LINMA2370 Project Part I : Modeling the transition dynamics of tropical rainforests

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1 Introduction

We depend on forests for our survival, from the air we breathe to the wood we use. Besides providing habitats for animals and livelihoods for humans, forests also offer watershed protection, prevent soil erosion and mitigate climate change. Hence it is vital to understand different factors that influence the state of the forests. Remote sensing is used to monitor tree cover distributions in different regions on the Earth. This is an important step towards building predictions about how forests might change due to various factors such as changes in rainfall, wildfires, man-made activities etc. Specially interesting data comes from the tropics (for example Amazon forest in South America, Congo forest or forests in Australasia). It is believed that the tropics exist in a delicate equilibrium and can be in one of the three states: 1. Forest with dense tree cover 2. Savannah with partial tree cover and partial grass cover 3. Treeless but grass covered Arid state.

One may wonder what factors determine if a given tropic will be in a forest state, savannah, or arid state. Data indicates that amount of rainfall is the key determinant on which of the 3 different states a given tropical forest exists in. Secondly, the forest fires are also important factors behind whether a given tropic is in 3 different forest states. Wildfires use grass as a fuel and keeps burning trees until they reach an equilibrium with a certain proportion of grass and tree cover. But, if the tree cover reaches a certain threshold value, then the grass cover is suppressed, which subsequently leads to lower fuel for the wildfires. Such low/minimal wildfires combined with high rainfall can lead the tropic to a forest state.

It is not trivial to understand how and why does a given tropic transition between the given three states of stability and how can we protect them from environmental and man-made changes. In this project we will build a dynamical system model that lets us understand the patterns of change in the tropics.

In the following tasks, we will build dynamical system model for the changes in tree cover density based on different phenomena that were described above.

(a) forest state (b) savannah state (c) arid state

Figure 1: Different forest states based on the tree and grass cover

We will start by building a simple model that only includes tree growth by means of reproduction, and then add layers of complexity to this basic model. We will simulate all of these models to understand how the amount of tree cover changes as a function of the amount of rainfall, and parameters that determine the heterogeneity of the landscape and the feedback between vegetation and the regional climate. The tasks are as follows :

1. For this task, the objective is to code a differential equation describing the rate of change in the tree cover, where the tree cover is denoted as T(expressed as a % of total cover, $0 \leq T \leq 100$). In natural settings, the primary factor that influences the growth of trees is the total number of trees in the area. If there are more trees, then there is more seed production, pollination etc., and hence will lead to further increase in tree cover. So, the rate of tree cover should be proportional to the number of trees. Moreover, the expansion rate (call it as r) is determined by the amount of rainfall (R, expressed as average rainfall in mm/day). Combined, the rate of tree growth with the above logic will be

   $$\frac{dT}{dt} = r(R)T$$

   where

   $$r(R) = r_m \frac{R}{h_R + R}.$$

   Here $'r'_m$ is the maximum tree cover expansion rate (in 1/yr), and $'h'_R$ is the amount of rainfall (in mm/day) where $'r(R)'$ is reduced by half of its maximum.

   Observe how the rate of tree growth is written as a function of the amount of average rainfall. However, one should consider that any given soil will have specific limitations of the amount of nutrients and the area of the land, hence constraining the maximum number of trees that can exist on a given area. Suppose that the maximum carrying capacity of the land in terms of the fraction of tree cover is k (in %, thus $0 \leq k \leq 100$). As the net tree cover approaches this k, the rate of tree growth saturates. We use a logistic growth equation to model this phenomenon, as follows:

   $$\frac{1}{T}\frac{dT}{dt} = r(R)(1 - \frac{T}{k})$$

- **a. (1 point)** Write code for the per-capita logistic growth equation for tree cover based on the above explanation. The function should be of the form `percapitaLogisticGrowth(R, T, params)` where R is the average rainfall, T is the current value of tree cover and r is the expansion rate. Note : When the amount of rainfall is not given explicitly, always consider a default value `R0 = 2.0` mm/day
- **b. (2 points)** Plot the per-capita rate of change in tree cover $\frac{1}{T}\frac{dT}{dt}$ on y-axis against the tree cover (T) in a range (0–100%] on the x-axis. Assume that the rainfall is constant (`R0 = 2.0 mm/day`), and the model parameters from Table 1.

