Problem 2. Grocery retailer.

A large, national grocery retailer tracks productivity and costs of its facilities closely. Data below were obtained from a single distribution center for a one-year period. Each data point for each variable represents one week of activity. The variables included are the number of cases shipped (X1), the indirect costs of the total labor hours as a percentage (X2), a qualitative predictor called holiday that is coded 1 if the week has a holiday and 0 otherwise (X3), and the total labor hours (Y).

i:	1	2	3	...	50	51	52
X_{i1}:	305,657	328,476	317,164	...	290,455	411,750	292,087
X_{i2}:	7.17	6.20	4.61	...	7.99	7.83	7.77
X_{i3}:	0	0	0	...	0	0	0
Y_i:	4264	4496	4317	...	4499	4186	4342

b. Obtain the diagonal element of the hat matrix. Identify any outlying X observations.

c. Management wishes to predict the total labor hours required to handle the next shipment containing X1 = 300,000 cases whose indirect costs of the total hours is X2 = 7.2 and X3 = 0 (no holiday in week). Construct a scatter plot of X2 against X1 and determine visually whether this prediction involves an extrapolation beyond the range of the data. Also, use (10.29) to determine whether an extrapolation is involved. Do your conclusions from the two methods agree?

d. Cases 16, 22, 43, and 48 appear to be outlying X observations, and cases 10, 32, 38, and 40 appear to be outlying Y observations. Obtain the DFFITS, DFBETAS, and Cook's distance values for each of these cases to assess their influence. What do you conclude?

e. Calculate Cook's distance Di for each case and prepare an index plot. Are any cases influential according to this measure?

In [59]:
```
import pandas as pd, numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
```

In [32]:
```
df = pd.read_csv('CH06PR09.txt', sep = '\s+', header =None, names=['Y','X1','X2','X3']
df.head()
```

Out[32]:

	Y	X1	X2	X3
0	4264	305657	7.17	0
1	4496	328476	6.20	0
2	4317	317164	4.61	0
3	4292	366745	7.02	0
4	4945	265518	8.61	1

In [33]:
```
x1= df['X1']
x2= df['X2']
x3= df['X3']
y= df['Y']
```

b. Obtain the diagonal element of the hat matrix. Identify any outlying X observations.

In [34]:
```
n = len(y)
p = 4
h = (2*p)/n
print('H =',h)
from statsmodels.stats.outliers_influence import OLSInfluence
import statsmodels.api as sm
import statsmodels.formula.api as smf
model = smf.ols('y ~ x1+x2+x3', data=df)
results = model.fit()

test_class = OLSInfluence(results)
dir(test_class)
test_class.hat_matrix_diag
```

Out[34]:
```
H = 0.15384615384615385
array([0.02258497, 0.06179963, 0.21887726, 0.05297322, 0.20632818,
       0.02712212, 0.02861964, 0.05635264, 0.04017169, 0.04826901,
       0.03011634, 0.04977033, 0.02761134, 0.06047246, 0.03756448,
       0.25542493, 0.03324965, 0.05104935, 0.02561758, 0.02491881,
       0.19360472, 0.25771995, 0.05677233, 0.07959049, 0.05613301,
       0.02189441, 0.0269728 , 0.06097409, 0.03684681, 0.04174658,
       0.03663401, 0.09602318, 0.04193292, 0.02517837, 0.04621057,
       0.06622225, 0.03108517, 0.03204566, 0.04903249, 0.03210502,
       0.04373193, 0.12395571, 0.28685861, 0.22002363, 0.11050577,
       0.03159426, 0.06494377, 0.28177664, 0.02446692, 0.03420197,
       0.10278142, 0.02754093])
```

