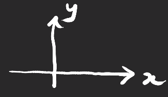
△ Cartesian geometry (vol-4) - Conic Sections { Circles, Parabola

Lecture-1 (6/May)



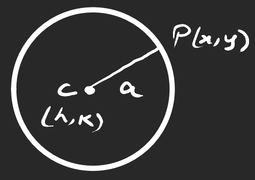
1. Circles

1.1 Eq^n of circles (cases)

I.

$*$ $|CP| = a = \sqrt{(x-h)^2+(y-k)^2} \Rightarrow \boxed{S \equiv (x-h)^2+(y-k)^2 = a^2}$ $*$

Circle — Eq^n of circle S (EOC)



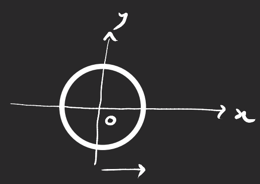
Centre = (h,k)

Radius = a

$y = ax^2+bx+c$

Cases (Constants)

I. $(h,k) = (0,0) \Rightarrow x^2+y^2=a^2$ EOC centre at origin



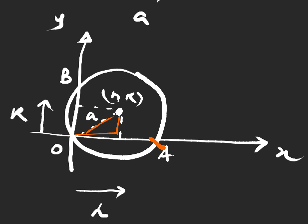
II. $(x,y) = (0,0)$ satisfies EOC $h^2+k^2=a^2$

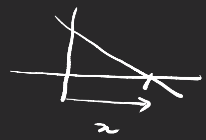
$(x-h)^2+(y-k)^2 = a^2 = h^2+k^2$

$x^2+y^2-2hx-2ky=0$ EOC pass. thru. O

OA = x intercept

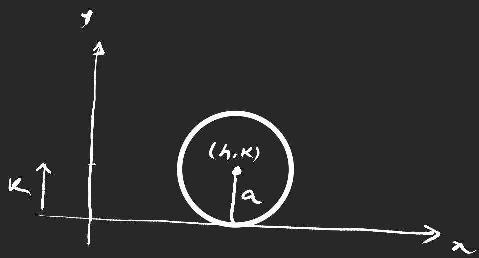
OB = y intercept





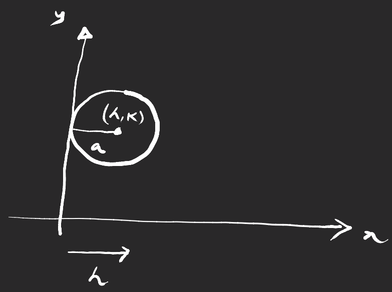
III. $k=a \Rightarrow (x-h)^2+(y-k)^2 = k^2$

$x^2+y^2-2hx-2ky+h^2=0$ EOC touches x axis



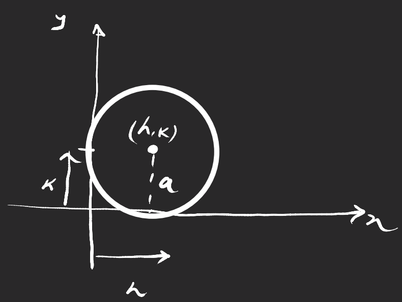
IV. $h=a \Rightarrow (x-h)^2+(y-k)^2 = h^2$

$x^2+y^2-2hx-2ky+k^2=0$ EOC touches y axis



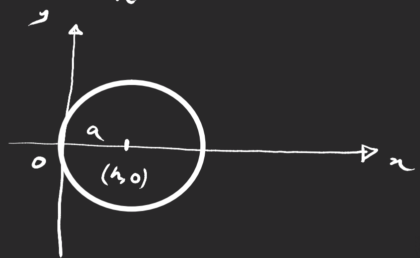
V. $k=h=a \Rightarrow (x-a)^2+(y-a)^2=a^2$

$x^2+y^2-2ax-2ay+a^2=0$ EOC touches both x & y



VI. $k=0$, $h=a \Rightarrow (x-a)^2+y^2=a^2$

$x^2+y^2-2ax=0$ EOC passes thru O, Centre on x axis



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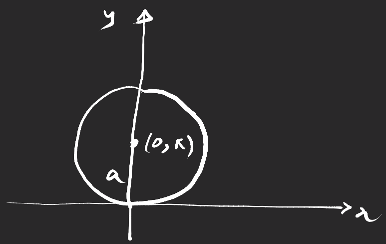
VII $h=0, \quad K=a \Rightarrow x^2+(y-a)^2=a^2$

$x^2+y^2-2ay=0$ — Eoc passes thru O, centred on y axis.



1.2 General form of EOC

* $S \equiv x^2+y^2+2gx+2fy+c=0 \Rightarrow x^2+2gx+g^2+y^2+2fy+f^2=g^2+f^2-c$

$(x+g)^2+(y+f)^2=f^2+g^2-c \leftarrow$ std eqn $(x-h)^2+(y-K)^2=a^2$

$\Downarrow$

$h=-g$

$K=-f$

$a=\sqrt{f^2+g^2-c}$

I (CTM)

* $x^2+y^2+2gx+2fy+c=0$ gen. eqn of circle

Centre $=(-g,-f)$

Rad. $=\sqrt{f^2+g^2-c}$

$f^2+g^2>c$ — Real circle

$f^2+g^2<c$ — Img. circle

$f^2+g^2=c$ — Rad $=0 \Rightarrow$ point

All the above cases can be studied

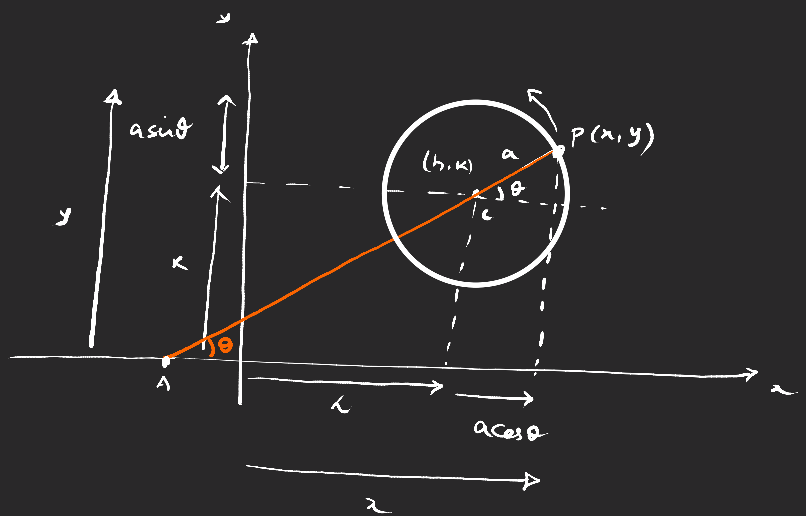
* $c=0 \Rightarrow x^2+y^2+2gx+2fy=0$, Rad $=\sqrt{f^2+g^2}$, cen $=(-g,-f)$ — circle passes thru. O
* $f=0$; $c=0 \Rightarrow x^2+y^2+2gx=0$ Rad $=g$; cent $=(-g,0)$ — centre on x axis
* $g=0$, $c=0 \Rightarrow$ . . . . . cent $=(0,-f)$ — " " y axis

1.3 Parametric form of EOC

* $\left.\begin{aligned} x &= h+a\cos\theta \\ y &= K+a\sin\theta \end{aligned}\right\} \Leftrightarrow (x-h)^2+(y-K)^2=a^2$

form 1 — E.O.C

if (C, rad) given — $\boxed{\text{Parameter}}$



* $x^2+y^2+2gx+2fy+c=0$

$(h,K)=(-g,-f)$, $a=\sqrt{f^2+g^2-c}$

$$\boxed{\begin{aligned} x &= -g+\sqrt{f^2+g^2-c}\,\cos\theta \\ y &= -f+\sqrt{f^2+g^2-c}\,\sin\theta \end{aligned}}$$

Parametric eqn of circle, form 2 — if eqn given

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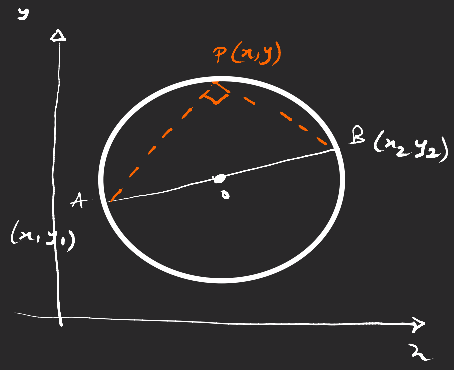
Lecture-2 (13 May) 1

1.4. Diameter form of circle



* AP : $m_1 = \dfrac{y-y_1}{x-x_1}$

  BP : $m_2 = \dfrac{y-y_2}{x-x_2}$

AP ⊥ BP (property of circle)

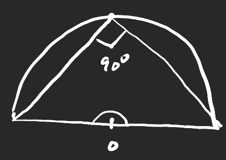
$\Downarrow$

$m_1 \cdot m_2 = -1$ (st. line)

$\Downarrow$

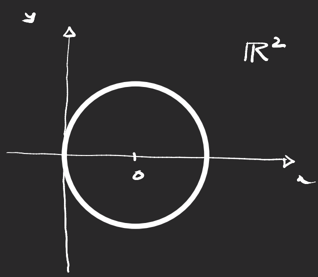
$$\frac{(y-y_1)}{(x-x_1)} \cdot \frac{(y-y_2)}{(x-x_2)} = -1$$

Properties of circle



* $\boxed{(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0}$ Diameter form of circle

1.5 Circles and Complex Numbers



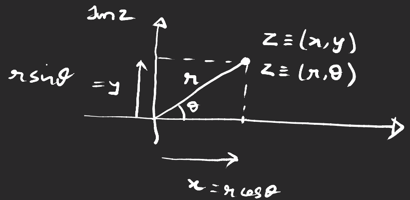
* $z = x+iy$, $\forall z \; \exists \; |z| : |z| = zz^* = \sqrt{x^2+y^2}$ 1st Approach

  $z^* = x - iy$

* $|z| = \sqrt{x^2+y^2} = r$ 2nd Approach

  $\tan\theta = \dfrac{y}{x}$

Argand plane $\mathbb{C}^1$

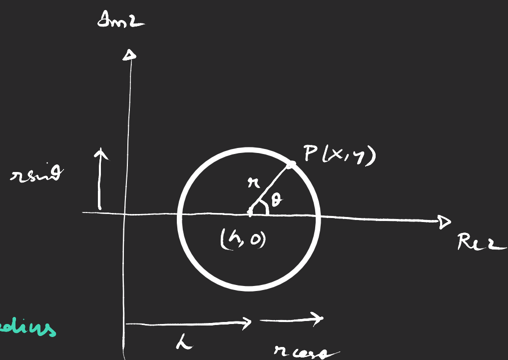


* $z = \underset{\text{1st}}{x+iy} = \underset{\text{2nd}}{r\overbrace{(\cos\theta + i\sin\theta)}^{=e^{i\theta}}} = \underset{\text{3rd}}{re^{i\theta}}$

  Cartesian Polar Eulerian

  $z^* = x - iy = r(\cos\theta - i\sin\theta) = re^{-i\theta}$

* $|z|^2 = x^2+y^2 = r^2 \longrightarrow$ Connection b/w CN & Circle



* $x = h + r\cos\theta$, $y = r\sin\theta$

Claim:

$\boxed{z = x+iy \Rightarrow |z-h| = r}$ EOC in $\mathbb{C}$ Center at (h,0), r: radius

- not quadratic in z
- Reduction of # of variable (dof)

$\Downarrow$

$|x+iy-h| = r \Rightarrow |\underbrace{(x-h)+iy}| = r \Rightarrow \sqrt{(x-h)^2+y^2} = r \Rightarrow \boxed{(x-h)^2 + y^2 = r^2}$

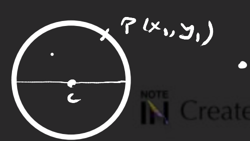
EOC in $\mathbb{R}$ on x axis

- quad. in x, y
- variables are double

1.6. Extra features

A. locate a point wrt a circle

* $x^2+y^2+2gx+2fy+c=0$, $P(x_1,y_1)$



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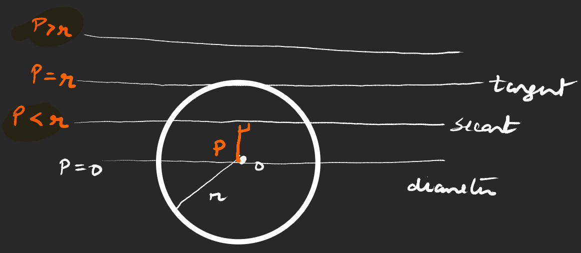
* $x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c = 0$ P is at the circle
> 0 P out circle
< 0 P inside the circle

B. tangency condⁿ using ⊥ distance

* line l : $p = r$ 'tangency condⁿ'

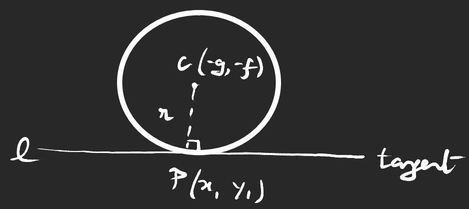


Lecture-3 (30 May)

C. Circles interacting with straight line (tangent/Normal)

* PC line $m = \dfrac{y_1+f}{x_1+g}$ Normal

PC ≡ $y - y_1 = \dfrac{y_1+f}{x_1+g}(x - x_1)$ Eqⁿ of Normal



* $m \cdot m_T = -1 \Rightarrow m_T = -\left(\dfrac{x_1+g}{y_1+f}\right) \Rightarrow y - y_1 = -\left(\dfrac{x_1+g}{y_1+f}\right)(x - x_1)$ Eqⁿ of tangent

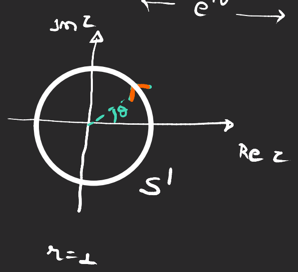
D. Real geometry associated with 'Group' (Remark)

* $C = x + iy = r(\cos\theta + i\sin\theta) = re^{i\theta} \xrightarrow{r=1} C = e^{i\theta}$ circle with a unit radius

$\leftarrow e^{i\theta} \rightarrow$

Representⁿ # 1 algebraic

* $C \equiv$



$\theta \in [0, 2\pi)$

Representation # 2 geometric

* Transformations : GROUPS (they are abstract)
||
operations done on any "object"
in such a way that a particular property
is preserved (symmetry)

Example : U(1) group — has a very interesting property

• S^1 has a U(1) invariance : Claim

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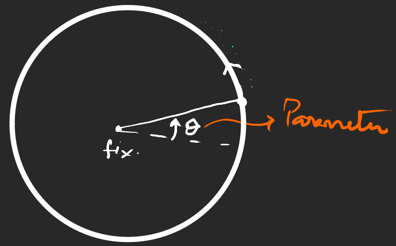
$U(1)$ = rotation transformation in a complex plane in such a way that $|AB| \equiv |r| =$ invariance



- numbers $\{\mathbb{R}, \mathbb{C}\}$ = Scalar
- Metrics $\{\mathbb{R}, \mathbb{C}\}$
- Group Abstract
  - vector
  - Tensor
  - Spinor
  - Geometrical

$U(1)$ group	$\longleftrightarrow$	S^1 geometry

$U(1)$ — U: Unitary; 1: 1-D

- Rep 1 : $c = e^{i\theta}$
- Rep 2 : c = circle (S^1)

$c \cdot c^* = 1$ (Identity)

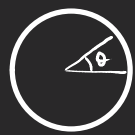
$\Downarrow$

group has an $\underbrace{\text{identity element}}_{\text{DO NOTHING ELEMENT}}$

$\downarrow$

'Lie groups.'

