PRIMITIVE GEOMETRY

| Lines / Angles / Δ / quadrilateral |

Lecture-1 (29/Jan) 2

1. Congruence (Math-9)

$s^2 = g_{ab}x^a x^b$ g_{ab} : Metric tensor

$s^2 \to x^2$, x'

* $\boxed{\text{Congruence} \equiv \text{transformation} : \underbrace{\text{distances and } \overset{\text{size}}{\text{Angles}}}_{\text{properties left unchanged}} = \text{invariant}}$ (sizes)

Scaling = spatial dilation : ∃ scale factor

scale factor = 1

Prop. that can change ⟶ Combⁿ of "Rigid Motion"
- Translation — $p \cdot x \to x' = x + a$
- → Rotations $R = \begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}_{2\times 2}$
- ↳ Reflections / Parity — $x \to x' = -x$

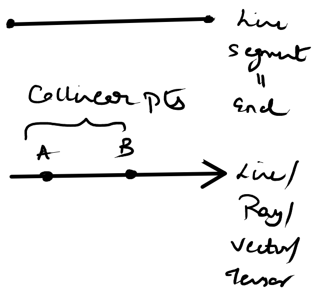
} Repositions

* Congruence = Equal   'Bastardization'

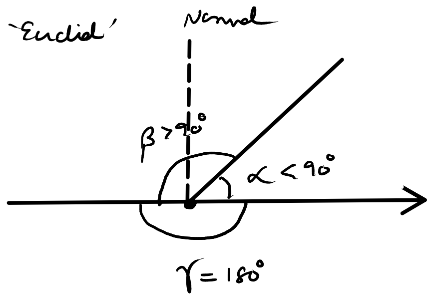
Objects of Geometry

* Point = that which has no part   300 bc 'Euclid'

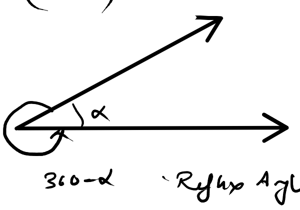
* Line = breadthless length   'Euclid'



* Angle = Inclination   'Euclid'

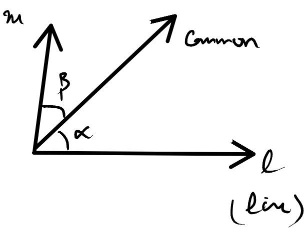


classifications (Math-9)



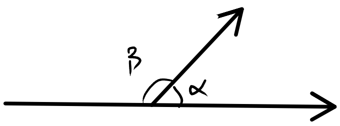
'Reflex Angle'

*



{α, β} : Adjacent Angle

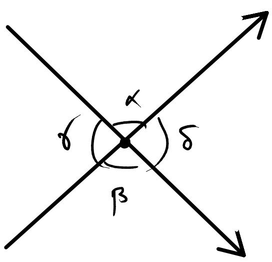
*



$\alpha + \beta = 180°$ (Linear pair / Supplementary)

$a + b = 90°$ (Complementary)

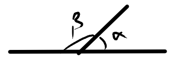
*



Intersecting Lines

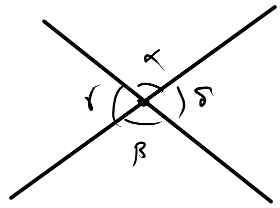
Non Intersecting
||
Parallel
(Euclidean geometry)

* T.1 : If a ray stands on a line, then the sum of 2 adjacent Angles is 180°



$\alpha+\beta=180^\circ$

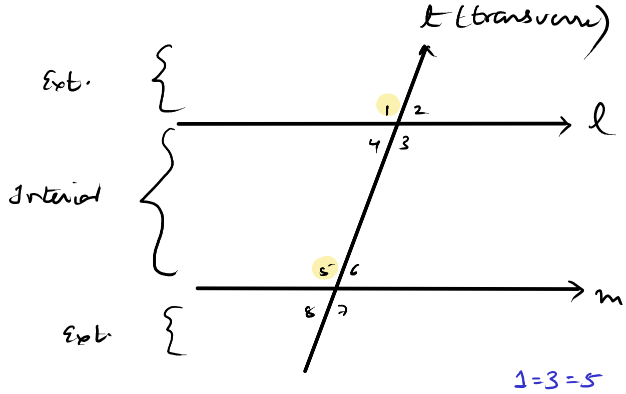
* T.2 : if $\underbrace{\text{2 lines Intersect each other,}}_{\text{Premise}}$ then VOA are equal (vertically opp. Angles)



Proof :

* $\cancel{\alpha}+\cancel{\gamma}=180^\circ=\cancel{\gamma}+\beta \Rightarrow \boxed{\alpha=\beta}$

Parallel Lines



* Corresponding Angles → {1,5}, {4,8}, {2,6}, {3,7}

* Alt. Int → {4,6}, {3,5}

  Alt. Ext → {1,7}, {2,8}

$1=3=5$

$4=6$

* Int. ∠ on the same side ≡ Consecutive Int = Co-interior → {4,5}, {6,3}

  ‖

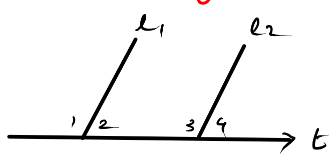
  Allied Angle

$\underbrace{5}_{3}+6=180^\circ \Rightarrow \boxed{3+6=180^\circ}$

Co-int are supplementary

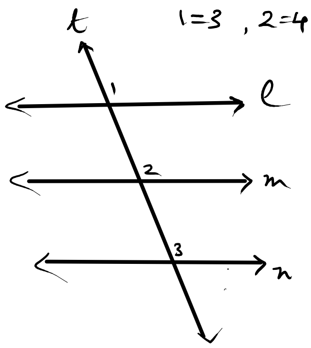
* T.3 : if transversal intersects 2 ‖ lines, each pair of corresponding Angles is Equal ≡ Corresponding Angle axiom

  \+ vice-versa



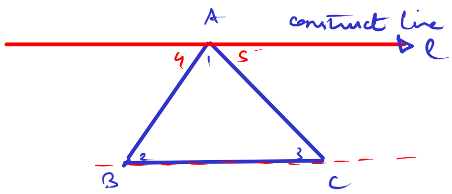
$1=3,\ 2=4$

* T.4 : if $m \parallel l$ & $l \parallel n$ ⟶ $m \parallel n$



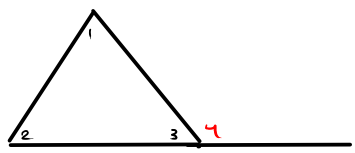
* T.5 : Angle sum property of a $\underbrace{\text{triangle}}_{\text{not non intersecting 3 lines}}$

Proof :



* $\underbrace{1+4+5}_{}=180^\circ$, with $4 = 2$, $5 = 3$ (Alt. Int) $\Rightarrow \boxed{1+2+3=180^\circ}$ ⇐ Euclidean space

