-DPSK system is derived with Meijer-G function. In addition, we proposed a novel method to produce a sequence both satisfies some specific distribution and some specific power spectrum density to simulate the turbulent atmosphere. Also the intensity scintillation and phase fluctuation to the performance of the system are researched respectively. The reason why the turbulence will affect performance of the system is given as discussed in Section IV, so in coherent FSO system measures should be taken to compensate the power of the signal especially when σ_R^2 is over 1.0. The statistical standard deviation of the received signal frequency should be compensated to less than $100MHz$ when the transmission rate is 1Gbit/s to guarantee the performance of the system. By comparing performance of MZI-DPSK and traditional PSK, the advantage of MZI-DPSK is presented.

Appendix A
Distribution Discussion of the Phase Fluctuation

Because three components of the electric field vectors of the light beam all subject to the same wave equation, electric field equation can be replaced by a scalar as [1]

$$\nabla^2 \tilde{u} + k_0^2 n^2(r)\tilde{u} = 0. \tag{20}$$

where $\tilde{u}$ is one component of the electric field in any direction, $n(r)$ is the refractive index of atmosphere at the position of r.

By Rytov transformation $\Psi = \ln \tilde{u}$, we get the Riccati formulation,

$$\nabla^2 \psi(r) + [\nabla \psi(r)]^2 + k_0^2 n^2(r) = 0. \tag{21}$$

We decompose $n(r)$ into the form $n(r) = 1 + n_1(r)$, where $n_1(r)$ is a perturbation. And we ignore high-order items of Ψ, then $\Psi = \Psi_0 + \Psi_1$, where Ψ_0 is the definite solution to the wave equation, and Ψ_1 is the perturbation solution to the wave equation. Additionally, light wave $\tilde{u}$ can be expressed by the form

$$\begin{cases} \tilde{u} = A\exp(jS) \\ \tilde{u}_0 = A_0 \exp(jS_0) \end{cases}, \tag{22}$$

where $\tilde{u}_0$ is the definite part of $\tilde{u}$, A and S are the amplitude and phase of $\tilde{u}$ respectively, while A_0 and S_0 is definite part of A and S. Therefore we can get the perturbation solution to the wave equation as

$$\Psi_1 = \Psi - \Psi_0 = \ln \frac{A}{A_0} + j(S - S_0), \tag{23}$$

where the imaginary part of Ψ_1 is the phase fluctuation $\Delta\phi = S - S_0$.

With scalar diffraction theory and forward scattering approximation, if we assume that non-uniform refractive-index caused by atmospheric waves on the scattering angle satisfies $\theta_0 = \lambda / l_0 \ll 1$, we can get the expression of phase fluctuation at the plane of $z = z'$

$$\Delta\phi = \mathrm{Im}\left\{ M \frac{k^2}{2\pi \tilde{u}_0(r)} \right\}, \tag{24}$$

where

$$M = \iiint\limits_V \frac{\exp\left\{ jk\left[(z - z') + \frac{|\rho - \rho'|}{2(z - z')} \right] \right\} n_1(r')\tilde{u}_0(r')d^3r'}{z - z'} dV$$

where $r = (\rho, z)$ is the expression of r in cylindrical coordinates, and V is volume of scatter. From (24), we know the phase fluctuation can be expressed as superposition of many independent contributions. According to central limit theorem, the phase fluctuation satisfies Gaussian distribution.

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