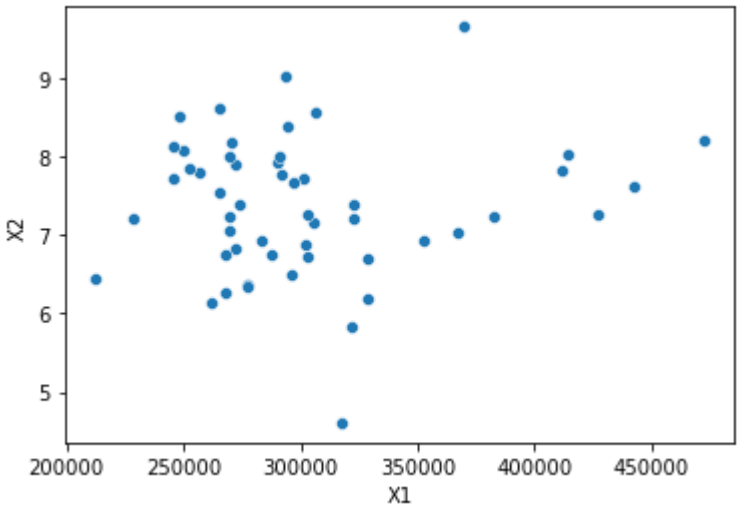
```
array([0.02258497, 0.06179963, 0.21887726, 0.05297322, 0.20632818,
       0.02712212, 0.02861964, 0.05635264, 0.04017169, 0.04826901,
       0.03011634, 0.04977033, 0.02761134, 0.06047246, 0.03756448,
       0.25542493, 0.03324965, 0.05104935, 0.02561758, 0.02491881,
       0.19360472, 0.25771995, 0.05677233, 0.07959049, 0.05613301,
       0.02189441, 0.0269728 , 0.06097409, 0.03684681, 0.04174658,
       0.03663401, 0.09602318, 0.04193292, 0.02517837, 0.04621057,
       0.06622225, 0.03108517, 0.03204566, 0.04903249, 0.03210502,
       0.04373193, 0.12395571, 0.28685861, 0.22002363, 0.11050577,
       0.03159426, 0.06494377, 0.28177664, 0.02446692, 0.03420197,
       0.10278142, 0.02754093])
```

- **Case 3: 0.21887726**
- **Case 5: 0.20632818**
- **Case 16: 0.25542493**
- **Case 21: 0.19360472**
- **Case 22: 0.25771995**
- **Case 43: 0.28685861**
- **Case 44: 0.22002363**
- **Case 48: 0.28177664**

=> The diagonal elements of the hat matrix of case 3 , 5, 16, 21, 22, 43, 44, and 48 exceed twice the mean leverage value. They are considered as outliers

c. Management wishes to predict the total labor hours required to handle the next shipment containing X1 = 300,000 cases whose indirect costs of the total hours is X2 = 7.2 and X3 = 0 (no holiday in week). Construct a scatter plot of X2 against X1 and determine visually whether this prediction involves an extrapolation beyond the range of the data. Also, use (10.29) to determine whether an extrapolation is involved. Do your conclusions from the two methods agree?

In [35]:
```
sns.scatterplot(x=x1, y= x2, data =df);
```



$$(\mathbf{X}'\mathbf{X})^{-1} = \begin{bmatrix} 1.8628 & -.0000 & -.1806 & .0473 \\ & .0000 & -.0000 & -.0000 \\ & & .0260 & -.0078 \\ & & & .1911 \end{bmatrix}$$

X'_new = [1 300,000 7.2 0] hnew,
new = .01829, no extrapolation

d. Cases 16, 22, 43, and 48 appear to be outlying X observations, and cases 10, 32, 38, and 40 appear to be outlying Y observations. Obtain the DFFITS, DFBETAS, and Cook's distance values for each of these cases to assess their influence. What do you conclude?

Cook's distance

In [82]:
```
import statsmodels.api as sm
import statsmodels.formula.api as smf

np.set_printoptions(suppress=True)
influence = results.get_influence()
cooks = influence.cooks_distance
print(cooks[0])
```

```
[0.00029593 0.02447558 0.00208065 0.00210645 0.02299628 0.00027356
 0.00906669 0.02148586 0.00992029 0.04935012 0.00679854 0.00075503
 0.00124502 0.03875147 0.00026066 0.07689508 0.01381267 0.00397261
 0.00389968 0.0107533  0.00876289 0.00077461 0.01124087 0.00742601
 0.00182761 0.00015415 0.00215776 0.00287149 0.00181756 0.00029833
 0.0092834  0.09975974 0.02661839 0.01796257 0.03245103 0.00424634
 0.00782168 0.03463803 0.00778751 0.03649915 0.00049195 0.01276193
 0.07921931 0.01341707 0.00242642 0.00265884 0.01898976 0.00549887
 0.00000234 0.02274285 0.05313716 0.00147602]
```

```
[0.00029593 0.02447558 0.00208065 0.00210645 0.02299628 0.00027356
 0.00906669 0.02148586 0.00992029 0.04935012 0.00679854 0.00075503
 0.00124502 0.03875147 0.00026066 0.07689508 0.01381267 0.00397261
 0.00389968 0.0107533  0.00876289 0.00077461 0.01124087 0.00742601
 0.00182761 0.00015415 0.00215776 0.00287149 0.00181756 0.00029833
 0.0092834  0.09975974 0.02661839 0.01796257 0.03245103 0.00424634
 0.00782168 0.03463803 0.00778751 0.03649915 0.00049195 0.01276193
 0.07921931 0.01341707 0.00242642 0.00265884 0.01898976 0.00549887
 0.00000234 0.02274285 0.05313716 0.00147602]
```

