In [1]:
```
import pandas as pd, numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
import math
```

9.10

Job proficiency. A personnel officer in a governmental agency administered four newly developed aptitude tests to each of 25 applicants for entry-level clerical positions in the agency. For purpose of the study, all 25 applicants were accepted for positions irrespective of their test scores. After a probationary period, each applicant was rated for proficiency on the job. The scores on the four tests (X1, X2, X3, X4) and the job proficiency score (Y) for the 25 employees were as follows:

Subject	Test Score				Job Proficiency Score
i	X_{i1}	X_{i2}	X_{i3}	X_{i4}	Y_i
1	86	110	100	87	88
2	62	97	99	100	80
3	110	107	103	103	96
...	...	...	...	...	...
23	104	73	93	80	78
24	94	121	115	104	115
25	91	129	97	83	83

a. Prepare separate stem-and-leaf plots of the test scores for each of the four newly developed aptitude tests. Are there any noteworthy features in these plots? Comment.

b. Obtain the scatter plot matrix. Also obtain the correlation matrix of the X variables. What do the scatter plots suggest about the nature of the functional relationship between the response variable Y and each of the predictor variables? Are any serious multicollinearity problems evident? Explain.

c. Fit the multiple regression function containing all four predictor variables as first-order terms. Does it appear that all predictor variables should be retained?

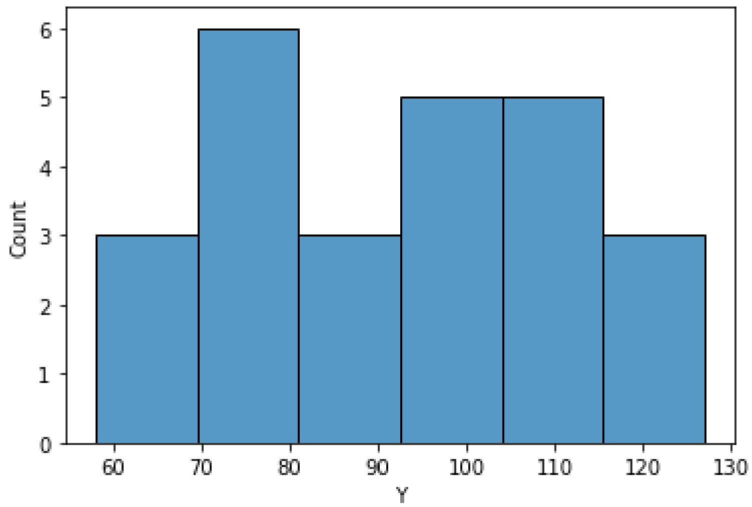
In [11]:
```
df = pd.read_csv('CH09PR10.txt', sep = '\s+', header =None, names=['Y', 'X1', 'X2', 'X
df.head()
```

Out[11]:

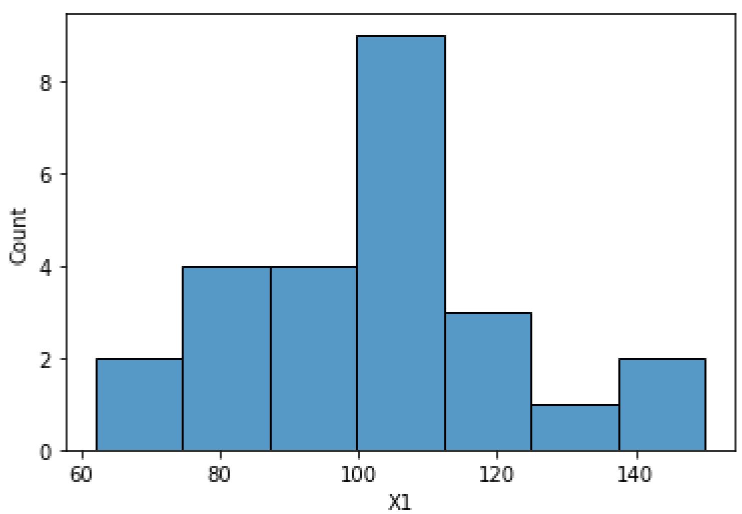
	Y	X1	X2	X3	X4
0	88.0	86.0	110.0	100.0	87.0
1	80.0	62.0	97.0	99.0	100.0
2	96.0	110.0	107.0	103.0	103.0
3	76.0	101.0	117.0	93.0	95.0
4	80.0	100.0	101.0	95.0	88.0

a. Prepare separate stem-and-leaf plots of the test scores for each of the four newly developed aptitude tests. Are there any noteworthy features in these plots? Comment.