S^1

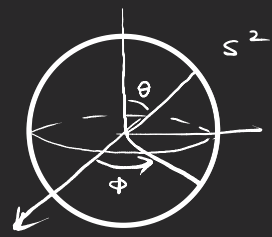


parameter = 1

$\downarrow$

$U(1)$

S^2



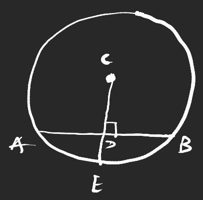
parameter = 2

$\downarrow$

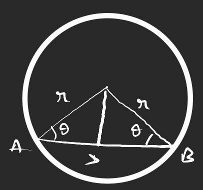
$U(2)$

1.7. fundamental features

1.

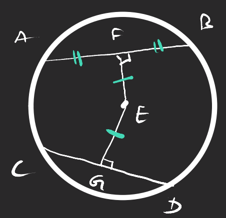


if $CE \perp AB \Rightarrow AD = BD$



Isosceles Δ

2.



$EF = EG \Rightarrow AB = CD$

(if $EF \perp AB \Rightarrow AF = FB$)

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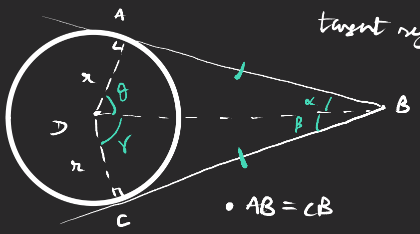
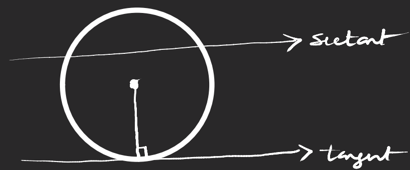
3.

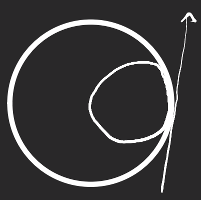


- $AB = CB$
- $\alpha = \beta$
- $\theta = \gamma$

$\left(\because \triangle DAB \cong \triangle DCB \text{ RHS} \right)$

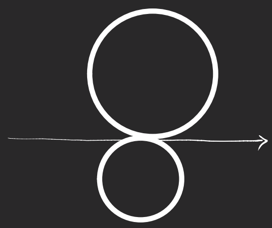


4. Tangent circles

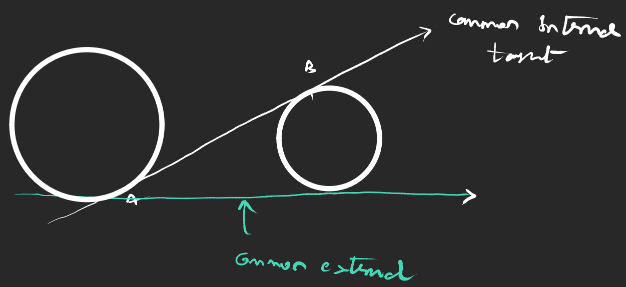


internally tangent circles

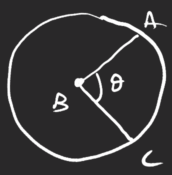
{ 2 circles are tangent }



Externally tangent circles

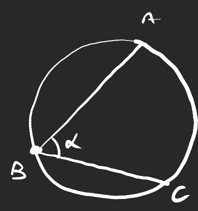


5.

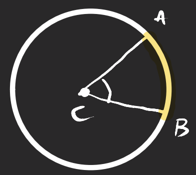


vertex = center

Central Angle ⟷ Inscribed Angle



vertex = on the circle



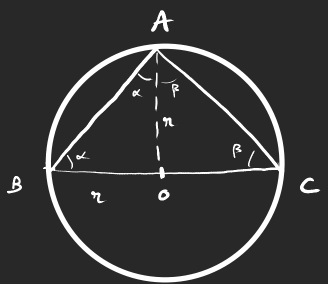
C: "vertex"

measure ↓ arc length ↓

$m(\angle ACB) \propto m(\widehat{AB})$

minor arc $< 180°$
major arc $> 180°$

6.

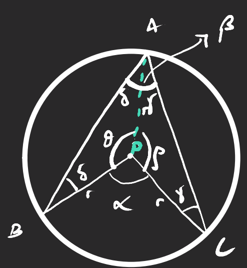


Angle subtended at circumference by semi circle is $90°$

$\because 2(\alpha+\beta) = 180° \Rightarrow \alpha+\beta = 90°$

Lecture-4 (1/Jun)

7.



- $\triangle AOB$: $2\delta + \theta = 180°$
- $\triangle AOC$: $2\gamma + \rho = 180°$
- $\theta + \rho + \alpha = 360°$

$\Rightarrow 2(\delta+\gamma) + \theta + \rho = 360°$

$\alpha = \underbrace{2(\delta+\gamma)}_{\beta} \Rightarrow \boxed{\alpha = 2\beta}$

⇓

$\boxed{\text{Central } \angle = 2(\text{Inscribed } \angle)}$ CTM

$\angle$ subtended by arc on centre ‖ $\angle$ subt. by arc on any pt on circumf.

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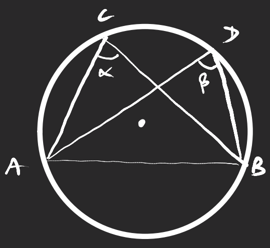
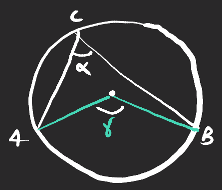
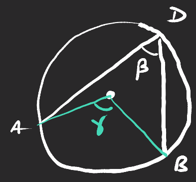
8.



Angles in the same segment $\boxed{\alpha = \beta}$

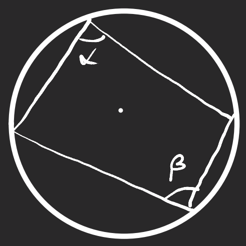




$\gamma = 2\alpha \quad \longleftrightarrow \quad \gamma = 2\beta \quad \Rightarrow \quad \boxed{\alpha = \beta}$

cyclic quadr.

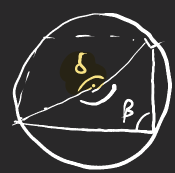
9.



$\alpha + \beta = 180^\circ$

I.

II.



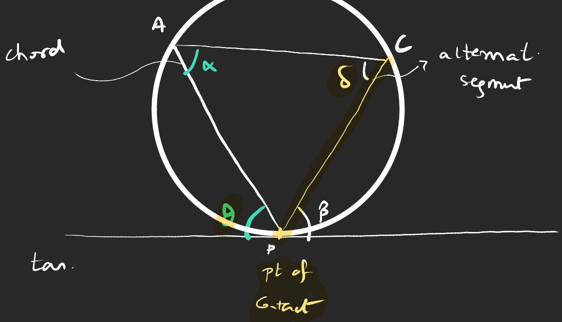
$\gamma = 2\alpha$

$\delta = 2\beta$

$\delta + \gamma = 360^\circ \Rightarrow 2(\alpha+\beta) = 360^\circ \Rightarrow \boxed{\alpha + \beta = 180^\circ}$

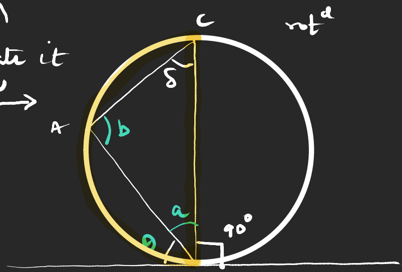
10. Alternate segment (tan-chord Thm.)

orig.



legit cheat!

rotate it ACW



Exploiting the sym. of a semicircle

- $a + \theta = 90^\circ$ (Linear pair)
- $b = 90^\circ$ ($\angle$ subt. by a SC)
- $a + b + \delta = 180^\circ$ (Angle sum.)

$\Downarrow$

$a + \delta = 90^\circ = a + \theta$

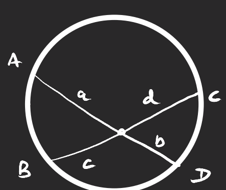
$\Downarrow$

$\boxed{\theta = \delta}$

TP $\boxed{\begin{matrix}\alpha = \beta \\ \theta = \delta\end{matrix}}$

Power Theorems (lengths)

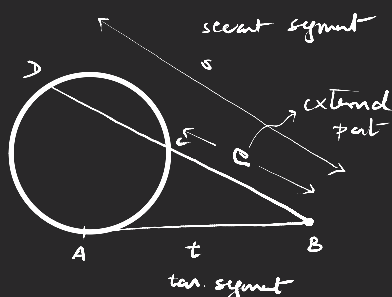
1.



chord-chord

$\boxed{a \cdot b = c \cdot d}$

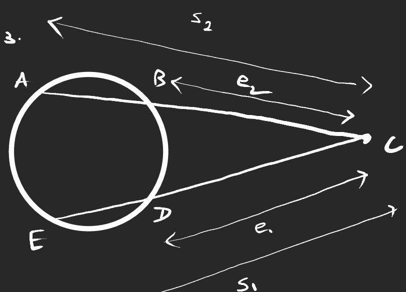
2



tan-sec.

$\boxed{t^2 = es}$

3.



sec-sec.

$\boxed{s_1 e_1 = s_2 e_2}$

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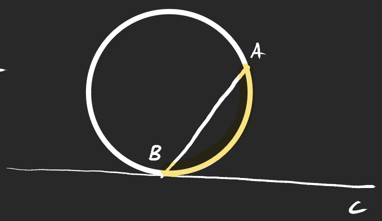
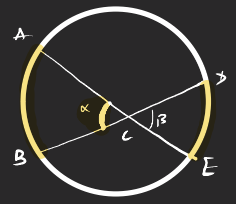
Angle Theorems

1. tan-chord $\angle$



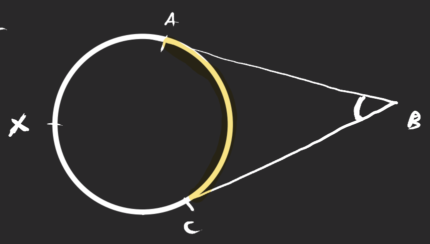
$m\angle ABC \propto \frac{1}{2} m(\widehat{AB})$

2. chord-chord $\angle$



$m\angle ACB \propto \frac{1}{2}\left(m(\widehat{AB}) + m(\widehat{DE})\right)$

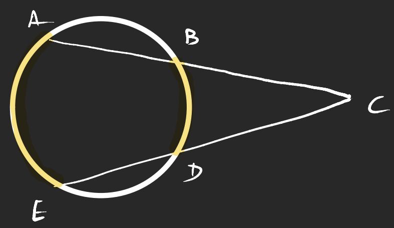
3. tan-tan $\angle$



$m(\angle B) \propto \frac{1}{2}\left(m(\widehat{AXC}) - m(\widehat{AC})\right)$

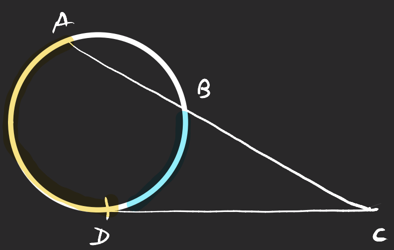
maj. minor

4. sec-sec. $\angle$



$m(\angle C) \propto \frac{1}{2}\left(m(\widehat{AE}) - m(\widehat{BD})\right)$

5. sec-tan. $\angle$



$m(\angle C) \propto \frac{1}{2}\left(m(\widehat{AD}) - m(\widehat{BD})\right)$

Lecture-5 (2/9m) 2

Practice

Q1 a°) $C_1 \equiv x^2 + (y+2)^2 = 9$ centre : $(0,-2)$, $r = 3$

a) $C \equiv x^2 + y^2 + 2x\sin\theta + 2y\cos\theta - 8 = 0$ centre , rad = ?

centre : $(-\sin\theta, -\cos\theta)$

radius : 3 units

b) $2x^2 + \underbrace{\lambda xy} + 2y^2 + (\lambda-4)x + 6y - 5 = 0$ centre , rad = ?

$\frac{\lambda}{2} = 0 \Rightarrow \boxed{\lambda = 0}$

centre : $\left(1, -\frac{3}{2}\right)$

rad $= \frac{\sqrt{23}}{2}$

- $(x-a)^2 + (y-b)^2 = r^2$
- $x^2 + y^2 + 2gx + 2fy + c = 0$

$C : (-g, -f)$

$r = \sqrt{g^2 + f^2 - c}$

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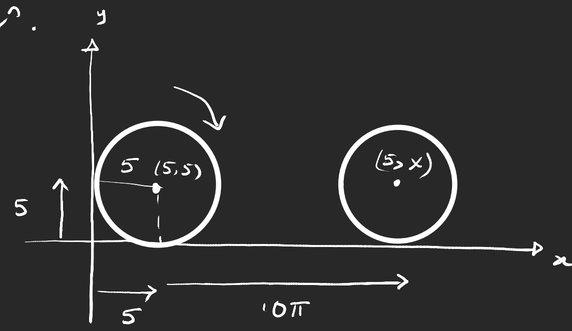
Q2. Circle of rad. 5 : touches coord. axes in the 1st quad.

If circle makes 1 complete roll on axis along +ve dirⁿ.

Eqⁿ in new position ?

$$(x-h)^2+(y-k)^2=r^2$$
$$\downarrow$$
$$(x-(5+10\pi))^2+(y-5)^2=5^2$$

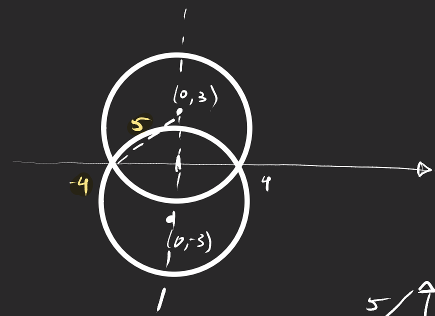


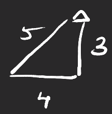
Q3. C : passes through 2 points on the x axis which are at distance of 4 from the origin

$r=5$ Eqⁿ = ?

$$(x-0)^2+(y\mp 3)^2=5^2$$
$$x^2+y^2\mp 6y-16=0$$

Centre : $(0,\pm 3)$



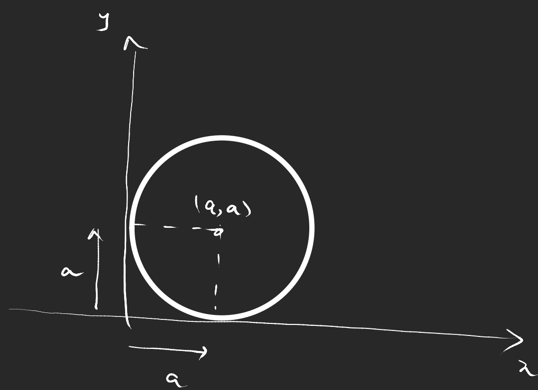


Q4. Circle C which touches the coordinate axes & whose centre lies on the line $lx+my+n=0$

Eqⁿ of C = ?

