Problem 3: Kidney function.

Creatinine clearance (Y) is an important measure of kidney function, but is difficult to obtain in a clinical office setting because it requires 24-hour urine collection. To determine whether this measure can be predicted from some data that are easily available, a kidney specialist obtained the data that follow for 33 male subjects. The predictor variables are serum creatinine concentration (X1), age (X2), and weight (X3).

Subject i	X_{i1}	X_{i2}	X_{i3}	Y_i
1	.71	38	71	132
2	1.48	78	69	53
3	2.21	69	85	50
...	...	...	...	...
31	1.53	70	75	52
32	1.58	63	62	73
33	1.37	68	52	57

Adapted from W. J. Shih and S. Weisberg, "Assessing Influence in Multiple Linear Regression with Incomplete Data," *Technometrics* 28 (1986), pp. 231–40.

a. Fit the multiple regression function containing the three predictor variables as first-order terms. Obtain the variance inflation factors. Are there indications that serious multicollinearity problems exist here? Explain.

b. Obtain the residuals and plot them separately against $\hat{Y}$ and each of the predictor variables. Also prepare a normal probability plot of the residuals. Discuss.

c. What is added-variable plot? How is it used for? Prepare separate added-variable plots against e(X1|X2, X3), e(X2|X1, X3), and e(X3|X1, X2). Discuss.

In [1]:
```
import pandas as pd, numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
```

In [2]:
```
df = pd.read_csv('CH09PR15.txt', sep = '\s+', header =None, names=['Y','X1','X2','X3']
df.head()
```

Out[2]:

	Y	X1	X2	X3
0	132.0	0.71	38.0	71.0
1	53.0	1.48	78.0	69.0
2	50.0	2.21	69.0	85.0
3	82.0	1.43	70.0	100.0
4	110.0	0.68	45.0	59.0

In [3]:

```
x1= df['X1']
x2= df['X2']
x3= df['X3']
y= df['Y']
```

a. Fit the multiple regression function containing the three predictor variables as first-order terms. Obtain the variance inflation factors. Are there indications that serious multicollinearity problems exist here? Explain.

In [5]:

```
import statsmodels.api as sm
import statsmodels.formula.api as smf
model = smf.ols('y ~ x1+x2+x3', data=df)
results = model.fit()
results.summary()
```

Out[5]:

OLS Regression Results

Dep. Variable:	y	**R-squared:**	0.855
Model:	OLS	**Adj. R-squared:**	0.840
Method:	Least Squares	**F-statistic:**	56.92
Date:	Sun, 04 Dec 2022	**Prob (F-statistic):**	2.88e-12
Time:	18:22:40	**Log-Likelihood:**	-127.93
No. Observations:	33	**AIC:**	263.9
Df Residuals:	29	**BIC:**	269.8
Df Model:	3		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	120.0473	14.774	8.126	0.000	89.832	150.263
x1	-39.9393	5.600	-7.132	0.000	-51.393	-28.486
x2	-0.7368	0.141	-5.211	0.000	-1.026	-0.448
x3	0.7764	0.172	4.517	0.000	0.425	1.128

Omnibus:	2.889	**Durbin-Watson:**	2.349
Prob(Omnibus):	0.236	**Jarque-Bera (JB):**	2.452
Skew:	-0.658	**Prob(JB):**	0.293
Kurtosis:	2.768	**Cond. No.**	639.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Coefficients

- b0 = 120.0473
- b1 = -39.9393
- b2 = -0.7368
- b3 = 0.7764

Regression function: Y_hat = beta[0] + beta[1]*X1 + beta[2]*X2 + beta[3]*X3

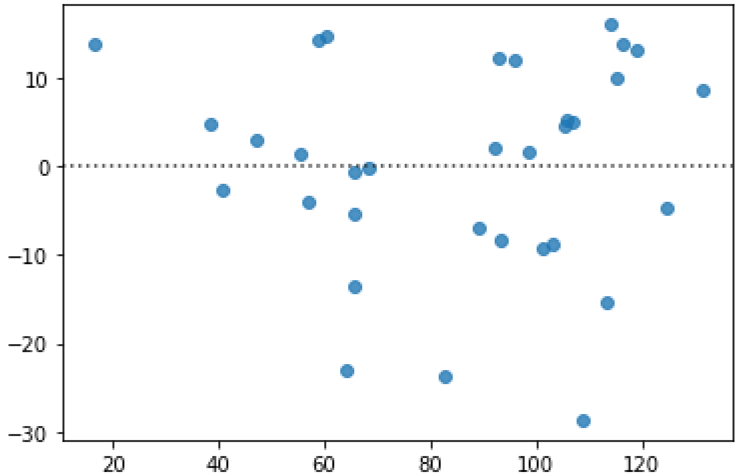
Y_hat = 120.0473 + (-39.9393)X1 + (-0.7368)X2 + (0.7764)X3

The condition number is not large, 639. . This might not indicate that there are not strong multicollinearity or other numerical problems.

