ITDSIU21095_RA_LAB2

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1 Problem 1:

[1]:
```
import pandas as pd, numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
import scipy.stats as stats
import statsmodels.api as sm
import statsmodels.formula.api as smf
```

1.0.1 a.

[2]:
```
df = pd.read_csv('1.19.txt', sep = '\s+')
df.head()
# Y = GPA
# X = ACT
```

[2]:

	Y	X
0	3.897	21
1	3.885	14
2	3.778	28
3	2.540	22
4	3.028	21

[3]:
```
X = df['X']
Y = df['Y']
X_bar = np.mean(X)
Y_bar = np.mean(Y)
X_err = X - X_bar
Y_err = Y - Y_bar
print(X_bar, Y_bar)
X_err.head()
```

```
24.725 3.0740500000000006
```

[3]:
```
0    -3.725
1   -10.725
2     3.275
3    -2.725
4    -3.725
Name: X, dtype: float64
```

[4]:
```
A = np.sum(X_err*Y_err)
B = np.sum(X_err**2)
print(A,'\n',B)
```

```
92.40565
 2379.925
```

[5]:
```
b1 = A / B
b0 = Y_bar - b1*X_bar
print(b1, b0)
```

```
0.038827126905259614 2.1140492872674566
```

[6]:
```
X_hat = 28
n = len(X)
Y_hat = b0 + b1* X_hat
resid = Y - Y_hat
print(np.sum(resid))
print((np.sum((resid - np.mean(resid))**2)))
print(np.var(resid, ddof=1)*(n-1))
```

```
-15.259060873767076
49.405453699999995
49.4054537
```

[7]:
```
SSE = np.sum((Y - Y_hat)**2)
MSE = SSE / (n-2)
print(MSE, "\n", SSE)
```

```
0.43513371347099783
 51.34577818957774
```

[8]:
```
s2_Yh= MSE * (1/n + ((X_hat-X_bar)**2)/sum(((X-X_hat)**2)))
s_Yh = np.sqrt(s2_Yh)
print(s2_Yh,'\n' ,s_Yh)
```

```
0.004898838859112428
 0.06999170564511503
```

[9]:
```
t = stats.t.ppf(q = 1- 0.05/2, df=n-2)
t
```

[9]:
```
1.9802722492407059
```

[10]:
```
L = Y_hat - t * s_Yh
U = Y_hat + t * s_Yh
print('L =',L)
print('U =',U)
```

```
L = 3.0626062082486802
U = 3.339811472980771
```

95 percent interval estimate of the mean freshman GPA for students whose ACT test score is 28 that is between 3.06 and 3.34

1.0.2 b.

[11]:
```
s2_pred = MSE + s2_Yh
s_pred = np.sqrt(s2_pred)
s_pred
```

[11]: 0.6633494948593164

[12]:
```
L2 = Y_hat - t * s_pred
U2 = Y_hat + t * s_pred
print('L =',L2)
print('U =',U2)
```

```
L = 1.8875962443969811
U = 4.51482143683247
```

Mary Jones freshman GPA using a 95 percent prediction interval that is between 1.9 and 4.5

1.0.3 c.

The prediction interval is larger than the confidence interval and that is expected because the variance of the prediction interval is larger and therefore the interval should be wider.

1.0.4 d.

[13]:
```
W2 =2* stats.f.ppf(q = 1- 0.05,dfn = 2, dfd=n-2)
W2
```

[13]: 6.146180682334335

[14]:
```
W = np.sqrt(W2)
W
```

[14]: 2.4791491851710608

[15]:
```
L3 = Y_hat - A * s_Yh
U3 = Y_hat + A * s_Yh
```

```
print('L =',L3)
print('U =',U3)
```

```
L = -3.2664202141307976
U = 9.668837895360248
```

Confident band when Xh= 28: 3.027688960595906 and 3.3747287206335455

No, the confident band is a bit wider at this point Xh = 28 than the confident interval in part (a) because it is not just presenting the confidence intervals at a single Xh, it is presenting the confidence intervals for entire regression line.

2 Problem 2:

2.0.1 a.

