Lecture-1 (27/12/2021)

statements in math saying to itself — Gödel — 'incompleteness theorems'

$\uparrow$

$2+2=4$
$--p--q----$
$\exists x \; \forall y$

Formal system
Meaningless / symbol

$\cong$
Isomorphism
"Essentially the same"

'I' — self-reference

$\uparrow$

Particles / molecules / proteins / atom
C, O, N, H

Primitive / meaningless (non-self)

* Non-Self $\xrightarrow[\text{HOW}]{?}$ Self

* Tools for thinking
→ Isomorphism
→ Recursion
→ Paradox
→ Infinity
→ Formal system

Isomorphism

Isomorphism b/w skateboard & car:
1. Same wheels
2. Carry person

* $\exists$ Map (transformation) : structure is preserved. (Invariant)

Recursion

* Recursion = Protocols / Repeat till "Final Case"

* Fibonacci series : $a_1, a_2, a_3, \ldots$: $1, 1, 2, 3, 5, 8, 13 \ldots$

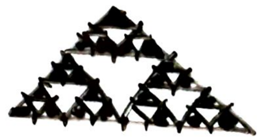
$\Updownarrow$

A.P: $\boxed{a_n = a_{n-1} + a_{n-2}}$; n ≥

$n=3$: $a_3 = a_2 + a_1 \Leftrightarrow a_2 = 1+1$

→ defined in terms of itself

* Sierpinski's Gasket
(Fractal)
||
Fraction
||
Broken = Fragments



* $\mathbb{Q} = \{ \frac{p}{q} : q \neq 0, p, q \in \mathbb{Z} \}$

Broken

Whole 1, 5, 9, 7
(Assumption)

Paradox

* Quine's classification
→ Veridical — appear absurd (but) T ; $\underbrace{\text{verdad}}$ = True
→ Falsidical (Fallacy) $1=0$
→ $\boxed{\text{Antinomy}}$

* Zeno's Paradox :

$\{ \ldots \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1$

* division by 0 : $a = b \Rightarrow a^2 = ab \Rightarrow \underbrace{a^2 - b^2}_{\frac{(a+b)\cancel{(a-b)}}{(a-b)}} = \underbrace{ab - b^2}_{\frac{b\cancel{(a-b)}}{(a-b)}}$

Fallacy

$\underbrace{a}_{b} + b = b \Rightarrow 2\cancel{b} = \cancel{b} \Rightarrow 2 = 1$

'Undefined'

Lecture-2 (28/12/2021)

* $1 = \sqrt{1} = \underbrace{\sqrt{(-1)(-1)}} = \sqrt{-1}\cdot\sqrt{-1} = i\cdot i = -1$

$\sqrt{xy} = \sqrt{x}\,\sqrt{y}$ iff atleast one of x, y is +ve

* $i = \sqrt{-1} = (-1)^{1/2} = \underbrace{(-1)^{2/4} = \left((-1)^2\right)^{1/4}} = (1)^{1/4} = 1$

$a^{bc} = (a^b)^c \quad \forall\, a, b, c \in \mathbb{R}^+$

* Antinomy of liar : 'This statement is not True'

T $\Downarrow$ F

F $\Downarrow$ T

[H.W.] 5-6 lines opinion/thoughts/Creative/Novel

Gödel's take

$\boxed{\text{I am not provable}}$

* Barber's paradox : (1 Barber) Barber's rule - he shaves all those who don't shave themselves.

who shaves Barber

[HW]

Russell's/Zermelo : Paradox in set theory

Infinity

* $\exists$ diff. degrees of Infinity 'Cantor diagonal Argument'

Formal System

(M, I, U)

I. The MU puzzle

* Formal sys : M-I-U sys.
* Rules : 1. ....I $\longrightarrow$ ....IU

ex: MI $\longrightarrow$ MIU

2. ....X $\longrightarrow$ XX

X= ex: MI $\longrightarrow$ MII, MIU $\longrightarrow$ MIUIU

MU $\longrightarrow$ MUU

MUM $\longrightarrow$ MUMUM

MUM $\longrightarrow$ MUMM

3. ....III.... $\longrightarrow$ ...U....

ex: UMIIIMU $\longrightarrow$ UMUMU

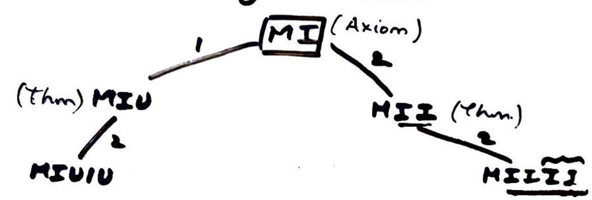
MIIII $\longrightarrow$ MIU

MIIII $\longrightarrow$ MUI

4. ...UU.... $\longrightarrow$ ......

drop double U

* Game/Puzzle : Try to make $\boxed{MU}$ starting from $\boxed{MI}$ } challenge!! HW.



Std. Def.

* String = ordered sequence
* Axiom = Starting pt / Primitive / 'seems' self-evident
  Free / Pre-given
* Theorem = String which Results at the End of Derivation / produced.
* Rules of Inference = Rules of produced
  = symbol-shunting rules
* Derivation = line-by-line demonstration

II. PQ-system

* Formal system : p, q, - (hyphen) p q -
* Axiom : $xp-qx$ $\quad x = \{-----\ldots\}$ String of hyphens

  A: --p-q----

* Rule : $xpyqz \longrightarrow xpy-qz-$ $\quad x, y, z = \{-----\ldots\}$

  ex: $\underbrace{--}_{x}p\underbrace{---}_{y}q\underbrace{-}_{z} \longrightarrow --p----q--$

  $--p-q--- \longrightarrow --p--q----$ [HW]

Isomorphism ??

Lectures (29/12/2021)

* "This sentence is not true"
  is → (To be)
  (ask linguistic questions ?)

* Formal sys : p, q, -
  Axiom : xp-qx , x = { - - . . }
  Rule : xpyqz → xpy-qz-

* $\underbrace{--}_{2}\underbrace{p}_{+}\underbrace{-}_{1}\underbrace{q}_{=}\underbrace{---}_{3}$ → $\underbrace{--}_{2}\underbrace{p}_{+}\underbrace{--}_{2}\underbrace{q}_{=}\underbrace{----}_{4}$

* --p--q---- ⇒ $\boxed{\text{pq-theorem} \cong \text{Addition}}$

* p ⇔ plus
  q ⇔ equal
  - ⇔ 1
  -- ⇔ 2
  } Interpretation
  Addition as process

* Interpretation = symbol-word correspondence.
  (Not unique)

* Interpretation (Choice)

  Meaningless
  Theorems ≇ Reality
  p ⇔ Horse
  q ⇔ happy
  - ⇔ Apple
  --p--q---

  Meaningful
  Theorem ≅ Reality (patchwork of)
  p ⇔ +
  q ⇔ =
  - ⇔ 1
  Meaningful → Well-formed seq
  ↓
  Grammar

Meaning

$\boxed{\text{Active}}$
language
learn new words (meaning)
↓
Create new statements

$\boxed{\text{Passive}}$
Formal system
p ⇔ +
q ⇔ =
- ⇔ 1
} "meaningful" Interpretation
↓
2+2+2 = 6
--p--p--q------ (Not well-formed)
(Just portion of reality)
No Creation of New statement
∃ requirement of Formality

* Difference (Ref MU puzzle)

Human
Complaint/Boredom
Will Notice Facts
Can't act unobservant (incapable)
$\boxed{\text{Ability to EXIT the system}}$
↓
Meta-thinking
Intelligent Mode

Machine
No complaints
Not stop
No insight
follow facts
No ability to EXIT the system
(Within the system)
Machine mode

* $\boxed{\text{Formal System}}$
∃ symbol
Rules
Axioms

$\boxed{\text{Reality}}$
∃ Elementary particle
laws of physics
Axioms: initial configuration of particle

Lecture 4 (30/12/2021)

Algebra = to bind
Fractal = to break
To fraction

Formal Sys : $x \in$ variable, $a \in \mathbb{R}$, $=$
$\{+, -, /, \times\}$, $\{(), [\,], \{\,\}\}$
(ex) $x \in$ var, Number system

Rules : 1. Algebraic Expression = various combination acc. to rules

2. Term = parts (separated by $+, -$)

Types of Expression

Monomial
3, $2x$, $\sqrt{5}x^3z^4$
constant term

Binomial
$x+y$, $\sqrt{2}x^4 + z^{1/3}$

Trinomial
$x+y+z$

Polynomial

3. Factor = parts made by product $(\times)$

(ex) $7x \to 7, x$ } Factors

(ex) $3xy + 7z \to 3xy \to 3, x, y$ } factors
$\to 7z \to 7, z$ } factor

(ex) $-xy + 3 \to -xy \to -1, x, y$ } Factor
$\to 3 \to 3$

(ex) $-ac + b^2 + 7 \to -ac \to -1, a, c$
$\to b^2 \to b, b$
$\to 7 \to 7$ } Factor

4. Coefficient $\neq$ constant Part | $x^0 = 1$, $y^0 = 1$

choice based on simplification

D(Coeff of a Factor) = all parts except the factor (remain)

(ex) $5x = x \cdot 5$
$\to$ coeff of $x = 5$
$\to$ $\boxed{\text{Coeff of } 5 = x}$

$x\sqrt{2}$ ✓

$\boxed{\sqrt{2}x}$

Bad Notation

(ex) $5xy$
$\to$ coeff of $y = 5x$
coeff of $y^0 = 5x$
$\to$ coeff of $5y = x$
$\to$ coeff of $5xy = 1$

(ex) $-8xy - 7 \to$ coeff of $8 = -xy$

* $8x^4y - 7x^3yz + \frac{4}{3}x^2yz^2 - \pi xyz$

Coeff of $\frac{2}{3}$ in term $\frac{4}{3}x^2yz^2$ $= \frac{2}{3}x^2yz^2$

5. Like term = term : variable parts remain same

(ex) $\underline{5x^2y} + 7xy^2 - 3xy - \underline{4x^2y}$
L L

6. $\underbrace{\text{Evaluation}}_{\text{assignment of value}}$ of Algebraic Expression

{ Mathematical Equality (=)

* $x = 6 + \sqrt{2}$, $y = \sqrt{2}$, $z = 2 + \pi$
$8x^4y - 7x^3yz + \frac{4}{3}x^2yz^2 - \pi xyz \overset{HW}{=} ?$

Lecture 5 (31/12/2021)

* Formal sys: $x \in$ var, Number theory $\boxed{a \in \mathbb{R}}$
* Rules:

1. Algebraic Expression = comb^n accord to rules
2. Terms $(+, -)$ { Monomial, Binomials
3. Factor $(\times)$
4. Coefficients of particular factor
5. Like terms = terms : variable parts remain same
6. Evaluation of Algebraic Exp. $(=)$
   a value is required (pre-given)
7. Operations on Alg. Exp. { Addition, Subtraction, Multiplication
8. Identities $(=)$
9. Factorization
10. Division of Alg Exp.
11. Algebraic Exp + "=" $\longrightarrow$ $\boxed{\text{Algebraic Eq}^n}$

Addition / Subtraction

* "Add" Coeff. of like terms

$x + y = x - (-y)$

Practice (warm-up) Grade 7

* $2x - \{5y - (x - 2y)\} = 3x - 7y$
* $-m - [m + \{m + n - 2m - (m - 2n)\} - n] = -2n$

$\underbrace{m + n - 2m - (m - 2n)}_{-2m + 3n}$

* $3x^2z - 4yz + 3xy - \{x^2z - (x^2z - 3yz) - 4yz - 7z\}$

$= 3x^2z - 3yz + 3xy + 7z$

* $15x - [8x^3 + 3x^2 - \{8x^2 - (4 - 2x - x^3) - 5x^3\} - 2x]$

$= -12x^3 + 5x^2 + 19x - 4$

confidence booster

* $-3(a+b) + 4(2a - 3b) - (2a - b) = 3a - 14b$
* $-5(a+b) + 2(2a - b) + 4a - 7 = 3a - 7b - 7$
* $5 + [x - \{2y - (6x + y - 4) + 2x^2\} - (x^2 - 2y)]$

$= -3x^2 + 7x + y + 1$

* $4(a^2 + b^2 + 2ab) - [4(a^2 + b^2 - 2ab) - \{-b^3 + 4(a - 3)\}]$

$4(\cancel{a^2} + \cancel{b^2} + 2ab) - 4(\cancel{a^2} + \cancel{b^2} - 2ab) + \{-b^3 + 4a - 12\}$

$8ab + 8ab - b^3 + 4a - 12 = 16ab - b^3 + 4a - 12$

(Level 1-2)

Lecture 6 (1|1|2022)

Addition / Subtraction of Alg. Exp (Grade 8)

* $\left(5x^2-\frac{1}{3}x+\frac{5}{2}\right)+\left(-\frac{1}{2}x^2+\frac{1}{2}x-\frac{1}{3}\right)+\left(-2x^2+\frac{1}{5}x-\frac{1}{6}\right)$

  $=\frac{5}{2}x^2+\frac{11}{30}x+2$

* $2+\left(\frac{2y}{3}-\frac{5y^2}{3}+\frac{5y^3}{2}\right)+\left(-\frac{4}{3}+\frac{2y^2}{3}-\frac{y}{2}\right)+\left(\frac{5y^3}{3}+3y^2+3y+\frac{6}{5}\right)$

  $=\frac{25}{6}y^3+2y^2+\frac{19}{6}y+\frac{28}{15}$

* $\left(\frac{12}{5}x^2yz-\frac{3}{5}xyz+\frac{2}{3}x^2y\right)-\left(\frac{3}{2}x^2y+\frac{4}{5}y-\frac{1}{3}x^2yz\right)$

  $=\frac{41}{15}x^2yz-\frac{5}{6}x^2y-\frac{3}{5}xyz-\frac{4}{5}y$

* $\left(\frac{7}{2}-\frac{x}{3}-\frac{x^2}{5}\right)-\left(\frac{9}{2}+\frac{x}{2}+\frac{3}{5}x^2+\frac{7}{4}x^3\right)$

  $=-\frac{7}{4}x^3-\frac{4}{5}x^2-\frac{5}{6}x-1$

Multiplication of Alg. Exp $\begin{cases} M\times M \\ M\times B \\ B\times B \end{cases}$

* Number theory ⇒ Rules of Exponents ($a^m a^n = a^{m+n}$)

  ⇒ $a\cdot(b+c)=a\cdot b+a\cdot c$

  $(a+b)(c+d)=ac+ad+bc+bd$

Type 1

* $\left(\frac{-8}{5}x^2yz^3\right)\cdot\left(\frac{-3}{4}xy^2z\right)=\frac{6}{5}x^3y^3z^4$
* $\left(\frac{3}{14}x^2y\right)\left(\frac{7}{2}x^4y\right)=\frac{3}{4}x^6y^2$
* $(-x)(x^2yz)\left(-\frac{3}{7}xyz^2\right)=\frac{3}{7}x^4y^2z^3$
* $(4s^2t)(3s^3t^3)(2st^4)(-2)=-48s^6t^8$