C : $(x-a)^2+(y-a)^2=a^2$; Centre (a,a) → $la+ma+n=0$
$$\downarrow$$
$$a=?$$

Plug & chug.



$$* \quad x^2+y^2+\frac{2nx}{l+m}+\frac{2ny}{l+m}+\frac{n^2}{(l+m)^2}=0$$

$$(l+m)^2(x^2+y^2)+2n(l+m)(x+y)+n^2=0$$

Q5. C : passes through the point $(2,0)$ whose centre is the limit of the point of intersection of

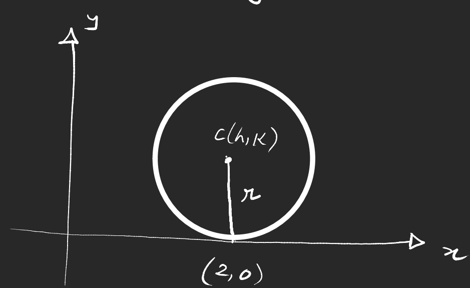
2 lines $L_1\equiv 3x+5y=1$, $L_2\equiv(2+c)x+5c^2y=1$ as $c\to 1$ Eqⁿ of circle

HW

$$(h,k)=\left(\frac{2}{5},\frac{-1}{25}\right) \qquad r=\frac{\sqrt{1601}}{25}$$

$$(x-h)^2+(y-k)^2=r^2 \leftarrow$$

$$\left(x-\frac{2}{5}\right)^2+\left(y+\frac{1}{25}\right)^2=\frac{1601}{625}$$



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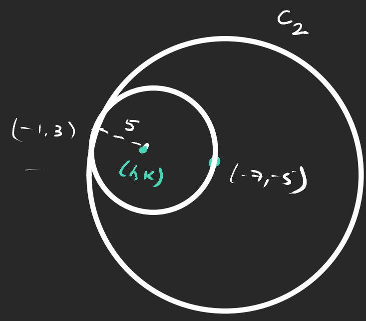
Lecture-6 (3/Jun) 1.40

Q6. C_1: rad = 5 lies within a circle $C_2 \equiv x^2+y^2+14x+10y-26=0$ & touches the given circle at a point $(-1,3)$

Eqⁿ of circle C_1 = ?



* C_2: centre $(-7,-5)$

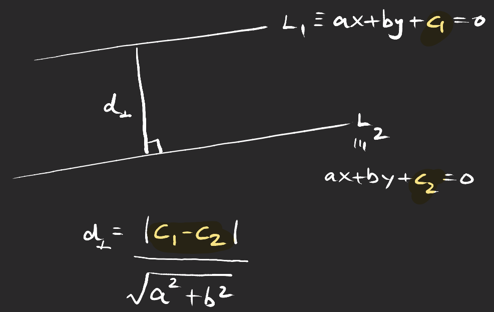
Rad = 10

* $(h,k) = (-4,-1)$ ⟹ $(x+4)^2+(y+1)^2 = 5^2$

rad = 5

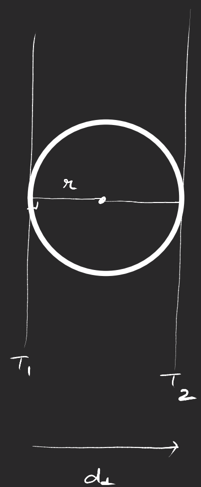
Q7. $T_1 \equiv 3x-4y+4=0$, $T_2 \equiv 6x-8y-7=0$ } tangents to a circle

Rad = ?



$$d_\perp = \frac{|c_1-c_2|}{\sqrt{a^2+b^2}}$$

$d_\perp$ ⟹ $\boxed{r = 3/4}$



Q8. $C \equiv x^2+y^2+3x+4y-1=0$; $L \equiv y=2x+m$ is diameter to C, $m = ?$

Centre = $\left(-\frac{3}{2}, -2\right)$

$m = 1$

Q9 C: passes thru. the pts. $(1,-2)$ & $(4,-3)$

Centre lies on $3x+4y=7$

Eqⁿ of circle C = ?

'Circles + linear eqⁿ'

$C \equiv x^2+y^2+2gx+2fy+c=0$ centre $(-g,-f)$

- $C(1,-2)$ ⟹ $5+2g-4f+c=0$
- $C(4,-3)$ ⟹ $25+8g-6f+c=0$
- $(-g,-f) \longrightarrow 3(-g)+4(-f)=7$

$c = \frac{11}{3}$

Centre: $\left(-\frac{47}{15}, \frac{9}{15}\right)$

Eqⁿ of circle

$15(x^2+y^2)-94x+18y+55=0$

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HW

10. circle $r=3$, centre lies on line $y=x-1$

Eqⁿ of circle if it passes thru $(7,3)$

11. St. Line $L \equiv \frac{x}{a}+\frac{y}{b}=1$ cuts axes at A & B

Eqⁿ of circle passing thru. $(0,0)$, A & B = ?

12. C : Passes thru. $(1,0)$ & $(0,1)$ w/ smallest possible radius.

Eqⁿ of circle = ?

Lecture-7 (4 Jun) 1.10

Q13. $(x\cos\beta + y\sin\beta - A)^2 + (x\sin\beta - y\cos\beta - B)^2 = \xi^2$ ⟶ a) Radius of circle ? (Algebra)

if β changes : the locus of its centre is again a circle

b) find its centre & radius

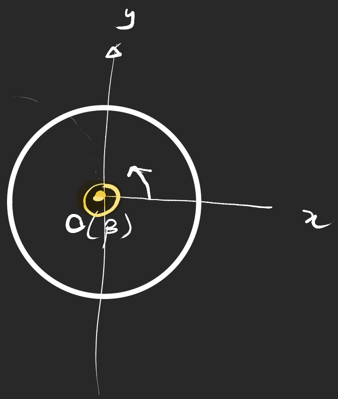
a) $(x\cos\beta + y\sin\beta - A)^2 + (x\sin\beta - y\cos\beta - B)^2 = \xi^2$

* $x^2\cos^2\beta + y^2\sin^2\beta + A^2 + 2xy\cancel{\cos\beta\sin\beta} - 2yA\sin\beta - 2xA\cos\beta +$
$x^2\sin^2\beta + y^2\cos^2\beta + B^2 - 2xy\cancel{\sin\beta\cos\beta} + 2yB\cos\beta - 2xB\sin\beta = \xi^2$

$$x^2+y^2-2x\underbrace{(A\cos\beta+B\sin\beta)}_{g}-2y\underbrace{(A\sin\beta-B\cos\beta)}_{f}+\underbrace{A^2+B^2-\xi^2}_{c}=0$$

Centre : $(A\cos\beta + B\sin\beta,\ A\sin\beta - B\cos\beta)$

Radius $= \sqrt{g^2+f^2-c} = \sqrt{\cancel{A^2}+\cancel{B^2}-\cancel{A^2}-\cancel{B^2}+\xi^2} = \xi = \text{const}$



b) $C(p,q) \equiv C(A\cos\beta + B\sin\beta,\ A\sin\beta - B\cos\beta)$

$$\left.\begin{aligned} p &= A\cos\beta + B\sin\beta \\ q &= A\sin\beta - B\cos\beta \end{aligned}\right\} \quad p^2+q^2 = A^2+B^2$$

$$\left\{ x^2+y^2 = \underbrace{A^2+B^2}_{\text{Const}} \right\}$$

Circle

Centre $(0,0)$

Radius $=\sqrt{A^2+B^2}$

Remember : When we calculate locus of a point

- Eqⁿ in h, k
- Replace $(h,k) \to (x,y)$
- Eqⁿ of locus of a pt

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Q17. $C \equiv x^2+y^2-2x+4y-p=0$

$A(2,8)$: it is an interior point of C which neither touches nor intersects the axes

p is constraint : value ?

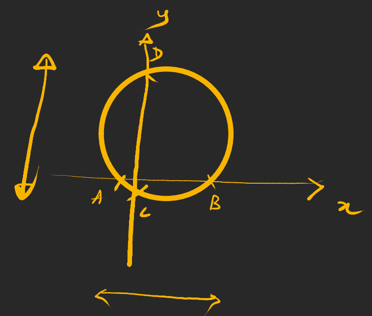
New Concept

* Intercepts cut by the circle on axis

$C: x^2+y^2+2gx+2fy+c=0$

x intercept $= ?$ $\xrightarrow{y=0}$ $x^2+2gx+c=0 \Rightarrow x_{1,2} = \dfrac{-2g \pm \sqrt{4g^2-4c}}{2} = -g \pm \sqrt{g^2-c}$

$$\boxed{x_{int} \equiv x_2 - x_1 = \left|2\sqrt{g^2-c}\right|}$$



y intercept $= ?$ $\xrightarrow{x=0}$ $y_{1,2} = -f \pm \sqrt{f^2-c}$

Lecture-8 (6/Jan) 2

HW discussion

HW

10. circle $r=3$, centre lies on line $y=x-1$

Eqⁿ of circle if it passes thru $(7,3)$

* $C \equiv x^2+y^2=a^2$ given 1 (won't work!)

excluded based on given Cond"

* $C \equiv (x-h)^2+(y-k)^2=a^2$ given 2 centre (h,k), $r=a$ ✓

* $C \equiv x^2+y^2+2gx+2fy+C=0$ given 3 (too expanded)

* Centre $= (h,k)$: satisfies $y=x-1 \Rightarrow \boxed{k=h-1} \Rightarrow$ Centre $(h, h-1)$

$C \equiv (x-h)^2+(y-k)^2=a^2 \Rightarrow (x-h)^2+(y-h+1)^2=9$

$(7,3)$ satisfies C $\Rightarrow (7-h)^2+(4-h)^2=9 \Rightarrow$

$h = ?$ 4 / 7, k 3 / 6

$k=3 \longrightarrow (x-4)^2+(y-3)^2=9$

$k=6 \longrightarrow (x-7)^2+(y-6)^2=9$

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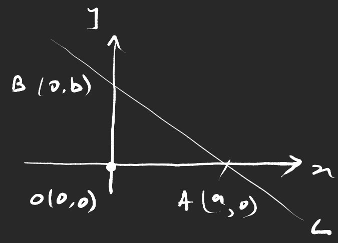
11. St. Line $L \equiv \frac{x}{a}+\frac{y}{b}=1$ cuts axes at A & B

Eqⁿ of circle passing thru. (0,0), A & B = ?



* $C \equiv (x-h)^2+(y-k)^2=a^2$ given 1

* $C \equiv x^2+y^2+2gx+2fy+c=0$ given 2 ✓

A, B, O satisfy C → C(A) Eqⁿ 1; → C(B) Eq 2; → C(O) Eq 3 } ⇒ solving 3 eqⁿ

- 3 linear
- 3 quad. eqⁿ → 3 linear ↓ 3 variables

* C(A) ⇒

C(B) ⇒

C(O) ⇒ . . . $C \equiv x^2+y^2-ax-by=0$

12. C : Passes thru. (1,0) & (0,1) w/ smallest possible radius.

Eqⁿ of circle = ?

* $C \equiv (x-h)^2+(y-k)^2=a^2$ given 1

$C \equiv x^2+y^2+2gx+2fy+c=0$ given 2 ✓ → $r=\sqrt{g^2+f^2-c}$, Centre $(-g,-f)$

C : A(1,0), B(0,1) satisfied $\xrightarrow{C(A)}$, $\xrightarrow{C(B)}$ $\boxed{\begin{aligned}1+2g+c=0\\1+2f+c=0\end{aligned}}$ → $g=-\frac{(1+c)}{2}$, $f=-\frac{(1+c)}{2}$

* C : $r=r_{min}$

$r=\sqrt{f^2+g^2-c}=\sqrt{2\left(\frac{1+c}{2}\right)^2-c}=\sqrt{\frac{1+c^2+\cancel{2c}-\cancel{2c}}{2}}=\sqrt{\frac{1+c^2}{2}}$

$\because c^2 \geq 0$

Approach 1

At $c=0$ ⇒ $\boxed{r=r_{min}=\frac{1}{\sqrt{2}}}$

⇓

$g=-\frac{1}{2}$, $f=-\frac{1}{2}$

} ⇒ $\boxed{C \equiv x^2+y^2-x-y=0}$

Approach 2

* r_{min} ⇒ $\frac{dr}{dc}=0$ ⇒ $\frac{d}{dc}(1+c^2)=2c=0$ ⇒ $\boxed{c=0}$

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Q17. $C \equiv x^2+y^2-2x+4y-p=0$

$A(2,8)$: it is an interior point of C which neither touches nor intersects the axes

p is constant : value ?