[16]:
```
df1 = pd.read_csv('CH01PR28.txt', sep='\s+', header=None, names=['crime_rate',␣
 ↪'percentage'])
df1.head()
# Y = crime rate
# X = percentage
```

[16]:

	crime_rate	percentage
0	8487	74
1	8179	82
2	8362	81
3	8220	81
4	6246	87

[17]:
```
X1 = df1['percentage']
Y1 = df1['crime_rate']
X1_bar = np.mean(X1)
Y1_bar = np.mean(Y1)
X1_err = X1 - X1_bar
Y1_err = Y1 - Y1_bar
print(X1_bar, Y1_bar)
X1_err.head()
```

```
78.5952380952381 7111.202380952381
```

[17]:
```
0   -4.595238
1    3.404762
2    2.404762
3    2.404762
4    8.404762
Name: percentage, dtype: float64
```

[18]:
```
A1 = np.sum(X1_err*Y1_err)
B1 = np.sum(X1_err**2)
```

```python
print(A,B)
```

```
92.40565 2379.925
```

[19]:
```python
b1_2 = A1 / B1
b0_2 = Y1_bar - b1_2*X1_bar
print(b1_2, b0_2)
```

```
-170.57518863868833 20517.599945150243
```

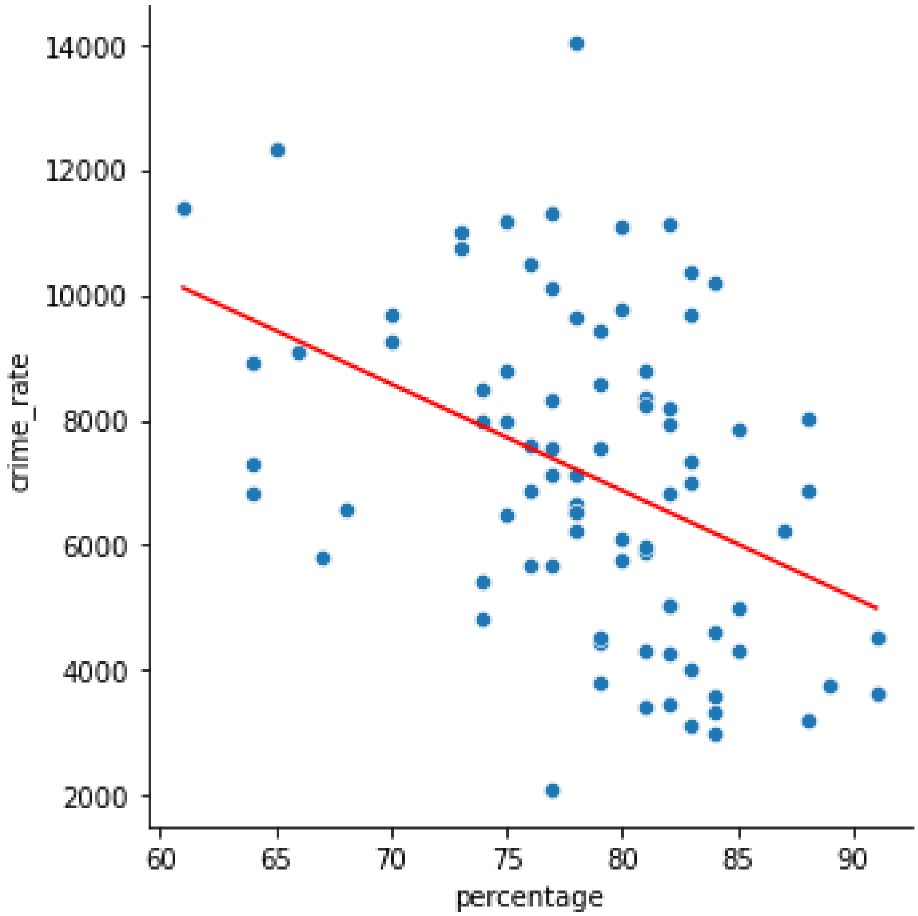
[20]:
```python
model = smf.ols('Y1 ~ X1', data=df1)
results = model.fit()
results.summary()
```

[20]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                            OLS Regression Results
==============================================================================
Dep. Variable:                     Y1   R-squared:                       0.170
Model:                            OLS   Adj. R-squared:                  0.160
Method:                 Least Squares   F-statistic:                     16.83
Date:                Sat, 08 Oct 2022   Prob (F-statistic):           9.57e-05
Time:                        22:49:06   Log-Likelihood:                -770.43
No. Observations:                  84   AIC:                             1545.
Df Residuals:                      82   BIC:                             1550.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept   2.052e+04   3277.643      6.260      0.000     1.4e+04     2.7e+04
X1          -170.5752     41.574     -4.103      0.000    -253.280     -87.871
==============================================================================
Omnibus:                        2.224   Durbin-Watson:                   1.495
Prob(Omnibus):                  0.329   Jarque-Bera (JB):                2.229
Skew:                           0.360   Prob(JB):                        0.328
Kurtosis:                       2.655   Cond. No.                     1.01e+03
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The condition number is large, 1.01e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
"""
```

[21]:
```python
sns.relplot(x='percentage', y='crime_rate', data=df1)
# z = np.linspace(20, 120, 20)
```