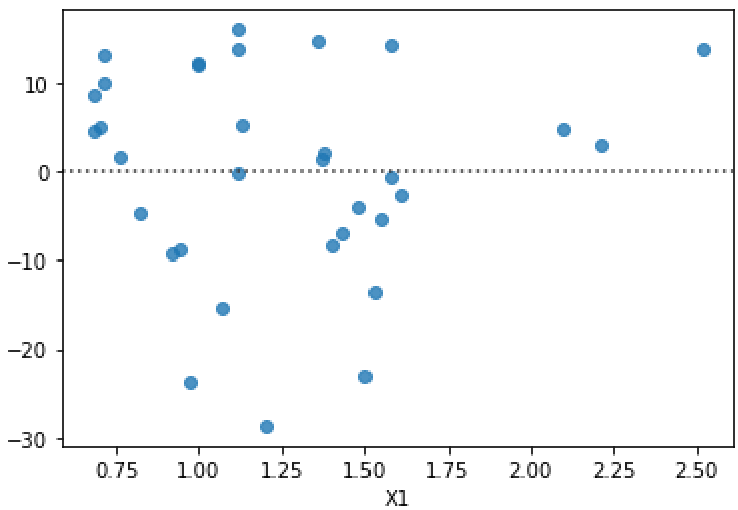
b. Obtain the residuals and plot them separately against $\hat{Y}$ and each of the predictor

variables. Also prepare a normal probability plot of the residuals. Discuss.

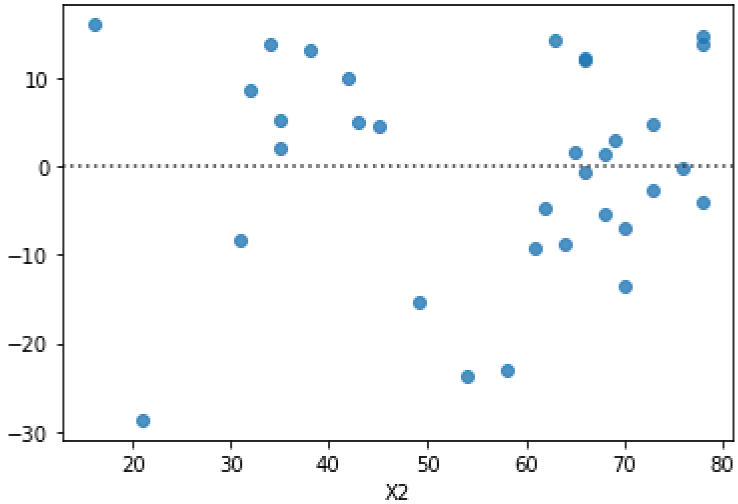
In [7]:
```
resid = results.resid
Y_hat = results.predict()
sns.residplot(x=Y_hat, y=resid, data=df);
```



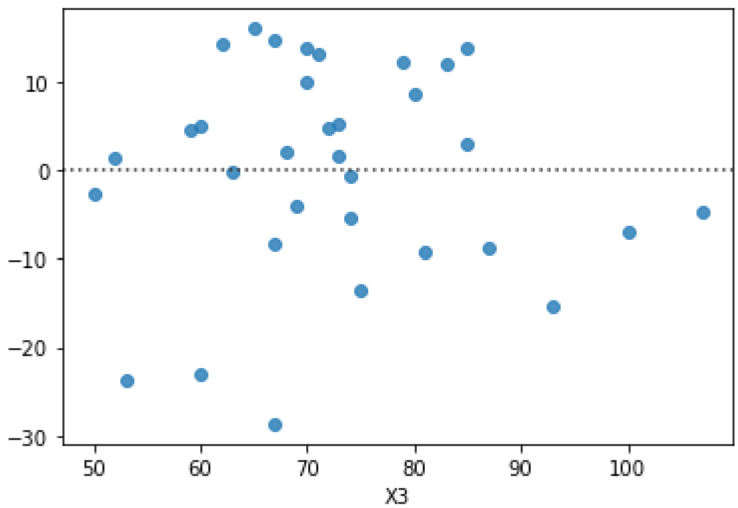
In [8]:
```
# x1
sns.residplot(x=x1, y=resid, data=df);
```



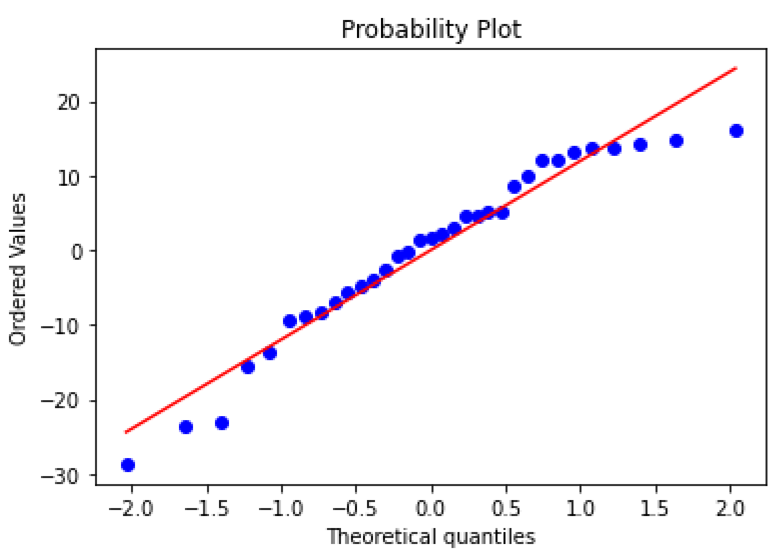
In [9]:
```
# x2
sns.residplot(x=x2, y=resid, data=df);
```