  $(20x^{10}y^{20}z^{30})(10xyz)^2=2000x^{12}y^{22}z^{32}$

* $(-3x^2y)(4xy^2z)(-xy^2z^2)\left(\frac{4}{5}z\right)=\frac{48}{5}x^4y^5z^4$

Type 2

* $2x\underbrace{(3x+5y)=(3x+5y)}2x=6x^2+10xy$

  $ab=ba$ (Commutative)
  Abelian

* $\left(\frac{-3a^2b}{5}\right)\left(\frac{2a}{3}-b\right)=-\frac{2a^3b}{5}+\frac{3a^2b^2}{5}$
* $\frac{7}{2}s^2t(s+t)=\frac{7}{2}s^3t+\frac{7}{2}s^2t^2$
* $\left(3x-\frac{4}{5}y^2x\right)\frac{1}{2}xy=\frac{3}{2}x^2y-\frac{2}{5}x^2y^3$
* $3y(2y-7)-3(y-4)-63=6y^2-24y-51$

  $\xrightarrow{y=-2}$ $=21$

* $3a(a+b+c)-2b(a+b+c)-4c(-a+b+c)$

  $3a^2-2b^2-4c^2+7ac+ab-6bc$

* $x^2(1-3y^2)+x(xy^2-2x)-3y(y-4x^2y)$

  $=-x^2+10x^2y^2-3y^2$

* $4st(s-t)-6s^2(t-t^2)-3t^2(2s^2-s)+2st(s-t)$

  $=-3st^2$

Lecture 7 (2/1/2022)

Multiplication of Alg. Exp. (B×B)

* $(2x+3y)(4x-5y) = 8x^2+2xy-15y^2$
* $\{2m+(-n)\}\{-3m+(-5)\} = -6m^2-10m+3mn+5n$
* $\left(y+\frac{2}{7}y^2\right)(7y-y^2) = 7y^2+y^3-\frac{2}{7}y^4$
* $\frac{1}{3}(6x^2+15y^2)(6x^2-15y^2) = 12x^4-75y^4$
* $(2x+3y)(3x+4y)-(7x+3y)(x+2y) = 6y^2-x^2$
* $(2x^2-4x+5)(x^2+3x-7) = 2x^4+2x^3-21x^2+43x-35$

8. Identities (Part 1)

* $3x+2=11$ True only for $x=3$ (Not an Id.)
* $x+2x=3x$ True $\forall x \in \mathbb{R}$ (Trivial Identity)
*

$\neq$ } ≡ LEAF Category/concept

↑

Forget differences

⇓

Equality (=)

Std.

* $(x\pm y)^2 = (x\pm y)(x\pm y) = x^2+y^2\pm 2xy$ I, II
* $(x+y)(x-y) = x^2-y^2$ III
* $(x+a)(x+b) = x^2+\underset{\text{Sum}}{(a+b)}x+\underset{\text{Pdt.}}{ab}$ (Imp)

* $\left(y+\frac{y^2}{2}\right)^2 = y^2+y^3+\frac{y^4}{4}$
* $(a^2+b^2)(-a^2+b^2) = b^4-a^4$
* $(-a+c)\underbrace{(-a-c)}_{-(a+c)} = -(-a+c)(a+c) = (a-c)(a+c) = a^2-c^2$
* if $x+\frac{1}{x}=4$, $x^2+\frac{1}{x^2}=?$, $x^4+\frac{1}{x^4}=?$

$\downarrow$

$\left(x+\frac{1}{x}\right)^2 = 4^2 \Rightarrow x^2+\frac{1}{x^2}+2 = 16 \Rightarrow \left(x^2+\frac{1}{x^2}\right)^2 = (14)^2$

$\downarrow$

$x^4+\frac{1}{x^4} = 194$

Worst Alt.

* $\underbrace{\frac{x}{1}+\frac{1}{x}}_{\frac{x^2+1}{x}} = 4 \Rightarrow x^2+1 = 4x \Rightarrow \boxed{x^2-4x+1=0}$

* if $x+\frac{1}{x}=9$, $x^2+\frac{1}{x^2}=53$, $x-\frac{1}{x}=?$
* $\underbrace{\left(x+\frac{1}{x}\right)^2}_{(9)^2}+\underbrace{\left(x-\frac{1}{x}\right)^2}_{?} = 2x^2+2\frac{1}{x^2} = 2\underbrace{\left(x^2+\frac{1}{x^2}\right)}_{53} = 106$

$\left(x-\frac{1}{x}\right)^2 = 25 \Rightarrow \boxed{x-\frac{1}{x} = \pm 5}$

* If $x^2+\frac{1}{x^2}=27$, $x+\frac{1}{x}=?$, $x-\frac{1}{x}=?$ HW

$(x+y)^n = ?$ 'Binomial Theorem' (Combinatorics)

Lecture 8 (3/1/2022)

8 Identities (Part 1)

* $\left(x-\frac{1}{x}\right)\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}\right)\left(x^4+\frac{1}{x^4}\right) = x^8-\frac{1}{x^8}$

* $2a^2+2b^2+2c^2-2ab-2bc-2ca \overset{\text{T.P}}{=} (a-b)^2+(b-c)^2+(c-a)^2$

* if $a^2+b^2+c^2-ab-bc-ca=0 \xrightarrow{\text{T.P}} a=b=c$

$\downarrow$ Imp.

$$\underbrace{(a-b)^2}_{+}+\underbrace{(b-c)^2}_{+}+\underbrace{(c-a)^2}_{+ve}=0 \Rightarrow \left.\begin{matrix} a-b=0 \\ b-c=0 \\ c-a=0 \end{matrix}\right\} \Rightarrow a=b=c$$

8.1 Identities (Part 2) (Grade 9)

* $(x+y)^2 = x^2+y^2+2xy \xrightarrow{y\to -y} (x-y)^2 = x^2+y^2-2xy$
* $(x+y)(x-y) = x^2-y^2$
* $(\underbrace{x+y}_{\equiv \omega}+z)^2 = (\omega+z)^2 = \omega^2+z^2+2\omega z$

$= (x+y)^2+z^2+2(x+y)z$

$= x^2+y^2+z^2+2xy+2yz+2zx$ □ 'Sq. of Trinomial'

* $(x+y)^3 = (x+y)(x+y)^2 = (x+y)\{x^2+y^2+2xy\}$

$= x^3+y^3+xy^2+2x^2y+yx^2+2xy^2$

$= x^3+y^3+3xy^2+3x^2y = x^3+y^3+3xy(x+y)$ 'cube of Binomial'

$\downarrow y\to -y$

$(x-y)^3 = x^3-y^3+3xy^2-3x^2y = x^3-y^3-3xy(x-y)$ □

* $(x+y)^3 = \underbrace{x^3+y^3}+3xy(x+y)$

$x^3+y^3 = (x+y)^3-3xy(x+y) = (x+y)\{(x+y)^2-3xy\}$

* $x^3+y^3 = (x+y)\{x^2+y^2+2xy-3xy\}$

$= (x+y)(x^2+y^2-xy)$ □

* $(x-y)^3 = x^3-y^3-3xy(x-y)$

$x^3-y^3 = (x-y)(x^2+y^2+xy)$ □

* $(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

$\rightarrow x^3+y^3+z^3+\cancel{xy^2}+\cancel{xz^2}-\cancel{x^2y}-xyz-\cancel{zx^2}$

$+\cancel{yx^2}+yz^2-\cancel{xy^2}-\cancel{y^2z}-xyz$

$+zx^2+\cancel{zy^2}-xyz-yz^2-\cancel{z^2x}$

$= x^3+y^3+z^3-3xyz$

* $x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$ □

if $\boxed{x+y+z=0}$ Cond^n

$\downarrow$

$\boxed{x^3+y^3+z^3=3xyz}$ Conditional Idt.

Lecture 9 (4|1|2022)

* $\left(\frac{1}{4}a-\frac{1}{2}b+1\right)^2 = \frac{a^2}{16}+\frac{b^2}{4}-\frac{1}{4}ab-b+\frac{a}{2}+1$

* if $a^2+b^2+c^2=20$, $a+b+c=0$; $ab+bc+ca=-10$

* if $a^2+b^2+c^2=250$, $ab+bc+ca=3$, $a+b+c=\pm16$

* $(\underbrace{a+b+c}_{x})^2-(\underbrace{a-b-c}_{y})^2 = 2a\cdot2(b+c)=4a(b+c)$

* if $x+y=12$, $xy=27$; $x^3+y^3=756$

$(x+y)^3 = x^3+y^3+3xy(x+y)$

* if $x+\frac{1}{x}=7$; $x^3+\frac{1}{x^3}=322$

Imp

* $(4x+2y)^3-(4x-2y)^3 = 16y^3+192x^2y$

$(a+b)^3=a^3+b^3+3ab(a+b)$
$(a-b)^3=a^3-b^3-3ab(a-b)$
$=2(b^3+3a^2b)$

Imp

* if $a+b=10$, $a^2+b^2=58$; $a^3+b^3=370$

* $(x+y)^3 = x^3+y^3+3xy(x+y)$, $(x+y)^2=x^2+y^2+2xy$

$= x^3+y^3+\frac{3}{2}(x+y)\{(x+y)^2-x^2-y^2\}$, $xy=\frac{(x+y)^2-x^2-y^2}{2}$

$(x+y)^3 = x^3+y^3+\frac{3}{2}(x+y)^3-\frac{3}{2}(x^2+y^2)$

$\boxed{-\frac{1}{2}(x+y)^3+\frac{3}{2}(x^2+y^2)=x^3+y^3}$

* if $x^2+\frac{1}{x^2}=7$, $x^3+\frac{1}{x^3}=?=18$

$\left(x+\frac{1}{x}\right)^2 = \underbrace{x^2+\frac{1}{x^2}}_{7}+2 \Rightarrow x+\frac{1}{x}=3$

Imp

* if $x^4+\frac{1}{x^4}=47$, $x^3+\frac{1}{x^3}=18$

* $(6m-n)(36m^2+6mn+n^2)-(3m+2n)^3$
$=189m^3-9n^3-54m^2n-36mn^2$

Method 1

* multiply

Method 2

* Multiply ; $(a+b)^3 = a^3+b^3+3ab(a+b)$ ✓

Method 3

* $a^3+b^3 = (a+b)^3-3ab(a+b)$
$=(a+b)\{a^2+b^2-ab\}$

* $a^3-b^3=(a-b)\{a^2+b^2+ab\}$ ✓

Lecture 10 (5|1|2022)

* if $a+b=7$, $ab=12$, $a^2-ab+b^2=?=13$

* if $a-b=4$, $ab=45$, $a^3-b^3=?=604$

$a^3-b^3=(a-b)\{\underbrace{a^2+b^2}_{(a-b)^2+3ab}+ab\}$

'Level-6'

* $30^3+20^3-50^3=3\cdot30\cdot(20)(-50)$ | $(a+b+c)=0$, $a^3+b^3+c^3=3abc$

* $(28)^3+(-15)^3+(-13)^3=3(28)(-15)(-13)$

* $(2x-y+3z)(4x^2+y^2+9z^2+2xy+3yz-6xz)$

$=8x^3-y^3+27z^3+18xyz$

* if $(a+b+c)=15$, $a^2+b^2+c^2=83$

$a^3+b^3+c^3-3abc=?=180$

Level-7

* if $x+y+z=-1$, $xy+yz+zx=-1$, $xyz=-1$

$x^3+y^3+z^3=?$ | $(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$

$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

$\rightarrow(x+y+z)\left((x+y+z)^2-3(\overbrace{xy+yz+zx}^{-1})\right)$

$=-1\{1+3\}$

* $x^3+y^3+z^3=-4-3=-7$

* $\dfrac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}=$ HW

9. Factorization

* $5xy \xrightarrow{\text{Factors}} 1, 5, x, y, 5xy, xy$

* $HCF(21a^3b^2, 35a^5b^5)=7a^3b^5$ (# Ref theory)

* $6x^3+8x^2y=2x^2(3x+4y)$ | $B \xrightarrow{F} M\cdot B$ (Type 1)

* $3x+7=7\left(\frac{3x}{7}+1\right)$ | $\frac{a+b}{c}=\frac{a}{c}+\frac{b}{c}$

$=75\left(\frac{3x}{75}+\frac{7}{75}\right)$

* $8(5x+9y)^2+12(5x+9y)$ | $B \xrightarrow{F} B\cdot T$ Type-2

$4(5x+9y)\{2(5x+9y)+3\}$

$=4(5x+9y)\{10x+18y+3\}$

* $(y-x)a+(x-y)b$

* $2a+6b-3(a+3b)^2$

* $(x+y)(2x+3y)-(x+y)(x+1)$

Lecture 11 (6/1/2022)

* $\dfrac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$

$a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$N \equiv x^3+y^3+z^3 = 3xyz = 3(a^2-b^2)(b^2-c^2)(c^2-a^2)$

$D \equiv g^3+h^3+w^3 = 3ghw = 3(a-b)(b-c)(c-a)$

$\dfrac{N}{D} = \dfrac{(a+b)\cancel{(a-b)}(b+c)\cancel{(b-c)}(c+a)\cancel{(c-a)}}{\cancel{(a-b)}\cancel{(b-c)}\cancel{(c-a)}} = (a+b)(b+c)(c+a)$

9. Factorization (contd.) Grade 8

* $x^2+xy+8x+8y = (x+8)(x+y)$
* $n-7+7lm-lmn = (n-7)(1-lm)$
* $ax^2+by^2+bx^2+ay^2 = (a+b)(x^2+y^2)$
* $a^2x^2+(ax^2+1)x+a = (ax^2+1)(x+a)$
* $a^3x+a^2(x-y)-a(y+z)-z = (a+1)(a^2x-ay-z)$
* $6ab-b^2+12ac-2bc = (6a-b)(b+2c)$

sums	← products
a^2+b^2+2ab	$(a+b)^2$
a^2+b^2-2ab	$(a-b)^2$
a^2-b^2	$(a+b)(a-b)$

Flavor change

* $(3a-b)^2-9c^2 = (3a-b)^2-(3c)^2 = (3a-b+3c)(3a-b-3c)$
* $3a^4-48b^4 = 3\underbrace{\left((a^2)^2-(4b^2)^2\right)}_{3(a^2-4b^2)(a^2+4b^2)} = 3(a+2b)(a-2b)(a^2+4b^2)$

$\dfrac{3a^4-48b^4}{3a^2-12b^2} = (a^2+4b^2)$ (why to do it!)

