Problem 2

In [1]:
```
import pandas as pd, numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
```

In [2]:
```
df1 = pd.read_csv('CH01PR19.txt', sep = '\s+', header =None, names=['Y','X1'])
```

In [3]:
```
df2 = pd.read_csv('CH08PR16.txt', sep = '\s+', header =None, names=['X2'])
```

In [4]:
```
Y = df1['Y']
X1 = df1['X1']
X2 = df2['X2']
df = pd.DataFrame({'Y':Y,'X1':X1,'X2':X2})
df.head()
```

Out[4]:

	Y	X1	X2
0	3.897	21	0
1	3.885	14	1
2	3.778	28	0
3	2.540	22	1
4	3.028	21	0

In [5]:
```
x1 = df1['X1']
x2 = df2['X2']
y = df1['Y']
```

a. Explain how each regression coefficient in model (8.33) is interpreted here

In [13]:
```
import statsmodels.api as sm
import statsmodels.formula.api as smf
model = smf.ols('y ~ x1+x2', data=df)
results = model.fit()
results.summary()
```

Out[13]:

OLS Regression Results

Dep. Variable:	y	**R-squared:**	0.077
Model:	OLS	**Adj. R-squared:**	0.062
Method:	Least Squares	**F-statistic:**	4.914
Date:	Sat, 19 Nov 2022	**Prob (F-statistic):**	0.00893
Time:	23:08:33	**Log-Likelihood:**	-112.19
No. Observations:	120	**AIC:**	230.4
Df Residuals:	117	**BIC:**	238.7
Df Model:	2		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	2.1984	0.339	6.488	0.000	1.527	2.870
x1	0.0379	0.013	2.949	0.004	0.012	0.063
x2	-0.0943	0.120	-0.786	0.433	-0.332	0.143

Omnibus:	26.256	**Durbin-Watson:**	1.820
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	45.317
Skew:	-0.977	**Prob(JB):**	1.44e-10
Kurtosis:	5.291	**Cond. No.**	151.

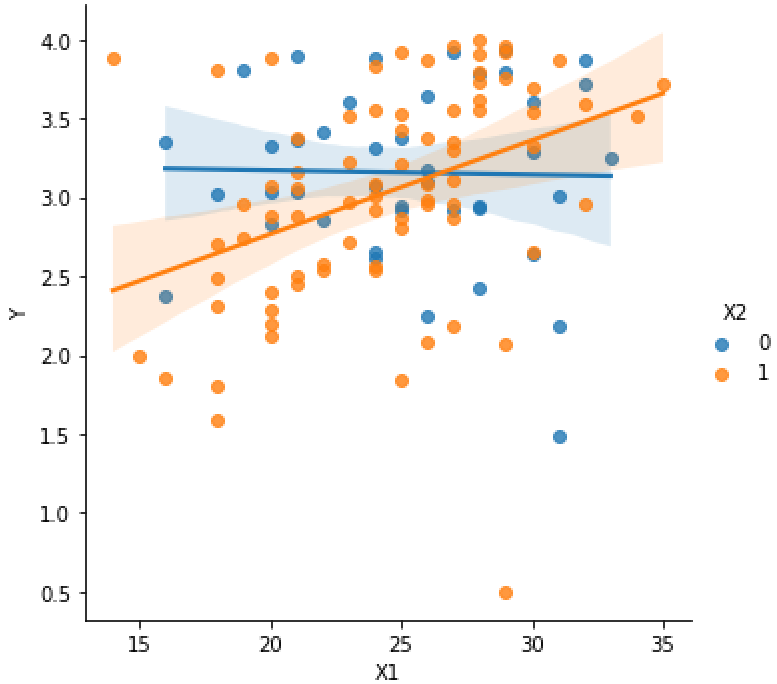
Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

- b1 = 0.0379; for every unit increase or decrease in X1, mean Y is increase or decrease by 0.0379 units.
- b2 = -0.0943; for every unit increase or decrease in X2, mean Y is increase or decrease by 0.0943 units.

b. Fit the regression model and state the estimated regression function.

In [14]:

```
sns.lmplot(x='X1',y='Y',hue='X2',data=df)
plt.show()
```



Coefficients

- b0 = 2.1984
- b1 = 0.0379
- b2 = -0.0943

Regression function: Y_hat = beta[0] + beta[1]*X1* + *beta[2]*X2

Y_hat = 2.1984 + (0.0379)X1 + (-0.0943)X2

c. Test whether the X2 variable can be dropped from the regression model; use α = .01. State the alternatives, decision rule, and conclusion.