Formula sheet
1. Std. Eqn
2. gen. eqn
3. x_{int}, y_{int} length cut by C

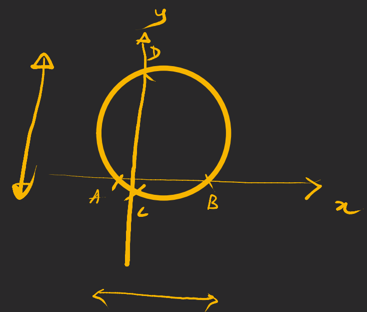
New Concept

1. Intercepts cut by the circle on axis

$C: x^2+y^2+2gx+2fy+c=0$

x intercept = ? $\xrightarrow{y=0}$ $x^2+2gx+c=0 \Rightarrow x_{1,2} = \dfrac{-2g \pm \sqrt{4g^2-4c}}{2} = -g \pm \sqrt{g^2-c}$

$\boxed{x_{int} \equiv x_2 - x_1 = |2\sqrt{g^2-c}|}$ length of x intercept



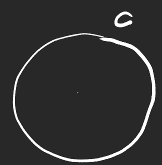
y intercept = ? $\xrightarrow{x=0}$ $y_{1,2} = -f \pm \sqrt{f^2-c}$

$\boxed{|y_{int}| = |y_2-y_1| = |2\sqrt{f^2-c}|}$ length of y intercept

* $C \equiv x^2+y^2-2x+4y-p=0$

$A(2,8)$: $\cancel{4}+64-\cancel{4}+32-p<0 \Rightarrow \boxed{p>96}$ — (1)
inside

$p \in (96, \infty)$



$A(x_1, y_1)$

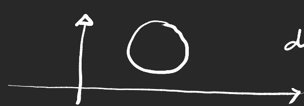
$C(A) = 0$ on
>0 outside
<0 inside

$g=-1$, $c=-p$

* $|x_{int}| = 2\sqrt{1+p}$ $\xrightarrow[DNE]{\therefore x_{int}}$ $1+p<0 \Rightarrow \boxed{p<-1}$ — (2)

$f=4$,
$|y_{int}| = 2\sqrt{4+p}$ $\xrightarrow[DNE]{}$ $4+p<0 \Rightarrow \boxed{p<-4}$ — (3)

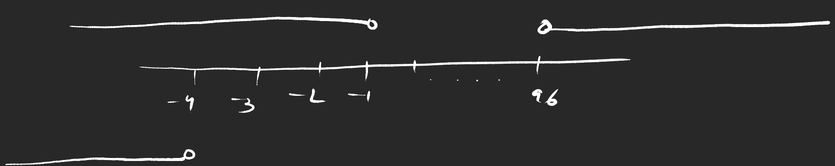
} not wanted



demand

demand: Inside + not int^ x + not int^ y

$\Downarrow$



$\underbrace{\hspace{5cm}}$

$\boxed{p = \phi}$ $\Rightarrow$ No possible solution exists for p

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HW

- circles RD Sharma – Ex-1, 2, 3   Deadline = Tuesday

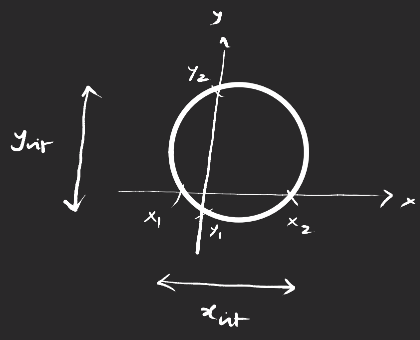
Lecture-9 (8/Jan) 'Dark side of circle' 1:45

Intercepts cut by the circle on axes

* $C \equiv x^2+y^2+2gx+2fy+c=0$ $r=\sqrt{g^2+f^2-c}$

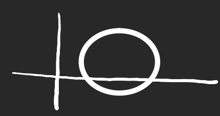
$x_{int} = |2\sqrt{g^2-c}| = |2\sqrt{r^2-f^2}|$

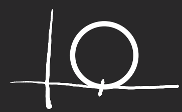
$y_{int} = |2\sqrt{f^2-c}|$

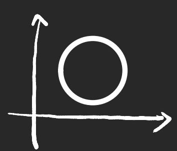


Remarks

- if C cuts x axis at 2 distinct pts. $\Rightarrow g^2-c>0$
- if C touches x axis at 1 pt $\Rightarrow g^2=c$
- if C touches y axis at 1 pt $\Rightarrow f^2=c$
- C lies completely above/below x axis $\Rightarrow$ x_{int} DNE
  $x_{int} \notin \mathbb{R}$
  $\Downarrow$
  $g^2-c<0$

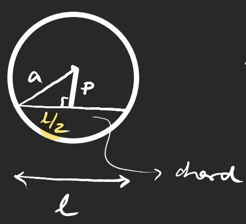






- $\boxed{\text{length of chord} = l = 2\sqrt{a^2-p^2}}$

  p: $\perp$ dist. from centre to chord

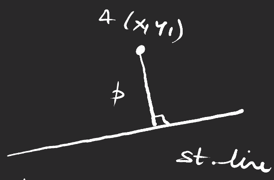


$a^2 = p^2 + \frac{l^2}{4}$

Q15. C : centre (3, −1)

cuts off a chord of length 6 on a line $L \equiv 2x-5y+18=0$

Eq^n of circle C = ?



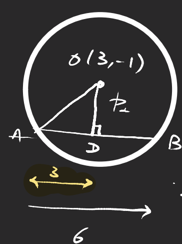
$L \equiv ax+by+c=0$

$p = \frac{|L(A)|}{\sqrt{a^2+b^2}} = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

* $C \equiv (x-3)^2+(y+1)^2 = a^2$

a ← unknown. $= (OA)^2 = p_2^2 + 9 = 29+9 = \boxed{38}$

$p_2 = \frac{|2(3)-5(-1)+18|}{\sqrt{4+25}} = \frac{|29|}{\sqrt{29}} = \sqrt{29}$



∵ circle thm.

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Q16. $C_1 \equiv x^2+y^2-2x-2y+1=0$

$C_2 \equiv x^2+y^2-16x-2y+61=0$

$L \equiv y = 2x + a \equiv L(a)$

$\uparrow$ Parameter

find the range of Parameter a : L lies b/w the circles C_1 & C_2 without intersecting / touching either C_1, C_2

* things needed to know
  - straight lines
  - "Extra features"
  - fundamental Eq^n of Circle

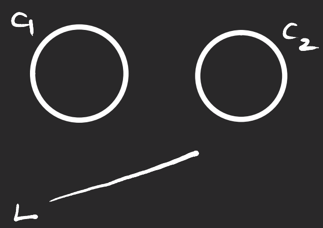
Lecture-10 (10/Jun) 2

Q16. $C_1 \equiv x^2+y^2-2x-2y+1=0$

$C_2 \equiv x^2+y^2-16x-2y+61=0$

$L \equiv y = 2x + a \equiv L(a)$

$\uparrow$ Parameter



find the range of Parameter a : $\overbrace{\text{L lies b/w the circles } C_1 \text{ \& } C_2}^{\text{given}}$ without intersecting / touching either C_1, C_2

* $C_1 \equiv (x-1)^2 + (y-1)^2 = 1$ Centre $= (1,1)$, $r_1 = 1$

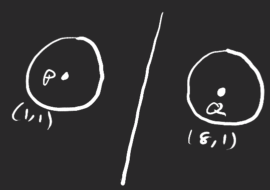
$C_2 \equiv (x-8)^2 + (y-1)^2 = 4$ Centre $= (8,1)$, $r_2 = 2$

$L \equiv y - 2x - a = 0$

St. line

$P(x_1, y_1)$ $Q(x_2, y_2)$

I.



$\left.\begin{array}{l} L(1,1) = 1-2-a \\ L(8,1) = 1-16-a \end{array}\right\}$ $\xrightarrow{\text{opp side Cond}^n}$ $L(P)L(Q) < 0$

$\Downarrow$

$\boxed{(1-2-a)(1-16-a) < 0}$

$\Downarrow$

$a \in (\ ?\ )$

same side same sign

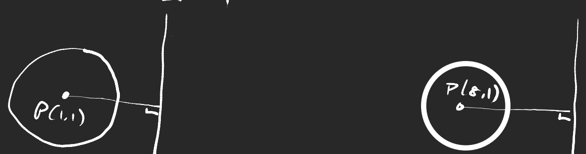
$\underbrace{L(P) < 0 \ \ L(Q) < 0 \quad \quad > 0 \ \ > 0}$

$L(P)L(Q) > 0$

- $\boxed{L(P)L(Q) < 0}$ opp side opp. sign.

II.

$r_2 > r_1$



circle

$r_2 > r_1$

not Touching

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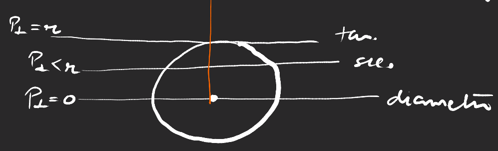
$y-2x-a=0$ $\qquad\qquad$ $P_\perp > r_2$

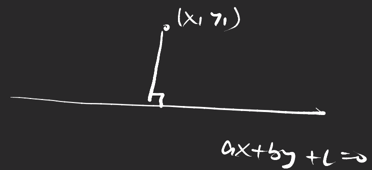
$\boxed{P_\perp > r_1 \Rightarrow \dfrac{|1-2-a|}{\sqrt5} > 1}$ $\qquad$ $\boxed{P_\perp > r_2 \Rightarrow \dfrac{|1-16-a|}{\sqrt5} > 2}$

$\Downarrow$

$a \in$



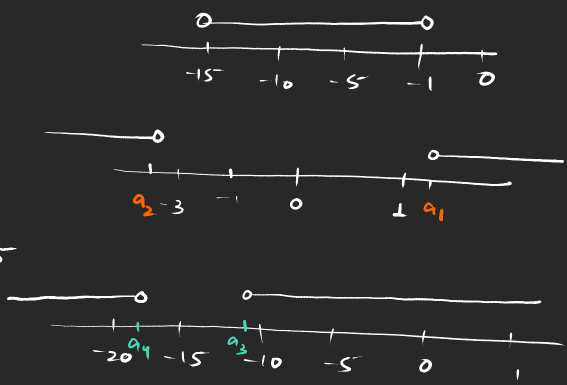
st line

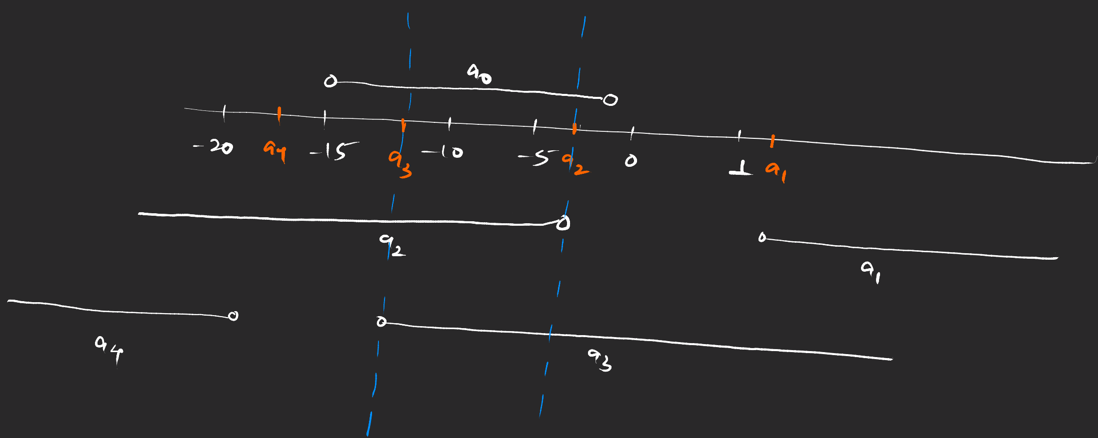


$$P_\perp = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

Solving 3 inequalities :

* $(-1-a)(-15-a) < 0 \Rightarrow a \in (-15, -1)$ $\quad a_0$
* $|-1-a| > \sqrt5 \Rightarrow \underbrace{a > \sqrt5 - 1}_{1.23}$ (a_1; 2.23) or $\underbrace{a < -\sqrt5 - 1}_{-3.23}$ (a_2)
* $|-15-a| > 2\sqrt5 \Rightarrow \underbrace{a > 2\sqrt5 - 15}_{-10.}$ (a_3; 4.47) or $\underbrace{a < -2\sqrt5 - 15}_{-19}$ (a_4)





$\boxed{a \in (a_3, a_2)}$

$\boxed{a \in \left(2\sqrt5 - 15, -\sqrt5 - 1\right)}$

Q17. $C \equiv x^2 + y^2 = 10$

$L \equiv x\sqrt5 + 2y = 3\sqrt5$

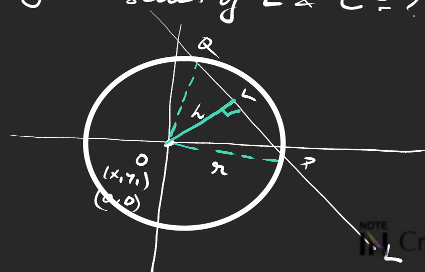
Ar.(Δ) formed by the line joining the origin to points of intersection of L & C = ?

$Ar(\Delta) = \frac{1}{2}(QP)\cdot(OL)$

$\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} = h$ $\qquad$ $r = \sqrt{10}$ $\qquad$ $r^2 = h^2 + LP^2$

$2(LP)$

$Ar(\Delta) = 5$

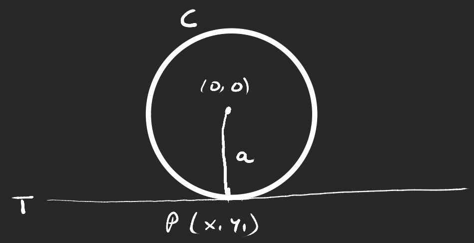


1.8 Some More Extra feelings

A Tangent line of Circle (theorems)

a) Eq^n of the tangent $(T=0)$

i) tan. at the pt (x_1, y_1) on $C \equiv x^2+y^2=a^2$



* P lies on C $\Rightarrow x_1^2+y_1^2=a^2$

* Eq^n of 1 pt form $\Rightarrow y-y_1 = m_T(x-x_1) \Rightarrow y-y_1 = -\frac{x_1}{y_1}(x-x_1)$

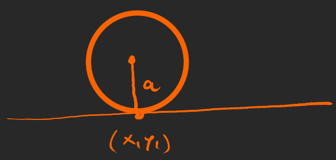
$m_T \cdot \underbrace{m_N}_{\frac{y_1}{x_1}} = -1 \Rightarrow m_T = -\frac{x_1}{y_1}$

$y y_1 - \underbrace{y_1^2} = -x_1 x + x_1^2 \Rightarrow x x_1 + y y_1 = \underbrace{x_1^2+y_1^2}_{a^2}$

$\boxed{T \equiv x x_1 + y y_1 = a^2}$

$\updownarrow$

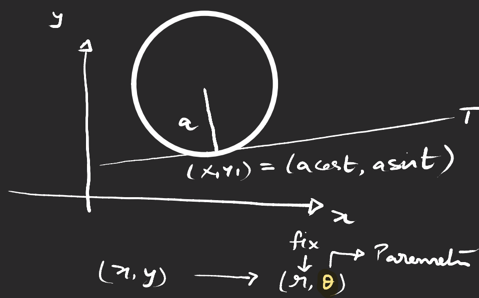
$T \equiv x x_1 + y y_1 - a^2 = 0$



Parameter
$\downarrow$

ii.1) tangent at the pt. $P(a\cos t, a\sin t)$
$\qquad\qquad\qquad\qquad x_1 \qquad y_1$

$C \equiv x^2+y^2=a^2$

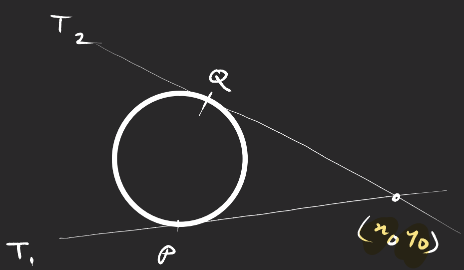


* $\because T \equiv x x_1 + y y_1 = a^2 \Rightarrow \cancel{a} x \cos t + \cancel{a} y \sin t = a^{\cancel{2}}$

$\Downarrow$

$\boxed{T \equiv x\cos t + y \sin t = a}$

ii.2) Point of intersection of tangents at the pt $P(\alpha), Q(\beta)$
(Parameters over α)



* $T_1(\alpha) \equiv x\cos\alpha + y \sin\alpha = a \quad \} \cos\beta$

$T_2(\beta) \equiv x\cos\beta + y\sin\beta = a \quad \} \cos\alpha$

* $\cancel{x\cos\alpha\cos\beta} + y\sin\alpha\cos\beta = a\cos\beta$
$\cancel{x\cos\alpha\cos\beta} + y\sin\beta\cos\alpha = a\cos\alpha$

$$y = \frac{a(\cos\beta-\cos\alpha)}{\sin\alpha\cos\beta-\sin\beta\cos\alpha} = \frac{a(\cos\beta-\cos\alpha)}{\sin(\alpha-\beta)} = \frac{a\,\cancel{2}\sin\frac{\alpha+\beta}{2}\,\cancel{\sin\frac{(\alpha-\beta)}{2}}}{\cancel{2\sin\frac{\alpha-\beta}{2}}\cos\frac{\alpha-\beta}{2}}$$

C	D	$\frac{C+D}{2}$	$\frac{C-D}{2}$

$S+S = 2SC$
$S-S = 2CS$
$C+C = 2CC$
$C-C = 2SS^*$

$\sin 2\theta = 2\sin\theta\cos\theta$

* $y = \dfrac{a\sin\left(\frac{\alpha+\beta}{2}\right)}{\cos\frac{\alpha-\beta}{2}}$; $x = \dfrac{a\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}$

HW

$$x = \frac{a-y\sin\alpha}{\cos\alpha} = \frac{a - \dfrac{a\sin\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}\sin\alpha}{} = a\left\{\cos\frac{\alpha-\beta}{2} - \sin\frac{\alpha+\beta}{2}\sin\alpha\right\}$$

Plug & chug!

