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Free-space optical channel simulator for weak-turbulence conditions

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Free-space optical (FSO) communication may be severely influenced by the inevitable turbulence effect that results in channel gain fluctuations and fading. The objective of this paper is to provide a simple and effective simulator of the weak-turbulence FSO channel that emulates the influence of the temporal covariance effect. Specifically, the proposed model is based on lognormal distributed samples with a corresponding correlation time. The simulator is based on the solution of the first-order stochastic differential equation (SDE). The results of the provided SDE analysis reveal its efficacy for turbulent channel modeling.

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1. INTRODUCTION

Free-space optical (FSO) communication is a widely researched field with a well-established and rigorous theoretical foundation and extensive experimental validation [1]. The high level of interest in this topic is related to its many advantages, despite the drawbacks of transmission through turbulent channels. The turbulence results from stochastic temperature heterogeneities and pressure changes along the optical propagation path. As a result, the received optical signal experiences random fading that may affect certain communication characteristics, such as achievable bit rate and bit error rate (BER). The fading statistics have a non-Gaussian distribution and non-trivial second-order statistics, such as power spectral density and the covariance function. A testbed that simulates realistic channel conditions may significantly ease the design of optical communication links.

Most of the proposed FSO channel simulation methods are based on some kind of low-pass filtering and non-linear (exponent function) processing of a white Gaussian signal. Epple [2] suggested the use of first- or second-order Bessel filters with tunable cut-off frequencies. Kim *et al.* [3] used Butterworth filtering for this task. In his work, the spectral and temporal parameters were defined by choosing the appropriate cut-off frequency and the order of the filter. In [4], the use of a Gaussian approximation of the Kolmogorov spectrum was applied for the filter design. Recently, the use of a sophisticated method based on fractal phase screen generation has been used for the modeling of the spatial signal's propagation [5]. In addition, a laboratory setup for turbulence simulation was proposed [6].

Stochastic differential equations (SDEs) are a powerful modeling and simulation tool for stochastic processes that may make use of higher-order statistics. Currently, the most celebrated application of SDEs is in the field of financial mathematics, where it is used for the pricing of stock options (for example, the renowned Black–Scholes formula). However, some research in a field of SDE modeling of radio-frequency channels has been presented in the past [7]. Currently, the numerical SDE solution may be implemented by one line of code in numerical software, such as Mathematica or Matlab [8], thus making SDEs an attractive modeling tool.

This paper proposes the use of an SDE for modeling fading channels under weak-turbulence conditions, based on the technique that was first proposed in [9] and later summarized in [7]. The results in [7,9] were largely theoretical, rather than numerical. The main contribution of this paper is the presentation of a numerical analysis of the theoretical model outlined in [7,9]. The results of the numerical analysis were compared to experimental [10] and analytical models [1] of fading statistics based on lognormal distributions. Notwithstanding its simplicity, the method presented here displayed typical performance results when modeling typical channel conditions.

2. THEORY

A. Turbulence Theory

1. Scintillation Statistics

The scintillation index, σ_I^2, describes fluctuations (scintillations) in the received optical power, I, as measured by a point receiver and is defined as [1, Ch. 8, Eq. (9)]

$$\sigma_I^2 = \frac{\langle I^2 \rangle}{\langle I \rangle^2} - 1, \tag{1}$$

where $\langle\rangle$ stands for averaging over time.

Different models for scintillation statistics may be found in the literature for different conditions [1,11]. The lognormal distribution is typically used to describe weak-turbulence conditions ($\sigma_I < 1$), and its probability density function (PDF) is given by [11, Eq. (4.29)]:

$$f_x(x; \mu, \sigma^2) = \frac{1}{x\sqrt{2\pi\sigma_I^2}} \exp\left\{-\frac{[\ln(x) - \ln(\mu)]^2}{2\sigma_I^2}\right\}, \tag{2}$$

where $\mu = I_0$ is the average intensity.