If ∃ Morenem → Do it

* $x^4-y^4 = (x+y)(x-y)(x^2+y^2)$
* $16a^2b-\dfrac{b}{16a^2} = b\left((4a)^2-\dfrac{1}{(4a)^2}\right) = b\left(4a+\dfrac{1}{4a}\right)\left(4a-\dfrac{1}{4a}\right)$
* $100(x+y)^2-81(a+b)^2 = (10(x+y))^2-(9(a+b))^2$

  $\rightarrow (10x+10y+9a+9b)(10x+10y-9a-9b)$
* $x^4-81y^4 = (x+3y)(x-3y)(x^2+9y^2)$ (Level 2)
* $x^8-y^8 = (x-y)(x+y)(x^2+y^2)(x^4+x^4)$

$x^4+y^4 = (x^2)^2+(y^2)^2+2x^2y^2-2x^2y^2$
$= (x^2+y^2)^2-2x^2y^2$

* $x^4+y^4 = (x^2+y^2)^2-(\sqrt{2}xy)^2 = (x^2+y^2+\sqrt{2}xy)(x^2+y^2-\sqrt{2}xy)$

$x^8-y^8 = (x-y)(x+y)(x^2+y^2)(x^2+y^2+\sqrt{2}xy)(x^2+y^2-\sqrt{2}xy)$

$\left.\begin{array}{l} * \ a^{12}x^4-a^4x^{12} \\ * \ a^2-b^2-a-b \end{array}\right\}$ HW

$(a+b)(a-b) = a^2-b^2$

Lecture-12 (7/11/2022) | Sums $\to$ Prods

9. Factorization (Contd.) 'Grade 8'

* $a^2-b^2-a-b=(a+b)(a-b)-(a+b)$
$=(a+b)\{a-b-1\}$

| if $\exists$ Moreness $\to$ Do it

* $a^4-2a^2b^2+b^4=(a^2-b^2)^2=(a-b)^2(a+b)^2$
* $x^4-(x-z)^4=(z^2-2zx+2x^2)(2x-z)z$
* $a^2+b^2-2(ab-ac+bc)=\underbrace{a^2+b^2-2ab}_{(a-b)^2}+\underbrace{2ac-2bc}_{2c(a-b)}$

$\to (a-b)\{a-b+2c\}$

flavor change

* $a^2\underbrace{-1+2x-x^2}_{-(x^2-2x+1)}=a^2-\underbrace{(1-x)^2}_{(x-1)^2}=(a+1-x)(a-1+x)$
* $(2a+3b)^2+2(2a+3b)(2a-3b)+(2a-3b)^2=16a^2$
* $4(x+y)^2-28y(x+y)+49y^2=(2x-5y)^2$

$(x-2)^2=x^2+4-4x$

$(2-x)^2=4+x^2-4x$

$(x-1)^2=x^2+1-2x$ | $(x^2+1)^2=x^4+1+2x^2$

* $1-2ab-(a^2+b^2)=(1+a+b)(1-a-b)$
* $x^4+x^2+1=(x^2)^2+x^2+1$
$=(x^2+1)^2-x^2=(x^2+1+x)(x^2+1-x)$
* $x^2+8x+15=x^2+8x+16-1=(x+4)^2-1^2$ ← Imp
$=(x+5)(x+3)$
* $x^4+4=(x^2)^2+2^2$
$=(x^2+2)^2-(2x)^2=(x^2+2+2x)(x^2+2-2x)$
* $x^{16}-y^{16}+x^8+y^8=(x^8)^2-(y^8)^2+x^8+y^8=(x^8+y^8)(x^8-y^8)+(x^8+y^8)$
$=(x^8+y^8)(x^8-y^8+1)$
* $(p+q)^2-(a-b)^2+p+q-a+b=(p+q+a-b+1)(p+q-a+b)$

flavor change (Quad. Exp.) | Sums $\to$ products

* $(x+a)^2=x^2+a^2+2xa$
* $(x+a)(x+b)=x^2+\underbrace{ax+bx}_{(a+b)x \text{ sum}}+\underbrace{ab}_{\text{product}}$

Mid term split (Mechanism)

* $x^2-7x+12=(x+a)(x+b)$

$\downarrow$

$x^2-(a+b)x+ab \Rightarrow \left.\begin{array}{l} a+b=-7 \\ ab=12 \end{array}\right\}$ $\boxed{\text{Trial/Error}}$ $a=-3$, $b=-4$

* $x^2-7x+12=(x-3)(x-4)$
* $x^2+5x-36=(x+9)(x-4)$

Completing squares

* $ax^2+bx+c=a\left(x^2+\frac{b}{a}x+\frac{c}{a}\right)$ coeff of $x^2=1$

$\to a\left(x^2+\left(\frac{b}{a}\right)x+\frac{c}{a}+\left(\frac{b}{2a}\right)^2-\left(\frac{b}{2a}\right)^2\right)$ add/sub $\frac{1}{2}$(coeff of x)

$=a\left(\left(x+\frac{b}{2a}\right)^2+\underbrace{\left(\frac{c}{a}-\frac{b^2}{4a^2}\right)}_{\left(\frac{4ac-b^2}{4a^2}\right)}\right)$

* $ax^2+bx+c=a\left\{\underbrace{\left(x+\frac{b}{2a}\right)^2-\left(\frac{b^2-4ac}{4a^2}\right)}\right\}$
(Sums)

$x^2-y^2=(x+y)(x-y)$
(factors)

Lecture-13 (8/1/2022)

$(x+a)(x+b) = x^2 + (a+b)x + ab$

* $-2x^2 - 3x + 2 = (x+2)(1-2x)$

Mechanism of Mid term split $(a \neq 1)$

$$ax^2+bx+c = l(x-\alpha)(x-\beta)$$
$$= l\{x^2 - (\alpha+\beta)x + \alpha\beta\}$$
$$= lx^2 - l(\alpha+\beta)x + l\alpha\beta$$

* Coeff of x^2 : $a = l$

Coeff of x : $-l(\alpha+\beta) = b \Rightarrow \boxed{\alpha+\beta = -\frac{b}{a}}$ sum

Coeff of const : $\underbrace{l}_{a}\alpha\beta = c \Rightarrow \boxed{\alpha\cdot\beta = \frac{c}{a}}$ product

* $\underbrace{ax^2+bx}_{-(-bx)}+\overset{a\alpha\beta}{c} = a(x-\alpha)(x-\beta)$

$-a(\alpha+\beta)x$

* Conventional def of split (Coeff. of x)

Trial / Error

Split it into 2 parts : $\sum(\text{parts}) = b$

$\prod(\text{parts}) = ac$

* $12x^2 - \underbrace{23yx}_{-15y-8y} + 10y^2 = (4x-5y)(3x-2y)$

* $\underbrace{(2x+3y)}_{=u}^2 - 5(2x+3y) - 14 = u^2 - \underbrace{5u}_{-7u+2u} - 14$

* $6x^2 + 35xy - 6y^2 = (6x-y)(x+6y)$

* $3m^2 + 24m + 36 = 3\{m^2 + 8m + 12\}$ → Mid term split won't work

$\rightarrow 3\{m^2 + 8m + 12 + \left(\frac{8}{2}\right)^2 - \left(\frac{8}{2}\right)^2\}$

$3\{(m+4)^2 + 12 - 16\} = 3\{(m+4)^2 - 2^2\}$

$= 3(m+6)(m+2)$

* $y^2 + 6y + 8$

* $4y^2 - 8y + 3$

* $6 - x - 2x^2$

Lecture-14 (9/1/2022)

9. Factorization (Adv.): Grade 9

* $2p(a-b)+3q(5a-5b)+4r(2b-2a)$
$= (a-b)(2p+15q-8r)$

* $(x^2+3x)^2-5(x^2+3x)-y(x^2+3x)+5y$
$= (x^2+3x)\{x^2+3x-5\}-y\{x^2+3x-5\}$
$= (x^2+3x-5)(x^2+3x-y)$

* $a^3x+a^2(x-y)-a(y+z)-z$
$\underbrace{a^3x+a^2x}-\underbrace{a^2y-ay}-\underbrace{az-z} = (a+1)\{a^2x-ay-z\}$
$a^2x(a+1)-ay(a+1)-z(a+1)$

* $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8zx$
$(-\sqrt{2}x)^2+y^2+(2\sqrt{2}z)^2-2\sqrt{2}xy+4\sqrt{2}yz-8zx$
$= (-\sqrt{2}x+y+2\sqrt{2}z)^2$

* $(-x+y+z)^2 = (-x)^2+y^2+z^2-2xy+2yz-2zx$

* $4a^2+12ab+9b^2-8a-12b$
$= (2a+3b)(2a+3b-4)$ | $(a+b)^2 = a^2+b^2+2ab$

* $\left(5x-\frac{1}{x}\right)^2 \pm 2\cdot 2\left(5x-\frac{1}{x}\right)+2^2 = \left(5x-\frac{1}{x}\pm 2\right)^2$

* $4(x+y)^2-28y(x+y)+49y^2 = (2x-5y)^2$

* $x^4+5x^2+9 = (x^2)^2+5x^2+3^2+6x^2-6x^2$
$= (x^2+3)^2-x^2 = (x^2+3+x)(x^2+3-x)$

* $x^4+4x^2+3 = (x^2)^2+4x^2+4-1$
$= (x^2+2)^2-1 = (x^2+3)(x^2+1)$

* $(\underbrace{x^2-4x}_{=y})(\underbrace{x^2-4x}_{=y}-1)-20 = y(y-1)-20$

Substitution (Tricks)

$= y^2-y-20 = (y-5)(y+4) = (x^2-4x-5)(x^2-4x+4)$

* $5\sqrt{5}x^2+30x+8\sqrt{5} = 5\sqrt{5}x^2+10x+20x+8\sqrt{5}$
$= 5x(\sqrt{5}x+2)+4\sqrt{5}(\sqrt{5}x+2)$
$= (\sqrt{5}x+2)(5x+4\sqrt{5})$

* $7\sqrt{2}x^2-10x-4\sqrt{2} = 7\sqrt{2}x^2-14x+4x-4\sqrt{2}$
$7x(\sqrt{2}x-2)+2\sqrt{2}(\sqrt{2}x-2) = (7x+2\sqrt{2})\underbrace{(\sqrt{2}x-2)}_{\sqrt{2}(x-\sqrt{2})}$

$\rightarrow (7\sqrt{2}x+4)(x-\sqrt{2})$ ✓

Lecture-15 (10/1/2022)

9. Factorization (Adv.): Grade 9

* $(x+y)(x-y)^2 = \underbrace{(x+y)(x-y)}_{x^2-y^2}(x-y) = (x^2-y^2)(x-y)$

* $x^3+y^3+z^3-3xyz = \underbrace{(x+y+z)}_{=0}(x^2+y^2+z^2-xy-yz-zx)$

$x^3+y^3+z^3 = 3xyz \Rightarrow \dfrac{x^3+y^3+z^3}{xyz} = 3$

* $\dfrac{x^2}{yz}+\dfrac{y^2}{xz}+\dfrac{z^2}{xy}=3$

Another Expression

* $x^3+y^3+z^3-3xyz = (x+y+z)\{x^2+y^2+z^2-xy-yz-zx\}$

$2(x^2+y^2+z^2-xy-yz-zx) = \underbrace{(x-y)^2}_{x^2+y^2-2xy}+\underbrace{(y-z)^2}_{y^2+z^2-2yz}+\underbrace{(z-x)^2}_{z^2+x^2-2zx}$

* $x^3+y^3+z^3-3xyz = \frac{1}{2}(x+y+z)\{(x-y)^2+(y-z)^2+(z-x^2)\}$ ✓

* $x^2+y^2+z^2-xy-yz-zx = \frac{1}{2}\{(x-y)^2+(y-z)^2+(z-x)^2\} = 0$

$\Downarrow$

$x-y = y-z = z-x = 0$

$\boxed{x=y=z}$

* $8\underbrace{(a+1)}_{=x}{}^2+2(a+1)\underbrace{(b+2)}_{=y}-15(b+2)^2$

$8x^2+\underbrace{2yx}_{12yx-10xy}-15y^2 = (2a+3b+8)(4a-5b-6)$

Flavor change

* $(2a+1)^3+(a-1)^3 = 3a((2a+1)^2-(2a+1)(a-1)+(a-1)^2)$ $\quad x^3+y^3=(x+y)(x^2-xy+y^2)$

$\to 3a\{4a^2+1+4a-(2a^2-2a+a-1)+a^2+1-2a\}$

$= 3a\{3a^2+3a+3\} = 9a(a^2+a+1)$

* $(x+1)^3-(x-1)^3 = 2(3x^2+1)$

* $(x+1)^3+(x-1)^3 = 2x(x^2+3)$

* $a^6-b^6 = (a+b)(a-b)(a^2-ab+b^2)(a^2+ab+b^2)$

* $a^7+ab^6 = a(a^2+b^2)(a^4+b^4-a^2b^2)$

* $x^{12}-y^{12}$

* x^9-y^9

* x^3+3x^2+3x-7

* x^6-7x^3-8

Lecture 16 (11|1|2022)

9. Factorization (Adv.) Contd:

* $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$
* $(x+y)^3 = x^3+y^3+3xy(x+y) = x^3+y^3+3x^2y+3xy^2$
* $x^3+y^3 = (x+y)(x^2-xy+y^2)$
* $x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$
* $x^3+3x^2+3x-7 = (x+1)^3-1-7 = (x+1)^3-2^3$
  $= (x-1)(x^2+4x+7)$
* $x^6-7\underbrace{x^3}_{\equiv y}-8 = y^2-7y-8 = (y-8)(y+1)$
  $= (x^3-2^3)(x^3+1) = (x-2)(x+1)(x^2+4+2x)(x^2+1-x)$
* $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p = \left(3p-\frac{1}{6}\right)^3$
* $\underbrace{(x+y)^3}_{\equiv a} - \underbrace{(x-y)^3}_{\equiv b} - 6y\underbrace{(x^2-y^2)}_{6y(x-y)(x+y)}$

  $\rightarrow a^3-b^3-3ab(a-b) = (a-b)^3 = 8y^3$

  Subs. Trick
  $x+y=a$
  $x-y=b$
  $2y = a-b$

* $\overset{(\sqrt{2}x)^3}{2\sqrt{2}x^3} + 3\sqrt{3}y^3 + \sqrt{5}(5-3\sqrt{6}xy)$
  $= (\sqrt{2}x+\sqrt{3}y+\sqrt{5})(2x^2+3y^2+5-\sqrt{6}xy-\sqrt{15}yz-\sqrt{10}x)$
* $(x-2y)^3+(3z-x)^3+(2y-3z)^3 = 3(x-2y)(2y-3z)(3z-x)$
* $p^3(q-r)^3+r^3(p-q)^3+q^3(r-p)^3$
  $= 3pqr(q-r)(p-q)(r-p)$

official
* $x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

Proof
* $\underbrace{x^3+y^3}+z^3-3xyz = \underbrace{(x+y)}_{=k}{}^3-3xy(x+y)+z^3-3xyz$
  $= k^3+z^3-3xyk-3xyz$
  $= (k+z)(k^2-kz+z^2)-3xy(k+z)$
  $= (k+z)(k^2-kz+z^2-3xy) = \text{RHS}$ □

  $k = x+y$

* $8a^3+b^3+12a^2b+6ab^2$
* $a^3-8+b^3+3a^2b+3ab^2$
* $a^3-8b^3-64c^3-24abc$
* $2\sqrt{2}a^3+8b^3-27c^3+18\sqrt{2}abc$
* $(a+b)^3+(b+c)^3+(c+a)^3-3(a+b)(b+c)(c+a)$ factors
  || T.P
  $2(a^3+b^3+c^3-3abc)$
* $(x-a)^3+(x-b)^3+(x-c)^3-3(x-a)(x-b)(x-c) = ?$
  s.t "such that" $a+b+c = 3x$
  Evaluation
* $a^3+6ap+p^3-8 = ?$: $p = 2-a$

Lecture 17 (12/1/2022)

1a Division of Alg Exp.

a. Polynomial ≡ Expressions : Powers $\in \mathbb{Z}^+$ of var.