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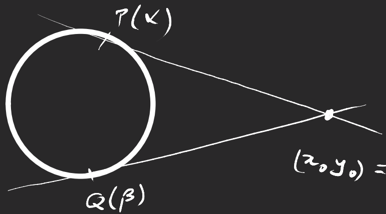
$\cos\alpha$ $\cos\alpha \cdot \cos\frac{\alpha-\beta}{2}$



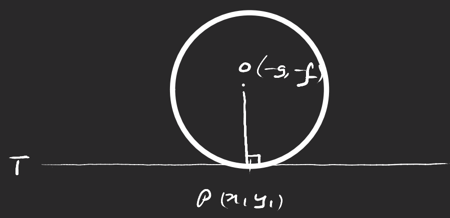
*

$(x_0, y_0) = \left(\dfrac{a\sin\frac{\alpha+\beta}{2}}{\cos\left(\frac{\alpha-\beta}{2}\right)}, \dfrac{a\cos\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}} \right)$ pt of intersection

Lecture-11 (11 Jun) 1.5

(iii) Eqn of tangent at $P(x_1, y_1)$ on $C \equiv x^2+y^2+2gx+2fy+c=0$

steps same as those done in (i)



* $y-y_1 = m_T(x-x_1)$ $\qquad m_T \cdot m_N = -1$

$m_T = \dfrac{-(x_1+g)}{y_1+f}$ $\qquad m_N = \dfrac{y_1+f}{x_1+g}$

$(y-y_1)(y_1+f) = -(x_1+g)(x-x_1)$

$yy_1 + yf - y_1^2 - y_1 f = -x_1 x + x_1^2 - gx + gx_1$

$\underbrace{x_1^2+y_1^2}_{= -2gx_1-2fy_1-c} - xx_1 - yy_1 - gx + gx_1 - fy + fy_1 = 0$ — (1)

$\left\{ \because P \text{ satisfies } C \quad C(P) \equiv x_1^2+y_1^2+2gx_1+2fy_1+c=0 \right.$

$-gx_1 - fy_1 - xx_1 - yy_1 - gx - fy - c = 0$

$$\boxed{xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c = 0}$$

HW

(iv) if $L \equiv y = mx + c$ touching $C \equiv x^2+y^2=a^2$

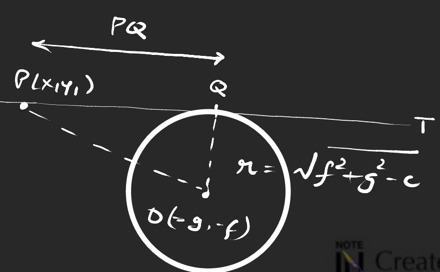
then $c = \pm a\sqrt{1+m^2}$ & contact pts. $\left(\mp \dfrac{am}{\sqrt{1+m^2}}, \mp \dfrac{a}{\sqrt{1+m^2}}\right)$ or $\left(\mp \dfrac{a^2 m}{c}, \pm \dfrac{a^2}{c}\right)$

Eqn of tangent $y = mx \pm a\sqrt{1+m^2}$

(v) $C \equiv (x-h)^2 + (y-k)^2 = a^2$

Eqn of tangent with slope m : $(y-k) = m(x-h) \pm a\sqrt{1+m^2}$

B. length of tangent from $P(x_1, y_1)$ $(\sqrt{C_1})$



* $C \equiv x^2+y^2+2gx+2fy+c=0$

$PO^2 = r^2 + PQ^2 \Rightarrow PQ = \sqrt{(x+g)^2 + (y+f)^2 - (f^2+g^2-c)}$

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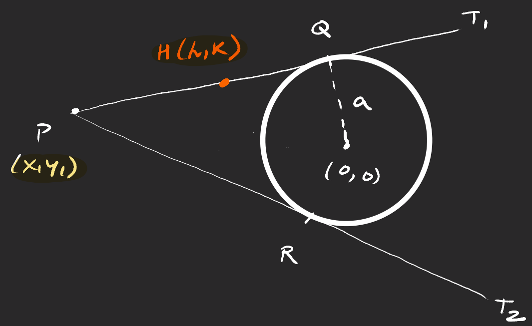
$$PQ = \sqrt{x_1^2+y_1^2+2gx_1+2fy_1+c} \equiv \sqrt{C_1}$$

Circle expression evaluated at pt 1

c. Combined equation of pair of tangent



* $C \equiv x^2+y^2=a^2$, $P \equiv (x_1, y_1)$

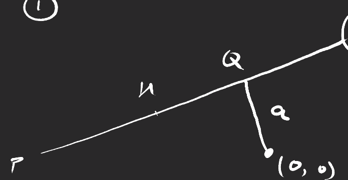
* Let (h,k) be any pt on either of tangents from P

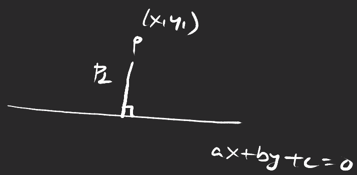
Eqⁿ line PH : $y-y_1 = \dfrac{k-y_1}{h-x_1}(x-x_1)$

$$y(h-x_1) - y_1h + \cancel{y_1x_1} = x(k-y_1) - kx_1 + \cancel{y_1x_1}$$

$$\boxed{y(h-x_1) - x(k-y_1) + kx_1 - hy_1 = 0} \quad \text{—— ①}$$

Eqⁿ of "tangent"





$$\boxed{P_\perp = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}}$$

* $P_\perp = \dfrac{|0(h-x_1) - 0(k-y_1) + kx_1 - hy_1|}{\sqrt{(k-y_1)^2+(h-x_1)^2}} = a$

$$\boxed{(kx_1 - hy_1)^2 = a^2\{(k-y_1)^2+(h-x_1)^2\}} \quad \text{—— ②} \quad \text{Condⁿ satisfied by } (h,k)$$

* $(h,k) \longrightarrow (x,y) \Rightarrow$ Eqⁿ of locus.

$$(yx_1 - xy_1)^2 = a^2\{(x-x_1)^2+(y-y_1)^2\} \quad \text{eqⁿ of locus —— ③}$$

Simplify & that's it. (Algebra)

* $a^2\{x^2+x_1^2-2x_1x + y^2+y_1^2-2yy_1\} = y^2x_1^2 + x^2y_1^2 - 2x_1y_1xy$

$$x^2(a^2-y_1^2) + y^2(a^2-x_1^2) - 2a^2x_1x - 2a^2y_1y + \underbrace{a^2x_1^2+a^2y_1^2}_{a^2(x_1^2+y_1^2)} + 2x_1y_1xy = 0$$

$$\underbrace{x^2(a^2-y_1^2)+y^2(a^2-x_1^2)+a^2(x_1^2+y_1^2)} = -2x_1y_1xy + 2a^2x_1x + 2a^2y_1y \quad \text{—— ④}$$

$$-\{x^2(y_1^2-a^2) + y^2(x_1^2-a^2) - a^2(x_1^2+y_1^2)\}$$

x^2 y^2 a^2 a^2 x_1^2 y_1^2

Trick : $(x^2+y^2-a^2)(x_1^2+y_1^2-a^2) = x^2(y_1^2-a^2) + x_1^2x^2 + y^2(x_1^2-a^2) + y^2y_1^2 - a^2(x_1^2+y_1^2) + a^4$

$$= \underbrace{x^2(y_1^2-a^2) + y^2(x_1^2-a^2) - a^2(x_1^2+y_1^2)} + \underbrace{(\checkmark\checkmark\checkmark)}$$

Lecture-12 (12 Jun) 1.5

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Jan ④

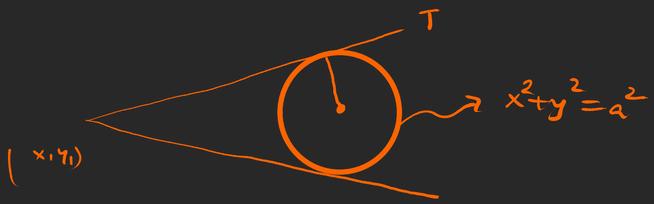
* $x^2(y_1^2-a^2)+y^2(x_1^2-a^2)-a^2(x_1^2+y_1^2) = 2x_1y_1xy - 2a^2x_1x - 2a^2y_1y$

$(x^2+y^2-a^2)(x_1^2+y_1^2-a^2) - x_1^2x^2 - y_1^2y^2 - a^4 = 2x_1y_1xy - 2a^2x_1x - 2a^2y_1y$ Play. trick.

$$(x^2+y^2-a^2)(x_1^2+y_1^2-a^2) = \underbrace{x^2x_1^2+y^2y_1^2+a^4-2a^2x_1x-2a^2y_1y+2x_1y_1xy}_{(xx_1+yy_1-a^2)^2}$$

$$\boxed{(x^2+y^2-a^2)(x_1^2+y_1^2-a^2) = (xx_1+yy_1-a^2)^2} \longleftrightarrow (C\cdot C_1 = T^2)$$

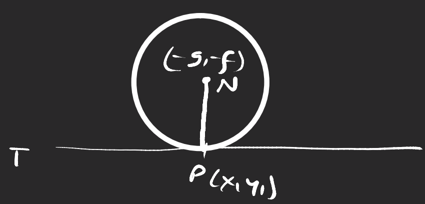
Combined eqⁿ of pair of tang.



Note: $CC_1 = T^2$ holds true even for $C \equiv x^2+y^2+2gx+2fy+c=0$

B. Normal of circle

* $C \equiv x^2+y^2+2gx+2fy+c=0$, $P(x_1,y_1)$



Eqⁿ of Normal

$$\boxed{N \equiv y(x_1+g) - x(y_1+f) + fx_1 - gy_1 = 0}$$

$\downarrow$ $\left.\begin{matrix} g=0 \\ f=0 \end{matrix}\right\}$ $x^2+y^2=a^2$

$$\boxed{x_1y - y_1x = 0}$$

* if circle is parametrized by $\theta \Rightarrow (x,y) = (a\cos\theta, a\sin\theta)$

$P(x_1,y_1) \longrightarrow P(a\cos\theta, a\sin\theta)$

$\boxed{N \equiv x\sin\theta - y\cos\theta = 0}$ $\longrightarrow$ " $m = \frac{y}{x} = \tan\theta$ "

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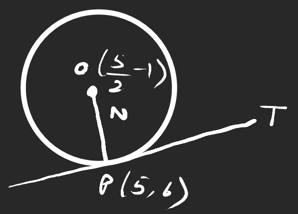
L touches the C_1 — values of a & b = ?

Lecture-13 (13/Jun) 120

Q18 $C \equiv x^2+y^2-5x+2y-48=0 \longrightarrow O\left(\frac{5}{2},-1\right)$

$P \equiv (5,6)$

$N \equiv ?$



$(y-6) = m_N(x-5) = \frac{14}{5}(x-5)$

$m_N = \frac{6+1}{5-\frac{5}{2}} = \frac{14}{5}$

$5y-30 = 14x-70 \Rightarrow \boxed{14x-5y-40=0}$

Q19 $C_1 \equiv x^2+y^2-2x=3$

$C_2 \equiv x^2+y^2-4y=6$

$x^2+y^2+2gx+2fy+c=0$

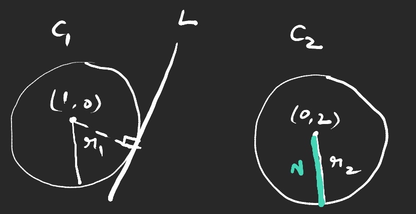
$L \equiv ax+by=2 \;;\; a,b \neq 0$

$\boxed{N \text{ of } C_2 \equiv L}$ ✓

$r=\sqrt{f^2+g^2-c}$

$\boxed{L \text{ touches the } C_1}$ ✓

values of a & b = ?