2. The logistic function above describes only one aspect of rate of change in the tree cover. There is another important effect that influences the rate of tree growth called the 'nursing effect'. According to nursing effect, if the tree cover decreases, it will cause a decrease in the protection of plants and seedlings. Such a loss in protection/nursing will cause a reduction in the rate of tree cover. In addition, if the tree cover falls below a certain limit, the rate of change will be drastically reduced as the seedlings will not have much protective cover from the nursing/protective trees. To model this nursing effect, assume that the per-capita rate of change in tree cover will be negatively proportional (with a multiplicative constant $-m_n$) to a 'nursing function'. The nursing function should be lower at higher tree cover, and once the tree cover falls below a certain threshold (call it h_n) the nursing function value should increase steeply. This steep increase in nursing function value will lead to a steep decrease in the rate of change due to the negative proportionality. To model the effect of steep increase in the loss function at low levels of tree cover, we use a Monod function (described below). Nursing effect using the Monod function can be modeled as:

$$\frac{1}{T}\frac{dT}{dt} = -m_n \frac{h_n}{T + h_n}.$$

   - **a. (2 points)** Write code for the overall per-capita rate of change in tree cover by combining the logistic growth equation from question no. 1a and the nursing effect described above. This function should of the form `percapitaLogisticNursingeffects(R, T, params)`.
   - **b. (2 points)** Plot the per-capita rate of change in tree cover $\frac{1}{T}\frac{dT}{dt}$ by inlcuding both logistic and nursing effects on y-axis against the tree cover in a range $(0 - 100\%]$ on the x-axis. Compare this plot with that of the pure logistic model from question 1b. Answer why is the percapita rate of change for the combined logistic and nursing effect model so different from the pure logistic effect model at lower tree cover. Assume that the rainfall is constant (`R0 = 2.0mm/day`), and the model parameters from Table 1.

3. The final important term to model the rate of change in tree cover is due to 'fire mortality effect'. This effect determines what is the decrease in the

rate of change in tree cover if there was a wildfire as a function of the net tree cover of the tropic. If the tree cover falls below a certain threshold, which will be replaced by grass , then the wildfires will have more grass as fuel to burn. The per-capita rate of change in tree cover is negatively proportional to the 'fire mortality function' (with a proportionality constant $-m_f$). This is similar to the case of nursing effect we considered above. However, there is a small difference here. Here we will additionally use a constant p, that will determine how rapidly does the fire mortality function change once a low tree cover is reached. i.e., p will determine the slope of change in the fire mortality function. Such effects can be modeled using sigmoidal Hill functions. Fire mortality effect should be modeled as

$$\frac{1}{T}\frac{dT}{dt} = -m_f \frac{h_f^p}{T^p + h_f^p}$$

Where h_f determines the tree cover below which the fire mortality increases steeply, and T is the % tree cover.

- **a. (1 points)** Write code for the per-capita rate of change in tree cover due to fire mortality function only. This function should of the form `percapitaFireeffect(R, T, params)`.
- **b. (2 points)** Write code for the per-capita rate of change in tree cover due to combined logistic, nursing and fire mortality effects. This function should of the form `percapitaLogisticNursingFireeffects(R, T, params)`.
- **c. (2 points)** Take the plot from question 2b. On this figure, plot the negative of pure fire mortality function with the per-capita rate of change in tree cover on y-axis and the associated tree cover $(0-100\%]$ on the x-axis. i.e., plot two separate curves on the same figure where one curve describes the per capita rate of tree cover change as

  $$f(T) = r(R)(1 - \frac{T}{k}) - m_n \frac{h_n}{T + h_n}$$

  and the second curve describes negative of the pure fire mortality function, i.e.,

  $$g(T) = m_f \frac{h_f^p}{T^p + h_f^p}.$$

  Assume that the rainfall is constant (`R0 = 2.0mm/day`), and the model parameters from Table 1.
- **d. (3 points)** How many points of intersection do you see between the plot of function $g(T)$ and the plot of function $f(T)$ i.e., the plot that considered both logistic growth and nursing effects? What is the approximate value of the overall per capita rate of change in tree cover (i.e., `percapitaLogisticNursingFireeffects(R, T, params)`) at these points of intersection.

