LINMA2370 Project Part II : Analyzing the transition dynamics of tropical rainforests

Hari Kalidindi

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1 Problem

The equation for the tree cover dynamics developed in Part I reads:

$$\frac{dT}{dt} = r(R)T(1-\frac{T}{k}) - m_n T\frac{h_n}{T+h_n} - m_f T\frac{h_f^p}{T^p+h_f^p},$$

where

$$r(R) = r_m\frac{R}{h_R+R}.$$

1. **Analyze the tree cover dynamics.**
   - **a.** Prove that the interval $[0, 100]$ is invariant for T under this dynamics.
   - **b.** Derive the (implicit) equation defining the set of equilibria $(\overline{T}, \overline{R})$. Compute the Jacobian analytically around each equililibrium $(\overline{T}, \overline{R})$.
   - **c.** Compute and plot the graph of the equilibria $(\overline{T}, \overline{R})$, describing the tree cover at rainfall values in the range of $[0 - 5\text{mm/day}]$. Plot which equilibria are attractive and which are repulsive. Hint: You can use numpy roots to find the solutions of the analytical equation, $\frac{dT}{dt} = 0$.
2. **Extended model of tree cover dynamics**. In Part I we modeled the tree cover dynamics as a function of the current tree cover (T in %), and used the amount of rainfall (R in mm/day) as one of the input parameter. However, in reality there is a phenomenon called 'vegetation-rainfall feedback' that increases the amount of rainfall locally where there are high number of trees. This increase in local rainfall due to the forest feedback occurs in addition to the default global level of rainfall that

occurs independently. Once rainfall changes due to this local feedback, it will further influence the rate of change in tree cover. Hence, the tree cover dynamics should be modeled as a two-dimensional dynamical system to include the effect of the rate of change in the rainfall as a function of the tree cover. Below we describe both these equations:

The tree cover evolves as:

$$\frac{dT}{dt} = r(R)T(1 - \frac{T}{k}) - m_n T \frac{h_n}{T + h_n} - m_f T \frac{h_f^p}{T^p + h_f^p},$$

while the amount of rainfall is determined by:

$$\frac{dR}{dt} = r_R((R_{\text{constant}} + b\frac{T}{k}) - R).$$

Let us interpret the parameters. r_R is the maximum rate toward equilibrium for rainfall. R_{constant} is the default amount of rainfall in the absence of any amount of tree cover in the tropic. b determines the amount of increase in the rainfall based on what is the percentage of tree cover (T) relative to the maximum carrying capacity (k). Intuitively, in case the local vegetation-rainfall feedback is assumed to be absent, then $b = 0$. In that case, if we initialize the rainfall dynamics at $R = 0$mm/day, the final amount of rainfall would reach R_{constant} as the time increases. We thus have two variables R and T, while the input parameter is R_{constant}.

- **a.** Write code to simulate the two-dimensional equations for tree cover and rainfall dynamics.
- **b.** Simulate the two-dimensional tree-cover, rainfall dynamics for different pairs of initial values $(T_0, R_0, R_{\text{constant}})$, where T_0 is the initial tree-cover ranging from $[0 - 100\%]$, and R_0 is the initial rainfall ranging from $[0 - 5\text{mm/day}]$. Assume that for all simulations, the default rainfall R_{constant} is the same as R_0. Similar to question 4b in Project 1, run the simulations for 600 years. Now collect the equilibrium points (i.e., $T(600)$) at each starting pair (R_0, T_0) into a variable called fps. These are the set of equilibria that are derived empirically by simulating the differential equation. Plot the fps where y-axis corresponds to the final tree cover (value of $'T(600)'$ at $t = 600$ yr) and the x-axis corresponds to the amount of rainfall ($'R'$ value). In other words, plot the graph of equilibria from simulating the new two-dimensional treecover dynamics.
- **c.** Compare the (R_0, T_0) graph of equilibria from above to the (R_0, T_0) plot from question 4b in Project 1. What is the difference between these two plots and what observation can be made about the two-dimensional system compared to the one-dimensional case from Project 1?

3. **Learning how to use, analyze and visualize GeoSpatial remote sensing data**. In this task, you will get the current tree cover and expected amount of rainfall data for the next 30 years from three different tropical regions on the planet (South America, Africa, and Australasia). Essentially, you will receive .mat files that contain data about the amount of tree cover in the three different locations from the year 2014, data about the expected amount of rainfall over a period of 30 years between 2014-2044, and the latitudes and logitudes of the geospatial regions of interest. Note that the .mat files can be opened from python scipy library (scipy.io), and do not need matlab for this. **HELP:** To help you get started with how to visualize and use the geospatial data, we have uploaded a python notebook script. To use this script, one needs to install the following python softwares/packages: Anaconda, Cartopy.

   - **a.** The task is to use the given geospatial data and simulate the tree cover dynamics that you have developed above for a period of 30 years from 2014 - 2044. You are required to visualize how the tree cover changes between 2014 and 2044 on the world map. Note, the data is in the form of matrices, where each item in the matrix correspond to a different pixel/region on the world map. Hence, to simulate the forest cover across the map, you need to run different simulations corresponding to each pixel of the world map.
   - **b.** If you simulated the model correctly, then you would observe that the tree cover changes in the year 2044 in the Amazon forest (South America) are different from that of Congo (Africa). Such differences exist because those forests were at different initial tree cover levels in 2014. By using the hysteresis plot from question 2 ($T-R$ diagram), describe why there are such differences in how the Amazon forest changes compared to the Congo forest. [Hint: Explain in terms of which (T, R) states are reached by the Congo and Amazonian tropics in 2044, and why].

4. **LAST QUESTION:** In this question, we would like to model human action in terms of direct deforestation (tree cutting) or reforestation (tree planting). We ask you to model this action by a suitable change of the (one-dimensional or two-dimensional) equation, through a new input parameter embedding this action. We ask you to analyse the action theoretically and/or numerically, and possibly using the data and visualizing tools above to assess the (positive or negative) impact of human action in scenarios that you choose. You have here a lot of freedom to develop your analysis in the direction you want. Please justify your choices. Be concise: do not explore every possible direction!

This project is inspired from a recent research article about the hysteresis of the tropical forests in 21st century. The data about the tropical forest cover and rainfall are also provided by the authors from this research article. If you

params	description	default value	units
$R0$	default amount of rainfall	2.0	`mm/day`
r_m	maximal rate of tree cover expansion rate	0.3	`1/yr`
h_R	rainfall value where r is reduced by half	0.5	`mm/day`
m_n	maximal loss rate due to nursing effect	0.15	`1/yr`
h_n	tree cover below which rate of loss increases steeply (nursing effect)	10	%
m_f	maximal rate of loss due to fire mortality	0.11	`1/yr`
h_f	tree cover below which rate of loss increases steeply (fire mortality)	60	%
p	Hill function exponent	7	
k	Maximal carrying capacity	90	%
b	local feedback parameter	2	`mm/day`
r_R	Maximal rainfall rate toward equilibrium	1	`1/yr`

Table 1: List of parameter values for modeling the dynamics of tree cover and rainfall

are interested to know more about such research, you can refer to the following research article:

"Hysteresis of tropical forests in the 21st century." Nature communications (2020) by Arie Staal et al.,

Practical information

Questions : Feel free to contact me at hari.kalidindi@uclouvain.be to ask questions or set up an appointment.

Attention : You must do all the writing in groups of two students. Never share your production.

Submission : Each group is required to submit a zip containing its report (pdf) and its codes for Friday 9^{th} December, 8.30 am on Moodle.

Language : All reports are equally accepted in French and English