* Polynomial of 1 var

$\frac{2}{3}x + \frac{1}{5}$ linear (deg 1)

$\sqrt{5}x^2 + \frac{1}{3}x - 5$ Quad (2)

$\frac{3}{2}x^3 + \frac{1}{\sqrt{2}}x^2 + 5$ Cubic (3)

Not polynomial

$x + \frac{1}{x}$

$3x^2 - \frac{1}{x} + \frac{4}{x^2}$

$3 - 2\sqrt{xy}$

$\sqrt{x} + \sqrt{y}$

$x^2 + 2\sqrt{x} + 1$

$\frac{3}{2+x}$

Division (Easy Cancellations)

* $\frac{-15a^2bc^3}{3ab} = -5ac^3$

$\frac{a+b+c}{d} = \frac{a}{d} + \frac{b}{d} + \frac{c}{d}$

* $\frac{9m^5 + 12m^4 - 6m^2}{3m^2} = 3m^3 + 4m^2 - 2$ (Type 1 : $\frac{M}{M}$)

* $\frac{24x^3y + 20x^2y^2 - 4xy}{2xy} = 12x^2 + 10xy - 2$ (Type 2 : $\frac{P}{M}$)

* $\frac{\frac{2}{3}a^2b^2c^2 + \frac{4}{3}ab^2c^3 - \frac{1}{5}ab^2c^3}{\frac{1}{2}abc} = \frac{4}{3}abc + \frac{8}{3}bc^2 - \frac{2}{5}bc^2$

$\frac{a/b}{c/d} = \frac{a}{b} \cdot \frac{d}{c}$

Division (Factorization)

* $\frac{ax^2 + (b+ac)x + bc}{x+c} = \frac{(ax+b)\cancel{(x+c)}}{\cancel{(x+c)}}$

* $\frac{a^4 - b^4}{a-b} = (a+b)(a^2+b^2)$

* $\frac{a^{12} + a^6b^6 + b^{12}}{a^6 - a^3b^3 + b^6} = \frac{(a^6)^2 + 2a^6b^6 + (b^6)^2 - a^6b^6}{a^6 - a^3b^3 + b^6}$

$\frac{(a^6+b^6)^2 - (a^3b^3)^2}{a^6 - a^3b^3 + b^6} = a^6 + b^6 + a^3b^3$

* $\frac{x^{4a} + x^{2a}y^{2b} + y^{4b}}{x^{2a} + x^a y^b + y^{2b}} = x^{2a} + y^{2b} - x^a y^b$

Division (Long) of Polynomial

* $\frac{x^3 - 4x^2 + 7x - 2}{x - 2} = ?$

divisor) Divident (Q

D, N, R

divident = div. × Q + R

(Rule)
Method 1
Steps

1. divide 1st term of N by 1st term of D (IT)
2. Multiply (IT) by D
3. Subtract whatever you get

$$x-2\,\big)\,x^3 - 4x^2 + 7x - 2\,\big(\,x^2 - 2x + 3$$
$$(-)\;x^3 - 2x^2$$
$$-2x^2 + 7x - 2$$
$$(-)\;-2x^2 + 4x$$
$$3x - 2$$
$$(-)\;3x - 6$$
$$4$$

* $x^3 - 4x^2 + 7x - 2 = \underbrace{(x-2)}_{\text{Div}}\underbrace{(x^2 - 2x + 3)}_{Q} + \underbrace{4}_{R}$

$= x^3 - 2x^2 + 3x - 2x^2 + 4x - 6 + 4$

$= x^3 - 4x^2 + 7x - 6 + 4$

Method 2 : (Next Class)

* $\frac{2x^4 + 8x^3 + 7x^2 + 4x + 3}{x+3} \overset{?}{=} (2x^3 + 2x^2 + x + 1)$

* $$x+3\,\big)\,2x^4 + 8x^3 + 7x^2 + 4x + 3\,\big(\,2x^3 + 2x^2 + x + 1$$
$$(-)\;2x^4 + 6x^3$$
$$2x^3 + 7x^2 + 4x + 3$$
$$(-)\;2x^3 + 6x^2$$
$$x^2 + 4x + 3$$
$$(-)\;x^2 + 3x$$
$$x + 3$$
$$x + 3$$
$$0$$

Lecture-18 (13/1/2022) | $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

* $a^3+6ap+p^3-2^3 = a^3+p^3+(-2)^3+\underbrace{3(2ap)}_{-3(a)(p)(-2)}$

$\rightarrow \underbrace{(a+p-2)}_{0}(a^2+p^2+4-ap+2p+2a)=0$

$(\because p=2-a)$

1a Division of Alg. Exp. (Contd.) {Polynomials

* $\dfrac{x^3-6x^2+11x-6}{x^2-4x+3} = ? = (x-2)$

$$\begin{array}{r|l}
x^2-4x+3 & x^3-6x^2+11x-6 \quad (x-2 \\
 & x^3-4x^2+3x \\
 & (-) \\
 & -2x^2+8x-6 \\
 & -2x^2+8x-6 \\
 & 0
\end{array}$$

Method 2

* $\dfrac{x^3-4x^2+7x-2}{(x-2)} = ?$

$$\begin{array}{r|l}
 & x^2-2x+3 \\
x-2 & x^3-4x^2+7x-2 \\
 & x^3-2x^2 \\
 & -2x^2+7x-2 \\
 & -2x^2+4x \\
 & 3x-2 \\
 & 3x-6 \\
 & 4
\end{array}$$

* $x^3-4x^2+7x-2 = \underbrace{x^2(x-2)}_{x^3-2x^2} \underbrace{-2x(x-2)}_{-2x^2+4x} + \underbrace{3(x-2)}_{3x-6}+4$

* $x^3-4x^2+7x-2 = (x-2)(x^2-2x+3)+4$

Can factor "anything" out provided that approp. "Expand" terms are added to make it Equal

(Trick)

* $\dfrac{6y^5+15y^4+16y^3+4y^2+10y-35}{3y^2+5}$

$$\begin{array}{r|l}
 & 2y^3+5y^2+2y-7 \\
3y^2+5 & 6y^5+15y^4+16y^3+4y^2+10y-35 \\
 & -\;6y^5 \quad +10y^3 \\
 & 15y^4+6y^3+4y^2+10y-35 \\
 & -\;15y^4 \quad +25y^2 \\
 & 6y^3-21y^2+10y-35 \\
 & -\;6y^3 \quad +10y \\
 & -21y^2 \quad -35 \\
 & -21y^2 \quad -35 \\
 & 0
\end{array}$$

Method 2 $(3y^2+5)$

* $6y^5+15y^4+16y^3+4y^2+10y-35 = \underbrace{2y^3(3y^2+5)}_{6y^5+10y^3} + \underbrace{5y^2(3y^2+5)}_{15y^4+25y^2} + \underbrace{2y(3y^2+5)}_{6y^3+10y} \underbrace{-7(3y^2+5)}_{-21y^2-35}+0$

* $x^2+1 \mid x^4+x^3+8x^2+ax+b$, $a, b = ?$

* $x^4+x^3+8x^2+ax+b = (x^2+1)\underbrace{(x^2+x+7)}_{Q}+\underbrace{x(a-1)+b-7}_{R}$

$x^4+x^3+7x^2+x^2+x+7$

$x^4+x^3+8x^2+x+7$

* Compare. $x(a-1)+(b-7)=0x+0$

Coeff of x: $a-1=0 \Rightarrow a=1$

Coeff of const: $b-7=0 \Rightarrow b=7$

Lecture-14 (14/1/2022)

10.a. Polynomial (grade-9)

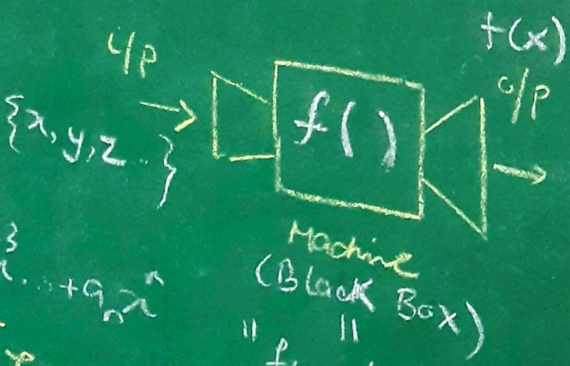
* Polynomial ≡ Alg Exp : Power $\in \mathbb{Z}^+$

* $f(x) = a_0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + \dots + a_n x^n$, $x \in \mathbb{R}, \mathbb{C}$, $n \in \mathbb{Z}^+$

"f of x" ≠ "f multiplied by x"   'Polynomial'

$\{a_0, a_1, \dots a_n\}$ = coeff.

highest(n) = degree



* $\underbrace{f(x)}_{\text{Proper}} = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \dots + a_n x^n$

$= \sum_{i=0}^{n} a_i x^i$

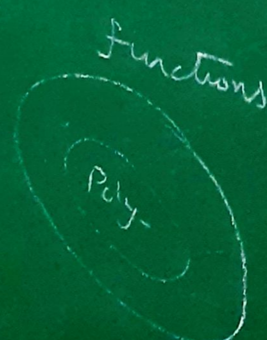
"Power series expansion of a function"

{Taylor series Expⁿ of a function

* $n = 0 \Rightarrow$ Constant poly.

$n = 1 \Rightarrow$ linear poly

$n = 2 \Rightarrow$ Quadratic poly.



* $f(x) = 8x^2 - 3x + 7$, $x \in \mathbb{R}$

$f(-1) = f$ at $(x = -1) = 8(-1)^2 - 3(-1) + 7 = 18$

'value of Polynomial'

Lecture-20 (16|1|2022)

20! Polynomials (Grade 9/10)

* Poly ≡ Alg Exp : Powers $\in \mathbb{Z}^+$

* $f(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n$, $n \in \mathbb{Z}^+$

"f of x" $= \sum_{i=0}^{n} a_i x^i$ $\quad x \in \mathbb{R}$

(Just a Notation)

$\left\{ \begin{array}{l} n^{th} \text{ deg. polynomial} \\ a_i - \text{coeff of } x^i \end{array} \right.$

* $f(x) = \sum_{i=0}^{n} a_i x^i \left\{ \begin{array}{l} n=0 \Rightarrow f(x) = a_0 (x)^0 = \text{const} \\ n=1 \Rightarrow f(x) = a_0 + a_1 x \ \text{'linear Poly.'} \\ n=2 \Rightarrow f(x) = a_0 + a_1 x + a_2 x^2 \ \text{'Quad Polyn.'} \\ n=3 \Rightarrow f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \ \text{Cubic Poly.} \end{array} \right.$

α. Value of a Poly.

* $f(x)$ = Polynomial of x

$\downarrow$ $x = a$ (pre-given value)

$\underbrace{f(x=a)}_{\text{replace } x \text{ by } a \text{ "everywhere"}} \equiv f(a) = $ Value of Poly. (Evaluation)

* if $f(x) = 0 \longrightarrow x = \alpha$

value of Poly is 0

" Root | Zero of a Poly.

Soln to an Eqn $f(x) = 0$

{∃ presupposed Soln}

* $f(x) = x^3 - 6x^2 + 11x - 6$ Cubic

$f(x=0) = f(0) = -6$ $\quad x = 0$ Pre given Value

$f(x=1) = f(1) = 0 \Rightarrow x = 1$ Special Value

$f(2) = 0 \Rightarrow x = 2$ is a Root|Zero

{a zero|Root| soln of $f(x)$}

$f(3) = 0 \Rightarrow x = 3$ is a Root|Zeros

* $f(x) = 2x^3 - 13x^2 + 17x - 12 \longrightarrow f(2) = -14$

* $f(x) = 2x^3 - 3x^2 + 7x - 6$

$f(1) = 0$

* $g(x) = 6x^3 - 11x^2 + Kx - 20$

if $g\left(\frac{4}{3}\right) = \frac{-16 + 4K}{3} - 20 = 0 \Rightarrow 4K = 76 \Rightarrow K = 19$

* $h(x) = 2x^3 - 5x^2 + ax + b$

if $h(1) = h(0) = 0 \longrightarrow a, b = ?$

* $p(x) = ax$ $\quad a \neq 0$

Roots = ? $\Rightarrow p(x) = 0 = ax \Rightarrow \boxed{x = \frac{0}{a} = 0}$

Remarks

* $x = \alpha$ Root|Zero of Poly {Presupposed Soln} $\longrightarrow f(\alpha) = 0$ "Stupid" problem

* $f(x) = 0$ Equation $\longrightarrow x = \alpha_1, \alpha_2 \dots$ Solution "Interesting" problem

Lecture-21 (17/1/2022)

10. Polynomials (Grade 9/10)

* $f(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n = \sum_{i=0}^{n} a_i x^i$, $i \in \mathbb{Z}^+$

deg of poly; index; $\to$ coeff. $\in \mathbb{R}$, $x \in \mathbb{R}$

* $f(x=\alpha) = \sum_{i=0}^{2} a_i (\alpha)^i = a_0 + a_1\alpha + a_2\alpha^2$ = Const # (Value of poly)

* $f(x) = 0$ (Equation) $\longrightarrow x = \alpha_1, \alpha_2$

Solutions / zeros / Roots of the polyno.

'Simple' theorems.