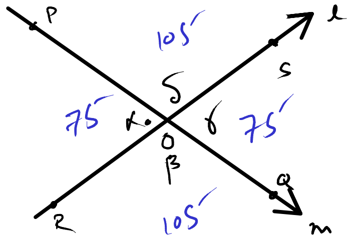
* T.6 : Exterior Angle of a triangle =: $\boxed{4=1+2}$



$1+2+3=180^\circ$, $3+4=180^\circ$

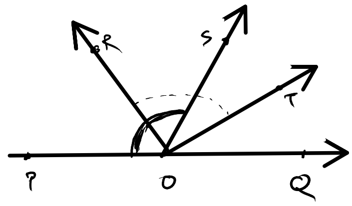
$1+2+\cancel{180}-4=\cancel{180} \Rightarrow \boxed{1+2=4}$

Q1



$\dfrac{\angle POR}{\angle ROQ} = \dfrac{5}{7}$, $\alpha, \beta, \gamma, \delta = ?$

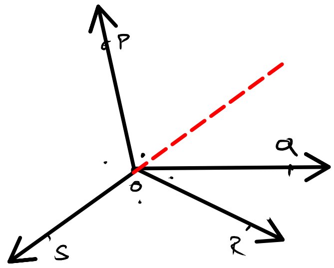
Q2



OR / OT is angle bisector of $\angle POS$, $\angle SOQ$ respectively

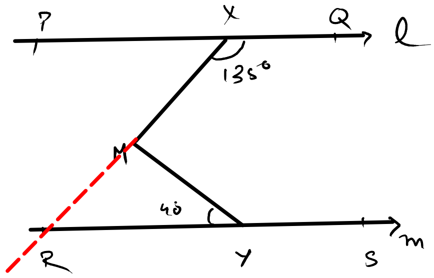
$\angle POS = x$, $\angle ROT = ?$ $= 90^\circ$

Q3



PT : $\angle POQ + \angle QOR + \angle SOR + \angle POS = 360^\circ$

Q4

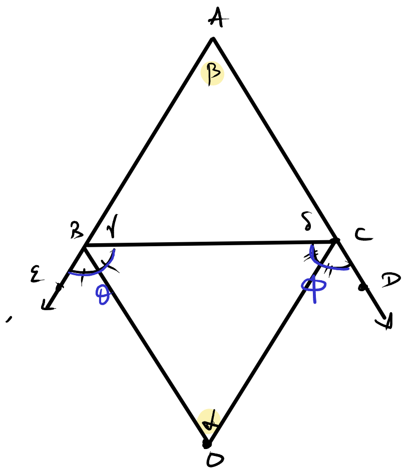


$PQ \parallel RS$, $\angle XMY = ?$ $= 85^\circ$

HW

Q5

6/10



BO / CO are bisectors of $\angle CBE$ & DCB

$\underbrace{\angle BOC}_{\alpha} = 90^\circ - \frac{1}{2} \underbrace{\angle BAC}_{\beta}$ P.T.

$\alpha = 90^\circ - \frac{1}{2}\beta \Rightarrow \alpha = \alpha(\beta)$

$2\alpha = 180 - \beta \Rightarrow \underbrace{2\alpha + \beta} = 180^\circ$

Angle sum prop :

$\beta + \gamma + \delta = 180^\circ$

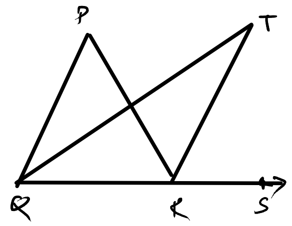
$\dfrac{\theta + \varphi}{2} + \dfrac{\alpha}{1} = 180^\circ \Rightarrow \theta + \varphi + 2\alpha = 360^\circ \Rightarrow \beta + 2\alpha = 180^\circ$

$\left.\begin{aligned} \theta + \gamma &= 180 \\ \varphi + \delta &= 180^\circ \end{aligned}\right\} \Rightarrow \theta + \varphi + \underbrace{\gamma + \delta}_{180 - \beta} = 360^\circ \Rightarrow \theta + \varphi = \beta + 180$

Lecture-2 (30/Jan) 1.5

HW

Q6



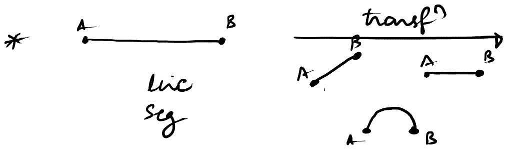
$\triangle PQR$, if bisector of $\angle PQR$ & $\angle PRS$ meets at T

PT: $\angle QTR = \frac{1}{2}\angle QPR$

NCERT-9 / Exemplar-9
↓
Ex-3/2/1
last ques. to 1st

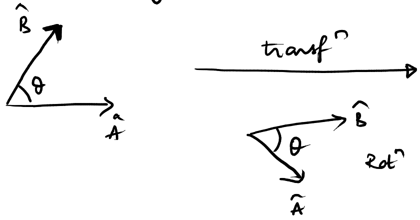
2. Congruence in Elementary geometric objects (Math-9)

a. Congruence of line segments



$|AB|$ = invariant

b. Congruence of Angles

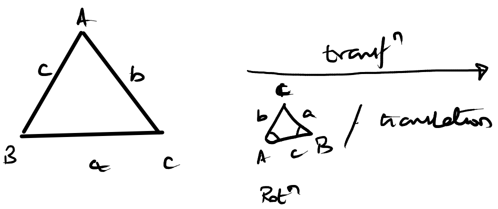


Trig.
↓
θ = invariant

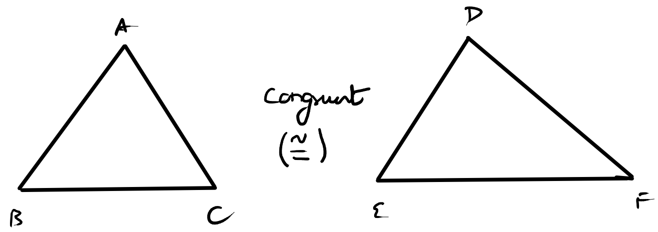
c. Cong. of Circle

diameter = invariant

d. Congruence of triangles



sides = invariant
Angles = invariant



$\Leftrightarrow$

1-1 correspondence

$\boxed{AB \to DE \;:\; \text{length} = \text{inv.}}$	$\equiv$	shorthand $\boxed{AB = DE}$
$AC \to DF$: "		
$BC \to EF$: "		

1-1 corresp.

