Problem 1.

Bottle return. A carefully controlled experiment was conducted to study the effect of the size of the deposit level on the likelihood that a returnable one-liter soft-drink bottle will be returned. A bottle return was scored 1, and no return was scored 0. The data to follow show the number of bottles that were returned (Y.j) out of 500 sold (n j) at each of six deposit levels (X j, in cents):

j:	1	2	3	4	5	6
Deposit level X_j:	2	5	10	20	25	30
Number sold n_j:	500	500	500	500	500	500
Number returned $Y_{.j}$:	72	103	170	296	406	449

An analyst believes that logistic regression model is appropriate for studying the relation between size of deposit and the probability a bottle will be returned.

a. Plot the estimated proportions $pj = Yj/nj$ against Xj . Does the plot support the analyst's belief that the logistic response function is appropriate?

b. Find the maximum likelihood estimates of $\beta 0$ and $\beta 1$. State the fitted response function.

c. Obtain a scatter plot of the data with the estimated proportions from part (a), and superimpose the fitted logistic response function from part (b). Does the fitted logistic response function appear to fit well?

d. Obtain $exp(b1)$ and interpret this number.

e. What is the estimated probability that a bottle will be returned when the deposit is 15 cents?

f. Estimate the amount of deposit for which 75 percent of the bottles are expected to be returned.

In [1]:
```
import pandas as pd, numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
```

In [2]:
```
df = pd.read_csv('CH14PR11.txt', sep = '\s+', header =None, names=['Xj','nj','Yj'])
df.head()
```

Out[2]:

	Xj	nj	Yj
0	2.0	500.0	72.0
1	5.0	500.0	103.0
2	10.0	500.0	170.0
3	20.0	500.0	296.0
4	25.0	500.0	406.0

In [3]:
```
X= df['Xj']
n= df['nj']
Y= df['Yj']
```

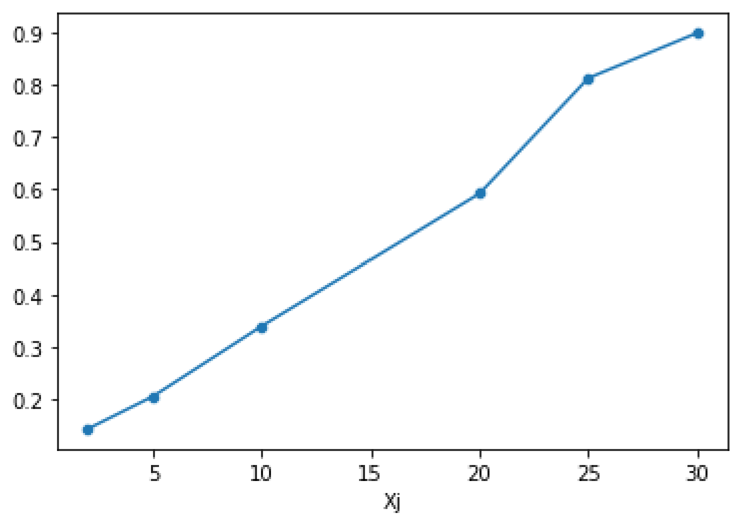
a. Plot the estimated proportions $pj = Yj/nj$ against Xj . Does the plot support the analyst's belief that the logistic response function is appropriate?

In [4]:
```
p = Y/n
```

In [5]:
```
sns.scatterplot(X, p)
sns.lineplot(X, p)
```

```
C:\Users\PC\anaconda3\lib\site-packages\seaborn\_decorators.py:36: FutureWarning: Pas
s the following variables as keyword args: x, y. From version 0.12, the only valid po
sitional argument will be `data`, and passing other arguments without an explicit key
word will result in an error or misinterpretation.
  warnings.warn(
C:\Users\PC\anaconda3\lib\site-packages\seaborn\_decorators.py:36: FutureWarning: Pas
s the following variables as keyword args: x, y. From version 0.12, the only valid po
sitional argument will be `data`, and passing other arguments without an explicit key
word will result in an error or misinterpretation.
  warnings.warn(
```

Out[5]: <AxesSubplot:xlabel='Xj'>



=> the plot support the analyst's belief that the logistic response function is appropriate

b. Find the maximum likelihood estimates of $\beta 0$ and $\beta 1$. State the fitted response function.

