RA_LAB3_ITDSIU21095

October 15, 2022

1 Problem 1

[1]:
```
import pandas as pd, numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
import scipy.stats as stats
import statsmodels.api as sm
import statsmodels.formula.api as smf
```

[15]:
```
df = pd.read_csv('CH01PR27.txt', sep = '\s+', header = None, names = ['the␣
↪measure of muscle mass','age'])
df.head()
```

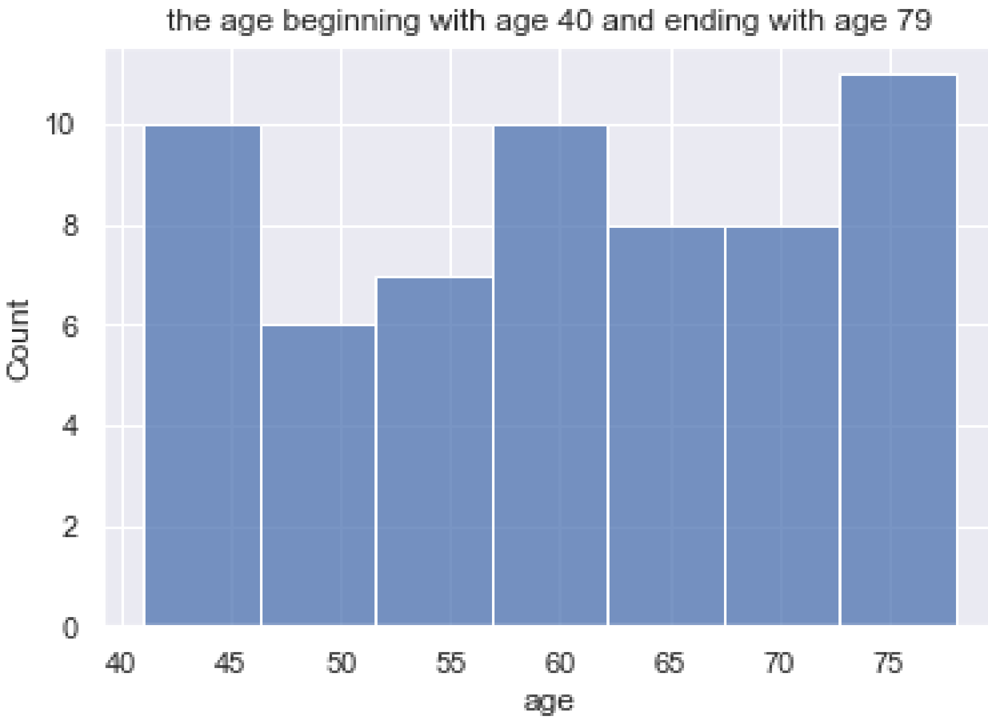
[15]:

	the measure of muscle mass	age
0	106	43
1	106	41
2	97	47
3	113	46
4	96	45

1.1 a.

Prepare a histogram for the ages Xi. What information does your plot provide? Is this plot consistent with the random selection of women from each 10-year age group? Explain.

[23]:
```
sns.set_theme()
sns.histplot(data=df, x = 'age').set(title='the age beginning with age 40 and␣
↪ending with age 79');
```



The information the plot provide is a nutritionist randomly selected 15 women from each 10-year age group, beginning with age 40 and ending with age 79. We can noticed that there are a slightly different distribution between these group. However this difference can be accepted and this plot can be consistent with the random selection of women from each 10-year age group.

1.2 b.

Obtain the residuals and prepare a normal probability plot of the residuals. Does the distribution of the residuals appear to be symmetrical?