$A \to D$: measure = inv.	$\equiv$	$\boxed{\angle A = \angle D}$
$B \to E$: "		
$C \to F$: "		

Ordered Eq^n

$\triangle ABC \cong \triangle DEF$

$\Downarrow$ by cpct

Angles	sides
$A = D$	$AB = DE$
$B = E$	$BC = EF$
$C = F$	$AC = DF$

Master formula

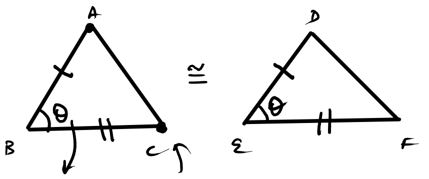
CPCT

"Corresponding parts of Congruent triangle"

d.1 Congruence Rules / Thms

- SAS
- ASA
- AAS

(End pts

* 

inclination end pt

(fixed)

via SAS $\xrightarrow{}$ 'ordered'

$\neq$

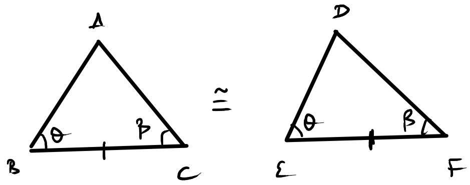
~~ASS~~
~~SSA~~

AB = DE (side - S)
∠B = ∠E (Angle - A)
BC = EF (side - S)

$\Downarrow$

$\boxed{CPCT}$

Axiom

* 

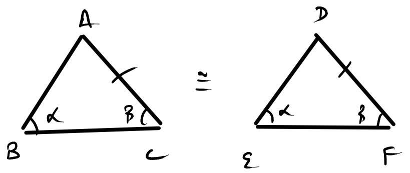
via ASA $\xrightarrow{}$ ordered

Hint

Proof
- AB = DE
- AB > DE
- AB < DE

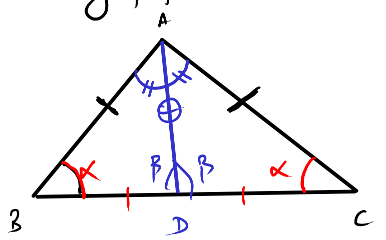
$\boxed{HW}$

To be sub. on Sun

* 

AAS $\xrightarrow{}$ ordered

d.2 Interesting properties

* 

$\Longleftrightarrow$ $\boxed{\angle B = \angle C}$ Isosceles Δ thm.

Isosceles Δ

$\Downarrow$

AB = AC (S)

proof: Construct - Angle bisector AD

proof - In ΔABD, ΔACD

AB = AC (given)

∠BAD = ∠CAD (by Construction / Angle bisec.)

AD = AD (Common)

by SAS Cong. ΔABD ≅ ΔACD

$\xrightarrow{cpct}$

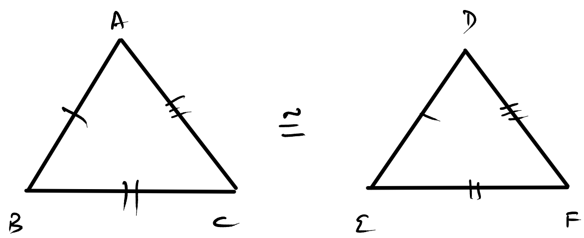
BD = CD

∠ADB = ∠ADC = ? = β

$\downarrow$ Linear pair

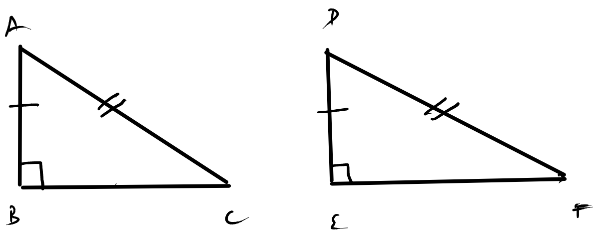
β + β = 180° ⇒ β = 90°

Perpendicular bisector

* 

SSS $\xrightarrow{}$ Congruency

proof HW

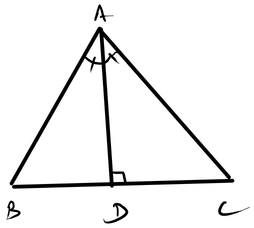
* 

RHS $\xrightarrow{}$ Congruency

Proof HW

Lecture-3 (31/Jan) 1.5

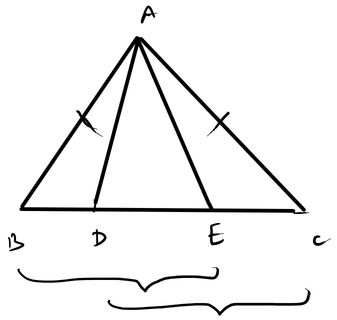
Q7



to prove AB = AC ⟶ Δ is isosceles.

ΔADB ≅ ΔADC (SAS)

Q8



~~BD + DE = DE + EC~~

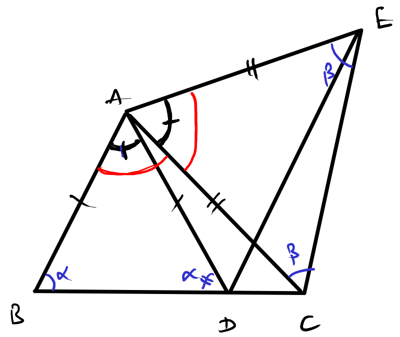
ΔABC isosceles : BE = CD

↓ To prove

$\boxed{AD = AE}$ ⟶ ΔADB ≅ ΔAEC $\xrightarrow{\text{cpct}}$ AD = AE

∠B = ∠C
AB = AC
BD = EC
} SAS

Q9



P.T
BC = DE ⟶ ΔABC , ΔADE

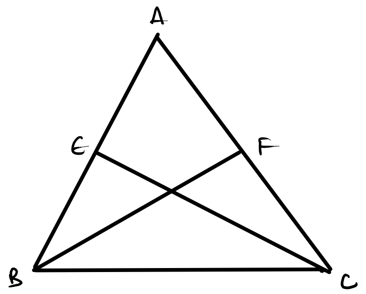
AB = AD (S)
AC = AE (S)

BAE = BAD + DAC + CAE

∠BAD = ∠CAE

$\underbrace{BAD + DAC}_{BAC} = \underbrace{CAE + DAC}_{DAE}$

Q10

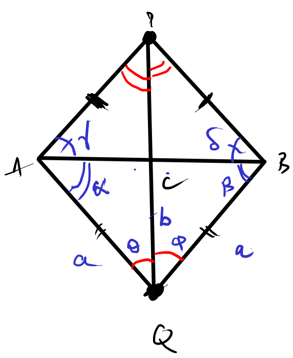


$\boxed{\Delta : \text{isosceles}}$

{E, F} mid pts ⟶ $\boxed{BF = CE}$

AB = AC (S)
∠A = ∠A (common)
AE = AF
} ΔBAF ≅ ΔCAE

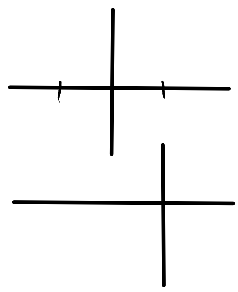
HW
Q11



AB is a line , {P, Q} : 2 pts

P.T : PQ is a perp. bisector of AB

'Bisect'