2. Temporal Covariance

The normalized temporal covariance function is given by [1, Ch. 8, Eq. (53)]:

$$b_I(\tau) = \frac{B_I(\tau)}{B_I(\tau = 0)}, \tag{3}$$

where $B_I(\tau)$ is the temporal covariance function. The correlation time, τ_c, may be used for covariance function characterization. Throughout the paper, the value of τ_c is given by solving Eq. (3) with respect to τ, as

$$b_I(\tau_c) = \exp(-1). \tag{4}$$

3. Power Spectral Density

The power spectral density (PSD) that corresponds to the temporal covariance function in Eq. (3) is given by [1, Ch. 8, Eq. (54)]:

$$S_I(\omega) = 4\int_0^\infty B_I(\tau)\cos(\omega\tau)\mathrm{d}\tau, \tag{5}$$

where ω is the angular frequency. We note that for the Kolmogorov spectrum, for $\omega < \omega_t$, the PSD expression, Eq. (5), is almost constant and decays as $\omega^{-8/3}$ for $\omega > \omega_t$ ($\cong$27 dB/decade), where ω_t is termed the characteristic frequency [1, Ch. 8.5.1].

B. Fading Statistics

The temporal changes in optical channel conditions are usually characterized by fading statistics [1,2].

1. Fading Probability

The probability of fade is defined as the percentage of time when the received power, I, is below some predefined threshold value, I_{thr}. Therefore, the probability of a fade is defined by the cumulative probability [1, Ch. 11, Eq. (23)]:

$$P(I \le I_{\text{thr}}) = \int_0^{I_{\text{thr}}} f_x(x)\mathrm{d}x. \tag{6}$$

The probability of fade for the lognormal distribution is given by the cumulative density function (CDF) [1, Ch. 11, Eq. (24)]

$$P(I \le I_{\text{thr}}) = \frac{1}{2}\operatorname{erfc}\left[-\frac{\sigma_I^2/2 - 0.23F_T}{\sqrt{2}\sigma_I}\right], \tag{7}$$

where [1, Ch. 11, Eq. (25)]

$$F_T = 10\log_{10}\left(\frac{\langle I \rangle}{I_{\text{thr}}}\right) \tag{8}$$

is the fade threshold parameter.

2. Level-Crossing Rate

The level-crossing rate (LCR) is defined as the number of negative (or positive) crossings of a prescribed threshold level and characterizes the expected number of fades per unit time, $\langle n(I_{\text{thr}})\rangle$. The expected number of fades is given by [1], Ch. 11, Eq. (34)]:

$$\langle n(I_{\text{thr}})\rangle = \nu_0 \exp\left[-\frac{(\sigma_I^2/2 - 0.23F_T)^2}{2\sigma_I^2}\right], \tag{9}$$

where ν_0 is termed as the quasi-frequency and is given by [1, Ch. 11, Eq. (35)]:

$$\nu_0 = \frac{1}{2\pi}\sqrt{\frac{b_2}{b_0}}, \tag{10}$$

where

$$b_2 = \frac{1}{2\pi}\int_0^\infty \omega^2 S_I(\omega)\mathrm{d}\omega, \tag{11a}$$

$$b_0 = \frac{1}{2\pi}\int_0^\infty S_I(\omega)\mathrm{d}\omega, \tag{11b}$$

and may be roughly approximated by the standard deviation of the PSD (treated as the PDF). Therefore, the value of $3\nu_0$ may be used as the maximum width of the spectrum [1, Section 11.3.2].

3. Average Fade Duration

The average fade duration (AFD) (also called the mean fade time), $\langle t(I_{\text{thr}})\rangle$, is the ratio between the fading probability and the level-crossing rate and is given by [1, Ch. 11, Eq. (39)]:

$$\langle t(I_{\text{thr}})\rangle = \frac{P(I \le I_{\text{thr}})}{\langle n(I_{\text{thr}})\rangle}, \tag{12}$$

where the corresponding quantities were defined in Eqs. (7) and (9).