[24]:
```
X = df['age']
Y = df['the measure of muscle mass']
X_bar = np.mean(X)
Y_bar = np.mean(Y)
X_err = X - X_bar
Y_err = Y - Y_bar
print(X_bar, Y_bar)
X_err.head()
```

```
59.983333333333334 84.96666666666667
```

[24]:
```
0   -16.983333
1   -18.983333
```

```
2   -12.983333
3   -13.983333
4   -14.983333
Name: age, dtype: float64
```

[25]:
```
A = np.sum(X_err*Y_err)
B = np.sum(X_err**2)
print(A,'\n',B)
```

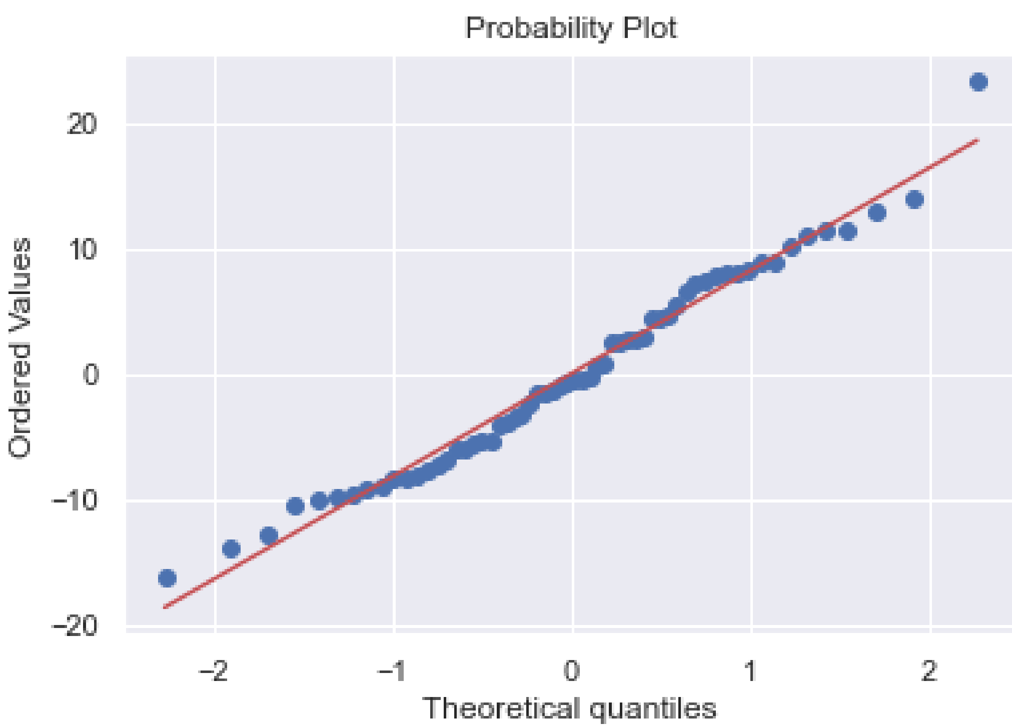
```
-9771.033333333333
 8210.983333333335
```

[26]:
```
b1 = A / B
b0 = Y_bar - b1*X_bar
print(b1, b0)
```

```
-1.1899955141385823 156.34656425641265
```