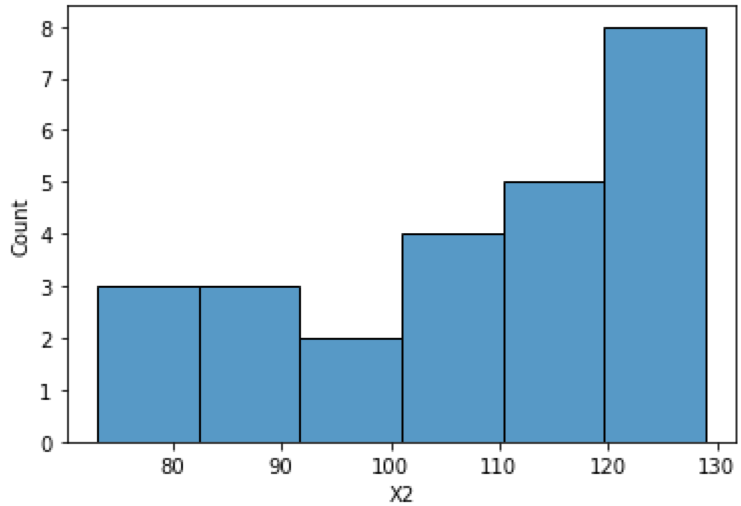
In [12]:
```
sns.histplot(data=df['Y']);
```



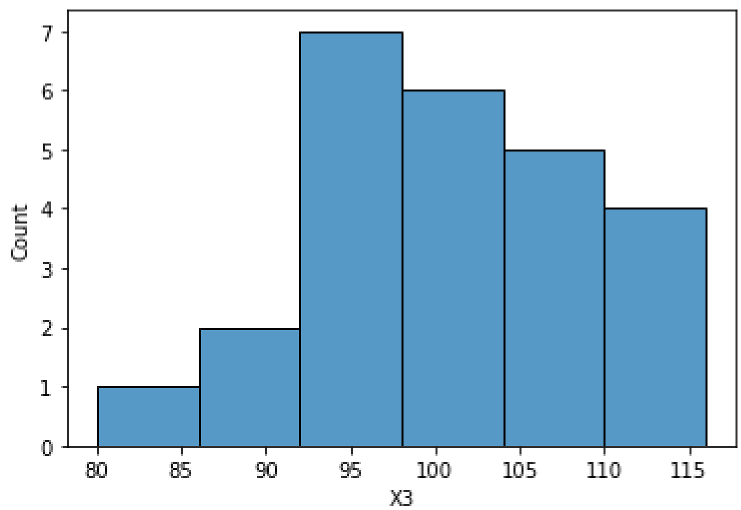
In [13]:
```
sns.histplot(data=df['X1']);
```



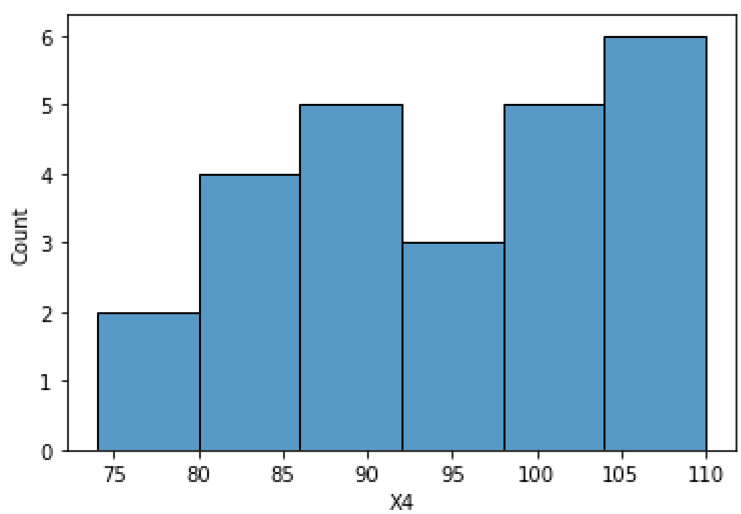
In [14]:
```
sns.histplot(data=df['X2']);
```