1. $f(x)$ poly, $a_i \in \mathbb{Z}$, $a_n = 1$

if $\exists$ integral root of $f(x) \Rightarrow$ Root = one of the factors of the const part (a_0)

{Integral Root Thm — Thm 1}

* $f(x) = x^3 + 2x^2 - 11x - 12$

$\exists$ integral root of $f(x) \Rightarrow$ Root = one of the factors of $a_0 = 12$

$= \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$f(3) = 0$

* $f(x) = x^3 - 6x^2 + 11x - 6 = 0 \Rightarrow$ Root = one of the fact of 6

1, 2, 3

* if $\exists$ Rational roots of $f(x) \Rightarrow$ $\left(\frac{b}{c}\right)$ simple fraction

b = factor of a_0

c = ,, ,, a_n

{Rational Root Thm. — Thm 2}

* $f(x) = 6x^3 + 5x^2 - 3x - 2$

$\exists$ Rational Root $\Rightarrow \left(\frac{b}{c}\right)$

b = factor (2) $= \pm 1, \pm 1$

c = factor (6) $= \pm 1, \pm 2, \pm 3, \pm 6$

1st Root = -1

* $\deg(f(x)) = n \Rightarrow \exists$ atmost n real roots

* $\boxed{\text{Zeros of Poly} \Rightarrow \text{Solve } f(x) = 0}$ } Thm. 3 (Poly. Eqn) Thm 4

β. Relationship b/w Roots

* $f(x) = ax + b \xrightarrow{\text{root}} f(x) = 0 = ax + b$

$\Downarrow$

$x = \alpha = -\frac{b}{a}$ — Root of linear poly

$$\boxed{x = -\frac{\text{Constant Part}}{\text{Coeff of } x}}$$

* $f(x) = 6x^2 - x - 2 \xrightarrow[?]{(\alpha, \beta) \text{ roots}} f(x) = 0$

$6x^2 - x - 2 = 0 \Rightarrow 6x^2 + 3x - 4x - 2 = 0$

$(ax^2 + bx + c = 0)$

$\Downarrow$

$3x(2x+1) - 2(2x+1) = 0$

$(3x-2)(2x+1) = 0$

$\Downarrow$

$3x - 2 = 0 \Rightarrow x_1 = 2/3 = \alpha$

$2x + 1 = 0 \Rightarrow x_2 = -\frac{1}{2} = \beta$

* $\alpha + \beta = \frac{1}{6} = -\frac{b}{a}$

$\alpha\beta = -\frac{1}{3} = \frac{c}{a}$

Proof

* $f(x) = ax^2 + bx + c \xrightarrow[(\alpha, \beta)]{\text{root}} ax^2 + bx + c = 0$

$x = \alpha$

$x = \beta$

* $ax^2 + bx + c = l\{(x-\alpha)(x-\beta)\}$

$ax^2 + bx + c = lx^2 - l(\alpha+\beta)x + l\alpha\beta$

Compare

Coeff of x^2: $l = a$

Coeff of x: $-l(\alpha+\beta) = b \Rightarrow \boxed{\alpha + \beta = -\frac{b}{a}}$

Coeff of const x^0: $c = l\alpha\beta \Rightarrow \boxed{\alpha\beta = \frac{c}{a}}$

* $f(x) = ax^2 + bx + c = lx^2 - l(\alpha+\beta)x + l\alpha\beta$

$= l\{x^2 - (\alpha+\beta)x + \alpha\beta\}$

Lecture 22 (22/1/2022)

10' Polynomial (grade 10)

β. Relationship b/w Roots.

* $f(x) = ax+b \xrightarrow[\text{zeros}]{\text{root}} \underbrace{f(x)}_{ax+b} = 0 \Rightarrow x = -\frac{b}{a}$

linear poly — linear eqn

* $f(x) = ax^2+bx+c \xrightarrow[(\alpha,\beta)]{\text{root}} \underbrace{f(x)}_{ax^2+bx+c} = 0 \Rightarrow \boxed{\alpha+\beta = -\frac{b}{a} \quad \alpha\beta = \frac{c}{a}}$

Quad poly

* $f(x) = x^2+7x+12$ roots = ? = (α, β)

Root $\Rightarrow f(x) = 0 \Rightarrow x^2+7x+12 = 0$ ✓

$\Downarrow$

$\boxed{\alpha\beta = 12, \ \alpha+\beta = -7}$

* $x^2+\underbrace{7x}_{(4+3)x}+12 = \underbrace{x^2+4x}_{x(x+4)}+\underbrace{3x+12}_{3(x+4)} = (x+4)(x+3)$

$x^2+7x+12 = (x+4)(x+3) = 0 \Rightarrow x+4 = 0 \Rightarrow \alpha = -4$ or $x+3 = 0 \Rightarrow \beta = -3$

* $f(x) = 4\sqrt{3}x^2+5x-2\sqrt{3}$ root = ? (α, β)

Root $\Rightarrow f(x) = 0 \Rightarrow 4\sqrt{3}x^2+5x-2\sqrt{3} = 0$

$\Downarrow$

$\alpha = \frac{\sqrt{3}}{4}, \ \beta = -\frac{2}{\sqrt{3}}$

$\alpha+\beta = \frac{\sqrt{3}}{4} - \frac{2}{\sqrt{3}} = \frac{3-8}{4\sqrt{3}} = \frac{-5}{4\sqrt{3}} = -\frac{b}{a}$

$\alpha\beta = -\frac{1}{2}$

* $f(x) = abx^2+(b^2-ac)x-\underbrace{bc}$ Roots: ?

Roots $\Rightarrow f(x) = 0$

* $abx^2+(b^2-ac)x-bc = 0 \Rightarrow \boxed{\alpha = \frac{c}{b}, \ \beta = -\frac{b}{a}}$

$\alpha\beta = -\frac{c}{a} = \frac{\text{constant}}{\text{coeff of } x^2}$

$\alpha+\beta = \frac{ac-b^2}{ab} = -\frac{(\text{coeff of } x)}{\text{coeff of } x^2}$

* $f(x) = ax^2+bx+c \longrightarrow (\alpha, \beta)$ roots

* $\alpha^2+\beta^2 = \frac{b^2-ac}{a^2}$, $\alpha+\beta = -\frac{b}{a}$, $\alpha\beta = \frac{c}{a}$

* $\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{b^2-2ac}{ac}$

* $\alpha^3+\beta^3 = \frac{3abc-b^3}{a^3}$

* $\frac{1}{\alpha^3}+\frac{1}{\beta^3} =$

* $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha} =$

* $\alpha^4+\beta^4 =$ HW

* $\frac{\alpha^2}{\beta^2}+\frac{\beta^2}{\alpha^2} =$

* $(a+b)^3 = a^3+b^3+3ab(a+b)$

$\alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$= \left(-\frac{b}{a}\right)^3 - \frac{3c}{a}\left(-\frac{b}{a}\right) = -\frac{b^3}{a^3}+\frac{3cba}{a^3} = \frac{3abc-b^3}{a^3}$

Lecture – 23 (23/1/2022)

Factorisation doubt (sheet)

$(a+b)^3 = a^3+b^3+3ab(a+b)$ $\quad [3a^2b+3ab^2]$

$a^3 \pm b^3 = (a\pm b)(a^2 \mp ab + b^2)$

* $x^3 - 7x + \underbrace{6 - 14}_{-8} + 14 = \underbrace{x^3 - 2^3} - 7x + 14$

$= (x-2)(x^2+2x+4) - 7(x-2)$

$= (x-2)\Big[\underbrace{x^2+2x-3}_{x^2+3x-x-3}\Big] = (x-2)(x-1)(x+3)$

$x(x+3)-1(x+3)$

* $x^4 + 4 = (x^2)^2 + 2^2 = \underbrace{(x^2+2)^2}_{x^4+4+4x^2} - 4x^2 = (x^2+2)^2 - (2x)^2$

$= (x^2+2-2x)(x^2+2+2x)$

* $f(x) = a_0 + a_1x + a_2x^2 + \dots a_nx^n$ $\quad$ n^{th} deg. poly

Expression $\quad x, y, z \in \mathbb{R}$

$\downarrow$

$f(\alpha) = a_0 + a_1\alpha + \dots a_n\alpha^n$ $\quad$ Exp.

* $f(\vartheta) = a_0 + a_1\vartheta + a_2\vartheta^2 + \dots a_n\vartheta^n$

* $f(x) = 2x^2 + x + 4$

$f(x) = ax^2 + bx + c$

$f(y) = \alpha y^2 + \beta y + \gamma$

$f(\alpha) = a\alpha^2 + b\alpha + c$

$f(\text{Rohan}) = a(\text{Rohan})^2 + b(\text{Rohan}) + c$

$\boxed{\text{Conventional (dogmatic)}}$

Var: x, y, z

Const: $2, 4, a, b, c, \alpha, \beta, \gamma$, Rohan

$\boxed{x = \text{Rohan}}$

$\boxed{\text{"formal system" (unorthodox)}}$

$\nexists$ Var-Constant dichotomy (dualism)

* $\boxed{f(x=a) = 0 \Rightarrow \text{Root} \,|\, \boxed{\text{Zero}}}$ Algebraic meaning of 'Root'

* Graphical meaning ??

8. Curve sketching of Polynomial

Coordinate geometry (grade 9)

* Descartes / Euclid / Newton

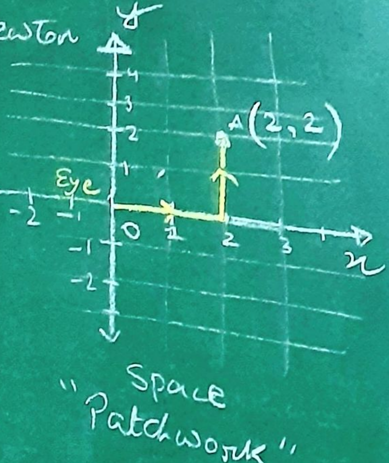
* Cartesian geometry

Coordinate geometry

Euclidean geometry

Coord. Sys

Frame of Reference



* Abstract | Mathematical tool

* $\boxed{\text{Representation} \neq \text{Reality}}$

* $x \in \mathbb{R}$, $y \in \mathbb{R}$

* (x, y) "Coordinates"

Lecture – 24 (24/1/2022)

γ. Curve sketching of Polynomial

Coordinate geometry (Grade 9/10)

* " $\mathbb{R}$ is order-isomorphic to ("is essentially same) linear continuum of geometry" (~ st. line)

1 D World (1 set of Real #) $\mathbb{R}$

Real Number line (descartes')

$\mathbb{R}^1$ 'Cantor–Dedekind Axiom'

* Pos.-val assignment : # var. needed = 1 (x)

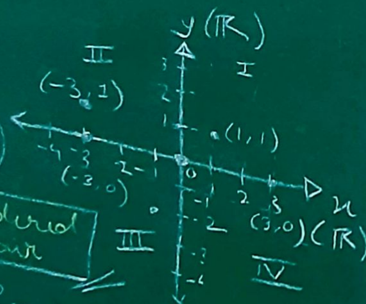
$x \in \mathbb{R}$ ($\mathbb{R}^1 \equiv$ 1-D Real space)

* Sym : translation in x

$\mathbb{R}^2$

* Pos-val assignment

var needed = 2 (x, y)



* $(3,2) \neq (2,3) \Rightarrow$ Ordered Pair

* $x \in \mathbb{R}$, $y \in \mathbb{R}$ ($\mathbb{R} \times \mathbb{R} \equiv \mathbb{R}^2 \equiv$ 2-D space)

* Sym. transl^n + Rotation in xy plane

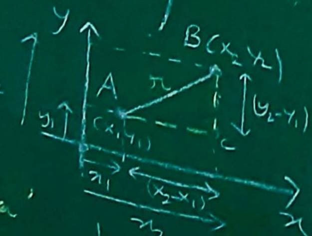
Distance formula

* $\text{Dist}(AB) \equiv |AB|$

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

'Distance formula'

$|AB|^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 = r^2$



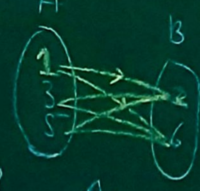
$H = \sqrt{P^2 + B^2}$

Set / Relation (grade-11)

* $A = \{1, 3, 5\}$, $B = \{2, 6\}$

* $A \times B \equiv \{(1,2), (1,6), (3,2), (3,6), (5,2), (5,6)\}$

"read by" 'Cartesian pdt of A, B'



Arrow diagram

'diff. kinds of products'

Number pdt
Vector pdt
Set pdt
Matrix pdt

* A, B, × (Cartesian pdt operator)

* (a, b) = Ordered pair (O.P) (duplet) : $a \in A$, $b \in B$

* $A \times B = \{(a,b) : a \in A, b \in B\}$

"Collection of all the O.P"

* $(a,b) \neq (b,a) \Rightarrow$ "ordered"

* A, B, C, ×

(a, b, c) – ordered Triplet : $a \in A$, $b \in B$, $c \in C$

* $A \times B \times C = \{(a,b,c) : a \in A, b \in B, c \in C\}$

(a,b) — Pair / duplet ⟶ 2D "world" (Plane)

(a,b,c) — triplet ⟶ 3D "world" (Volume)

(a,b,c,d) — quadruplet

⋮

$\underbrace{(a, b, c \ldots d)}_{n}$ – n-tuplet ⟶ nD "world"

* $A = \mathbb{R}$, $B = \mathbb{R}$, $C = \mathbb{R}$

* $\mathbb{R} = \{\ldots -10, 1, e, \pi, \ldots \infty\}$

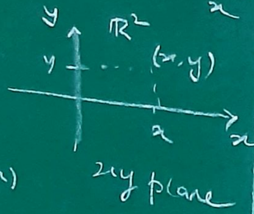
* $\mathbb{R} \times \mathbb{R} \equiv \mathbb{R}^2 = \{\ldots (-\pi, 0), (e, \pi) \ldots \infty\}$

$\mathbb{R} \times \mathbb{R} \times \mathbb{R} \equiv \mathbb{R}^3 = \{\ldots (1,3,5), (-7,6,3) \ldots\}$

Lecture-25 (29/1/2022)

Cartesian "Prdt" of sets

* A, B, $\times$ (Set Product op)
* $x \in \mathbb{R}$ — $\mathbb{R}^1$ 1D Eucl. line
* $x \in \mathbb{R}, y \in \mathbb{R}$ — $\mathbb{R}^2$ 2D Eucl. Plane
* (x, y) : ordered poin $(x,y) \neq (y,x)$

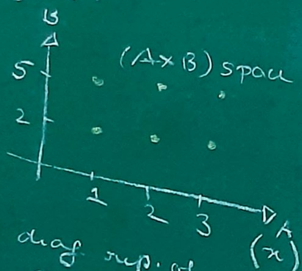


* $A \times B = \{(a, b) : a \in A, b \in B\}$

  $A = \{1, 2, 3\}$, $B = \{2, 5\}$

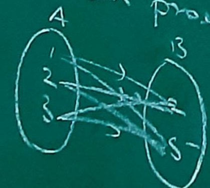
  $A \times B = \{(1,2), (1,5), (2,2), (1,5), (3,2), (3,5)\}$