[27]:
```
n= len(X)
Y_hat = b0 + b1 * X
resid = Y - Y_hat
```

[34]:
```
stats.probplot(resid, dist="norm", plot = plt);
plt.show();
```

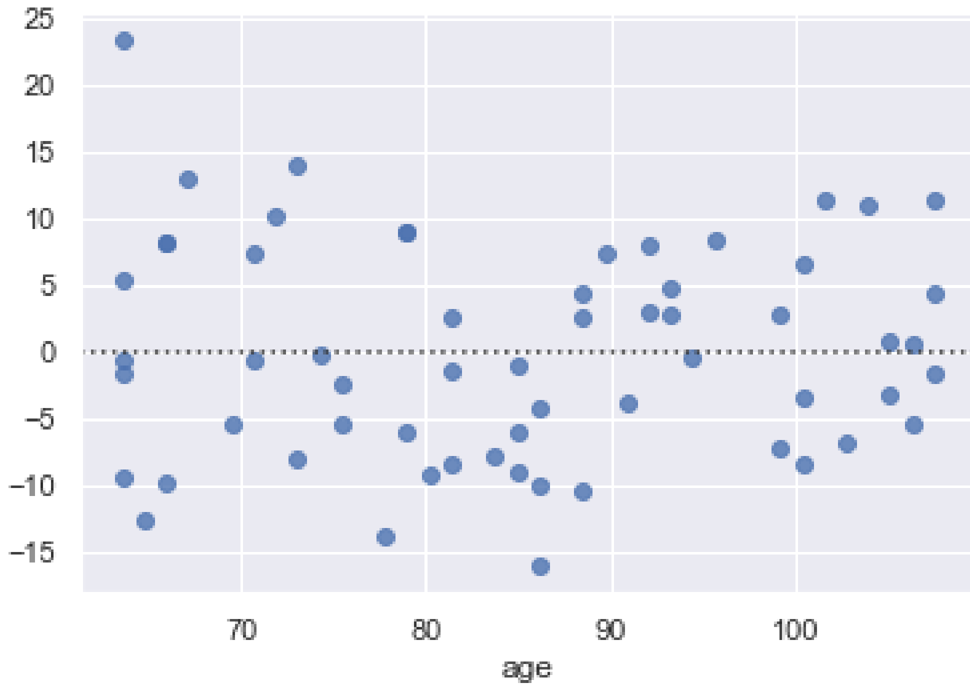


The distribution of the residuals appear to be symmetrical. However, the distribution in the right top connor and left bottom conner is not symmetrical.

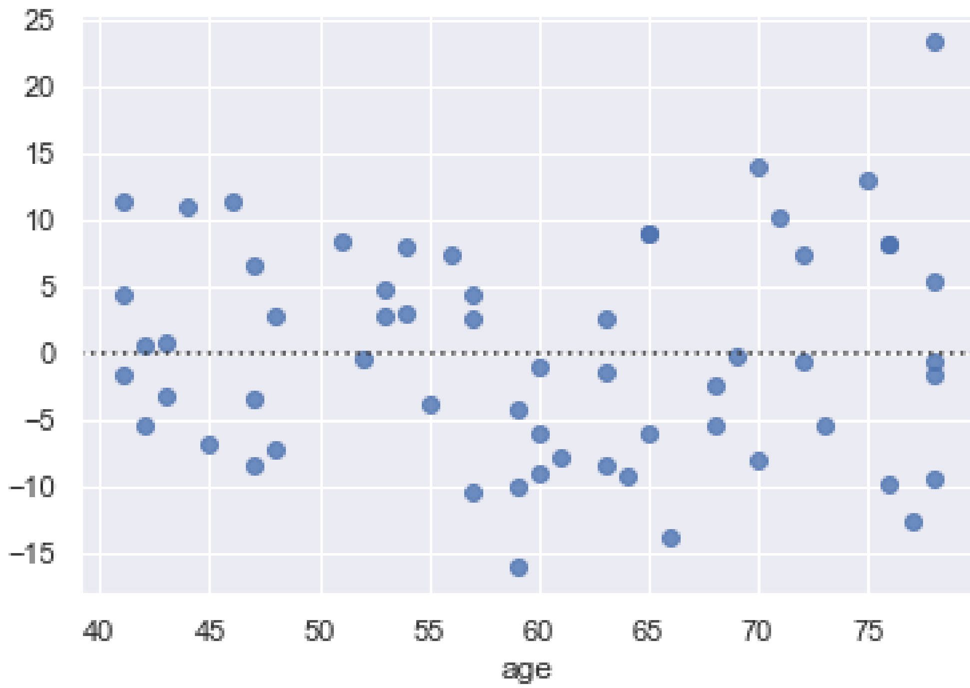
1.3 c.

Plot the residuals against Yi_hat and also against Xi on separate graphs to ascertain whether any departures from regression model (2.1) are evident. Do the two plots provide the same information? State your conclusions.