In [15]:
```
sns.histplot(data=df['X3']);
```



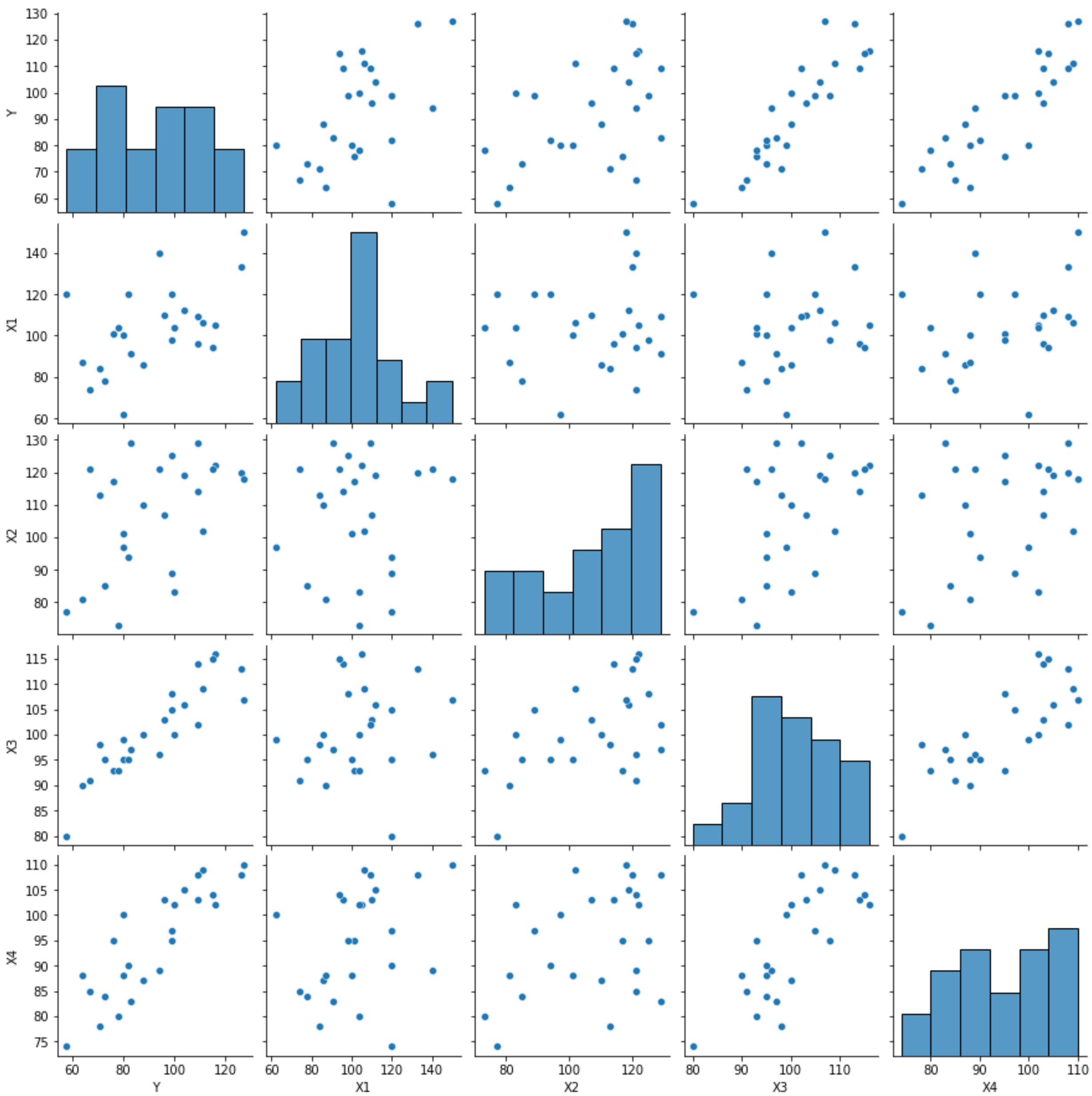
In [16]:
```
sns.histplot(data=df['X4']);
```