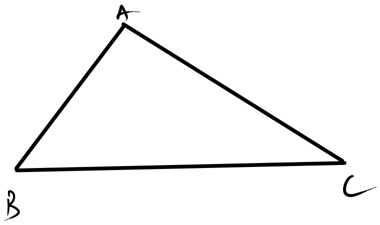
ΔPAQ ≅ ΔPBQ ⟶ ∠AQP = ∠BQP

AQ = BQ
AP = BP
PQ = PQ
} SSS

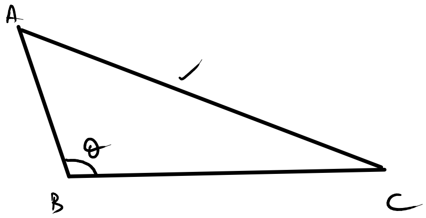
ΔACQ ≅ ΔBCQ ⟶ AC = BC ; ∠ACQ = ∠BCQ
↓
ACQ + BCQ = 180°

$\boxed{\angle ACQ = 90°}$

AQ = BQ
CQ = CQ
∠AQC = ∠BQC
} SAS

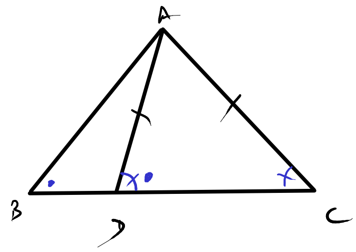
* 

$\longrightarrow$ $\boxed{\underbrace{AB+BC}_{\text{Any 2 sides}} > \underbrace{AC}_{3^{rd}\text{ side}}}$ Triangle side inequality

* 

$\longrightarrow$ Angle opp. to the longer side is larger

$\boxed{\text{if side = longest} \Rightarrow \theta = \text{Max.}}$

Q 

To prove
$AB > AD$

$AD = AC$

$\angle ADC = \angle ACD$

$\Delta ABD \longrightarrow \angle BAD + \angle DBA = \angle ADC$

$\Downarrow$

$\angle ADC > \underbrace{\angle ABD} \Rightarrow \angle ADC > \underbrace{\angle ABC}$

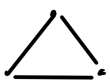
$\Downarrow$

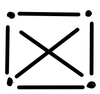
$AB > \underbrace{AC}_{AD}$ □

Lecture-4 (1/Feb) 1.5

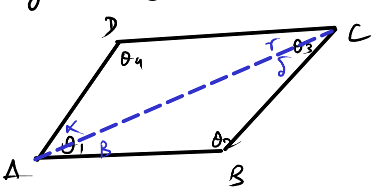
3. Quadrilaterals (Meth-9)

* = St line segment / 2-point correlation

*  = triangle / 3-point correlation

*  = Quadrilateral / 4-point correlation

a. Angle sum of quad

* 

$$\boxed{\sum_{i=1}^{4} \theta_i = 360^\circ}$$

Angle sum property of quad

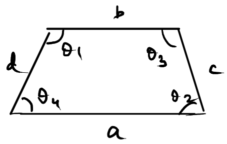
Proof:

* $\alpha + \theta_4 + \gamma = 180^\circ$

(+) $\beta + \theta_2 + \delta = 180^\circ$

$\theta_1 + \theta_2 + \theta_3 + \theta_4 = 360^\circ$

b. Types of Quadrilaterals (□ABCD)



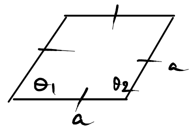
trapezium

$a = b,\ c = d$

$\theta_1 = \theta_2,\ \theta_3 = \theta_4$

$\downarrow$

$\parallel^{gm}$

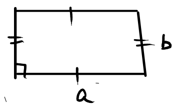


Rhombus

$\theta_1 = \theta_2$

$\downarrow$

Square

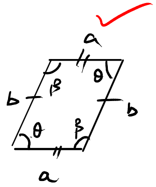


Rectangle

$a = b$

$\downarrow$

Square

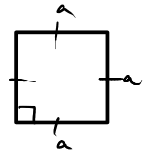


$\parallel^{gm}$

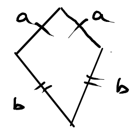
$\theta = 90^\circ$

$\downarrow$

Rectangle



Square



Kite

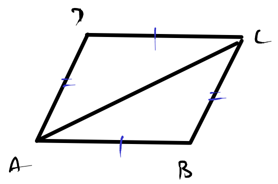
$a = b$

$\downarrow$

Rhombus

c. Parallelogram ($\parallel^{gm}$)

Thms

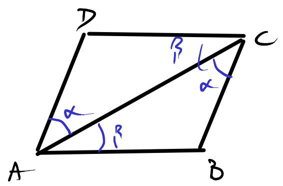
* 

$\{\parallel^{gm}, \text{diag}\} \longrightarrow$ diag divides $\parallel^{gm}$ in 2 congruent Δ

To prove: $\boxed{\Delta ABC \cong \Delta CDA}$ ordered eqn

Proof:

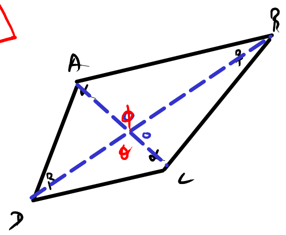
$AB = DC$
$\angle D = \angle B$
$BC = AD$
} SAS

* 

$\{\parallel^{gm}\} \longrightarrow$ Opp. sides are Equal

$\boxed{\angle DAC = \angle ACB}$
$AC = AC$
$\angle BAC = \angle DCA$
} $\xrightarrow{\Delta CAD \cong \Delta ACB}$ $CB = AD$ (by cpct)

HW

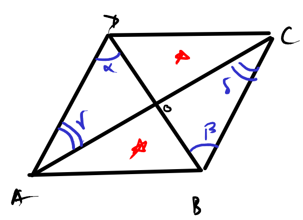
* 

$\{\text{quad}, \alpha, \beta\} \longrightarrow$ Quad $= \parallel^{gm}$

HW
Proof

ΔAOB, ΔDOC

$\theta = \phi$ (V.O.A)

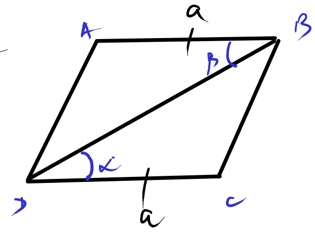
* 

$\{\parallel^{gm}, \text{diag1}, \text{diag2}\} \Longleftrightarrow$ diag 1 bisects diag 2

HW

$\Delta DAO \cong \Delta BCO$

$\alpha = \beta$
$\gamma = \delta$
$AD = BC$
} ASA $\xrightarrow{\text{cpct}}$ $DO = BO$ ✓ diag bisect

* 

$\{\text{Quad}, 1 \parallel \text{\& Equal side}\} \Longrightarrow \parallel^{gm}$

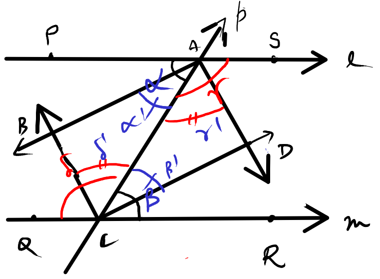
HW

$AB \parallel DC \rightarrow \alpha = \beta$
$DB = DB$
$AB = DC$
} SAS $\longrightarrow \Delta ABD \cong \Delta CDB$