[40]:
```
sns.residplot(x=Y_hat,y= resid, data=df);
```



[41]:
```
sns.residplot(x=X, y=resid, data=df);
```



The two plots provide the same information. Additionally, there is an outlier in the left top corner in plot 1 and in the right top corner in plot 2

1.4 d.

Assume that (3.10) is applicable and conduct the Breusch-Pagan test to determine whether or not the error variance varies with the level of . Use = 0.01. State the alternatives, decision rule, and conclusion. Is your conclusion consistent with your preliminary findings in part (c)?

[43]:
```python
model = smf.ols('Y ~ X', data=df)
results = model.fit()
results.summary()
```

[43]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                            OLS Regression Results
==============================================================================
Dep. Variable:                      Y   R-squared:                       0.750
Model:                            OLS   Adj. R-squared:                  0.746
Method:                 Least Squares   F-statistic:                     174.1
Date:                Sat, 15 Oct 2022   Prob (F-statistic):           4.12e-19
Time:                        10:21:59   Log-Likelihood:                -210.17
No. Observations:                  60   AIC:                             424.3
Df Residuals:                      58   BIC:                             428.5
```

```
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept    156.3466      5.512     28.363      0.000     145.313     167.381
X             -1.1900      0.090    -13.193      0.000      -1.371      -1.009
==============================================================================
Omnibus:                        1.213   Durbin-Watson:                   2.416
Prob(Omnibus):                  0.545   Jarque-Bera (JB):                1.208
Skew:                           0.319   Prob(JB):                        0.547
Kurtosis:                       2.726   Cond. No.                         319.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
"""
```

The alternative conclusions are: H0 : 1 == 0

Ha : 1 != 0

The decision rule is: If X^2 <= X^2_BP, conclude H0

If X^2 >= X^2_BP, conclude Ha

[46]:
```python
alpha = 0.01
resid_square = resid**2
model_1 = smf.ols('resid_square ~ X', data=df)
results_1 = model_1.fit()
results_1.summary()
```

[46]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                            OLS Regression Results
==============================================================================
Dep. Variable:           resid_square   R-squared:                       0.074
Model:                            OLS   Adj. R-squared:                  0.058
Method:                 Least Squares   F-statistic:                     4.615
Date:                Sat, 15 Oct 2022   Prob (F-statistic):             0.0359
Time:                        10:29:57   Log-Likelihood:                -349.29
No. Observations:                  60   AIC:                             702.6
Df Residuals:                      58   BIC:                             706.8
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
```

```
------------------------------------------------------------------------------
Intercept    -53.5326     56.015     -0.956      0.343    -165.659      58.593
X              1.9690      0.917      2.148      0.036       0.134       3.804
==============================================================================
Omnibus:                       66.573   Durbin-Watson:                   2.197
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              547.395
Skew:                           3.051   Prob(JB):                    1.36e-119
Kurtosis:                      16.481   Cond. No.                         319.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
"""
```

[48]:
```python
SSR_star= sum((results_1.fittedvalues - np.mean(resid_square))**2)
SSR_star
```

[48]: 31833.428951644546

[51]:
```python
X2_BP = (SSR_star/2) / ((SSE/n)**2)
X2_BP
```

[51]: 3.817124858502205

[82]:
```python
X2 = stats.chi2.ppf(q=1-0.01,df=1)
p =1- stats.chi2.cdf(x=3.817,df=1)
print(X2,p)
```

```
6.6348966010212145 0.050735001048005888
```