C. Stochastic Differential Equations

1. General Technique

The goal of this section is to provide a brief review of the technique for the synthesis of a predefined distribution, $p_x(x)$, with a predefined correlation time, τ_c, by a first-order SDE (Stratonovich form) [7, Eq. (7.3)]:

$$\frac{dx}{dt} = f(x) + g(x)\xi(t), \tag{13}$$

where the drift and diffusion functions are given by [7, Eq. (7.8)]:

$$f(x) = \frac{K}{2}\frac{d}{dx}\log p_x(x), \tag{14}$$

$$g(x) = \sqrt{K}. \tag{15}$$

Since both functions are time independent, it can be easily seen from the corresponding Fokker–Planck equation (FPE) [7, Eq. (5.265)] that the resulting solution is $p_x(x)$ distributed.

The use of statistical linearization is advised for the value of K [12, Ch. 7.2]. It is based on finding a slope α that minimizes the mean square error [7, Eq. (7.21)]:

$$\alpha = \arg\min_{\alpha} E\{(f(x) - \alpha x)^2\}. \tag{16}$$

By taking the derivative with respect to α and equating it to zero, the optimal value of the slope is found to be

$$\alpha_{\text{opt}} = \frac{E\{xf(x)\}}{E\{x^2\}}, \tag{17}$$

while the resulting diffusion value is given by solving

$$\alpha_{\text{opt}} = -\frac{1}{\tau_c}. \tag{18}$$

for K.

2. Application for Lognormal Distribution

By applying the technique above for the lognormal distribution given in Eq. (2), the resulting synthesis SDE is given by [9]:

$$f(x) = -\frac{K}{2\sigma_I^2 x}[\ln(x/I_0)] \tag{19}$$

$$g(x) = \sqrt{K}, \tag{20}$$

where K is given by

$$K = \frac{2I_0^2 \exp(\sigma_I^2)[\exp(\sigma_I^2) - 1]}{\tau_c}. \tag{21}$$

3. SIMULATION RESULTS AND DISCUSSION

The SDE solution was calculated by a Matlab toolbox for the numerical solution of SDEs [8] using the Euler method [13, Section 10.2]. The following discussion is based on a semi-analytical comparison of the stochastically generated results with the experimental results from a short-range turbulent-channel communication experiment [10]; the comparison is based on the theory outlined in the previous section (Section 2).

An example of the generated signal with $I_0 = 0.8$, $\sigma_I = 0.1$, $\tau_c = 17$ ms [as defined in Eq. (4)] and sampling frequency $f_s = 20$ kHz is presented in Fig. 1 (The supplementary Matlab and Mathematica source codes are can be found at the following link: http://bykhov.github.io/lognormal_sde/). These settings are matched with those of the experimental signal in [10]. Signals of length 25 s were used in order to verify that they approximately preserve the turbulent channel properties, as explained in the following subsections.

A. General Statistics

1. Probability Density Function

The most important property of a channel is the PDF of gain samples. In order to verify this property, we generated two signals with $\sigma_I = 0.1$ and 0.2 ($I_0 = 0.8, \tau_c = 17$ ms). The results are presented in Fig. 2, which shows the strong resemblance between the analytical PDF and the simulation results, except for at the highest x values at the right tail of the PDF. In order to quantify the fitting quality, the standard chi-squared goodness-of-fit test (chi2gof Matlab command) with a 0.5% insignificance level was applied to the log of the processes and showed the close resemblance between the theory and the simulation.

2. Auto-Covariance

Auto-covariance is a common channel property. Since we used the correlation time metric [Eq. (4)] in order to describe the auto-covariance, this metric was verified as follows. The normalized auto-covariance [1, Ch. 2, Eq. (13)] with five stochastically generated signals, together with their mean, and the experimental signal auto-covariance are presented in Fig. 3. The results show significant similarity between the covariance functions and the roughly similar correlation time, τ_c. In order to verify the robustness of the method, an additional plot with the arbitrarily chosen $\tau_c = 5$ ms is also presented. As expected from the above, the auto-covariance of the stochastically generated signal is very similar to that of the experimental measurement.