- **Case 16: 0.0769**
- **Case 22: 0.0008**
- **Case 43: 0.0792**
- **Case 48: 0.0055**
- **Case 10: 0.0494**
- **Case 32: 0.0998**
- **Case 38: 0.0346**
- **Case 40: 0.0365**

DFFITS

In [58]:
```
dffits = influence.dffits
print (dffits)
```

```
(array([-0.03406335,  0.31452483, -0.09030093, -0.09097388,  0.30122687,
         0.0327464 , -0.19090788, -0.29454043, -0.19909146,  0.45863297,
         0.16468907, -0.05441298, -0.06995876, -0.39974208,  0.03196095,
        -0.55399026,  0.23658593, -0.12512291,  0.12435794, -0.20892003,
        -0.18554251,  0.05508583, -0.21147763,  0.17115741,  0.08471422,
        -0.02457823,  0.09223019, -0.10624622,  0.08454026, -0.03419262,
        -0.19265267, -0.65107706,  0.33139277,  0.2732792 , -0.36689707,
        -0.12928618, -0.1768339 ,  0.38551766, -0.17575513,  0.3967203 ,
         0.04391492,  0.22441652,  0.56165186,  0.22969393,  0.09756525,
        -0.10239634, -0.27588168, -0.14684146,  0.00302436,  0.30677823,
        -0.46528358,  0.07619914]), 0.5547001962252291)
```

```
(array([-0.03406335,  0.31452483, -0.09030093, -0.09097388,  0.30122687,
         0.0327464 , -0.19090788, -0.29454043, -0.19909146,  0.45863297,
         0.16468907, -0.05441298, -0.06995876, -0.39974208,  0.03196095,
        -0.55399026,  0.23658593, -0.12512291,  0.12435794, -0.20892003,
        -0.18554251,  0.05508583, -0.21147763,  0.17115741,  0.08471422,
        -0.02457823,  0.09223019, -0.10624622,  0.08454026, -0.03419262,
        -0.19265267, -0.65107706,  0.33139277,  0.2732792 , -0.36689707,
        -0.12928618, -0.1768339 ,  0.38551766, -0.17575513,  0.3967203 ,
         0.04391492,  0.22441652,  0.56165186,  0.22969393,  0.09756525,
        -0.10239634, -0.27588168, -0.14684146,  0.00302436,  0.30677823,
        -0.46528358,  0.07619914]), 0.5547001962252291)
```

- **Case 16: -0.554**
- **Case 22: 0.055**
- **Case 43: 0.562**
- **Case 48: -0.147**
- **Case 10: 0.459**
- **Case 32: -0.651**
- **Case 38: 0.386**
- **Case 40: 0.397**

DFBETAS

In [49]:
```
dfbetas = pd.concat([pd.DataFrame(influence.dfbetas, columns = ['dfb_intercept', 'dfb_
print (dfbetas)
```

	dfb_intercept	dfb_pctmetro	dfb_poverty	dfb_single
0	-0.007459	-0.002714	0.006206	0.010689
1	0.167778	0.104486	-0.238319	-0.040585
2	-0.068934	-0.014358	0.085368	0.000549
3	0.009435	-0.066932	0.025312	0.019826
4	-0.048128	-0.082695	0.109284	0.244168
5	0.015085	0.001275	-0.014588	-0.008288
6	-0.079590	0.091650	0.011611	0.050624
7	-0.247737	0.090727	0.204290	0.034349
8	-0.133642	0.002884	0.134163	0.034015
9	0.364075	-0.104403	-0.314159	-0.063346
10	0.089702	-0.075212	-0.037264	-0.039532
11	-0.043534	0.012159	0.037881	0.007270
12	-0.038686	0.018090	0.025174	0.017251
13	0.247429	0.005724	-0.319297	0.116867
14	0.010756	0.012500	-0.017506	-0.006780
15	-0.247689	-0.059782	0.324815	-0.452100
16	0.157025	-0.088636	-0.099868	-0.049133
17	0.062243	0.018636	-0.094156	0.037392
18	-0.026298	-0.005698	0.048353	-0.043874
19	0.029409	0.022300	-0.072780	0.073071
20	-0.029541	0.069015	-0.010720	-0.162688
21	0.030423	-0.025287	-0.018702	0.044646
22	-0.109792	0.165314	0.003064	0.036262
23	-0.043157	-0.091669	0.120531	-0.039696
24	-0.006270	-0.050658	0.046720	-0.020785
25	-0.004113	-0.000529	0.002037	0.008053
26	0.024541	-0.040323	0.008184	-0.027020
27	0.007582	0.065726	-0.059360	0.025151
28	0.059946	-0.034894	-0.038450	-0.015942
29	-0.002098	0.020266	-0.013820	0.008970
30	0.020076	0.083893	-0.096184	0.057166
31	0.409541	0.091342	-0.570832	0.165206
32	-0.052422	-0.148650	0.186844	-0.094945
33	0.010502	0.094896	-0.042092	-0.085179
34	-0.017011	0.226607	-0.158940	0.092622
35	-0.112901	0.043900	0.092602	0.012459
36	-0.110065	0.028631	0.090040	0.038338
37	-0.099615	-0.082738	0.208365	-0.127009
38	0.045537	0.072878	-0.114512	0.049168
39	0.073799	-0.212059	0.093253	-0.111047
40	0.000971	0.028148	-0.015542	-0.009958
41	-0.082711	0.203604	-0.025029	-0.038122
42	-0.357797	0.133842	0.326180	0.356628
43	-0.153072	0.207164	0.051181	-0.039600
44	-0.063267	0.081261	0.025663	-0.021211
45	-0.055528	-0.005636	0.057179	0.022404
46	0.077563	-0.224277	0.036214	0.059928
47	0.044986	-0.093842	0.009014	-0.102215
48	-0.000330	0.000995	0.000090	-0.001017
49	-0.093733	-0.061904	0.178941	-0.100010
50	0.272088	-0.396715	-0.083162	0.099684
51	-0.013521	-0.013921	0.033098	-0.025961