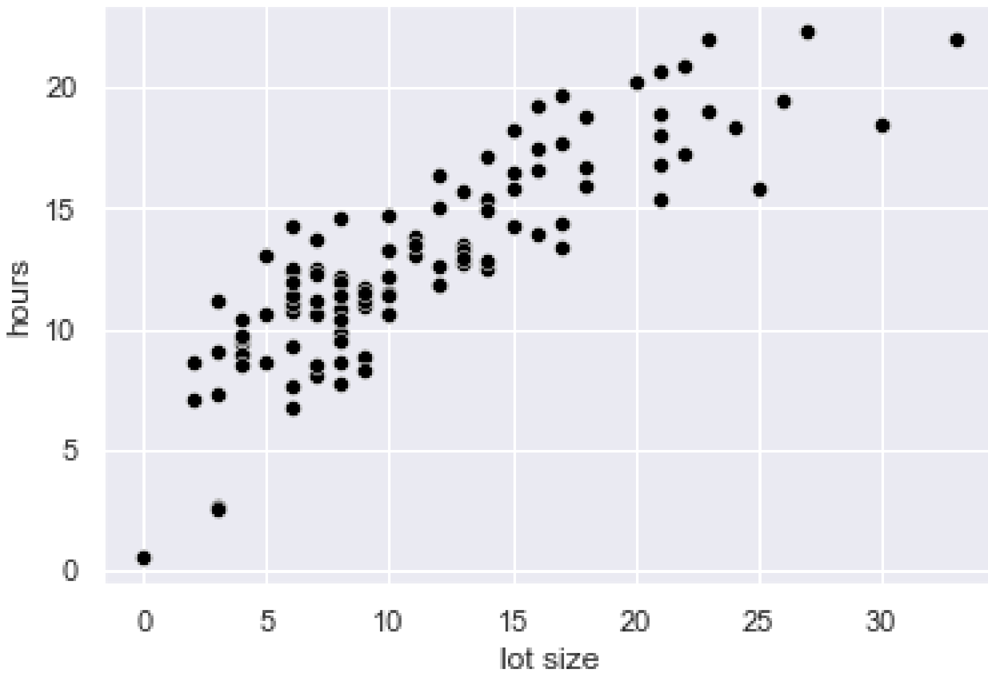
Conclusion: to control the alpha risk at 0.01, we require X^2(0.99;1) = 6.6. Since X^2_BP= 3.817 <= 6.63, we conclude H0, that the error variance is constant.

2 Problem 2:

[102]:
```python
df1 = pd.read_csv('CH03PR18.txt', sep = '\s+', header =None,␣
 ↪names=['hours','lot size'])
df1.head()
x= df1['lot size']
y= df1['hours']
x_new = np.sqrt(x)
```

2.1 a.

[72]:
```python
sns.scatterplot(x='lot size', y='hours', data=df1, color = 'black');
```



- A linear relation does not appear to be adequate here.
- The regression relation in the scatter-plot appears to be curvilinear here.
- The variability across different X levels appears to be fairly constant, thus a transformation to X is more appropriate here.

2.2 b.

[104]:
```
model = smf.ols('y ~ x_new', data=df1)
results = model.fit()
results.summary()
```

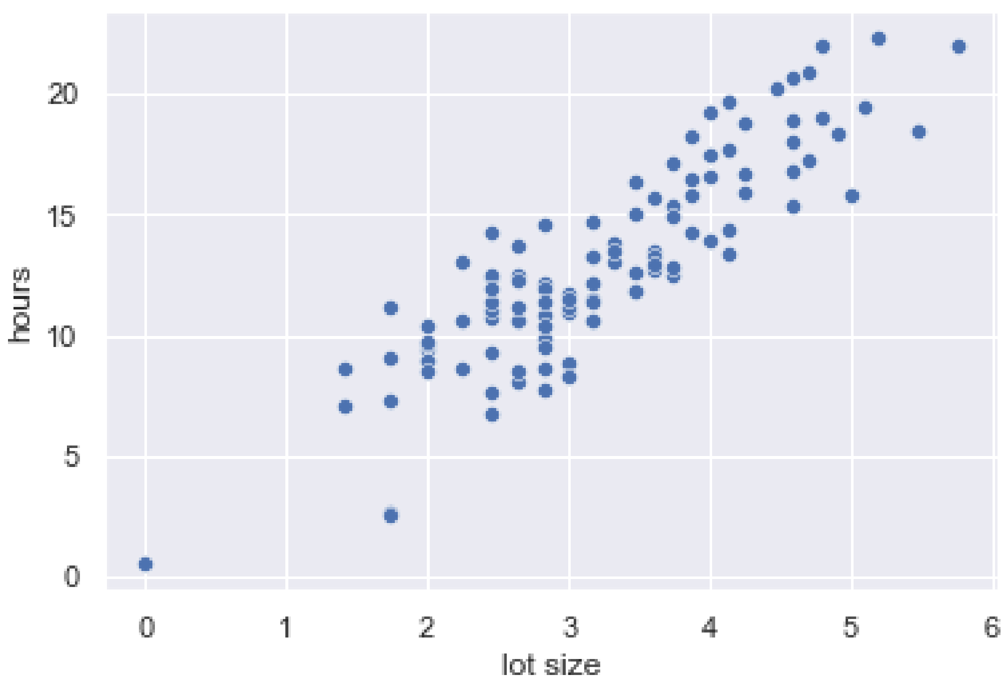
[104]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.770
Model:                            OLS   Adj. R-squared:                  0.768
Method:                 Least Squares   F-statistic:                     365.7
Date:                Sat, 15 Oct 2022   Prob (F-statistic):           1.29e-36
Time:                        11:56:43   Log-Likelihood:                -232.88
No. Observations:                 111   AIC:                             469.8
Df Residuals:                     109   BIC:                             475.2
Df Model:                           1
Covariance Type:            nonrobust
```