  $B \times A = \{(2,1), (2,2), (2,3), (5,1), (5,2), (5,3)\}$



* $A \times B \neq B \times A$

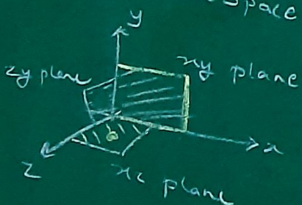
  'Non commutative Product'

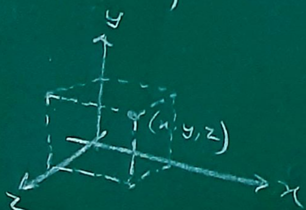


Geometry (3D)

* $\mathbb{R}^2 \times \mathbb{R} = \mathbb{R}^3 = \{(x, y, z) : x \in \mathbb{R}, y \in \mathbb{R}, z \in \mathbb{R}\}$

  3-D Eucl space

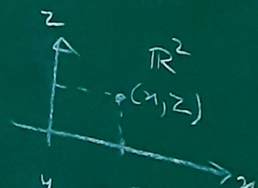


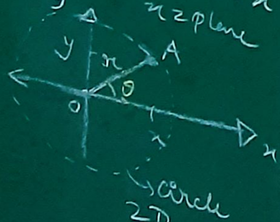


Diff. ways

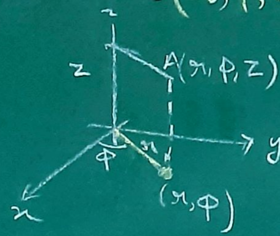
* Pos-val assignment

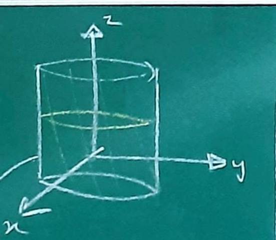
1. Euclidean rectangular (Cartesian) Coord.: (x, y)
2. Circular polar coord: (r, θ)



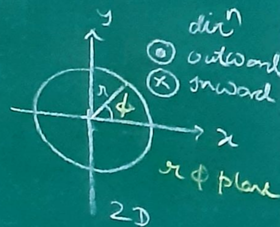


3. Cylindrical C.S (r, ϕ, z)





"Flat" (technical term)

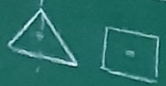


Lecture 26 (31/1/2022)

* set + "Binary Composition" (Math 12) => Group <=> Symmetry

Symmetry → Geometrical, Motion

Geometrical



Motion

{Insane Simplification}

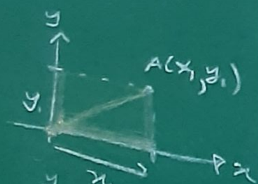
* degree of freedom = dof = "Conditional Indep"
* Constraint = Restriction

$\boxed{\text{Space = Judgement not a fact based on Symmetry (choice)}}$

Pos-val. assignment of an obj. (coord sys)

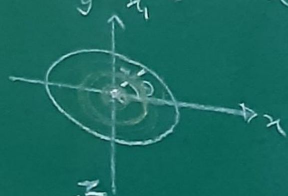
* Rectangular C.S (x, y)

$x \in (-\infty, \infty)$
$y \in (-\infty, \infty)$



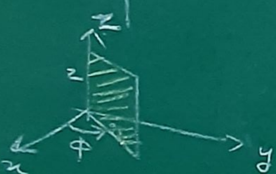
* Circular Polar C.S (r, θ)

$r \in (0, \infty)$
$\theta \in (0, 360^\circ]$

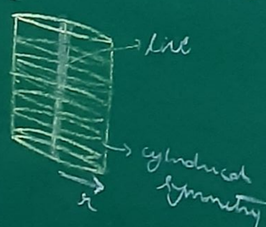


* Cylindrical C.S (r, ϕ, z)

$r \in (0, \infty)$
$\phi \in (0, 360^\circ]$
$z \in (-\infty, \infty)$

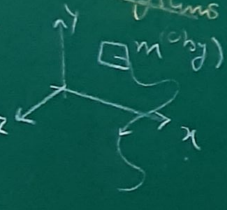


* "Becoming-cylinder of a line"



Physical Systems (3-D)

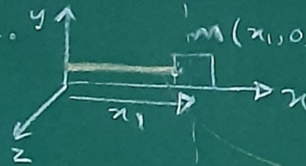
1.



dof = 3 = # dim. avail

Free-particle

2.



$\text{dof} = \underset{3D}{3} - \underset{\text{constraints}}{2} = \underset{\text{dir}^n\text{ of indep. Motion}}{1}$

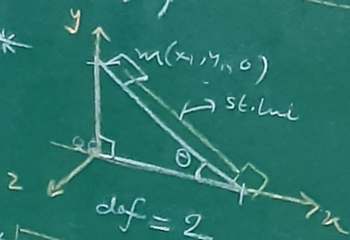
constraints = 2
$(y_1 = 0,\ z_1 = 0)$

→ curve / St.line / traj / Path / locus
{Comes after the Motion/Movement}

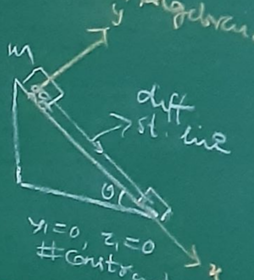
* ∄ traj. before Motion => Motion (Primary)
Space/Traj. (Secondary)

* (C.S) $\boxed{\text{Space = Past Representation}}$ (traj.) → Geometric, → Algebraic

*



dof = 2



$y_1 = 0,\ z_1 = 0$
# Constraint = 2
dof = 3 − 2 = 1

* $\boxed{\text{origin of C.S = Eye of the observer}}$ = The Rigid Body

* (Geometric) graph/plot/curve → Relation b/w x & y (Algebraic)
St. line (Past-Traj.) Math 11

* Relation b/w x & y (= Equation) → curve/traj./space
⇓
Curve tracing

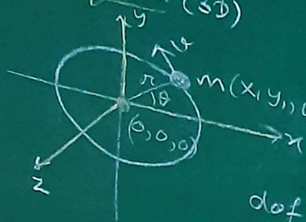
* Geometric rep^n → Algebraic rep. <=> Curve → Eq^n
Algebraic rep^n (= Eq^n) → Geometric rep (= curve) <=> Eq^n → Curve

lecture-27 (5|2|2022)

* Scientific Mania (Sickness / habit) ⟹ Symmetry
* Space = judgement
* Space = Part representation → Geometric (curve); → Algebraic (Equation)
* Origin = Eye of observer

Physicals system (3D)

4.



\# constraints ≡ K

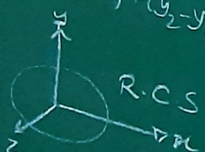
Constraints: $z_1 = 0$

$r = \sqrt{x_1^2 + y_1^2}$

\# constraint = 2

dof = 3 − 2 = 1 (indep. motion in θ dirⁿ)

* $|AB| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$ Distance formula 3D



R.C.S ⟶ Cir. Polar C.S

(x, y) changes

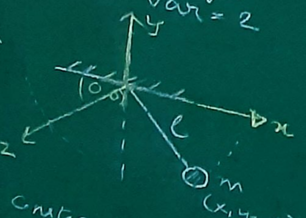
\# var = 2

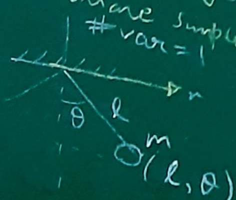
(r, θ): r → const, θ → changes

"Insane simplification"

\# var = 1

5.





Constraints: $z_1 = 0$

$l = \sqrt{x_1^2 + y_1^2} = \text{const}$

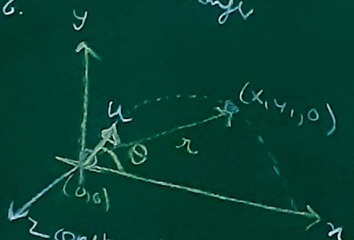
K = 2

dof = 3 − 2 = 1

(x, y) change

(r, θ): r → const, θ → var.

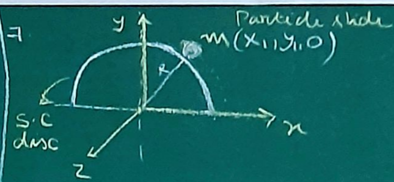
6.

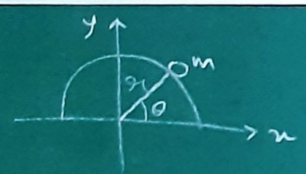


⟶ (r, θ) (indep.)

constraint: $z_1 = 0 \Rightarrow K = 1 \Rightarrow$ dof = 3 − 1 = 2

7.





constants: $z_1 = 0$

$R = \sqrt{x_1^2 + y_1^2}$

dof = 3 − 2 = 1

var: (x, y)

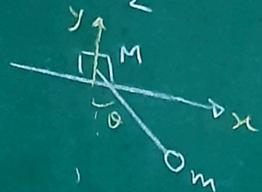
\# var = 2

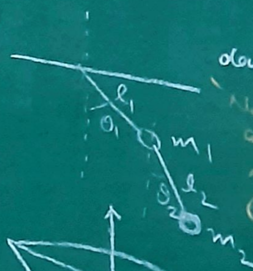
(r, θ): r → const, θ → var

r = R, Z = 0

\# var = 1

8.

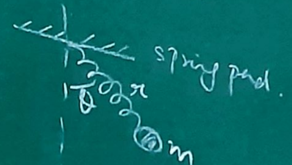




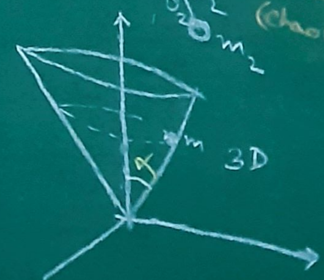
double pendulum

Not Easy analysis ~Reality (Chaotic)

9.

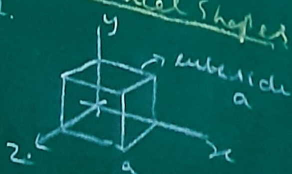


(r, θ) changes.



Mathematical Shapes

1.

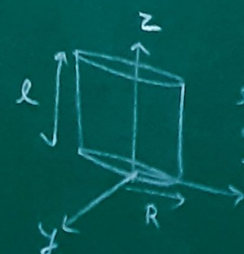


2.

$x \in (0, a)$

$y \in (0, a)$

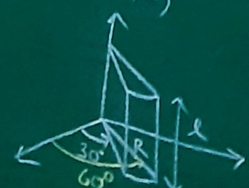
$z \in (0, a)$



$r \in (0, R)$

$\phi \in (0, 360°)$

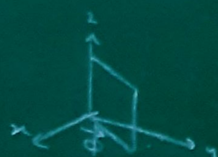
$z \in (0, L)$



$r \in (0, R)$

$\phi \in (0, 30°)$

$z \in (0, l)$



Lecture-28 (6|2|2022)

* Motion : Primary
  Space : secondary } → ∄ thing before Motion

* Space = Past Representation → Geometrical (curves) ↔ Algebraic (Equation) (Past, Trigo)

'Relation' b/w x, y

var = 2 (x, y)

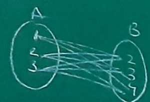
Relation (Math 11)

* A, B, × (Cartesian pdt), (a,b) : $a \in A, b \in B$ ordered pair (O.P)

* $A \times B = \{(a,b) : a \in A, b \in B\} \neq B \times A$ "collection of all o.p."

* $A = \{1,2,3\}, B = \{2,3,4\}$

$A \times B = \{(1,2),(1,3),(1,4),(2,2),(2,3),(2,4),(3,2),(3,3),(3,4)\} \neq B \times A$



Hand-Pick/choice/Judgement/Value assignment/Eval (Important)

* $R = \{(1,2),(2,3),(3,4)\} \subseteq A \times B$

$R = \{(x,y) : \underbrace{y = x+1}, x \in A, y \in B\}$ "Rule"/fixed/Pre-given

* $R^{-1} = \{(2,1),(3,2),(4,3)\} \subseteq B \times A$

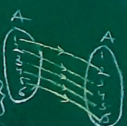
$\neq \frac{1}{R}$ "Inverse of R" (term Borrowed from # theory)

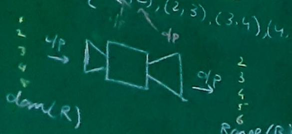
Number th. $a^{-1} = \frac{1}{a}$ inverse of $a \in \mathbb{Z}$

* $A = \{1,2,3,4,5,6\}$ "World"

$R = \{(x,y) : y = x+1, x \in A, y \in A\}$ Rule

$= \{(1,2),(2,3),(3,4),(4,5),(5,6)\}$

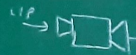




* $R = \{(a,b) : \exists \text{"rule"}, a \in A, b \in B\} \subseteq A \times B$ (i/p, o/p, Pre given)

$R = \{(a,b) : aRb, a \in A, b \in B\}$ Relation

"a relates to b"



* Domain = i/p = $dom(R) = \{a : (a,b) \in R\}$

Range = o/p = $Rang(R) = \{b : (a,b) \in R\}$

* $dom(R) = \{1,2,3,4,5\} = Range(R^{-1})$

$Range(R) = \{2,3,4,5,6\} = dom(R^{-1})$

* $R^{-1} = \{(2,1),(3,2),(4,3),(5,4),(6,5)\}$

* $R^{-1} = \{(b,a) : (a,b) \in R\} \subseteq B \times A$ (b: dom R^{-1}, a: range(R^{-1}))

'Inverse Relation'

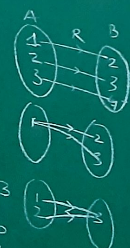
* $A = \{1,2,3\}, B = \{2,3,4\}$

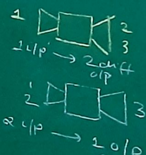
$R = \{(1,2),(2,3),(3,4)\}$

$= \{(x,y) : y = x+1, x,y \in A\} \subseteq A \times B$

$R_1 = \{(1,2),(1,3)\}$

$R_2 = \{(1,3),(2,3)\}$





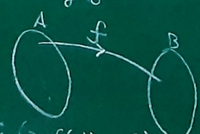
function

* $f : A \to B = \{(x, \underbrace{f(x)}_{y}), x \in A\}$

"f maps set A to set B"