=> We can conclude that there are no outliers in these variables because the values of X1,X2,X3,X4 are distributed quite evenly.

b. Obtain the scatter plot matrix. Also obtain the correlation matrix of the X variables. What do the scatterplots suggest about the nature of the functional relationship between the response variable Y and eachof the predictor variables? Are any serious multicollinearity problems evident? Explain

In [17]:
```
sns.pairplot(data=df);
```



In [18]:
```
matrix = df.corr()
print(matrix)
```

```
           Y        X1        X2        X3        X4
Y   1.000000  0.514411  0.497006  0.897064  0.869387
X1  0.514411  1.000000  0.102269  0.180769  0.326663
X2  0.497006  0.102269  1.000000  0.519045  0.396710
X3  0.897064  0.180769  0.519045  1.000000  0.782038
X4  0.869387  0.326663  0.396710  0.782038  1.000000
```

=> The scatter plot matrix shows that X1 and X2 appears to have a modest

linear association with Y (0.514410 and 0.497006, respectively). It's also shows that X3 and X4 have the strongest linear relationships with Y (0.897064 and 0.869387, respectively).Aside from that, X3 and X4 appear to have major multiconlinear issues (0.782038).

c. Fit the multiple regression function containing all four predictor variables as first-order terms. Does it appear that all predictor variables should be retained?

In [20]:

```
y = df['Y']
x1 = df['X1']
x2 = df['X2']
x3 = df['X3']
x4= df['X4']
import statsmodels.api as sm
import statsmodels.formula.api as smf
model = smf.ols('y ~ x1+x2+x3+x4', data=df)
results = model.fit()
results.summary()
```

Out[20]:

OLS Regression Results

Dep. Variable:	y	**R-squared:**	0.963
Model:	OLS	**Adj. R-squared:**	0.955
Method:	Least Squares	**F-statistic:**	129.7
Date:	Sun, 11 Dec 2022	**Prob (F-statistic):**	5.26e-14
Time:	22:14:06	**Log-Likelihood:**	-67.951
No. Observations:	25	**AIC:**	145.9
Df Residuals:	20	**BIC:**	152.0
Df Model:	4		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	-124.3818	9.941	-12.512	0.000	-145.119	-103.645
x1	0.2957	0.044	6.725	0.000	0.204	0.387
x2	0.0483	0.057	0.853	0.404	-0.070	0.166
x3	1.3060	0.164	7.959	0.000	0.964	1.648
x4	0.5198	0.132	3.940	0.001	0.245	0.795

Omnibus:	3.256	**Durbin-Watson:**	1.148
Prob(Omnibus):	0.196	**Jarque-Bera (JB):**	1.419
Skew:	0.139	**Prob(JB):**	0.492
Kurtosis:	1.867	**Cond. No.**	2.47e+03

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 2.47e+03. This might indicate that there are
strong multicollinearity or other numerical problems.

In [23]:

```
results.params
```

Out[23]:

```
Intercept   -124.381821
x1             0.295725
x2             0.048288
x3             1.306011
x4             0.519819
dtype: float64
```

Y= -124.381821+ 0.295725X1+0.048288X2+ 1.306011X3 + 0.519819X4

P-value = 5.26e-14 > 0.05, so we can conclude H0 => it appear that all predictor variables should be retained

9.18

Refer to Job proficiency Problems 9.10 and 9.11.

a. Using forward stepwise regression, find the best subset of predictor variables to predict job proficiency. Use α limits of .05 and .10 for adding or deleting a variable, respectively.

b. How does the best subset according to forward stepwise regression compare with the best subset according to the R2 a,p criterion obtained in Problem 9.11a?