```
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept      1.2547      0.639      1.964      0.052      -0.012       2.521
x_new          3.6235      0.189     19.124      0.000       3.248       3.999
==============================================================================
Omnibus:                        1.123   Durbin-Watson:                   2.098
Prob(Omnibus):                  0.570   Jarque-Bera (JB):                1.160
Skew:                          -0.151   Prob(JB):                        0.560
Kurtosis:                       2.600   Cond. No.                         12.3
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
"""
```

- The estimated linear regression function for the transformed data is slightly higher than it original.

2.3 c.

```
[105]: sns.scatterplot(x=x_new, y= y, data =df1);
```

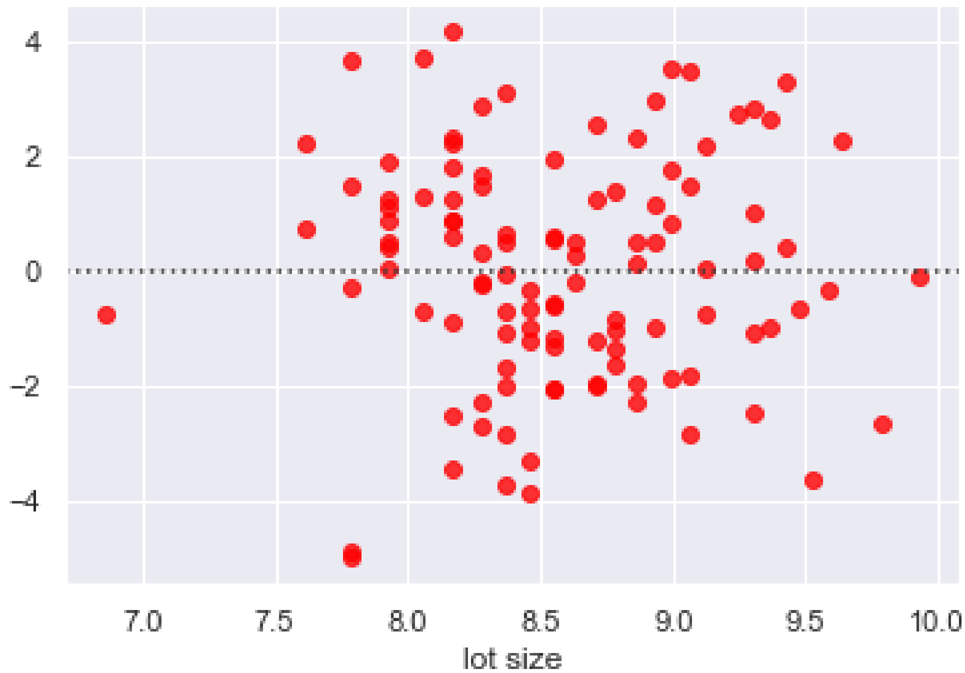


Conclusion: The scatter-plot shows a reasonably linear relation, the estimated regression line appears to be a good fit to the transformed data.