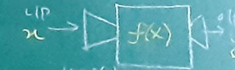
'function/Map' Special relation

* f is a rel^n which assigns to each element of A exactly 1 element of B



* $Dom\ f = \{x : (x, f(x)) \in f\}$; $Ran\ f = \{f(x) : (x, f(x)) \in f\}$

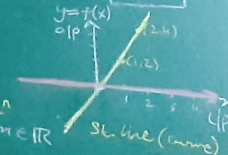
* $\boxed{y = f(x)}$



* $f : \mathbb{R} \to \mathbb{R}$

Real valued functions

Equation/Rel^n



* $y = f(x) = 2x, x \in \mathbb{R}$

$f : \mathbb{R} \to \mathbb{R} = \{(x, f(x)), x \in \mathbb{R}\}$

$= \{(0,0),(1,2),(2,4),(3,6),(4,8),(-3,-6) \ldots\}$

x	0	1	2	3
y	0	2	4	6

x=0 is the Zero of Poly f(x)

* Zero/Root : $(x_0, 0)$ $x = x_0$

"cut at x-axis"

$y = f(x) = 0 \Rightarrow x = \alpha$ is a Root/zero/cut at x axis

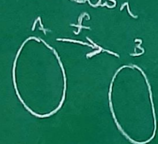
* A, B, ×, (a,b) : OP
* $A \times B = \{(a,b) : a \in A, b \in B\}$
* $R = \{(a,b) : \exists \text{ Rule}, a \in A, b \in B\}$
* $f : A \to B = \{(x, f(x)) ; x \in A\}$
* Becoming - ordered pair of the numbers (Elements)
* Becoming - Cartesian pdt of the ordered pair
* Becoming - Relation of the Cartesian pdt
* Becoming - function of the Relation
* Becoming - Inverse rel^n of the rel^n
* Becoming - Inv. fun^n of the fun^n
* Becoming - Equation of the curve
* Becoming - Geometric of the Algebraic
* Becoming - Algebraic of the Geometric
* Becoming - Zero/Root of the function

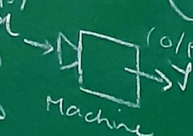
Lecture-29 (7/2/2022)

* $A, B, \times$ (Cartesian pdt.), (a,b) : OP
* $A \times B = \{(a,b) : a \in A, b \in B\}$
* $R = \{ (a,b) : \exists$ 'Rule', $a \in A, b \in B\}$ Relation

  (a: dom, b: Range)

  $R^{-1} = \{(b,a) : (a,b) \in R\}$ Inv. Rel^n
* $f : A \longrightarrow B = \{(x, f(x)) : x \in A\}$ function
* $f : \mathbb{R} \longrightarrow \mathbb{R}$





"Immanent" patterns of Becomings/change

* Becoming - Order pair of the Number
* Becoming - Cartesian pdt of the O.P
* Becoming - relation of the cartesian pt
* Becoming - function of the relation
* Becoming - Equation of the curve (Algebraic) (geometry)
  Becoming - root of the function

Function
- Set theory
  - 1-1 machine
  - Onto machine
  - Invertible machine
- Algebra
  - Polynomial Machine
  - Rational machine
  - Irrational machine
  - Piecewise machine {modulus, Signum, Greatest integer, Fractional part, least integer}
- (Beyond Algebra) Transcendental {Trigonometric / ITF, Logarithmic, Exponential}
- Number theory ($\mathbb{R}$, $\mathbb{C}$)
  - Analytic machine
  - Differentiable machine

(in relation to)

* $\boxed{\text{Function = Machine = transformation = change = Becoming}}$

Polynomial Machine / function / Map. (Poly. patterns of Becoming / Patterns of Becoming - polynomial)

* $y = f(x) = a_0 + a_1 x + a_2 x^2 + \cdots a_n x^n$, $n \in \mathbb{N}$, $a_i, x \in \mathbb{R}$

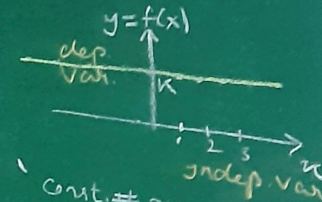
  Poly. fn^n in x

1. Constant Machine

* $y = f(x) = k$, $k \in \mathbb{R}$ Constant

Intuition 1

$f = \{(x, f(x)) : x \in \mathbb{R}\}$
$= \{(x, k) : x \in \mathbb{R}\}$
$= \{(1,k), (7,k), (11,k), (-1,k) \ldots\}$



'Const. # preserving transformation' (Becoming-number of the x)

Intuition 2

* $y = f(x) = k = k(x)^0$

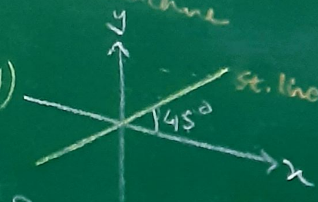
x	1	2	5	....
y	k	k	k	

2. Identity Machine / Stasis Machine / Equil^m machine

* $y = f(x) = x = I$ (x: indep Var)

  (Becoming-identical of the x)

* x (i/p) $\longrightarrow x$ (o/p) $\Rightarrow \nexists$ change $\Downarrow$ 'Equil^m'



'Equil^m preserving transformation'

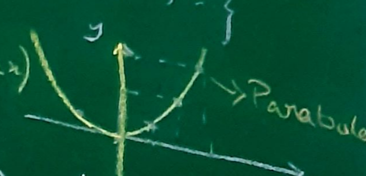
* $\exists$ 1 (Equil^m) : $a \times 1 = 1 \times a = a$ Remark (Number theory)

  $f = \{(x, x) : x \in \mathbb{R}\} = \{(1,1), (-5,-5) \ldots\}$

3. Square Machine

* $y = f(x) = x^2$ (Becoming-sq of the x)

  $f = \{(x, x^2) : x \in \mathbb{R}\}$
  $= \{(0,0), (1,1), (2,4), (3,9), (-1,1), (-2,4), (-3,9)\}$



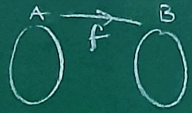
* $f(-x) = \{(x, (-x)^2) : x \in \mathbb{R}\} = f(x)$

* Symmetrical about y-axis

$\boxed{f(-x) = f(x) \Rightarrow \text{Even function / Machine}}$

'Sym about y preserving transf^n'

Lecture-30 (9/2/2022)

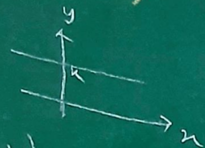


* $f : A \longrightarrow B = \{ (x, \underbrace{f(x)}) : x \in A \}$

* $\boxed{\text{Function = Machine = Transformation = change = Becoming}}$

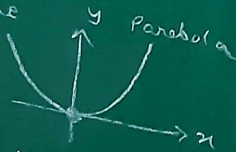
Polynomial Machine (Patterns of Becoming-Polynomial)

* $y = f(x) = a_0 + a_1 x + \quad a_n x^n$
* $y = f(x) = K$ Constant Machine $K \neq 0$.
  (Becoming-number of the var(x))
  'Constant # Preserving transformation'



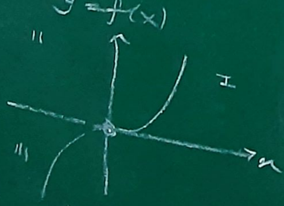
* $f(x) = 0 \Rightarrow \nexists x$ (Never switches off)
* $y = f(x) = x$ Identity Machine (1)
  (Becoming-identical of the var(x)) Equlm / Identity Preserving transformation'

* $y = f(x) = x^2$ Square Machine
* $\underbrace{P : x \to -x}_{\equiv \text{ Parity Transformation}} \Rightarrow f(-x) = f(x)$
  $\Downarrow$
  Even Machine
  'Parity preserving transfn'
  (Becoming-square of the var(x))



sym. about y- Preserving transfn

4. Cube Machine
* $y = f(x) = x^3$
  $f = \{ (x, x^3) : x \in \mathbb{R} \}$
  $= \{ (1, 1), (2, 8), (3, 27), (-1, -1), (-2, -8), (-3, -27) \}$ (Becoming-cube of the var(x))

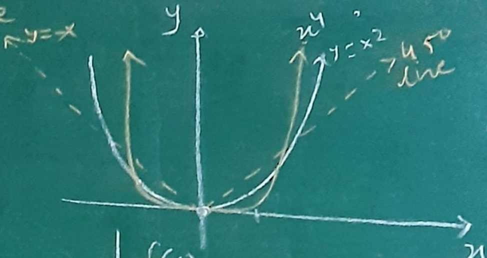


* sym. about opp quad. / "origin"
  $\boxed{P : x \longrightarrow -x \Rightarrow f(-x) = -f(x) \Rightarrow \text{Odd fun}^n}$
  $f(-x) = \{ (-x, \underbrace{(-x)^3}) : x \in \mathbb{R} \} = \{ (-x, -x^3) : x \in \mathbb{R} \}$
  $-f(x) = \{ -(x, x^3) \ {}^{-x^3} : x \in \mathbb{R} \}$

5. Even Power Machine
* $y = f(x) = x^{2n}$

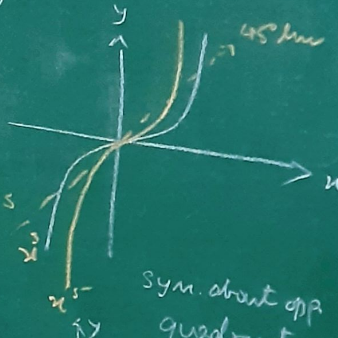
$y = x^2 \quad n = 1$
$y = x^4 \quad n = 2$
$y = x^6 \quad n = 3.$

$f = \{ (x, x^6) : x \in \mathbb{R} \}$
$= \{ (2, 64), (-2, 64) \ldots \}$



$\underbrace{f(x)}_{y} = 0 \Rightarrow \exists x$: machine switches off
symmetrical about y axis

6. Odd Power Machine
* $y = f(x) = x^{2n-1}$



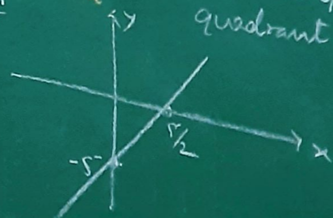
sym. about opp quadrant

7. Machines driving other machines
$\exists$ Couplings & Connections
* $f(x) = y = 2x - 5$

x	0	5/2
y	-5	0

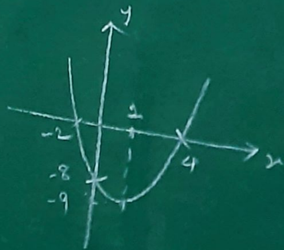
Switches off at $x = \frac{5}{2}$

'Plot relevant points' (0,0)



* $f(x) = x^2 - 2x - 8 = y$

x	0	1	-2	4
y	-8	-9	0	0



Switches off $x = -2, 4$

HW (Quad formulas Allowed)
* $f(x) = -3 - 2x - x^2$
* $f(x) = x^2 - 6x + 9$
* $f(x) = -4x^2 + 4x - 1$
* $g(x) = 2x^2 - 4x + 5$
* $h(x) = -3x^2 + 2x - 1$
* $g(x) = x^3 - 4x$
* $f(x) = x^3 - 2x^2$

Lecture-31 (12/2/2022)

7. Machines driving other machines

$\exists$ Couplings & Connections (Operators : $+, -, /, \times$)

Machines in themselves

* $y = f(x) = 2x - 5$
* $y = f(x) = ax + b$

'Stasis - machine connected to constant machine coupled with constant machine'

a. Quadratic Machine (Sq. machine)

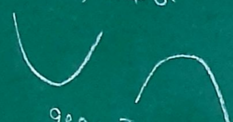
* $y = f(x) = ax^2 + bx + c$; $a, b, c \in \mathbb{R}$, $a \neq 0$.

Algebraic rep^n

* Root / Zero $\Rightarrow f(x) = 0$

$\Downarrow$

$\boxed{ax^2 + bx + c = 0}$



geometrical rep

* "Point"/state/Moment/ Becoming-0 of the machine : Pattern of Becoming-parabola of the machine RETURNS. It's the Return affirmed of Becoming.

{ Pattern of Becoming-parabola of the quadratic Machine

* $f(x) = x^2 - 2x - 8$

$\Downarrow$

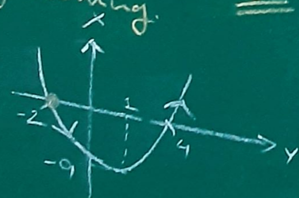
$y = x^2 - 2x - 8 = 0$

$x^2 - 4x + 2x - 8 = 0$

$x(x-4) + 2(x-4) = 0 \Rightarrow (x+2)(x-4) = 0$

$\Downarrow$

$x = -2, 4$



* Thm : A quadratic machine is that which can't have more than 2 patterns of Becoming-0 (roots)

Check

* $ax^2 + bx + c = 0$

let $\exists$ $x = \alpha, \beta, \gamma$ (3 distinct roots) $\alpha \neq \beta \neq \gamma$

* $x = \alpha \longrightarrow a\alpha^2 + b\alpha + c = 0$ —— ①

$x = \beta \longrightarrow a\beta^2 + b\beta + c = 0$ —— ②

$x = \gamma \longrightarrow a\gamma^2 + b\gamma + c = 0$ —— ③

* ② − ① $\Rightarrow a(\beta^2 - \alpha^2) + b(\beta - \alpha) = 0$

$a(\beta+\alpha)(\beta-\alpha) + b(\beta - \alpha) = 0$

$(\beta - \alpha)\{a(\beta+\alpha) + b\} = 0$

$\boxed{a(\alpha+\beta) + b = 0}$ $\because \alpha - \beta \neq 0$

* ③ − ① $\Rightarrow a(\underbrace{\gamma^2 - \alpha^2}_{(\gamma-\alpha)(\gamma+\alpha)}) + b(\gamma - \alpha) = 0 \Rightarrow (\gamma - \alpha)\{a(\gamma+\alpha) + b\} = 0$

$\Big\downarrow \gamma \neq \alpha$

$\boxed{a(\gamma+\alpha) + b = 0}$

* $a(\alpha+\beta) + b = 0$

$a(\gamma+\alpha) + b = 0$

$a(\cancel{\alpha} + \beta - \gamma - \cancel{\alpha}) = 0 \Rightarrow a(\beta - \gamma) = 0 \rightarrow a \neq 0$, $\beta - \gamma \neq 0$

$\Downarrow$

Can't have more than 2 roots - (α, β) $\Leftarrow$ assumption '$\exists\ \alpha, \beta, \gamma$' is wrong

Shreedhacharya (1024 ce) On patterns of becoming-0

* $ax^2 + bx + c = 0$

$a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) = 0 \Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$x^2 + \frac{b}{a}x + \frac{c}{a} + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 = 0$

* $\left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} - \frac{b^2}{4a^2} = 0 \Rightarrow \left(x + \frac{b}{2a}\right)^2 = \underbrace{\frac{b^2}{4a^2} - \frac{c}{a}}_{\frac{b^2 - 4ac}{4a^2}}$

* $x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

* $\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$ 'Becoming-0 of the quad. machine'

* $D \equiv b^2 - 4ac \Rightarrow x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$ $\rightarrow \alpha = \frac{-b + \sqrt{D}}{2a}$, $\rightarrow \beta = \frac{-b - \sqrt{D}}{2a}$