* L touches $C_1 \Rightarrow p_\perp = r_1 = \frac{|ax_1+by_1-2|}{\sqrt{a^2+b^2}} = \frac{|a-2|}{\sqrt{a^2+b^2}} = 2$

$r_1 = \sqrt{1^2-(-3)}$, $r_1 = 2$

$r_2 = \sqrt{4-(-6)} = \sqrt{10}$

$|a-2| = 2\sqrt{a^2+b^2}$ — (1) Cond^n satisfied by a & b

* L is Normal to C_2 $(ax+by-2=0) \longrightarrow$ L satisfies centre of C_2 $(0,2)$

$0+2b-2=0 \Rightarrow \boxed{b=1}$ — (2)

from 1

* $(a-2)^2 = 4(a^2+1) \Rightarrow$ $a = \cancel{0}, -\frac{4}{3}$

$\boxed{L \equiv -\frac{4}{3}x+y=2}$

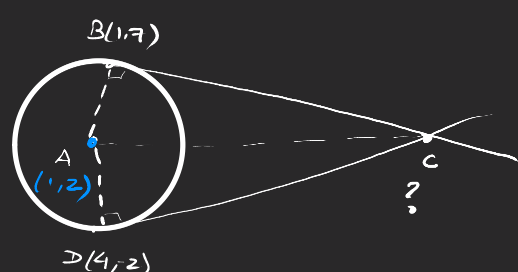
HW

Q20. $C \equiv x^2+y^2-2x-4y-20=0$

$B(1,7)$, $D(4,-2)$: lie on the circle C

Tangents are drawn from B & D, meet at C

Area $(\Delta BCD) = ?$



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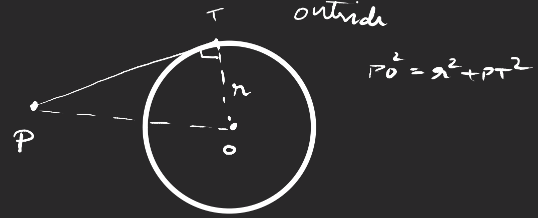
C. Power of a point $\Pi(p)$

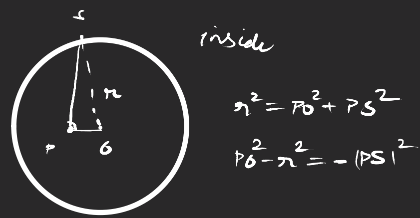
* Relative dist of a pt. from the circle

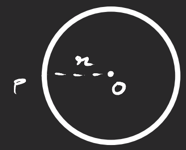
* $\Pi(P) = PO^2 - r^2 = |PT|^2$ if P outside $\Pi > 0$

$= -|PS|^2$ if P inside $\Pi < 0$

$= 0$ if P on the circle $\Pi = 0$







Lecture 13 (14/Jun) 2

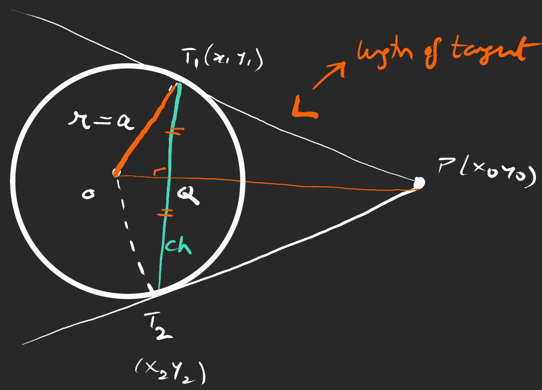
D. Chord of contact — eqn, length

* $C \equiv x^2+y^2=a^2$, $P(x_0 y_0)$ given

eqn of tan

* $PT_1 : xx_1 + yy_1 = a^2 \xrightarrow{P \text{ satisfied}} x_0x_1 + y_0y_1 = a^2$ ✓

$PT_2 : xx_2 + yy_2 = a^2 \xrightarrow[P \text{ satisf}]{} x_0x_2 + y_0y_2 = a^2$ ✓



$\boxed{\text{chord eqn : } T_1T_2 \equiv x_0x + y_0y = a^2}$

length of chord

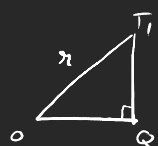
* $L(T_1T_2) = 2|T_1Q|$

$= 2\sqrt{r^2 - OQ^2}$

$= 2\sqrt{r^2 - \frac{r^4}{x_0^2+y_0^2}}$

$= 2r\sqrt{1 - \frac{r^2}{x_0^2+y_0^2}}$

$= 2r\frac{\sqrt{x_0^2+y_0^2-r^2}}{\sqrt{x_0^2+y_0^2}}$



$r^2 = T_1Q^2 + OQ^2$

$L \equiv x_0x + y_0y - r^2 = 0$

$OQ = p_L = \frac{|L(O)|}{\sqrt{x_0^2+y_0^2}} = \frac{|r^2|}{\sqrt{x_0^2+y_0^2}}$

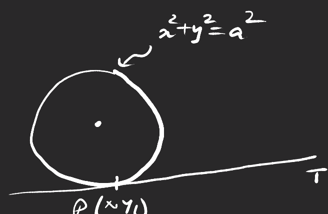
$OP = \sqrt{x_0^2+y_0^2} = L^2 + r^2$

$L = \sqrt{x_0^2+y_0^2-r^2}$

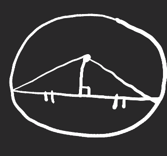
$L^2 = x_0^2+y_0^2-r^2$

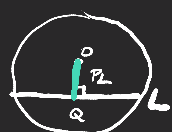
$\sqrt{L^2+r^2} = \sqrt{x_0^2+y_0^2}$

$\boxed{\text{Length of ch} = \frac{2Lr}{\sqrt{L^2+r^2}}}$

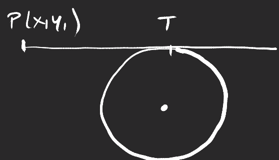


$T \equiv xx_1 + yy_1 - a^2 = 0$





$p_L = \frac{|L(O)|}{\sqrt{a^2+b^2}}$



$PT = \sqrt{C_1}$ length of tangent

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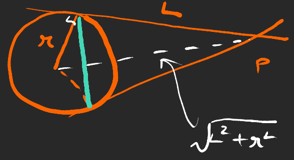
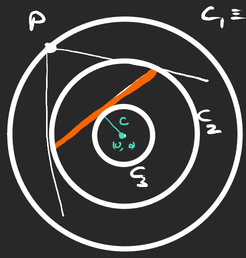


Q21. $C_1 \equiv x^2+y^2=a^2$

chord of contact drawn from C_1 to $C_2 \equiv x^2+y^2=b^2$

& touches $C_3 \equiv x^2+y^2=c^2$

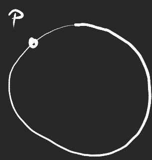
(a,b,c) : establish a relationship b/w parameters.



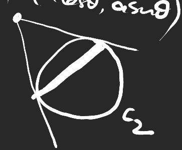
toolbox :

Going to Polar Coord

* P is on $C_1 \Rightarrow P(x,y) = P(a\cos\theta, a\sin\theta)$



* chord of contact to C_2 from P $\Rightarrow ch \equiv ax\cos\theta + ay\sin\theta = b^2$



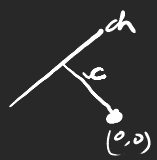
* ch is the line & c is $\perp$ dist. from origin $\Rightarrow c = \dfrac{|a(0)\cos\theta + a(0)\sin\theta - b^2|}{\sqrt{a^2\cos^2\theta + a^2\sin^2\theta}}$

$\Downarrow$

$c = \dfrac{b^2}{a} \Rightarrow \boxed{b^2 = ac}$

$\downarrow$

are in GP.



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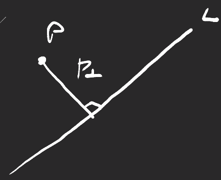
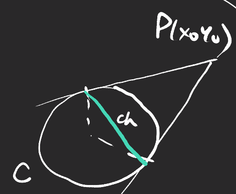


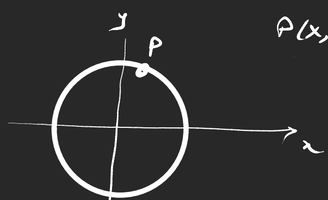
$P_\perp = \dfrac{|L(P)|}{\sqrt{a^2+b^2}}$

•



$ch \equiv xx_0 + yy_0 = a^2$

•



$P(x,y) \equiv P(a\cos\theta, a\sin\theta)$

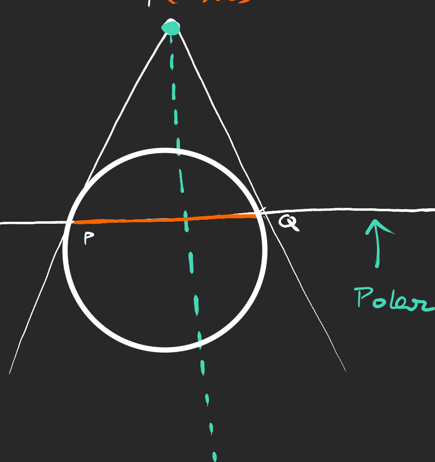
$\Downarrow$

$x^2+y^2=a^2$

E. Pole of a line / Polar line of a pt.

* $C \equiv x^2+y^2=a^2$, $A(x_1,y_1)$

given $A(x_1,y_1)$



* Polar of $\underbrace{pt.}_{A(x_1,y_1)}$ $\equiv$ st. line passes thru pt. of contact of tangents drawn from a given pt.

* Pole of any str. line $\equiv$ pt. of intersection of tangents at the pts. in which this st. line meets the circle.

Eq^n of Polar line = Eq^n of the chord of contact

$ch. \equiv xh + yk = a^2 \xrightarrow[A(x_1,y_1)]{\text{satisfies}} x_1h + y_1k = a^2$

Constraint eq^n on (h,k) { locus of (

$xx_0+yy_0=a^2$

Eq^n of chord

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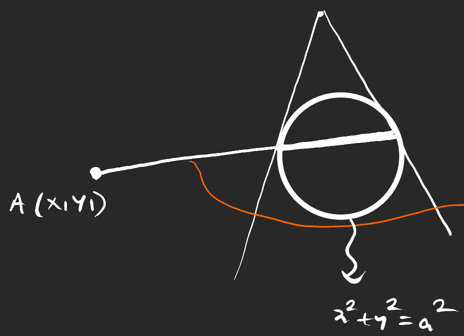
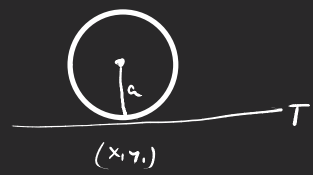
$(h,K) \rightarrow (x,y)$ $\{R(h,K)\}$



$\boxed{xx_1+yy_1=a^2}$ Eqⁿ of the polar.

$\Downarrow$

$T=0$



$T \equiv xx_1+yy_1=a^2$

Lecture 14 (15/Jun) 1.45

E. Pole & Polar (Recap / Clarification)

type given (Pole)

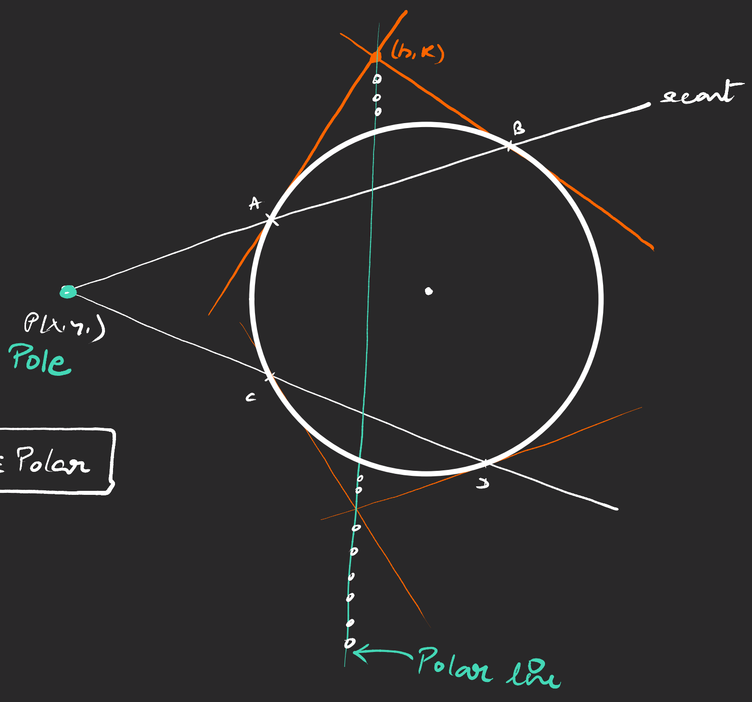
* $C \equiv x^2+y^2=a^2$, $P(x_1,y_1)$

from P secants are drawn

from intersections of secant with C,

tangents are drawn

$\boxed{\text{Locus of point of intersection of tangent line} \equiv \text{Polar}}$



* $(h,K) \longrightarrow$ AB chord of contact

$\Downarrow$

$xh+yK-a^2=0$

$\downarrow$ Passes thru A

$x_1h+y_1K-a^2=0$ Constraint eqⁿ on (h,K) $\longrightarrow$ $\boxed{T \equiv xx_1+yy_1-a^2=0}$

Polar line of Pole (x_1,y_1)

$\uparrow$

fixed

Type 2

- $L \equiv lx+my+n=0$
  $C \equiv x^2+y^2=a^2$ $\Big\}$?? pole of a given line wrt a circle $\Rightarrow$ L is polar

* $L \equiv lx+my+n=0 \longleftrightarrow L \equiv xx_1+yy_1-a^2=0$

$\Downarrow$

$$\frac{x_1}{l} = \frac{y_1}{m} = \frac{-a^2}{n} \Rightarrow \boxed{P(x_1,y_1) = \left(-\frac{a^2 l}{n}, -\frac{a^2 m}{n}\right)}$$

no need to memorize

System of Circles

Created by Notein

F. Direct & transverse common tangents

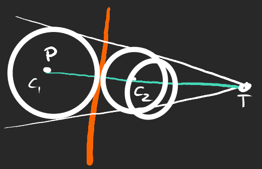
* 2 circles $C_1, C_2 \rightarrow (r_1, r_2)$

$C_1C_2 \equiv$ dist b/w their centres

3 situations

Both touch

Externally

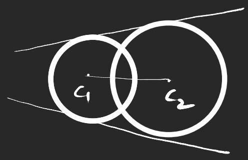


- 3 common tangents
- $C_1C_2 = r_1 + r_2$
- P divides C_1C_2 in $r_1 : r_2$ (internally)

Internally

- 1 common tangent
- $C_1C_2 = |r_1 - r_2|$
- P divides C_1C_2 in $r_1 : r_2$ externally

Circles intersect



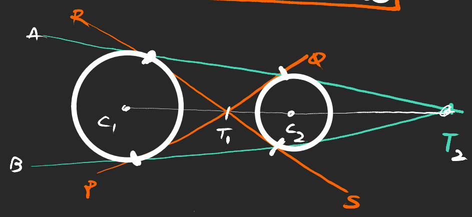
- 2 common tangent
- $|r_1 - r_2| < C_1C_2 < r_1 + r_2$

Circles will not intersect

C lies inside

- No common tan.
- $C_1C_2 < |r_1 - r_2|$

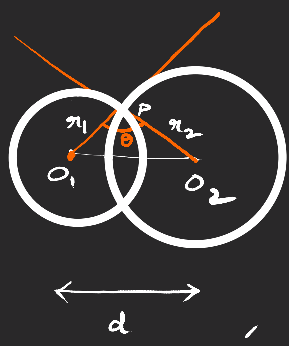
C lies outside



2 sets of tangent

Direct / external	Indirect / transverse
AT_2, BT_2	PQ, RS
$L = \sqrt{(C_1C_2)^2 - (r_1 - r_2)^2}$	$L = \sqrt{(C_1C_2)^2 - (r_1 + r_2)^2}$
T_2 divides C_1C_2 in $r_1 : r_2$ externally	T_1 divides C_1C_2 in $r_1 : r_2$ internally

G. Angle of intersection of 2 circles



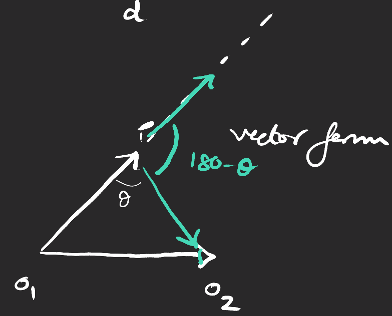
* 2 circles intersecting C_1, C_2

* $C_1 \equiv x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$, $r_1 = \sqrt{g_1^2 + f_1^2 - c_1}$