In [10]:
```
# x3
sns.residplot(x=x3, y=resid, data=df);
```



In [12]:
```
import scipy.stats as stats
stats.probplot(resid, dist="norm", plot = plt);
plt.show();
```



There is a slight increase in variability for ei vs. Y_hat, but overall it looks okay. The normal probability plot of the residuals looks fine (relatively straight). The plots of the residuals vs. each predictor all look appropriately "random."

c. What is added-variable plot? How is it used for? Prepare separate added-variable plots against e(X1|X2, X3), e(X2|X1, X3), and e(X3|X1, X2). Discuss.

"Added-variable plots" (also called "partial regression plots" or "adjusted variable plots") are refined residual plots that provide graphic information about the marginal importance of a predictor variable given the other variables already in the model.

In an added-variable plot, both the response variable Y and the predictor variable under investigation (say, X1) are both regressed against the other predictor variables already in the regression model and the residuals are obtained for each. These two sets of residuals reflect the part of each (Y and X1) that is not linearly associated with the other predictor variables.

In [26]:
```
model = smf.ols('x1 ~ x2+x3', data=df)
results = model.fit()
fig = sm.graphics.plot_partregress_grid(results)
```

```
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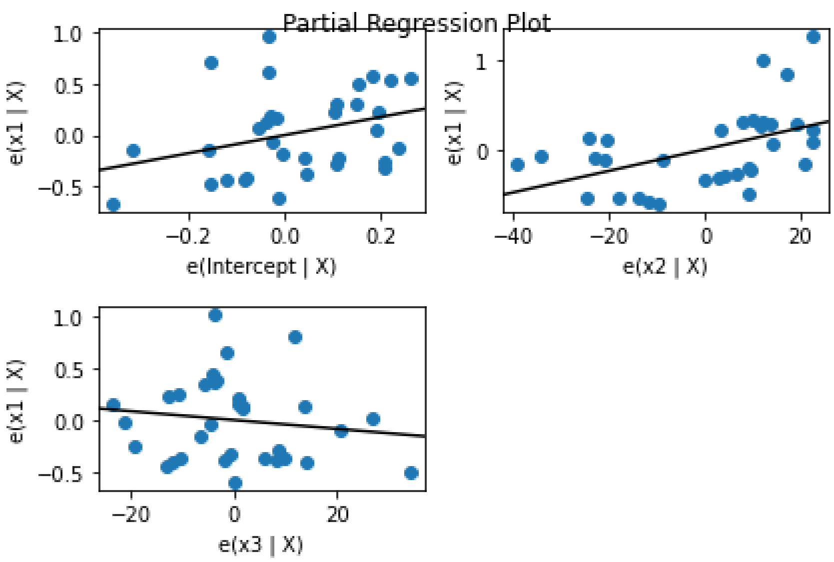
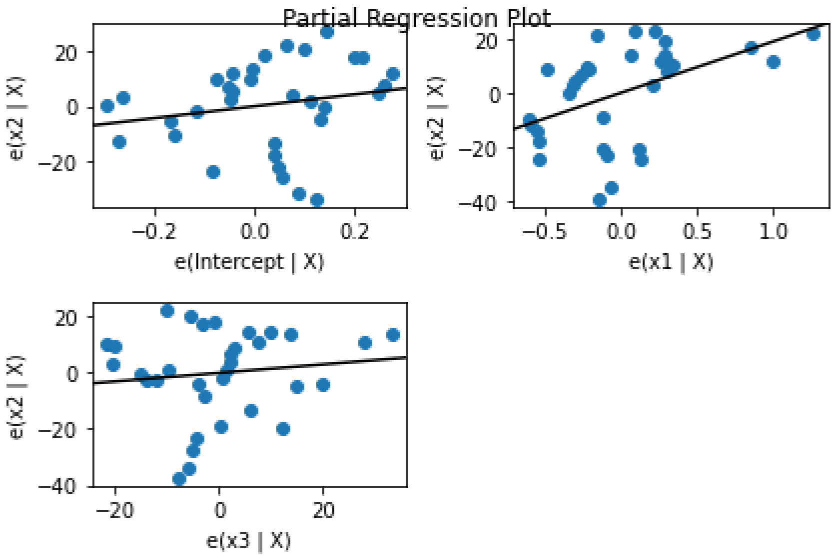


In [27]:
```
model = smf.ols('x2 ~ x1+x3', data=df)
results = model.fit()
fig = sm.graphics.plot_partregress_grid(results)
```

```
eval_env: 1
eval_env: 1
eval_env: 1
```



In [28]:

```
model = smf.ols('x3 ~ x1+x2', data=df)
results = model.fit()
fig = sm.graphics.plot_partregress_grid(results)
```

```
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eval_env: 1
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```