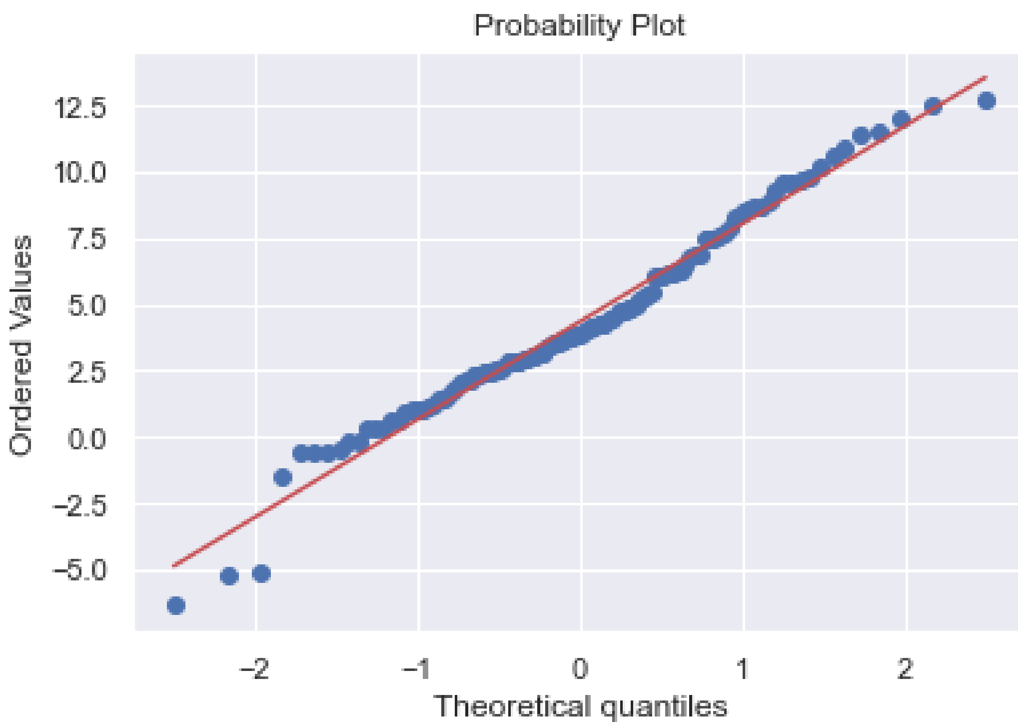
2.4 d.

[109]:
```
x_bar = np.mean(x)
y_bar = np.mean(y)
x_err = x - x_bar
y_err = y - y_bar
print(x_bar, y_bar)

a = np.sum(x_err*y_err)
b = np.sum(x_err**2)
print(a,'\n',b)

b1_2= a / b
b0_2 = y_bar - b1_2*x_bar
print(b1_2, b0_2)

y_hat = b0_2 + b1_2 * x
y_hat_new = b0_2 + b1_2 * x_new
resid1 = y - y_hat_new

sns.residplot(x=y_hat_new, y= resid1,data=df1,color='red');
```

```
11.36936936936937 12.926486486486487
2608.7040540540543
 4891.855855855856
0.5332749228355274 6.863486913347429
```



[110]:
```
stats.probplot(resid1, dist="norm", plot = plt);
plt.show();
```



Conclusion The normal probability plot shows that points fall reasonably close to a straight line, with very small tails deviating from the line, suggesting that the distribution of the error terms is approximately normal. These two plots above show a good fit between model and data points. Q - Q plot show the distribution of the residuals appear to be symmetrical.

2.5 e.

2.5.1 Y = 6.863486913347429 + 0.5333 X