4. We got all the necessary equations for modeling the rate of change in tree cover as a function of the actual amount of tree cover (in %), and the amount of average rainfall (in mm/day). Now the below set of questions needs you to simulate the differential equation at different levels of rainfall, and observe how the tree cover changes over time.

$$\frac{dT}{dt} = r(R)T(1 - \frac{T}{k}) - m_n T \frac{h_n}{T + h_n} - m_f T \frac{h_f^p}{T^p + h_f^p}.$$

- **a. (3 points)** Take the values of rainfall (R) in the range $[0, 5\,\text{mm/day}]$ with 100 equally spaced values in between. Furthermore, consider 5 different initial tree cover values (T_0) $[5, 25, 50, 75, 100\,\%]$. For each of the (R, T_0) pair, simulate the differential equation for the rate of change in tree cover for 600 years, and plot the trajectory of tree cover (T), in 5 separate figures (one figure each for 5 different initial tree cover values $'T_0'$). The x-axis should represent 'time (t)', the y-axis should represent 'rainfall value (R)' and z-axis should represent the value of tree cover (T) across time and rainfall value. Alternatively, instead of a 3D plot, you can also use 2D heatmaps where the tree cover will be visualized as different colors, while the x-axis represents the simulation time (600 years), and y-axis represents the amount of rainfall (R between [0-5 mm/day]).
- **b. (4 points)** You will observe that the tree covers at each starting pair (R, T_0) will reach an equilibrium value by the end of each simulation (at $t = 600\ \ \text{yr}$, call it as $'T(600)'$). Now collect the equilibrium points (i.e., $T(600)$) at each starting pair (R, T_0) into a variable called fps. These are all the fixed points that are derived empirically by simulating the differential equation. Plot the fps where y-axis corresponds to the final tree cover (value of $'T(600)'$ at $t = 600\ \ \text{yr}$) and the x-axis corresponds to the amount of rainfall ($'R'$ value). Once you plot the fixed point manifold on (T, R) plane above, then answer if this plot recovers all the fixed points of the dynamical system? If some fixed points are missing, then answer why they cannot be recovered.
  NOTE: each rainfall value (R) will be associated with multiple fixed points $'T(600)'$ at $t = 600$ yr based on the initial tree cover $T(0)$. So it is necessary to write a function that first identifies the fixed points reached for each $'R'$ over different values of $'T0'$. Once these unique fixed points are found, then plot each of the $'T(600)'$ for each $R \in [0 - 5\,\text{mm/day}]$ on the (T, R) plane.

NOTE 1 : Use explicit Runge-Kutta method of order 5(4) for all simulations. One unit of time in this simulation represents one year. This is available in scipy library "scipy.integrate.solve_ivp"

NOTE 2 : parameters with units "mm/day" in Table. 1 can be directly used in the differential equations without any need for conversion into "mm/year"

params	description	default value	units
$R0$	default amount of rainfall	2.0	`mm/day`
r_m	maximal rate of tree cover expansion rate	0.3	`1/yr`
h_R	rainfall value where r is reduced by half	0.5	`mm/day`
m_n	maximal loss rate due to nursing effect	0.15	`1/yr`
h_n	tree cover below which rate of loss increases steeply (nursing effect)	10	%
m_f	maximal rate of loss due to fire mortality	0.11	`1/yr`
h_f	tree cover below which rate of loss increases steeply (fire mortality)	60	%
p	Hill function exponent	7	
k	Maximal carrying capacity	90	%

Table 1: List of parameter values for modeling the dynamics of tree cover

Practical information

Questions : Feel free to contact me at hari.kalidindi@uclouvain.be to ask questions or set up an appointment.

Attention : You must do all the writing in groups of two students. Never share your production.

Submission : Each group is required to submit a zip containing its report (pdf) and its codes for Wednesday 16 november, 8.30 am on Moodle.

Language : All reports are equally accepted in French and English