```
sns.lineplot(x=X1, y=b0_2+b1_2*X1, color='red')
plt.show()
```



2.0.2 b.

[22]:
```
n1 = len(X1)
Y1_hat = b0_2 + b1_2* X1
resid1 = Y1 - Y1_hat
print(np.sum(resid1))
print((np.sum((resid1 - np.mean(resid1))**2)))
print(np.var(resid1, ddof=1)*(n1-1))
```

```
-5.093170329928398e-11
455273165.2925345
455273165.2925346
```

[23]:
```
SSE_1 = np.sum((Y1 - Y1_hat)**2)
MSE_1 = SSE_1 / (n1-2)
```

```
print(MSE_1, SSE_1)
```

```
5552111.771860177 455273165.2925345
```

[24]:
```
s2_b1_2= MSE_1/ B1
s_b1_2 = np.sqrt(s2_b1_2)
s_b1_2
```

[24]:
```
41.57432781088715
```

[25]:
```
t1 = stats.t.ppf(q = 1- 0.02/2, df=n1-2)
t1
```

[25]:
```
2.3726873452471393
```

[26]:
```
t_star = b1_2/s_b1_2
t_star
```

[26]:
```
-4.1028970910750235
```

Thus, |t_star| = 4.102 > t = 2.372, so we conclude Ha.

2.0.3 c.

[27]:
```
SSTO_1 = np.sum((Y1 - Y1_bar)**2)
print(SSTO_1)
```

```
548736107.5595237
```

[28]:
```
R2 = 1 - SSE_1/SSTO_1
R2
```

[28]:
```
0.17032402457104745
```

The coefficient of determination R-square is 0.1703

2.0.4 d.

[29]:
```
t2 = stats.t.ppf(q = 1-0.05/2, df= n1 -2)
t2
```

[29]:
```
1.9893185569368186
```

[30]:
```
L_d = b1_2 - t2* s_b1_2
U_d = b1_2 + t2* s_b1_2
print('L=',L_d)
print('U=',U_d)
```

```
L= -253.2797704450606
U= -87.87060683231607
```

We can conclude that the 95% confidence interval on the slope is from -253.28 to -87.87.

With confidence coefficient .95, we estimate that the mean Crime rate purity increases by somewhere between -253.28 and -87.87 for every each point increase in n the percentage of individuals in the county having at least a high-school diploma

2.0.5 e.

[31]:
```
Xh_2 = 75
Y1_hat2 = b0_2 + b1_2* Xh_2
Y1_hat2
```

[31]: 7724.460797248617

[32]:
```
SSE_2 = np.sum((Y1 - Y1_hat2)**2)
MSE_2 = SSE_2 / (n1-2)
print(MSE_2, SSE_2)
```

```
7077162.462351341 580327321.91281
```

[33]:
```
s2_Yh_1= MSE_2 * (1/n1 + ((Xh_2-X1_bar)**2)/sum(((X1-X1_bar)**2)))
s_Yh_1 = np.sqrt(s2_Yh_1)
print(s2_Yh_1,'\n' ,s_Yh_1)
```

```
112729.75474445238
 335.752520086525
```

[34]:
```
t_2 = stats.t.ppf(q = 1- 0.05/2, df=n-2)
t_2
```

[34]: 1.9802722492407059

[35]:
```
L_2 = Y1_hat2 - t_2*s_Yh_1
U_2 = Y1_hat2 + t_2*s_Yh_1
print('L =',L_2)
print('U =',U_2)
```

```
L = 7059.579399108639
U = 8389.342195388595
```

With the 95% confidence interval for the mean Crime rate purity when the percentage of individuals in the county having at least a high-school diploma is 75% that is between 7059.58 and 8389.34

For the population of all medium-sized counties in which 75% of individuals in the county having at least a high-school diploma, we are 95% certain that the mean Crime rate purity was between 7059 and 8389

[ ]:
```

```