In order to quantify the simulation performance, the variance of τ_c in 50 different realizations was calculated. Linear interpolation was applied in order to identify the $\exp(-1) \cong 0.36$ level crossing. The resulting standard deviation of the τ_c values

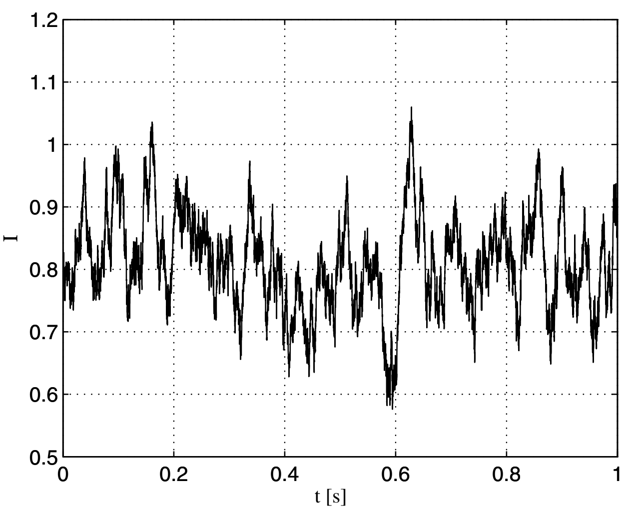

Fig. 1. Example of the SDE-generated signal.

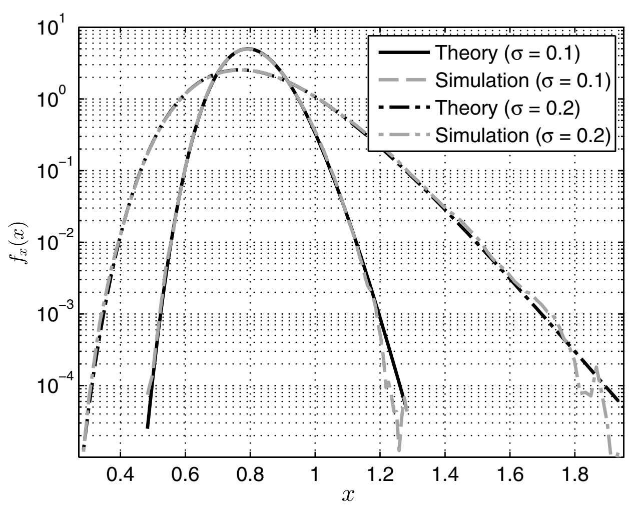

Fig. 2. PDFs of the SDE-generated signals for $\sigma_I = 0.1$ and 0.2 ($\mu = 0.8$).

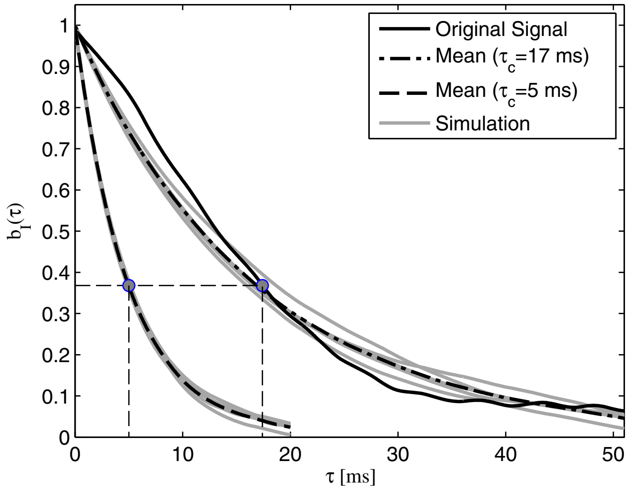

Fig. 3. Auto-covariance of the stochastically generated and experimentally measured signals (five examples and means are presented).