DFBETA_0:

- **Case 16: −0.2477**
- **Case 22: 0.0304**
- **Case 43: −0.3578**

- **Case 48: 0.0450**
- **Case 10: 0.3641**
- **Case 32: 0.4095**
- **Case 38: -0.0996**
- **Case 40: 0.0738**

DFBETA_1:

- **Case 16: −0.0598**
- **Case 22: -0.0253**
- **Case 43: 0.1338**
- **Case 48: -0.0938**
- **Case 10: -0.1044**
- **Case 32: 0.0913**
- **Case 38: -0.0827**
- **Case 40: -0.2121**

DFBETA_2:

- **Case 16: 0.3248**
- **Case 22: -0.0107**
- **Case 43: 0.3262**
- **Case 48: 0.0090**
- **Case 10: -0.3142**
- **Case 32: -0.5708**
- **Case 38: 0.2084**
- **Case 40: 0.0933**

DFBETA_3:

- **Case 16: -0.4521**
- **Case 22: 0.0446**
- **Case 43: 0.3566**
- **Case 48: -0.1022**
- **Case 10: -0.0633**
- **Case 32: 0.1652**
- **Case 38: -0.1270**
- **Case 40: -0.1110**

In [77]:
```
DFD = {'DFFITS':['-.554','.055','.562','-.147','.459','-.651','.386','.397'],
           "Cook's_D":['.0769','.0008','.0792','.0055','.0494','.0998','.0346','.0365'
    'DFBETA_0':['-.2477','.0304','-.3578','.0450','.3641','.4095','-0.0996','.0738'],
          'DFBETA_1':['-.0598','-.0253','.1338','-.0938','-.1044','.0913','-0.0827','-.
           'DFBETA_2':['.3248','-.0107','.3262','.0090','-.3142','-.5708','.2084','.09
           'DFBETA_3':['-.4521','.0446','.3566','-.1022','-.0633','.1652','-.1270','-.
          }
D = pd.DataFrame(DFD)
```

```
D.index = ['Case 16:','Case 22:','Case 43:','Case 48:','Case 10:','Case 32:','Case 38:
print(D)
```

```
         DFFITS Cook's_D DFBETA_0 DFBETA_1 DFBETA_2 DFBETA_3
Case 16:  -.554    .0769   -.2477   -.0598    .3248   -.4521
Case 22:   .055    .0008    .0304   -.0253   -.0107    .0446
Case 43:   .562    .0792   -.3578    .1338    .3262    .3566
Case 48:  -.147    .0055    .0450   -.0938    .0090   -.1022
Case 10:   .459    .0494    .3641   -.1044   -.3142   -.0633
Case 32:  -.651    .0998    .4095    .0913   -.5708    .1652
Case 38:   .386    .0346   -.0996   -.0827    .2084   -.1270
Case 40:   .397    .0365    .0738   -.2121    .0933   -.1110
```

e. Calculate Cook's distance Di for each case and prepare an index plot. Are any cases influential according to this measure?

Case 16: .161%, case 22: .015%, case 43: .164%, case 48: .042%, case 10: .167%, case 32: .227%, case 38: .152%, case 40: .157%