The alternatives:

- H0: β2 = 0
- Ha: β2 ≠ 0

The decision rule:

- If F∗ ≤ F(1−α; 1 , n-p), conclude H0
- If F∗ > F(1−α; 1 , n-p), conclude Ha

In [8]:
```
import statsmodels.api as sm
import statsmodels.formula.api as smf
model1 = smf.ols('y ~ x1', data=df)
results1 = model1.fit()
sse1 = np.sum((results1.fittedvalues - df.Y)**2)
ssr1 = np.sum((results1.fittedvalues - df.Y.mean())**2)
```

In [9]:
```
import statsmodels.api as sm
import statsmodels.formula.api as smf
model3 = smf.ols('y ~ x1+x2', data=df)
results3 = model3.fit()
```

In [10]:
```
sse12 = np.sum((results3.fittedvalues - df.Y)**2)
ssr12 = np.sum((results3.fittedvalues - df.Y.mean())**2)
```

In [11]:
```
ssrX2_X1 = ssr12 - ssr1
print('SSR(X2|X1)=',ssrX2_X1)
n = len(y)
Fstar = (ssrX2_X1/1 / (sse12/(n-3)))
print('F*=',Fstar)
import scipy.stats as stats
f = stats.f.ppf(q=1-0.01,dfn=1,dfd=n-3)
print('F=',f)
```

```
SSR(X2|X1)= 0.24071276344662396
F*= 0.6179313729082031
F= 6.856563808110685
```

For α = 0.01, we have F(0.99; 1,n-3) = 6.856. Since F∗ = 0.6179 ≤ 6.856, we conclude H0, that X2 can be dropped from the regression model that already contains X1.

d.Fit regression model (8.49) and state the estimated regression function. $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1} X_{i2} + \varepsilon_i$ **(8.49)**

In [15]:
```
import statsmodels.api as sm
import statsmodels.formula.api as smf
model1 = smf.ols('y ~ x1*x2', data=df)
results1 = model1.fit()
results1.summary()
```

Out[15]:

OLS Regression Results

Dep. Variable:	y	**R-squared:**	0.119
Model:	OLS	**Adj. R-squared:**	0.097
Method:	Least Squares	**F-statistic:**	5.244
Date:	Sat, 19 Nov 2022	**Prob (F-statistic):**	0.00198
Time:	23:09:15	**Log-Likelihood:**	-109.40
No. Observations:	120	**AIC:**	226.8
Df Residuals:	116	**BIC:**	237.9
Df Model:	3		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	3.2263	0.549	5.872	0.000	2.138	4.315
x1	-0.0028	0.021	-0.129	0.898	-0.045	0.040
x2	-1.6496	0.672	-2.454	0.016	-2.981	-0.318
x1:x2	0.0622	0.026	2.350	0.020	0.010	0.115

Omnibus:	28.768	**Durbin-Watson:**	1.936
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	63.541
Skew:	-0.948	**Prob(JB):**	1.59e-14
Kurtosis:	6.019	**Cond. No.**	454.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Coefficients

- b0 = 3.2263
- b1 = -0.0028
- b2 = -1.6496
- b3 = 0.0622

Regression function: Y_hat = beta[0] + beta[1]*X1* + *beta[2]*X2 + beta[3]*X1*X2

Y_hat = 3.2263 + (-0.0028)X1 + (-1.6496)X2 + (0.0622)X1X2

e. Test whether the interaction term can be dropped from the model; use α=.05. State thealternatives, decision rule, and conclusion. If the interaction term cannot be dropped fromthe model, describe the nature of the interaction effect.

The alternatives:

- $H0: \beta1=\beta2=\beta3=0$
- $Ha: \beta1 \neq 0, \beta2 \neq 0, \beta3 \neq 0$

The decision rule:

- If $F* \leq F(1-\alpha, p-1, n-p)$, conclude H0
- If $F* > F(1-\alpha, p-1, n-p)$, conclude Ha

In [16]:
```
p = 3
n = len(y)
F_star = (ssr12/(p-1)) / ((sse12)/(n-p))
print('F*=',F_star)
F = stats.f.ppf(q=1-0.05,dfn=p-1,dfd=n-p)
print('F=',F)
```

```
F*= 4.9141276865928445
F= 3.073762904449709
```

Since F∗ =4.914 > F=3.074, we conclude Ha that β1,β2, and β3 is significantly different from zero. To put it another way, the interaction term shouldn't be dropped from the model.