In [ ]:

9.22.

Refer to Job proficiency Problems 9.10 and 9.18. To assess externally the validity of the regression model identified in Problem 9.18, 25 additional applicants for entry-level clerical positions in the agency were similarly tested and hired irrespective of their test scores. The data follow.

Subject	Test Score				Job Proficiency Score
i	X_{i1}	X_{i2}	X_{i3}	X_{i4}	Y_i
26	65	109	88	84	58
27	85	90	104	98	92
28	93	73	91	82	71
...	...	...	...	...	...
48	115	119	102	94	95
49	129	70	94	95	81
50	136	104	106	104	109

a. Obtain the correlation matrix of the X variables for the validation data set and compare it with that obtained in Problem 9.10b for the model-building data set. Are the two correlation matrices reasonably similar?

b. Fit the regression model identified in Problem 9.18a to the validation data set. Compare the estimated regression coefficients and their estimated standard deviations to those obtained in Problem 9.18a. Also compare the error mean squares and coefficients of multiple de termination. Do the estimates for the validation data set appear to be reasonably similar to those obtained for the model-building data set?

c. Calculate the mean squared prediction error in (9.20) and compare it to MSE obtained from the model-building data set. Is there evidence of a substantial bias problem in MSE here? Is this conclusion consistent with your finding in Problem 9.21? Discuss.

d. Combine the model-building data set in Problem 9.10 with the validation data set and fit the selected regression model to the combined data. Are the estimated standard deviations of the estimated regression coefficients appreciably reduced now from those obtained for the model-building data set?

a. Obtain the correlation matrix of the X variables for the validation data set and compare it with that obtained in Problem 9.10b for the model-building data set. Are the two correlation matrices reasonably similar?

In [36]:
```
df1 = pd.read_csv('CH09PR22.txt', sep = '\s+', header =None, names=['Y', 'X1', 'X2', '
df1.head()
```

Out[36]:

	Y	X1	X2	X3	X4
0	58.0	65.0	109.0	88.0	84.0
1	92.0	85.0	90.0	104.0	98.0
2	71.0	93.0	73.0	91.0	82.0
3	77.0	95.0	57.0	95.0	85.0
4	92.0	102.0	139.0	101.0	92.0

In [37]:
```
matrix = df1.corr()
print(matrix)
```

```
           Y        X1        X2        X3        X4
Y   1.000000  0.537078  0.344774  0.888052  0.887939
X1  0.537078  1.000000  0.010571  0.177289  0.319639
X2  0.344774  0.010571  1.000000  0.343744  0.220764
X3  0.888052  0.177289  0.343744  1.000000  0.871447
X4  0.887939  0.319639  0.220764  0.871447  1.000000
```

In [38]:
```
matrix = df.corr()
print(matrix)
```

```
           Y        X1        X2        X3        X4
Y   1.000000  0.514411  0.497006  0.897064  0.869387
X1  0.514411  1.000000  0.102269  0.180769  0.326663
X2  0.497006  0.102269  1.000000  0.519045  0.396710
X3  0.897064  0.180769  0.519045  1.000000  0.782038
X4  0.869387  0.326663  0.396710  0.782038  1.000000
```

=> The two correlation matrices are reasonably similar. The correlation between X1 and X2 increases somewhat, as does the connection between X2 and Y, whereas the correlation between X2 and X4 decreases. The linear link between X3, X4 and Y grows, as do the multicollinear issues of X3 and X4.

b. Fit the regression model identified in Problem 9.18a to the validation data set. Compare the estimatedregression coefficients and their estimated standard deviations to those obtained in Problem 9.18a. Also compare the error mean squares and coefficients of multiple determination. Do the estimates for thevalidation data set appear to be reasonably similar to those obtained for the model-building data set?

model1

In [56]:
```
y = df1['Y']
x1 = df1['X1']
x2 = df1['X2']
x3 = df1['X3']
x4= df1['X4']
import statsmodels.api as sm
import statsmodels.formula.api as smf
model1 = smf.ols('y ~ x1+x3+x4', data=df1)
results1 = model1.fit()
results1.summary()
```