$AD = CB$

$\angle DAB = \angle BCD$

Q1



P.T: Bisectors of Interior Angle forms a Rectangle

HW

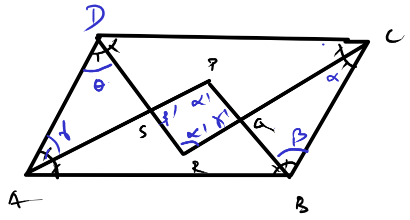
$\alpha=\beta \rightarrow \frac{\alpha}{2}=\frac{\beta}{2} \Rightarrow \alpha'=\beta'$

$\delta=\gamma \rightarrow \frac{\delta}{2}=\frac{\gamma}{2} \Rightarrow \delta'=\gamma'$

$\Delta BCA = \angle DAC \Rightarrow AB = CD$

$\alpha'+\gamma' = \delta'+\beta' = 90^\circ$

Q2



BISECTORS of ॥gm forms a Rectangle

* $DC \parallel AB \rightarrow \angle ADC + \angle DAB = 180^\circ \rightarrow \theta+\gamma = 90^\circ \rightarrow \theta+\gamma+DSA = 90^\circ$

$\boxed{DSA = 90^\circ} = \beta' \rightarrow$

$\gamma' = 90^\circ$

HW

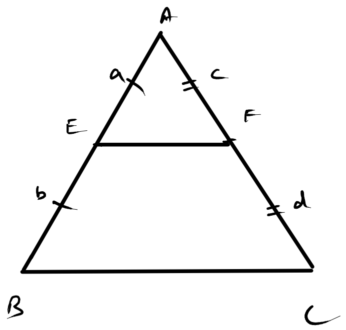
Rectangle ?

Square ?

Lecture-5 (9/Feb) 1.5

HW

*



$\boxed{\text{if } \begin{matrix} a=b \\ c=d \end{matrix} \Rightarrow EF \parallel BC}$

T.P

'Mid-point theorem'

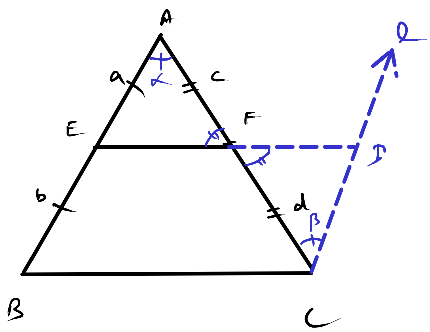
Later!!!

→ line seg. joining the mid pt of 2 sides of Δ is ∥ to the 3rd side

→

Lecture-6 (10/feb) 2

*



$ED \not\parallel BC$

$\boxed{\text{Construct } l : AB \parallel CD}$

$\Delta EAF \cong \Delta DCF \xrightarrow{cpct} EA = DC \xrightarrow{\because\ CB \parallel DC} BCDE$ ॥gm

$\left\{ \begin{matrix} AF = FC \\ \angle EAF = \angle FCD \\ \angle EFA = \angle CFD \end{matrix} \right\}$ ASA

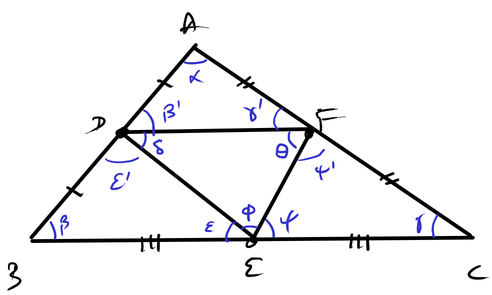
EB (given)

$ED \parallel BC$

⇓

$EF \parallel BC$

Q3.



$\{D, E, F\}$: Mid pts of AB, BC, AC

↓ T.P.

$\Delta ABC \xrightarrow[\text{into 4}]{\text{divided}}$ Congruent Triangles

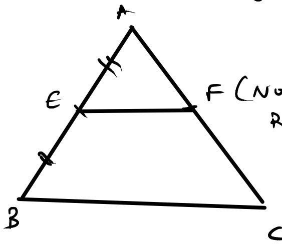
$\Delta DEF \cong \Delta DEB$

$\Delta DEF \cong \Delta CFE$

$\Delta DEF \cong \Delta FAD$

now

* The line drawn through the mid pt. of 1 side of a triangle, ∥ to another side bisects the 3rd side

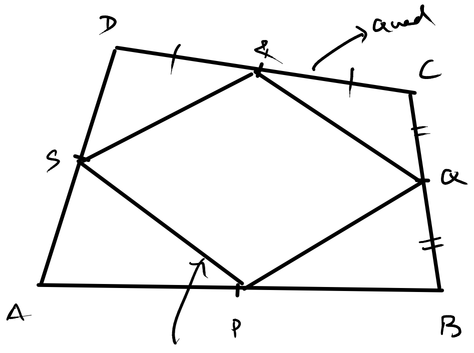


$EF \parallel BC \xrightarrow{T.P} AF = FC$

$\boxed{\text{'Converse of Mid pt. thm'}}$

now

Q4.



{P, Q, R, S} Mid pt.

$\boxed{\text{ABCD Quadrilateral} \longrightarrow \text{PQRS} \parallel gm}$

Interesting theorem

Construction Required!!

NCERT 9th : 6, 7, 8, 9 Start from Last Exercise !!!!! (Misc)

Misc. = diverse
= Manifold
= Heterogeneous
= Assorted

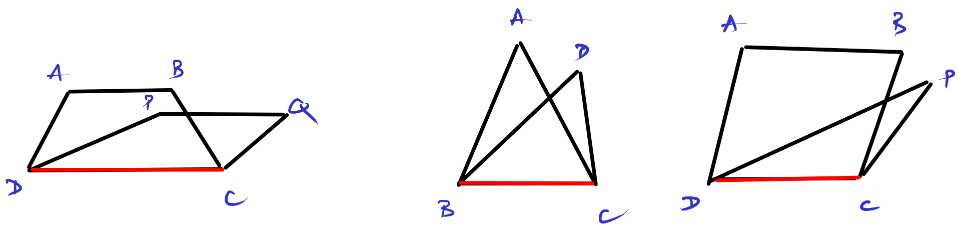
d. Congruency & Areas

* Congruence = transfⁿ : $\underbrace{\text{distances / Angles}}$ are invariant ⇒ Area is invariant

Transl.ⁿ Rotⁿ Reflection

SET 1

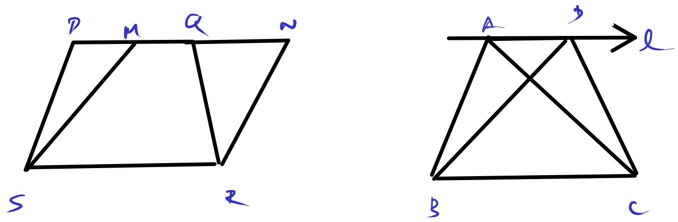
*

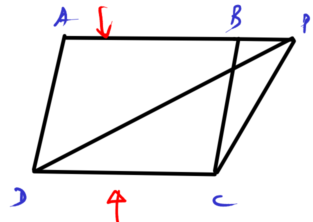


On Same Base
(Bounded from Below)

SET 2

*



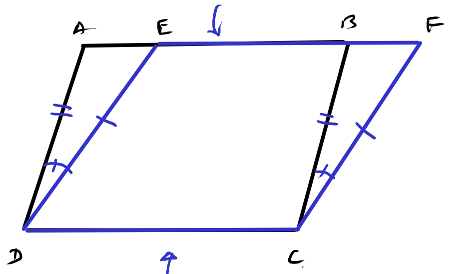


Between same Parallels
(Bounded from Above & below)

d.1 $\parallel^{gm}$ on same base & b/w same $\parallel$s

upper bound

*



$\parallel^{gm}$s on same base ; b/w same $\parallel$ lines are equal in Area

$\boxed{\{\parallel^{gm}s,\ \text{Upper bound, Lower bound}\} \longrightarrow \text{Ar.} = \text{invariant}}$

Theorem.