In [6]:
```
import statsmodels.api as sm
Xnew = sm.add_constant(df['Xj'])
#Y = df['Yj']

logit_model = sm.GLM(p, Xnew, family=sm.families.Binomial())
logit_results = logit_model.fit()
print(logit_results.summary())
```

```
                 Generalized Linear Model Regression Results
==============================================================================
Dep. Variable:                      y   No. Observations:                    6
Model:                            GLM   Df Residuals:                        4
Model Family:                Binomial   Df Model:                            1
Link Function:                  Logit   Scale:                          1.0000
Method:                          IRLS   Log-Likelihood:                -2.0908
Date:                Sat, 10 Dec 2022   Deviance:                     0.024363
Time:                        19:40:03   Pearson chi2:                   0.0246
No. Iterations:                     4   Pseudo R-squ. (CS):             0.3060
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const         -2.0766      1.897     -1.095      0.274      -5.795       1.642
Xj             0.1359      0.107      1.273      0.203      -0.073       0.345
==============================================================================
```

In [7]:
```
logit_results.params
```

Out[7]:
```
const   -2.076565
Xj       0.135851
dtype: float64
```

By plugging in β0 = -2.0766 and β1 = 0.135851, we get the Logistic model is:

π^ = [1 + exp(2.07656 − 0.13585 *X]^(−1)

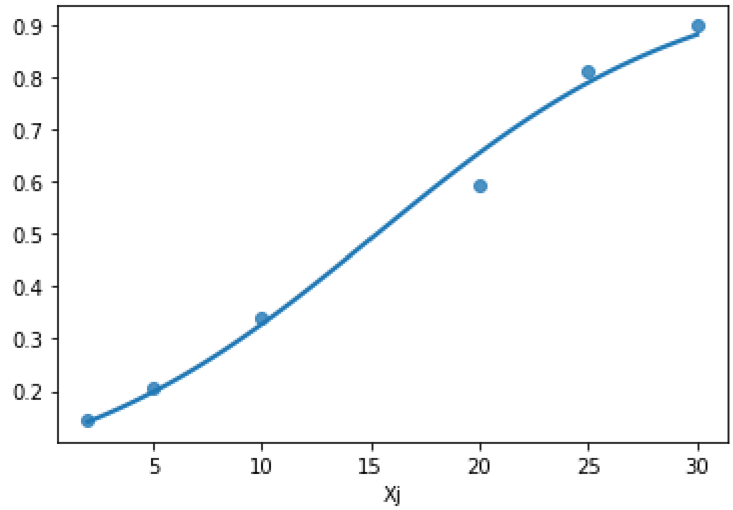
c. Obtain a scatter plot of the data with the estimated proportions from part (a), and super-impose the fitted logistic response function from part (b). Does the fitted logistic response function appear to fit well?

In [8]:
```
def logistic(a):
    return (np.exp(a) / (1 + np.exp(a)))

beta0 = logit_results.params[0]
beta1 = logit_results.params[1]

a = beta0 + beta1*df['Xj']
pi = logistic(a)

print(pi)
```

```
0    0.141260
1    0.198243
2    0.327821
3    0.654855
4    0.789133
5    0.880688
Name: Xj, dtype: float64
```

In [9]:
```
sns.regplot(x= X,y= p,data=df,logistic = True, ci =True)
```

Out[9]:
```
<AxesSubplot:xlabel='Xj'>
```



d. Obtain exp(b1) and interpret this number.

In [10]:
```
OR = np.exp(beta1)
print(OR)
```

```
1.1455109481658934
```

=> the odds of completing the task increase by 14.5 percent with each additional month of experience.

e. What is the estimated probability that a bottle will be returned when the deposit is 15 cents?

In [11]:
```
e = beta0 + beta1*15
pi = (np.exp(e) / (1 + np.exp(e)))
print(pi)
```

```
0.4903004704813551
```

=> the estimated probability that a bottle will be returned when the deposit is 15 cents is 0.4903

f. Estimate the amount of deposit for which 75 percent of the bottles are expected to be returned.

In [12]:
```
a = np.log(3)
d = (a - beta0) / beta1
d
```

Out[12]:
```
23.372533976602604
```

The amount of deposit for which 75 percent of the bottles are expected to be returned is 23.37