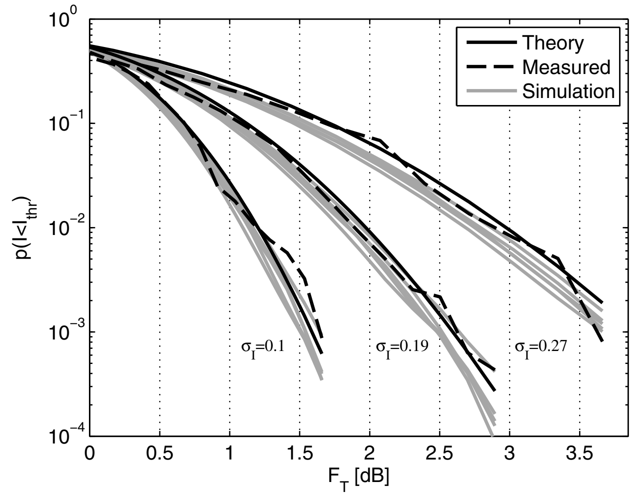

Fig. 5. Fading probability of the five stochastically generated signals together with analytical lognormal CDFs and the experimental measurement example.

is 1.15 ms, which is 6.8% of the experimental result's $\tau_c = 17$ ms value. With $\tau_c = 5$ ms, the standard deviation is 0.15 ms, which is 3% of the predefined τ_c value. Empirically, the better characteristics are probably the result of the higher ratio between the generated signal length and the correlation time.

3. Power Spectral Density

The PSDs of the experimental signal and the stochastically generated signal were evaluated using the Welch periodogram (Hamming window and 50% overlap) method, which results in smooth spectrum estimations (Fig. 4). Both signals can be approximately described by a sharp decay (≅27 dB/decade theoretical value), starting from some characteristic frequency f_T (Section 2.A.3).

B. Fading Statistics

The communication system design requires the mitigation of temporal channel gain changes that result in fading. A characterization of the fading statistics of the generated signal is provided in the following subsections.

1. Fading Probability

Since the fading probability is modeled by the CDF of the lognormal distribution, the simulation results are similar to the PDF in Section 2.B above. The analytical CDFs in accordance with Eq. (7), together with the stochastically generated and experimentally measured signals, are shown in Fig. 5 for $\sigma_I = 0.1$, 0219 and 0.27. The results are mainly in the left tail of the probability distribution (smallest values), and are therefore displayed with a logarithmic x-axis scale. As expected, the figure shows the similarity between the different graphs.

2. Level-Crossing Rate

In order to deal with the LCR, we first evaluated the quasi-frequency value of ν_0 of the signal by using the PSD variance technique [Eqs. (10) and (11)]:

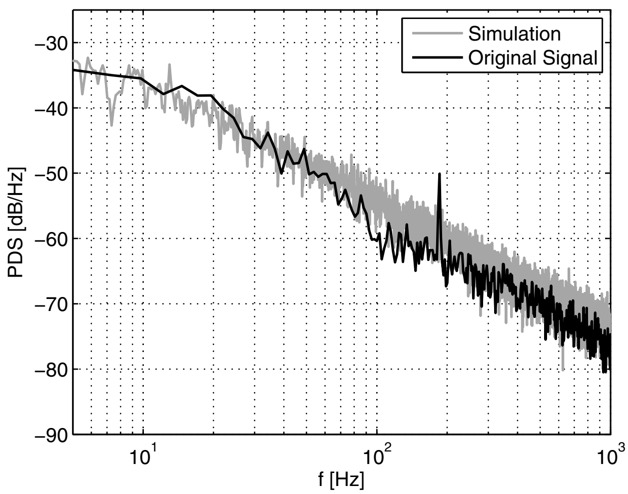

Fig. 4. PSD of a stochastically generated and an experimentally measured signal ($\mu = 0.8, \sigma_I = 0.2$). Both signals can be approximately described by a ≅27 dB/decade decay starting from some frequency f_T (Section 2.A.3).