ABCD , EBCD : 2 $\parallel^{gm}$

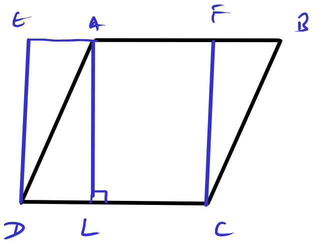
AB $\parallel$ DC , EF $\parallel$ DC

AD $\parallel$ BC , ED $\parallel$ BC

$\triangle ADE \overset{SAS}{\cong} \triangle BCF \implies \text{Ar.}(\triangle ADE) = \text{Ar.}(\triangle BCF)$

$\text{Ar}(\square ABCD) = \underbrace{\text{Ar.}(\triangle ADE) + \text{Ar}(\triangle EBCD)}_{\text{Ar}(\triangle BCF)} = \text{Ar.}(\square EFDC)$

Q1



$\parallel^{gm}$ ABCD
▭ EFDC
AL ⊥ DC

$\xrightarrow{P.T}$ 1. Ar. (ABCD) = ar. (EFDC)
2. Ar. (ABCD) = (DC) AL

Q2 If a Δ & a $\parallel^{gm}$ are on the same base & b/w same $\parallel$. P.T. $\text{Ar.}(\Delta) = \frac{1}{2}\text{Ar.}(\parallel^{gm})$

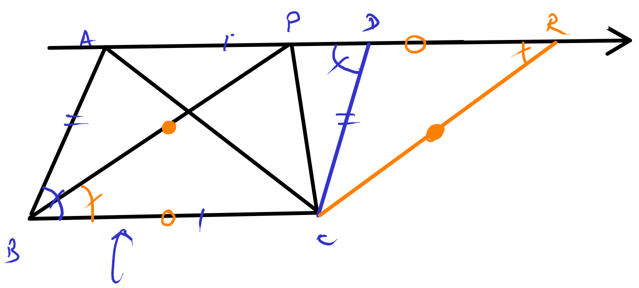
Lecture-7 (11/feb) 1.5

d.2 Triangles on same base & b/w same $\parallel$

Commit to Memory

Thm. CTM

*



$\boxed{\text{2 }\Delta\text{s on same base \& b/w same }\parallel\text{s are equal in Area}}$

$\text{Ar}(\triangle ABC) + \text{Ar}(\triangle ACD)$
$\parallel\parallel$
$\text{Ar}(\triangle ABC + \triangle ACD)$

* Construction CD $\parallel$ AB , CR $\parallel$ BP $\longrightarrow \underbrace{\square ABCD,\ \square PRCB}_{\text{Same base / Same }\parallel\text{s}} \implies \text{Ar.}(\square ABCD) = \underbrace{\text{Ar.}(\square PRCB)}_{\text{Ar}(\triangle PBC) + \text{Ar}(\triangle PCR)}$

⇓

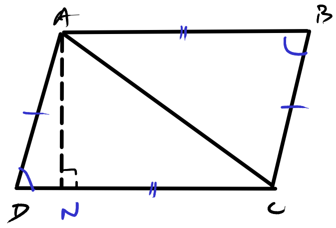
□ABCD : ∠ABC = ∠ADC, AD = BC, AB = DC $\quad \triangle ABC \cong \triangle CDA \longrightarrow \boxed{\text{Ar}(\triangle ABC) = \text{Ar}(\triangle CDA)}$

□PRCB : ∠PRC = ∠PBC, PR = BC, RC = PB $\quad \triangle PRC \cong \triangle CBP \longrightarrow \boxed{\text{Ar}(\triangle PRC) = \text{Ar}(\triangle CBP)}$

$\boxed{\text{Ar}(\triangle ABC) = \text{Ar}(\triangle PBC)}$

$\boxed{\text{Ar}(\triangle ACD) = \text{Ar}(\triangle PRC)}$

Q3



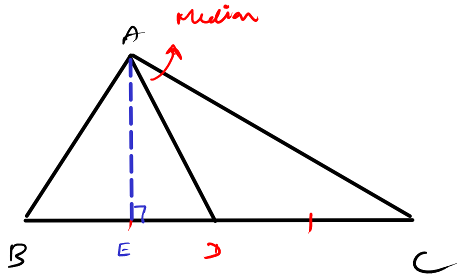
11th

$\boxed{Ar(\Delta ADC) = \frac{1}{2} \times \text{Base} \times \text{Height}}$

$$\left.\begin{array}{l} AD = BC \\ \angle ADC = \angle ABC \\ DC = AB \end{array}\right\} \Rightarrow \Delta ADC \overset{SAS}{\cong} \Delta CBA \Rightarrow Ar(\Delta ABC) = Ar(\Delta CBA)$$

$$\underbrace{Ar(\square ABCD)}_{(AN)\cdot(DC)} = Ar(\Delta ADC) + Ar(\Delta ABC) = 2(\Delta ABC) \Rightarrow Ar(\Delta) = \frac{1}{2} \times \text{base} \times \text{height}$$

Q4



RT : $\boxed{\text{Median of a triangle divides } \Delta \text{ into } 2\ \Delta s \text{ of Equal Areas.}}$

Theorem!

$$\left.\begin{array}{l} * \quad Ar(\Delta ABD) = \frac{1}{2}(AE)(BD) \\ Ar(\Delta ADC) = \frac{1}{2}(AE)\underbrace{(DC)}_{BD} \end{array}\right\} \Rightarrow Ar(\Delta ABD) = Ar(\Delta ADC) \checkmark$$

Lecture - 8 (5/Mar) 1.5

4. Similarity Transformation

* $x \xrightarrow{\text{scaling}} x' = ax$

Scale factor / proportionality const ; $a = f\left(\frac{b}{c}\right)$

* $x \xrightarrow{\text{transl}^n} x' = x + a$

* $\begin{pmatrix} x \\ y \end{pmatrix} \xrightarrow{\text{Rot}^n} \begin{pmatrix} x' \\ y' \end{pmatrix} = R \begin{pmatrix} x \\ y \end{pmatrix}$

Rot^n matrix 2x2

* $x \xrightarrow{\text{Refl}^n} x' = -x$

Congruency

Similarity Transf^n

* scalable congruency
  ||
  Similarity ≡ transf^n : distances ≠ invariant
  Angle = invariant

Deformation Not Allowed
||
stretching / twisting / crumpling / bending ⟷ without closing / opening holes, tearing, gluing, passing thru itself

if Allowed (removing structure)

Topological transformation

"location" / "look"

* All circles similar to each other
  " □ " " "
  " Equi. Δ " " "

All Rectangles Not similar $\frac{l_i}{b_i} \neq$ const.