Out[56]:

OLS Regression Results

Dep. Variable:	y	**R-squared:**	0.949
Model:	OLS	**Adj. R-squared:**	0.942
Method:	Least Squares	**F-statistic:**	130.0
Date:	Sun, 11 Dec 2022	**Prob (F-statistic):**	1.02e-13
Time:	23:13:22	**Log-Likelihood:**	-69.668
No. Observations:	25	**AIC:**	147.3
Df Residuals:	21	**BIC:**	152.2
Df Model:	3		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	-122.7671	11.848	-10.362	0.000	-147.406	-98.128
x1	0.3124	0.047	6.605	0.000	0.214	0.411
x3	1.4068	0.233	6.048	0.000	0.923	1.891
x4	0.4284	0.197	2.169	0.042	0.018	0.839

Omnibus:	1.376	**Durbin-Watson:**	1.251
Prob(Omnibus):	0.503	**Jarque-Bera (JB):**	0.842
Skew:	-0.449	**Prob(JB):**	0.657
Kurtosis:	2.944	**Cond. No.**	2.33e+03

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 2.33e+03. This might indicate that there are
strong multicollinearity or other numerical problems.

In [57]:
```
sse = np.sum((results1.fittedvalues - df1.Y)**2)
mse = sse/(n-4)
print('MSE=',mse)
```

```
MSE= 18.35493450991355
```

- Adj. R-squared: 0.942
- **Y= -122.7671+ 0.3124X1 + 1.4068X3 + 0.4284X4**
- MSE= 18.35493450991355

model2

In [58]:

```
y = df['Y']
x1 = df['X1']
x2 = df['X2']
x3 = df['X3']
x4= df['X4']
import statsmodels.api as sm
import statsmodels.formula.api as smf
model = smf.ols('y ~ x1+x3+x4', data=df)
results = model.fit()
results.summary()
```

Out[58]:

OLS Regression Results

Dep. Variable:	y	**R-squared:**	0.962
Model:	OLS	**Adj. R-squared:**	0.956
Method:	Least Squares	**F-statistic:**	175.0
Date:	Sun, 11 Dec 2022	**Prob (F-statistic):**	5.16e-15
Time:	23:13:27	**Log-Likelihood:**	-68.397
No. Observations:	25	**AIC:**	144.8
Df Residuals:	21	**BIC:**	149.7
Df Model:	3		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	-124.2000	9.874	-12.578	0.000	-144.734	-103.666
x1	0.2963	0.044	6.784	0.000	0.205	0.387
x3	1.3570	0.152	8.937	0.000	1.041	1.673
x4	0.5174	0.131	3.948	0.001	0.245	0.790

Omnibus:	2.687	**Durbin-Watson:**	1.203
Prob(Omnibus):	0.261	**Jarque-Bera (JB):**	1.314
Skew:	0.154	**Prob(JB):**	0.519
Kurtosis:	1.920	**Cond. No.**	2.10e+03

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 2.1e+03. This might indicate that there are
strong multicollinearity or other numerical problems.

In [59]:

```
sse = np.sum((results.fittedvalues - df.Y)**2)
mse = sse/(n-4)
print('MSE=',mse)
```

```
MSE= 16.580809888523795
```

- Adj. R-squared: 0.956
- **Y= 124.2000+ 0.2963X1 + 1.3570X3 + 0.5174X4**
- MSE= 16.580809888523795

Model 1 and Model 2 produce similar results; however, Model 1 has a larger MSE.

c. Calculate the mean squared prediction error in (9.20) and compare it to MSE obtained from the model-building data set. Is there evidence of a substantial bias problem in MSE here? Is this conclusion consistent with your finding in Problem 9.21? Discuss.

In [ ]:

```

```

d. Combine the model-building data set in Problem 9.10 with the validation data set and fit the selectedregression model to the combined data. Are the estimated standard deviations of the estimatedregression coefficients appreciably reduced now from those obtained for the model-building data set?

In [ ]:

```

```