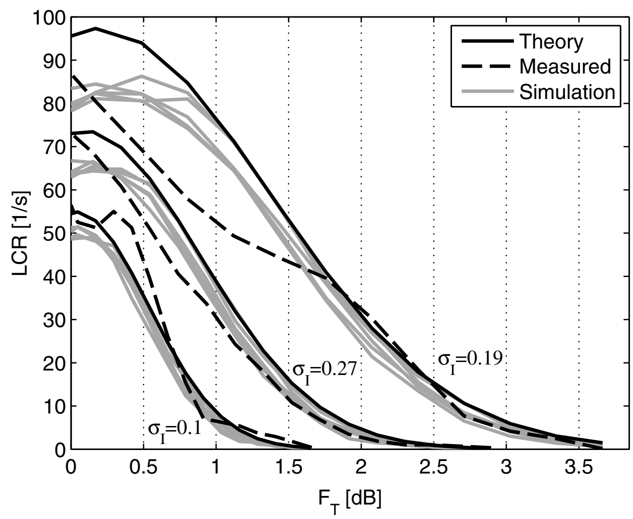

Fig. 6. LCR of five stochastically generated signals together with analytical LCRs and the experimental measurements example.

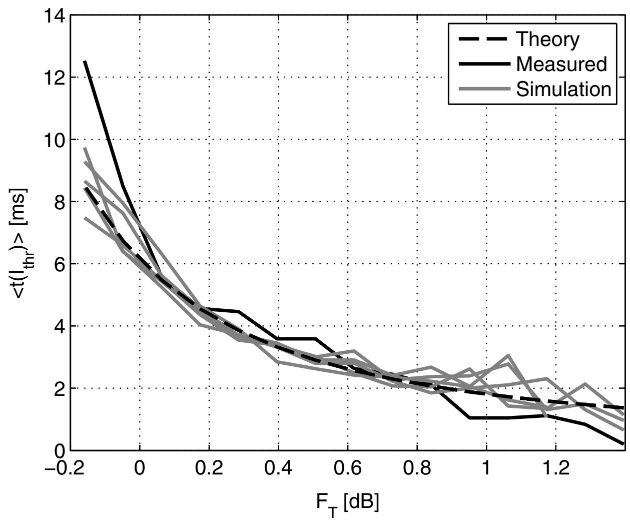

Fig. 7. AFD of five stochastically generated signals, together with analytical AFDs and an experimental measurement example.

$$\nu_0 \cong \left[\frac{\sum_m S_{yy}(f_m) f_m^2}{\sum_m S_{yy}(f_m)}\right]^{1/2}, \tag{22}$$

where $S_{yy}(f_m)$ is the value of a periodogram of the signal y at the frequency f_m, and the factor $1/2\pi$ in Eq. (10) vanishes because of the interchange between $\omega \to f$ (angular to ordinary frequency). Then, the generated signal was pre-filtered by a low-pass filter with a $3\nu_0$ cut-off frequency. The resulting LCR is presented in Fig. 6 for the same signals as in Fig. 5. The presented simulation results better fit the theory than the experimental ones. However, the average difference between the theory and the simulation is less than 6.5%.

C. Average Fade Duration

The corresponding AFD [Eq. (12)] of the signal with $\sigma_I = 0.1$ is presented in Fig. 7. The two additional σ_I value plots were omitted because of similar performance and overlapping. Both the LCR and the AFD display a significant resemblance between the theory and the simulation.

4. SUMMARY AND CONCLUSIONS

The SDE model provides a mathematically elegant and easy-to-implement solution for turbulent channel modeling. The SDE simulation results show the resemblance between the generated channel states and the theoretical predictions, as well as an example of the experimental measurements. However, the proposed method is based on the approximately exponential covariance function [7], and may be less effective under certain turbulence conditions.

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