All Ellipses Not similar $\frac{a_i}{b_i} \neq$ const

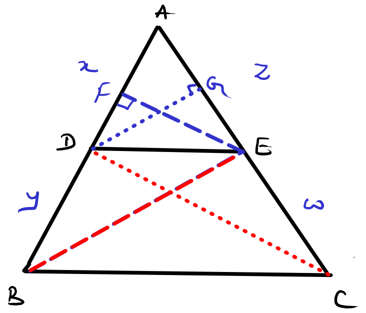
Lecture-9 (6/Mar) 1.5

* Thm 1 : Any $\triangle ABC$,

$$\boxed{\text{if } BC \parallel DE \Rightarrow \frac{x}{y} = \frac{z}{\omega}}$$

(You will see it later in life too)
→ Solid Metatrophy / CMP ~ lattice

Thales' theorem / Basic Proportionality Theorem (BPT)

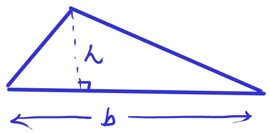


proof :

* $Ar(\triangle ADE) = \frac{1}{2}(AD)(FE) = \frac{1}{2}x(FE)$

$Ar(\triangle DBE) = \frac{1}{2}(DB)(FE) = \frac{1}{2}y(FE)$

$\Rightarrow \frac{Ar(\triangle ADE)}{Ar(\triangle DBE)} = \frac{x}{y}$ — ①

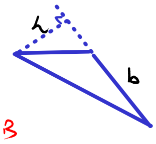


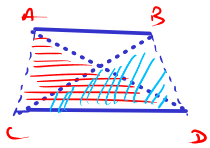
$A = \frac{1}{2} b \cdot h$

* $Ar(\triangle ADE) = \frac{1}{2}(AE)(DG) = \frac{1}{2}z(DG)$

$Ar(\triangle DEC) = \frac{1}{2}(EC)(DG) = \frac{1}{2}\omega(DG)$

$\Rightarrow \frac{Ar(\triangle ADE)}{Ar(DEC)} = \frac{z}{\omega}$ — ②



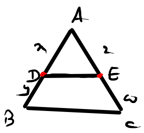


$AB \parallel CD$ CD Common base

$Ar(\triangle ACD) = Ar(\triangle BCD)$ 'thm'

* $Ar(\triangle DBE) = Ar(\triangle DEC)$ 'Thm.' 'Common DE' $\Rightarrow \underbrace{\frac{Ar(ADE)}{Ar(DBE)}}_{\frac{x}{y}} = \underbrace{\frac{Ar(ADE)}{Ar(DEC)}}_{\frac{z}{\omega}} \Rightarrow \frac{x}{y} = \frac{z}{\omega}$

Corollary



$\frac{x}{y} = \frac{z}{\omega}$ parts relational / \

* $\frac{x}{y} + 1 = \frac{z}{\omega} + 1$ (Comp.) $\Rightarrow \frac{x+y}{y} = \frac{z+\omega}{\omega} \Rightarrow \boxed{\frac{AB}{DB} = \frac{AC}{EC}}$ whole-part relation // \\

* $\frac{x}{y} = \frac{z}{\omega} \Rightarrow$ (Invert.) $\frac{y}{x} = \frac{\omega}{z} \rightarrow$ (Comp) $\frac{y}{x} + 1 = \frac{\omega}{z} + 1 \Rightarrow \boxed{\frac{AB}{AD} = \frac{AC}{AE}}$ // \\

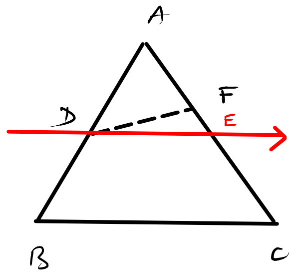
BPT Converse

* thm 2 : Any $\triangle ABC$, given $\frac{AD}{DB} = \frac{AF}{FC} \longrightarrow \underbrace{DF \parallel BC}$

|||

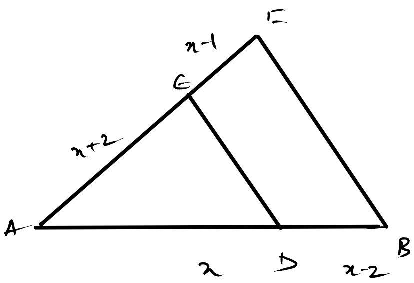
F and E are the same point

$\frac{AD}{DB} = \frac{AE}{EC}$



* $\frac{AE}{EC} = \frac{AF}{FC} \Rightarrow \frac{AE+EC}{EC} = \frac{AF+FC}{FC} \Rightarrow \frac{\cancel{AC}}{EC} = \frac{\cancel{AC}}{FC} \Rightarrow \boxed{EC = FC} \Rightarrow FE = 0 \Rightarrow F \,\&\, E$ are same $\Rightarrow DF \parallel BC$

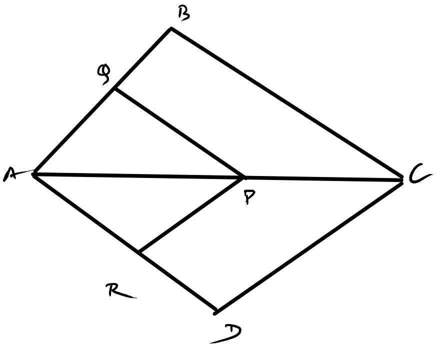
Q:



given $ED \parallel BC \rightarrow x = ? = 4$

$\frac{x}{x-2} = \frac{x+2}{x-1}$

Q2



QP ∥ BC , RP ∥ DC

P.T i) $\frac{AR}{AD} = \frac{AQ}{AB}$

ii) $\frac{QB}{AQ} = \frac{DR}{AR}$

$\triangle ABC$: $\frac{AQ}{QB} = \frac{AP}{PC}$

$\triangle ADC$: $\frac{AR}{RD} = \frac{AP}{PC}$

$\Rightarrow \frac{AQ}{QB} = \frac{AR}{RD}$

+1 Comp.

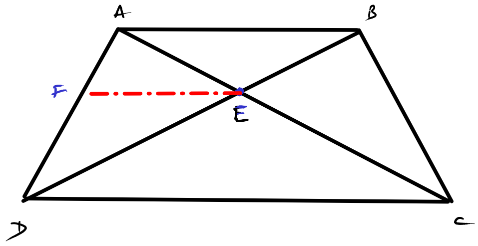
$\frac{AR}{AD} = \frac{AQ}{AB}$

Inv.