  $C_2 \equiv x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$, $r_2 = \sqrt{g_2^2 + f_2^2 - c_2}$

* $\theta \equiv$ angle b/w tangents of 2 circles at pt of intersection

consider it as a vector problem



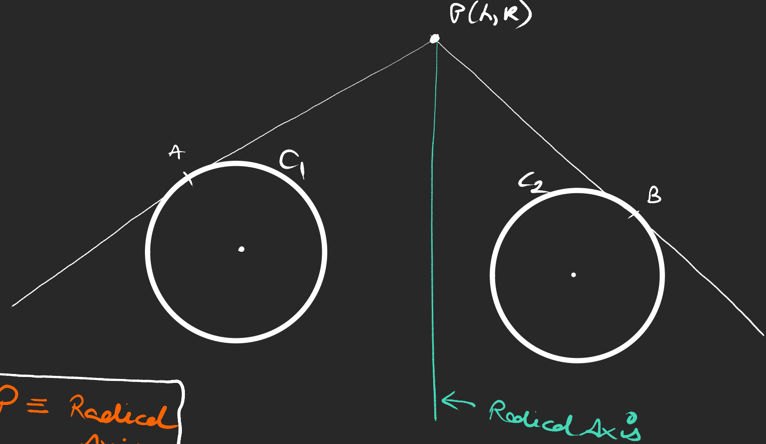
* $\vec{O_1O_2} = \vec{O_1P} + \vec{O_2P}$

$$|O_1O_2|^2 = |PO_1|^2 + |PO_2|^2 + 2\underbrace{\vec{PO_1} \cdot \vec{PO_2}}_{|PO_1||PO_2|\cos(180-\theta)}$$

$$d^2 = r_1^2 + r_2^2 - 2r_1r_2\cos\theta \Rightarrow \boxed{\cos\theta = \left(\frac{r_1^2 + r_2^2 - d^2}{2r_1r_2}\right)}$$

Created by Notein

* $\theta = 90^\circ$ ($\perp$) $\Rightarrow$ $\cos 90 = 0$ $\Rightarrow$ $\boxed{r_1^2 + r_2^2 = d^2}$ $\Rightarrow$ circles are orthogonal



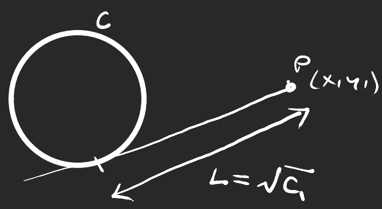
H. Radical Axis of the 2 Circles ($C_1 - C_2 = 0$)

* $C_1 \equiv x^2 + y^2 + 2g_1 x + 2f_1 y + c_1 = 0$

$C_2 \equiv x^2 + y^2 + 2g_2 x + 2f_2 y + c_2 = 0$

* $\boxed{P(h,k) \ : \ |PA| = |PB| \longrightarrow \text{the locus of } P \equiv \text{Radical Axis}}$

Eq^n of Radical Axis



* $C_1(P) \Rightarrow \sqrt{h^2 + k^2 + 2g_1 h + 2f_1 k + c_1} = |PA|$

$C_2(P) \Rightarrow \sqrt{h^2 + k^2 + 2g_2 h + 2f_2 k + c_2} = |PB|$

$\because |PA| = |PB|$

$2(g_1 - g_2)h + 2(f_1 - f_2)k + c_1 - c_2 = 0$ Constraint eq^n (h, k)

$(h, k) \longrightarrow (x, y)$

Shorthand

$\boxed{2x(g_1 - g_2) + 2y(f_1 - f_2) + c_1 - c_2 = 0 \ \equiv \ C_1 - C_2 = 0}$

Eq^n of Radical Axis

- $\boxed{\text{Arihant series}}$ — Algebra, Hall & Knight, SL Loney, CG, Trig.
- RD
- NCERT

Lecture-15 (16 Jun) 2

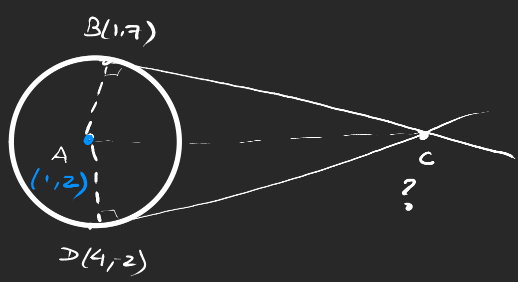
HW

Q1. $C \equiv x^2 + y^2 - 2x - 4y - 20 = 0$

$B(1,7)$, $D(4,-2)$: lie on the circle C

Tangents are drawn from B & D, meet at C

Area (ΔBCD) = ?



Q2. $C_1 \equiv x^2 + y^2 + 2ax + c^2 = 0$
$C_2 \equiv x^2 + y^2 + 2by + c^2 = 0$ } (a, b, c) : C_1 & C_2 touch each other

P.T $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$

Created by Notein

Q3. $C_1 \equiv x^2+y^2-4x-6y-12=0$

$C_2 \equiv x^2+y^2+6x+4y-12=0$

$C_3 \equiv x^2+y^2-2x-4=0$

C_4 : passes thru. the pts of intersection of C_1 & C_2

: cuts C_3 orthogonally

Eq^n of C_4 = ?

Q4. $C_1 \equiv x^2+y^2+3x+2y+1=0$

$C_2 \equiv x^2+y^2-x+6y+5=0$

$C_3 \equiv x^2+y^2+5x-8y+15=0$

a) Radical center ?

b) find eq^n of circle cuttg them orthogonally ?

2. $\boxed{\text{Bridge straight lines - circle - conics}}$

Situation 1

later

"gen. eq^n of conic section"

* $L \equiv ax^2+2hxy+by^2+2gx+2fy+c=0$ general eq^n of second degree

→ $a=b,\ h=0$ circle

→ $\Delta=0$ straight line

* $ax^2+2hxy+2gx=-by^2-2fy-c$ separate x & y

$a^2x^2+2ax(hy+g)=-aby^2-afay-ac$ multiply by a

$(ax+hy+g)^2-(hy+g)^2=-aby^2-2fay-ac$ perfect sq. in x

$(ax+hy+g)^2 = h^2y^2+g^2+2hgy-aby^2-2fay-ac$

$$a^2\left(x^2+\frac{2}{a}(hy+g)x+\left(\frac{hy+g}{a}\right)^2-\left(\frac{hy+g}{a}\right)^2\right)$$

$$a^2\left(\frac{x}{1}+\frac{hy+g}{a}\right)^2-(hy+g)^2$$

$$(ax+hy+g)^2-(hy+g)^2$$

$(ax+hy+g)^2 = y^2(h^2-ab)+2y(gh-af)+g^2-ac$

$ax+hy+g=\pm\sqrt{y^2(h^2-ab)+2y(gh-af)+(g^2-ac)}$

* $ax+hy+g=\pm\sqrt{(y-\alpha)^2}=\pm(y-\alpha)$ to get rid of sq. root in RHS

$ax+hy+g=y-\alpha$ → $ax+(h-1)y+g+\alpha=0$

$ax+hy+g=-y+\alpha$ → $ax+(h+1)y+g-\alpha=0$

$(h^2-ab)y^2+2(gh-af)y+(g^2-ac)$

$$y=\frac{-(gh-af)\pm\sqrt{4(gh-af)^2-4(h^2-ab)(g^2-ac)}}{2(h^2-ab)}$$

eq^n must have 2 same roots

the only way sq. root can go away

$$y=\frac{-(gh-af)}{2(h^2-ab)}=\alpha$$

→ $(y-\alpha)(y-\alpha)=(y-\alpha)^2$

Created by Notein

$D \equiv 4(gh-af)^2 - 4(h^2-ab)(g^2-ac) = 0$

discriminant

* $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents pair of straight lines

iff $(gh-af)^2 = (h^2-ab)(g^2-ac)$

$\cancel{g^2h^2} + a^2f^2 - 2ghaf = \cancel{g^2h^2} - h^2ac - abg^2 + a^2bc$

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$$\boxed{\Delta \equiv \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0}$$

pair of straight lines determinant condn

Situation 2

* $L_1 \equiv a_1x+b_1y+c_1=0$
  $L_2 \equiv a_2x+b_2y+c_2=0$

* $L_1L_2 \equiv (a_1x+b_1y+c_1)(a_2x+b_2y+c_2)=0$ Combined eqn of pair of S.L

$a_1a_2x^2 + a_1b_2xy + a_1c_2x + b_1a_2xy + b_1b_2y^2 + b_1c_2y + a_2c_1x + c_1b_2y + c_1c_2 = 0$

$a_1a_2x^2 + (a_1b_2+b_1a_2)xy + b_1b_2y^2 + (a_1c_2+a_2c_1)x + (b_1c_2+b_2c_1)y + c_1c_2 = 0$

$\boxed{ax^2+2hxy+by^2+2gx+2fy+c=0}$ 2nd degree gen. eqn

→ Combined eqn of pair of straight

can be factorized into 2 factors each having x & y & const (basic ingredients for straight line)

$\boxed{\text{Situation 1 is equivalent to Situation 2}}$

later

"gen. eqn of Conic section"

$L \equiv ax^2+2hxy+by^2+2gx+2fy+c=0$ general eqn of second degree
- → $a=b$, $h=0$ circle
- → $\Delta=0$ straight line

(Angle b/w 2 st. line)

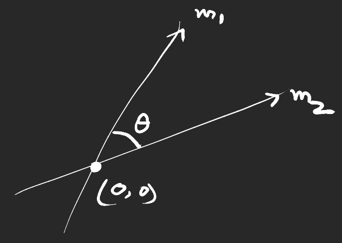
Created by Notein

2.1 Analysis of Homogeneous 2nd degree eqⁿ

for str. $\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = \begin{vmatrix} a & h & g \\ h & b & f \\ 0 & 0 & 0 \end{vmatrix} = 0$ ✓

* $g=0,\ f=0,\ c=0 \Rightarrow \boxed{ax^2+2hxy+by^2=0} \Rightarrow$ rep. 2 str. lines passing thru. origin

$\left\{ \begin{matrix} y = m_1 x \\ y = m_2 x \end{matrix} \right\}$



$\tan\theta = \left| \dfrac{m_1 - m_2}{1 + m_1 m_2} \right|$

factorise / perfect sq. in y

* $b\left(y^2 + \overbrace{\frac{2h}{b}}^{const} xy + \overbrace{\frac{a}{b}}^{const} x^2 \right) = 0$

$y^2 + \frac{2h}{b}xy + \left(\frac{h}{b}x\right)^2 - \left(\frac{h}{b}x\right)^2 + \frac{a}{b}x^2 = 0$

$\left(y + \frac{h}{b}x\right)^2 = \underbrace{\left(\frac{h^2}{b^2} - \frac{a}{b}\right)}_{\frac{h^2-ab}{b^2}} x^2 \Rightarrow y + \frac{h}{b}x = \pm \frac{\sqrt{h^2-ab}}{b} x \Rightarrow \boxed{y = \underbrace{\left(\frac{-h \pm \sqrt{h^2-ab}}{b}\right)}_{m_{1,2}} x}$

st. line thru. O

$\hookrightarrow y = m_1 x$

$\hookrightarrow y = m_2 x$

lecture-16 (17/Jun) 1.5

22 Jun Test st. line

* $y = \left(\dfrac{-h \pm \sqrt{h^2-ab}}{b} \right) x = m_{1,2} x$

Trick

II

$m_{1,2} = \dfrac{-h \pm \sqrt{h^2-ab}}{b}$

read off the value using I & II

$\boxed{B = h}$

$ab = 4AC \Rightarrow a\cancel{2A} = \overset{2}{\cancel{4}}AC \Rightarrow \boxed{C = \frac{a}{2}}$

$b = 2A \Rightarrow \boxed{A = \frac{b}{2}}$

Consider,

I

$Am^2 + Bm + C = 0 \Rightarrow m_{1,2} = \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$\Downarrow$

$m = m_{1,2} \Rightarrow m_1 + m_2 = -\dfrac{B}{A}$

$m_1 m_2 = \dfrac{C}{A}$

$m_1 + m_2 = -\dfrac{h}{b/2} \Rightarrow$

$m_1 m_2 = \dfrac{a}{b}$

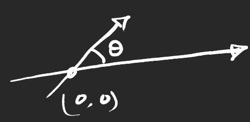
$$\boxed{\begin{matrix} m_1 + m_2 = -\dfrac{2h}{b} \\ m_1 m_2 = \dfrac{a}{b} \end{matrix}}$$

$\tan\theta = \left| \dfrac{m_1 - m_2}{1 + m_1 m_2} \right| = \pm \dfrac{\sqrt{(m_1+m_2)^2 - 4m_1m_2}}{1 + m_1 m_2}$

$\because (m_1 - m_2)^2 = m_1^2 + m_2^2 - 2m_1m_2 + 2m_1m_2 - 2m_1m_2$

$= (m_1+m_2)^2 - 4m_1m_2$

$$\boxed{\tan\theta = \pm \frac{2\sqrt{h^2-ab}}{a+b}}$$



* $\perp \Rightarrow \theta = 90° \Rightarrow \boxed{a+b=0}$

* $\parallel \Rightarrow \theta = 0° \Rightarrow \boxed{h^2 = ab} \longrightarrow$ Coincident $\because$ both pass thru O

Created by Notein

Comment

for $L \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

$\Downarrow$

2 st. line

$\rightarrow \perp$ $\boxed{a+b=0} \rightarrow$ coeff of x^2 + coeff of $y^2 = 0$ $\boxed{CTM}$

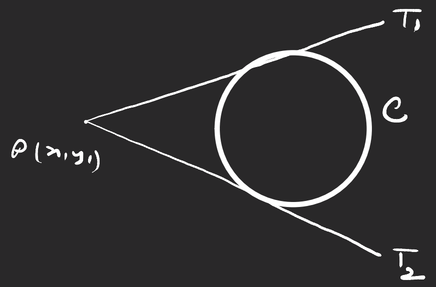
$\rightarrow \parallel \rightarrow L$ is converted to $ax^2+2hxy+by^2=0 \rightarrow (lx \pm my)^2 = 0$

$\rightarrow$ Coincident $\rightarrow L$ is converted to $(lx+my+n)^2=0$

Remark

1. On Eq^n of pair of tangent

* C, $P(x_1, y_1)$



$\boxed{CC_1 = T^2}$ Eq^n of pair of tangents.