$\frac{QB}{AQ} = \frac{DR}{AR}$

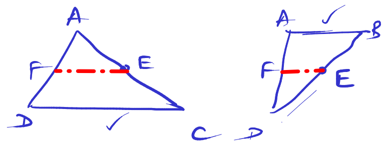
Lecture-10 (19/Mar) 1.5'

Q3. ∥ diag. bisect proportionally $\Rightarrow$ ABCD is $\underbrace{\text{trapezium}}$ AB ∥ DC 'Theorem'



* given : $\frac{AE}{EC} = \frac{BE}{ED}$

Construct : FE ∥ DC $\xrightarrow[\triangle ADC]{BPT}$ $\frac{AF}{FD} = \frac{AE}{EC} \overset{\text{given}}{=} \frac{BE}{ED}$



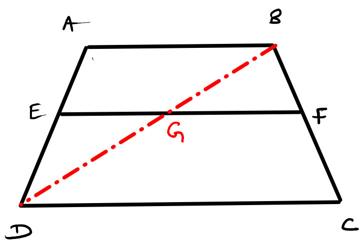
$\triangle ADB$ Converse of BPT

FE ∥ AB

⇓

AB ∥ DC → trapezium

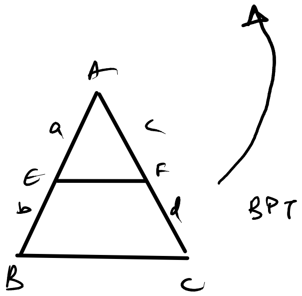
Q4 if EF ∥ AB ∥ CD $\Rightarrow$ To prove $\frac{AE}{ED} = \frac{BF}{FC}$ Thm.



* EG ∥ AB $\Rightarrow$ $\frac{AE}{ED} = \frac{BG}{GD}$ ($\triangle ABD$) BPT

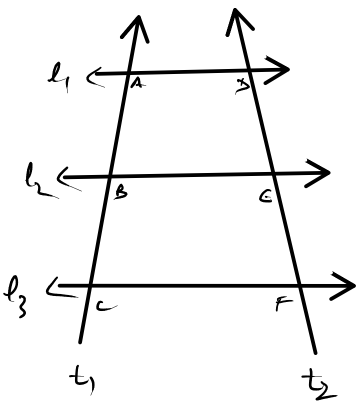
* GF ∥ DC $\Rightarrow$ $\frac{BG}{GD} = \frac{BF}{FC}$ ($\triangle BDC$)

$\Rightarrow \frac{AE}{ED} = \frac{BF}{FC}$



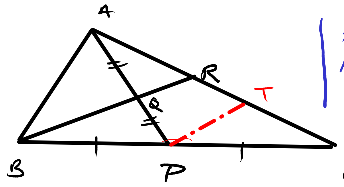
EF ∥ BC $\Rightarrow \frac{a}{b} = \frac{c}{d}$

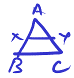
Q5. $\{l_1, l_2, l_3\}$: 3 ∥ lines
$\{t_1, t_2\}$: 2 transversal $\Rightarrow$ $\frac{AB}{BC} = \frac{DE}{EF}$



21%
20%

HW Q6. if $BP = PC$, $AQ = QP$ $\xrightarrow{T.P}$ $RA = \frac{1}{3} CA$





BPT $\Rightarrow$ $\frac{AX}{XB} = \frac{AY}{YC}$

Hint: Construct ∥ line ... $PT \parallel BR$

$\triangle BCR \xrightarrow{BPT} \frac{\cancel{PC}}{\cancel{PB}} = \frac{CT}{TR} \Rightarrow CT = TR$

$AC = AR + RT + TC$

$\boxed{AC = AR + 2RT} \Rightarrow 3AR = AC$

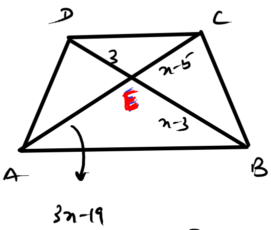
$\Downarrow$

$PT \parallel BR \Rightarrow PT \parallel QR \xrightarrow[BPT]{\triangle APT} \frac{\cancel{AQ}}{\cancel{QP}} = \frac{AR}{RT} \Rightarrow AR = RT$

$RA = \frac{1}{3} CA$ □

TEST M.M = 20

Q1



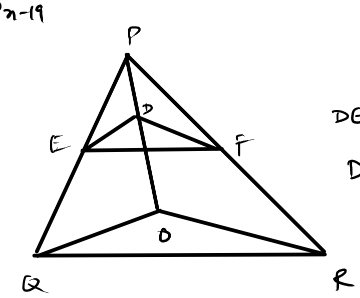
$AB \parallel DC$ trapezium $\rightarrow x = ?$

2.5 ' diag.s divide proportionally '

$x = 8$: $|DB| = 8$, $|AC| = 8$

$x = 9$: $|DB| = 9$, $|DB| = 12$

Q2

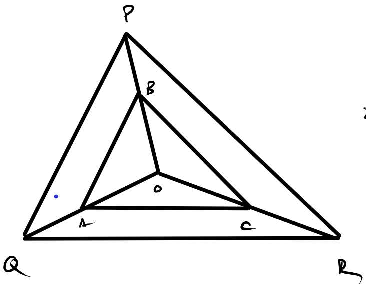


$DE \parallel OQ$
$DF \parallel OR$ $\xrightarrow{T.P}$ $EF \parallel QR$

5

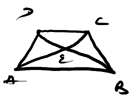
Result: 12.5

Q3



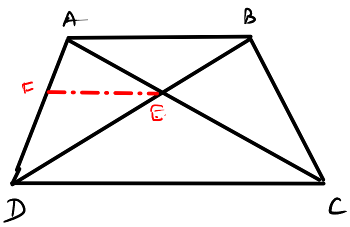
if $AB \parallel PQ$, $AC \parallel QR$ $\xRightarrow{T.P.}$ $BC \parallel PR$ 5

Q4 In a trapezium, the diagonals divide each other proportionally. Prove that!



$\frac{DE}{EB} = \frac{CE}{EA}$

Lecture-11 (23/Mar) 1.5



Trapezium

To prove: $\frac{AE}{EC} = \frac{EB}{ED}$

Proof: $\triangle ADC \xrightarrow{BPT} \frac{AF}{FD} = \frac{AE}{EC}$

$\triangle ABD \xrightarrow{''} \frac{AF}{FD} = \frac{BE}{ED}$

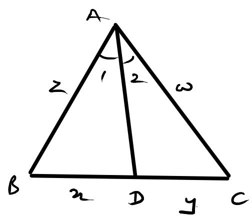
$\Rightarrow$ □

* $FE \parallel DC$
$FE \parallel AB$

Total M.M: 40

Interior Bisector Theorem

* if $\angle A$: bisected ($\Rightarrow 1 = 2$) then $\frac{x}{y} = \frac{z}{w}$



to be contd....