$\Downarrow$

$(x^2+y^2-a^2)(x_1^2+y_1^2-a^2) = (xx_1+yy_1-a^2)^2$ ✓

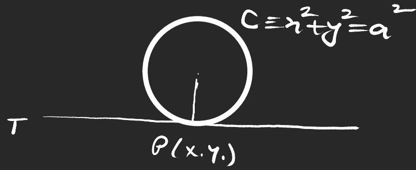
$\updownarrow$

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

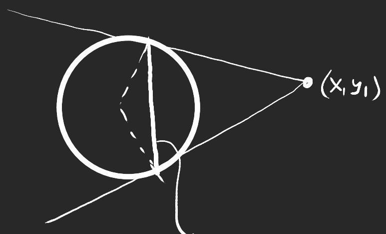
{gen. eq^n of 2^nd degree / Conic section / rep. 2 st. lines}

$\Delta = 0$

• reminder



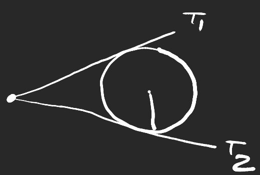
$T \equiv xx_1 + yy_1 - a^2 = 0$



$ch \equiv xx_1 + yy_1 - a^2 = 0$

ex $C = x^2+y^2+20x+20y+20=0$

$P(0,0) \equiv$ ⊥

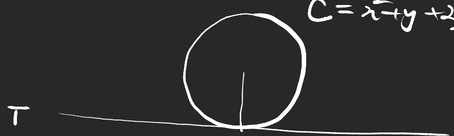


a) Eq^n of pair of tangents ?

$\downarrow$

* $\boxed{CC_1 = T^2} \longrightarrow 2x^2 + 5xy + 2y^2 = 0$

$C = x^2+y^2+2gx+2fy+c=0$



$T \equiv xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c = 0$

b) eq^n of tangents T_1, T_2 = ?

Eq^ns pass. thru origin $\begin{cases} y = m_1 x \\ y = m_2 x \end{cases}$

$2y^2 + 5xy + 2x^2 = 0 \Rightarrow y^2 + \frac{5}{2}xy + \left(\frac{5}{4}x\right)^2 - \left(\frac{5}{4}x\right)^2 + x^2 = 0$

$\left(y + \frac{5}{4}x\right)^2 = \left(\frac{25}{16} - 1\right)x^2 = \frac{9}{16}x^2$

$y + \frac{5}{4}x = \pm \frac{3}{4}x$ $\rightarrow y + 2x = 0 \equiv T_1$; $\rightarrow 2y + x = 0 \equiv T_2$

$ax^2+2hxy+by^2+2gx+2fy+c=0$

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = \begin{vmatrix} 2 & 5/2 & 0 \\ 5/2 & 2 & 0 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

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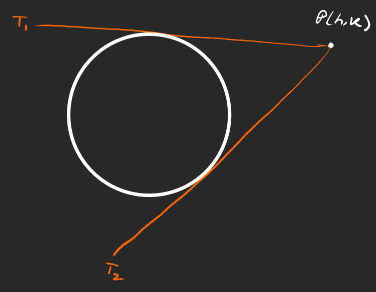
HW

1. $12x^2+7xy-10y^2+13x+45y-35=0$ —— check if it reps. 2 st. line
if yes, $\theta(L_1, L_2)=?$

2. $6x^2+2\lambda xy+12y^2+22x+31y+20=0 \longrightarrow \lambda=?$: eqn rep. 2 st. lines

Lecture-17 (18 Jun) 1.5

② Director Circle



* $C \equiv x^2+y^2-a^2=0$, $P(h,k)$

* Eqn of pair of tangents $\Rightarrow CC_1 = T^2$

$(x^2+y^2-a^2)(h^2+k^2-a^2) = (xh+yk-a^2)^2$ Combined/gen. 2nd degree/ rep. 2nd st. line

* $\boxed{\text{if } T_1 \perp T_2 \Rightarrow \text{locus of pt of intersection} = \text{Circle} = \text{Director Circle}}$

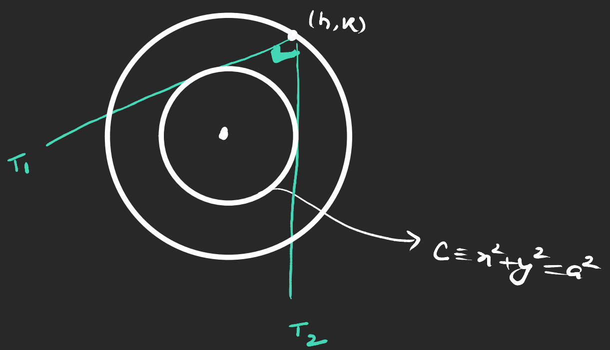
$\because \theta=90^\circ \Rightarrow$ "$a+b=0$" $\qquad ax^2+2hxy+by^2+2fy+2gx+c=0$

* $x^2(\cancel{h^2}+k^2-a^2)+y^2(h^2+\cancel{k^2}-a^2) \ldots = \cancel{x^2h^2}+\cancel{y^2}k^2 \ldots\ldots$ expand only for x^2 & y^2 terms

$x^2k^2+y^2h^2-a^2x^2-a^2y^2 \ldots\ldots = 0$

* "$a+b=0$" $\Rightarrow (k^2-a^2)+(h^2-a^2)=0 \Rightarrow h^2+k^2=(\sqrt{2}a)^2$ Constraint eqn

* locus of $(h,k) \Rightarrow \boxed{x^2+y^2=(\sqrt{2}a)^2}$



* Director circle = concentric circle whose radius is $\sqrt{2} \times$ rad. of circle

Comment

* $C \equiv x^2+y^2+2gx+2fy+c=0 \longrightarrow C_D \equiv x^2+y^2+2gx+2fy+c'=0$

$r=\sqrt{g^2+f^2-c}$

$r^2=g^2+f^2-c \xrightarrow{r \to \sqrt{2}r} (2r^2)=g^2+f^2-c' \Rightarrow 2(g^2+f^2-c)=g^2+f^2-c'$

$\Downarrow$

$\boxed{c'=2c-g^2-f^2}$

$$\boxed{C_D \equiv x^2+y^2+2gx+2fy+2c-g^2-f^2=0}$$

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HW $C_1 \equiv x^2+y^2-2x=1$, $C_2 \equiv x^2+y^2-2x=0$

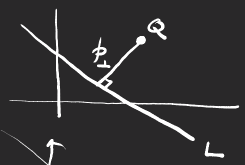
P moves on C_1, from P chord of contact is drawn (pt A, B) wrt C_2

find the locus of the circumcentre of the triangle CAB, C = centre of circle

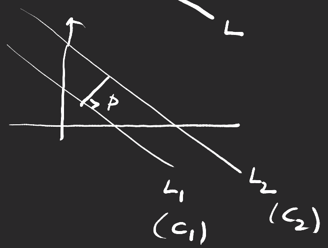
3. General Eqⁿ of Conic Section

* $L \equiv ax+by+c=0$

* $p_\perp = \dfrac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$



* $p_\perp = \dfrac{|c_1-c_2|}{\sqrt{a^2+b^2}}$



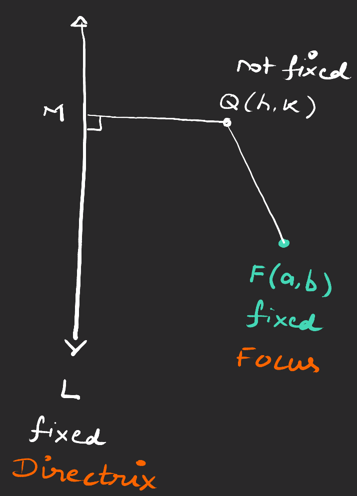
* $C \equiv x^2+y^2+2gx+2fy+c=0$

$r=\sqrt{g^2+f^2-c}$

Setup

* $L \equiv lx+my+n=0$ Directrix (fixed)

  $F(a,b)$ focus (fixed)



* $\because$ Q moves $\Rightarrow$ creation of shapes (locus of Q)

  $\downarrow$ to control we need a parameter } knob

  $\boxed{e = \text{eccentricity}}$ $e \in \mathbb{R}$

* Control for shape $\Rightarrow$ $\boxed{|QF| = e\,|MQ|}$ Constraint eqⁿ

Calculation

* $|QF|^2 = e^2|MQ|^2$

$$(h-a)^2+(k-b)^2 = e^2\left(\frac{|lh+mk+n|}{\sqrt{l^2+m^2}}\right)^2 = e^2\frac{|lh+mk+n|^2}{(l^2+m^2)}$$

for locus of (h,K)
$(h,K) \longrightarrow (x,y)$

$$(x-a)^2+(y-b)^2 = e^2\frac{|lx+my+n|^2}{(l^2+m^2)}$$

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* $(l^2+m^2)\{x^2+a^2-2ax+y^2+b^2-2by\} = e^2(l^2x^2+m^2y^2+n^2+2lmxy+2mny+2lnx)$

* $\overbrace{(l^2+m^2-e^2l^2)}^{A}x^2+\overbrace{(l^2+m^2-e^2m^2)}^{B}y^2+2\overbrace{(-l^2a-m^2a-e^2ln)}^{g}x+2\overbrace{(-l^2b-m^2b-e^2mn)}^{f}y+2\overbrace{(-e^2lm)}^{h}xy$

$$+\overbrace{(l^2a^2+l^2b^2+m^2a^2+m^2b^2-e^2n^2)}^{c}=0$$

* $\boxed{Ax^2+By^2+2gx+2fy+2hxy+c=0}$ { General eq^n of 2^{nd} degree
 " " " Conic sections

Cases

1. $A=B=1$; $h=0 \Rightarrow x^2+y^2+2gx+2fy+c=0 \Rightarrow$ Circle

$h=-e^2lm=0 \Rightarrow \boxed{e=0}$ Circle

$\because l,m\neq 0$

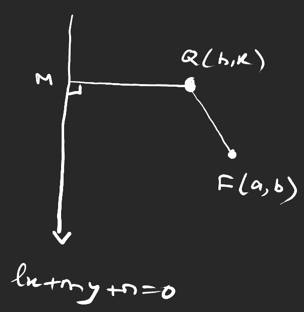
$h=0$, $A=B=1$	$e=0$	Circle

Lecture-18 (19/3m) 1.5

* $|QF|=e|QM|$ constraint eq^n

$\Downarrow$

$Ax^2+By^2+2gx+2fy+2hxy+c=0$



2. $\boxed{h^2=AB}$

$h=-e^2lm$

$A=l^2+m^2-e^2l^2=(1-e^2)l^2+m^2$

$B=l^2+m^2-e^2m^2=(1-e^2)m^2+l^2$

$\Downarrow$

$AB=\underbrace{(1-e^2)^2l^2m^2+m^2l^2}_{[(1-e^2)^2+1]l^2m^2}+(l^4+m^4)(1-e^2)=h^2=e^4l^2m^2$

* $[(1-e^2)^2+1]l^2m^2+(1-e^2)(l^4+m^4)=e^4l^2m^2+0(l^4+m^4)$

$1-e^2=0$ $\quad\because$ compare coeff. of l^4+m^4

$\Downarrow$

$e=\pm1 \longrightarrow \boxed{e=1}$

$\because |QF|=e|QM|$

* $|QF|=|QM|$

$h^2=AB \Rightarrow e=1$

quick calc. for eq^n of locus:

* $|QF|^2=|QM|^2$

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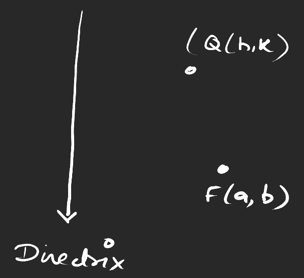
$$(h-a)^2+(k-b)^2 = \frac{|lh+mk+n|^2}{l^2+m^2} \longrightarrow (x-a)^2+(y-b)^2 = \frac{|lx+my+n|^2}{l^2+m^2}$$

$$(l^2+m^2)(x^2+a^2-2ax+y^2+b^2-2by) = \cancel{l^2x^2}+\cancel{m^2y^2}+n^2+2lmxy+2mny+2lnx$$

* $\boxed{Ax^2+By^2+2hxy+2gx+2fy+c=0}$ havoc!

Remark : At the level of geometry this algebraic eq^n did us no good.
It's no different than the gen. conic eq^n



Reason : the choice of Directrix $L \equiv lx+my+n=0$
the std. placements are not there.
(displaced conic section)
Problem with all conic section

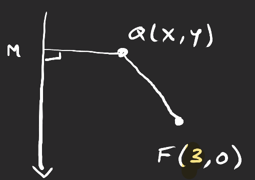
Solution : Analysis w/ easier directrix

Type I : $y^2 = 4ax$

ideas / concepts

$e=1$

*

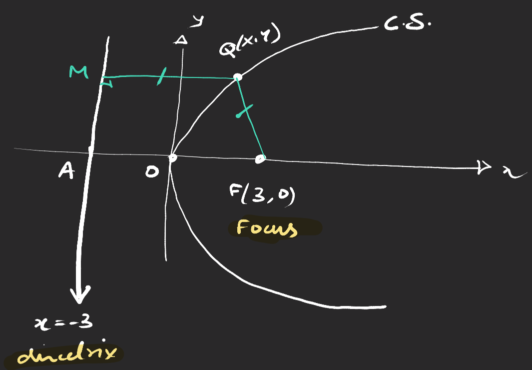


$L \equiv x+3=0$

$|QF| = |QM| \Rightarrow (x-3)^2+y^2 = \frac{|x+0y+3|^2}{1^2+0^2}$

$\cancel{x^2}+9-6x+y^2 = \cancel{x^2}+9+6x \Rightarrow \boxed{y^2=12x}$

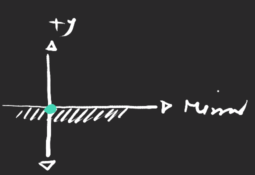
$C : \boxed{y^2=4(3)x}$ Easy to trace



- vertex $\equiv$ pt of intersection of CS & axes $= (0,0)$
- Axis $\equiv$ line passing thru. focus & $\perp$ to directrix $(y=0)$
- Double ordinate $\equiv$ chord $\perp$ to axis
- Latus Rectum $\equiv$ double ordinate passing thru focus
- focal length $\equiv$ dist. of any pt. on curve from the focus
- focal chord $\equiv$ chr. passes thru. focus

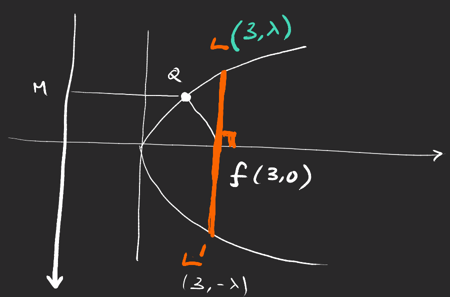
Curve tracing

- $x=0,\ y=0$ passes thru O



- symmetry in y $\left\{\begin{matrix} + \\ - \end{matrix}\right\}$ same x-y
- (More details done already)

*



$FL = FL' = \lambda$
$LL' = 2\lambda$
L satisfies C
$\Downarrow$
$\lambda^2 = 12(3) \Rightarrow \boxed{\lambda = \pm 6}$
$LL' = 12 \longrightarrow$ Latus-Rectum