Behaviour of Roots ($\mathbb{R}$)

* $\alpha - \beta = \frac{-\cancel{b} + \sqrt{D} - (-\cancel{b} - \sqrt{D})}{2a} = \frac{2\sqrt{D}}{2a} = \frac{\sqrt{D}}{a}$

* $\alpha - \beta = \frac{\sqrt{D}}{a} \xrightarrow[a \neq 0]{\boxed{D > 0}} \alpha - \beta \neq 0 \Rightarrow \boxed{\alpha \neq \beta}$ 2 distinct real roots

$\boxed{D < 0}$ ⚠ Trouble! $\nexists$ real roots

$\downarrow D = 0$

$\boxed{\alpha = \beta}$ 2 same real roots $\Rightarrow \alpha = -\frac{b}{2a} = \beta$

Lecture-32 (13/2/2022)

Q. Quadratic Machine

* $y = f(x) = ax^2+bx+c \longrightarrow f(x) = ax^2+bx+c = 0$

$$\downarrow$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \begin{cases}\to \alpha \\ \to \beta\end{cases}$$

Becoming -0 of the machine

* $D \equiv b^2-4ac$
  - $D>0$: $\alpha \neq \beta$
    - $a,b,c \in \mathbb{Q}$, D is perfect sq. $\to (\alpha,\beta) \in \mathbb{Q}$
    - $a,b,c \in \mathbb{R}$, D is perfect sq. $\to (\alpha,\beta) \in \mathbb{R}$
    - $D \neq$ perfect $\to (\alpha,\beta) \in \mathbb{Q}'$; $\underbrace{\alpha = 2+\sqrt{3},\ \beta = 2-\sqrt{3}}_{\text{Conjugate pair}}$
  - $D=0$: $\alpha = \beta = -\frac{b}{2a}$
  - $D<0$: $\sqrt{-1} \notin \mathbb{R} \to$ Complex # ($\mathbb{C}$); $\mathbb{R} \subset \mathbb{C}$

* $\alpha = \frac{-b+\sqrt{D}}{2a}$, $\beta = \frac{-b-\sqrt{D}}{2a}$

* $\alpha+\beta = -\frac{b}{a}$

* $\alpha\beta = \left(\frac{-b+\sqrt{D}}{2a}\right)\left(\frac{b+\sqrt{D}}{2a}\right)(-)$

$$= \frac{(b-\sqrt{D})(b+\sqrt{D})}{4a^2} = \frac{b^2-D}{4a^2} = \frac{b^2-(b^2-4ac)}{4a^2}$$

$$\boxed{\alpha\beta = \frac{c}{a}}$$

geometric Representⁿ: $\to D>0$, $\to D=0$, $\to D<0$

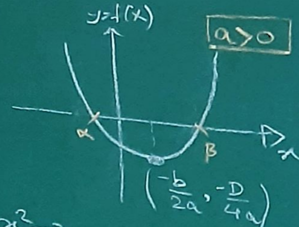
$D>0$

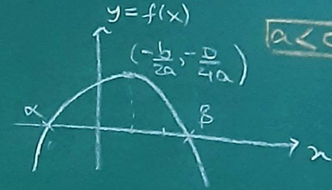
* $ax^2+bx+c=0 \begin{cases}\to \alpha \\ \to \beta\end{cases}$; $\alpha \neq \beta$ Becoming -0 of the machine / cuts at x axis

$\alpha = \frac{-b+\sqrt{D}}{2a}$, $\beta = \frac{-b-\sqrt{D}}{2a} = -\frac{(b+\sqrt{D})}{2a}$

* $ax^2+bx+c = (x-\alpha)(x-\beta) = 0$

$\alpha = \frac{-b+\sqrt{D}}{2a}$, $\beta = -\frac{(b+\sqrt{D})}{2a}$





* $x^2-2x-8=0$

$x^2-4x+2x-8=0$

$x(x-4)+2(x-4)=0$

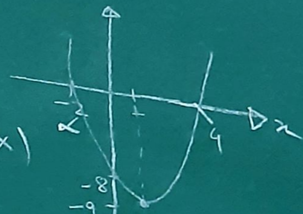
$\alpha=-2$, $\beta=4$ (cuts at x)

$a=1$

$b=-2$

$c=-8$

$D=36>0$



$y = x^2-2x-8 \xrightarrow{x=0} y=-8$ (cut at y)

{is it the minimum val.?} → Pre-Trigo, → Calculus

* $\alpha+\beta = -\frac{b}{a} \Rightarrow \frac{\alpha+\beta}{2} = -\frac{b}{2a}$ = Mid val ← symmetry; Min val of y

$$\boxed{\text{Vertex of Parabola } \left(-\frac{b}{2a}, -\frac{D}{4a}\right)}$$

* $y = a\left(-\frac{b}{2a}\right)^2 + b\left(-\frac{b}{2a}\right) + c = \frac{b^2}{4a} - \frac{b^2}{2a} + c$

$$= -\frac{b^2}{4a} + c = \frac{-b^2+4ac}{4a} = -\frac{D}{4a}$$

* vertex: $\left(\underset{x}{-\frac{b}{2a}}, \underset{y}{-\frac{D}{4a}}\right)$

$D>0$: $\boxed{a>0} \to -\frac{D}{4a} < 0 \Rightarrow y<0$ ∪

$a<0 \to -\frac{D}{4a} > 0 \Rightarrow y>0$ ∩

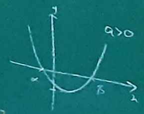
Lecture 3 (14/12/2022)

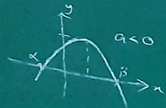
* $ax^2+bx+c=0 \longrightarrow x_{1,2} = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$ → α, → β

* $D = b^2-4ac$, $\alpha+\beta = -\dfrac{b}{a}$, $\alpha\beta = \dfrac{c}{a}$

$D>0$

*



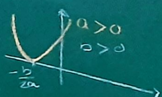


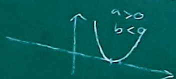
cuts = 2

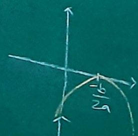
$D=0$

*

$\alpha = \beta = -\dfrac{b}{2a} \longrightarrow$ (minimum) Vertex (Zero) = Root







a<0 b>0

a<0
b<0

* vertex : $\left(-\dfrac{b}{2a}, 0\right)$

→ $a>0$ ∪

$x = -\dfrac{b}{2a}$

→ $b>0$ → $x<0$

→ $b<0$ → $x>0$

→ $a<0$ ∩

$x = -\dfrac{b}{2a}$ → $b>0 \to x>0$; → $b<0 \to x<0$

$D<0$ ∪ ∩

* $ax^2+bx+c=0 \longrightarrow$ (Switch off point) $\alpha = \dfrac{-b+\sqrt{D}}{2a}$, $\beta = \dfrac{-b-\sqrt{D}}{2a}$

$(\alpha, \beta) \notin \mathbb{R}$

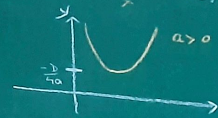
* $\boxed{\text{Switch pts. } \notin \mathbb{R}}$

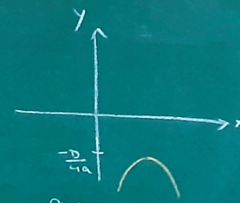
⇓

$\boxed{\nexists \text{ cuts at } x \text{ axis}}$

{Honest to God Cartesian}

* Vertex : $\left(-\dfrac{b}{2a}, -\dfrac{D}{4a}\right)$





* $a>0$ ↓ $y = -\dfrac{D}{4a} > 0$

$a<0$ ↓ $y = -\dfrac{D}{4a} < 0$

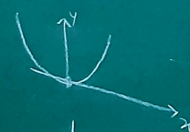
* $ax^2+bx+c=0 \not\Rightarrow (x-\alpha)(x-\beta)=0$ in $\mathbb{R}$ → $x=\alpha,\beta$

$\boxed{\text{Not factorizable in } \mathbb{R}}$

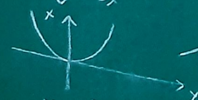
⇓

$x \notin \mathbb{R}$

* $y = x^2$



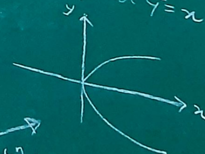
↓ $x \to y$, $y \to x$



$x = y^2$

rot^n by 90° xy plane

$x = y^2$ or $y^2 = x$



Practice

* $\dfrac{1}{x-2} + \dfrac{2}{x-1} = \dfrac{6}{x}$

$\dfrac{x-1+2x-4}{(x-2)(x-1)} = \dfrac{6}{x}$

$6\underbrace{(x-2)(x-1)}_{x^2-3x+2} = x(3x-5) \Rightarrow 3x^2-13x+12 = 0 \to \alpha = 3$, $\beta = 4/3$

$\dfrac{a}{b} + \dfrac{c}{d} = \dfrac{ad+cb}{bd}$

↓ ≠

$\dfrac{b}{a} + \dfrac{d}{c} = \dfrac{bc+ad}{ac}$

* $\dfrac{4}{x} - 3 = \dfrac{5}{2x+3} \Rightarrow -6x^2-6x+12 = 0$

⇓

$x^2+x-2=0$

→ $\alpha = -2$, $\beta = 1$

* $x^2 + \underbrace{2\sqrt{2}x}_{3\sqrt{2}-\sqrt{2}} - 6 = 0$

$x^2+3\sqrt{2}x-\sqrt{2}x-6=0$

$x(x+3\sqrt{2}) - \sqrt{2}(x+3\sqrt{2}) = 0 \Rightarrow \alpha = \sqrt{2}$, $\beta = -3\sqrt{2}$

Lecture-34 (19/2/2022)

Practice 1

$$ax^2+bx+c=0$$

* $\sqrt3 x^2+\underbrace{10x}_{3x+7x}+7\sqrt3=0 \Rightarrow \sqrt3 x(x+\sqrt3)+7(x+\sqrt3)=0$

$\Downarrow$

$x=-7/\sqrt3,\ -\sqrt3$

* $x^2-\underbrace{2ax}_{-\{(a+b)+(a-b)\}x}+a^2-b^2=0$

Method 1

$a^2-b^2=(a+b)(a-b)$

$x^2-(a+b)x-(a-b)x+(a+b)(a-b)=0$

$x(x-(a+b))-(a-b)(x-(a+b))=0$

$(x-a+b)(x-a-b)=0 \Rightarrow x=a+b,\ a-b$

Method 2

* $(x-a)^2-b^2=0 \Rightarrow (x-a-b)(x-a+b)=0$

* $4x^2-\underbrace{4a^2x}_{\{a^2-b^2+a^2+b^2\}}+\underbrace{(a^4-b^4)}_{(a^2-b^2)(a^2+b^2)}=0 \Rightarrow x_{1,2}=\frac{a^2+b^2}{2},\ \frac{a^2-b^2}{2}$

* $9x^2-\underbrace{9(a+b)x}_{-3\{(2a+b)+(a+2b)\}}+\underbrace{(2a^2+5ab+2b^2)}=0$

$2a^2+4ab+ab+2b^2$

$2a(a+2b)+b(a+2b)$

$(2a+b)(a+2b)$

$\Downarrow$

$9x^2-3\{(2a+b)+(a+2b)\}x+(2a+b)(a+2b)=0$

$9x^2-3(2a+b)x-3(a+2b)x+(2a+b)(a+2b)=0$

$x_{1,2}=\frac{2a+b}{3},\ \frac{a+2b}{3}$

* $x^2+\left\{\frac{a}{a+b}+\frac{a+b}{a}\right\}x+\underbrace{1}=0 \to x_{1,2}=-\frac{a+b}{a},\ -\frac{a}{a+b}$

$x^2+\frac{a}{a+b}x+\frac{a+b}{a}x+\frac{a}{a+b}\times\frac{a+b}{a}=0$ Extreme term split

* $x^2+3x-(a^2+a-2)=0 \to x_{1,2}=-(a+2),\ a-1$

* $\frac{1}{a+b+x}=\frac1a+\frac1b+\frac1x \to x_{1,2}=?$

$\frac{1}{a+b+x}-\frac1x=\frac1a+\frac1b$

$\frac{1}{a+b}\ne\frac1a+\frac1b$

$\frac{x-a-b-\cancel{x}}{x(a+b+x)}=\frac{a+b}{ab}\Rightarrow \frac{-\cancel{(a+b)}}{x(a+b+x)}=\frac{\cancel{a+b}}{ab}$

$x^2+(a+b)x+ab=0$

$x^2+ax+bx+ab=0\Rightarrow x_{1,2}=-a,-b$

* $x=\cfrac{1}{2-\cfrac{1}{2-\cfrac{1}{2-x}}}$

$x\ne2$

$\cfrac{1}{\frac1a}=a$

$x=\cfrac{1}{2-\cfrac{1}{\left(\frac{4-2x-1}{2-x}\right)}}=\cfrac{1}{\frac21-\left(\frac{2-x}{3-2x}\right)}$

$x=\cfrac{1}{\frac{6-4x-2+x}{3-2x}}=\frac{3-2x}{4-3x}$

$4x-3x^2=3-2x\Rightarrow 3x^2-6x+3=0$ ✓

$\downarrow$

$x^2-2x+1=0\Rightarrow(x-1)^2$

$\Downarrow$

$x_{1,2}=1,1$

Lecture-35 (20/12/2022)

Practice 2 (special)

1. Infinitely nested Radicals

* $x = a^n$ $\quad n \in \mathbb{R} \to \mathbb{Z} \to$ "Power"
$\qquad\qquad\qquad \to \mathbb{Q} \to$ Surd / radicals

* $x = \sqrt{2+\underbrace{\sqrt{2+\sqrt{2+\sqrt{2+\cdots}}}}_{x}} = 2$

$x = \sqrt{2+x} \Rightarrow x^2 = 2+x \Rightarrow \boxed{x^2 - x - 2 = 0}$

$x^2 - 2x + x - 2$

$x_{1,2} = 2, -1 \Leftarrow x(x-2)+1(x-2) = 0$

* $x = \sqrt{n+\sqrt{n+\sqrt{n+\sqrt{n}}}}$

$x = \sqrt{n+x} \Rightarrow x^2 - x - n = 0$

$x = \frac{1}{2}\left(1+\sqrt{1+4n}\right)$

* $x = \sqrt{n-\sqrt{n-\sqrt{n-\sqrt{n}}}}\cdots$

$x = \sqrt{n-x} \Rightarrow x^2 + x - n = 0 \Rightarrow x = \frac{1}{2}\left(-1+\sqrt{1+4n}\right)$

* $\sqrt{x+\sqrt{x-\sqrt{x+\sqrt{x-\cdots}}}} = 2$

$\sqrt{x+\sqrt{x-\underbrace{\sqrt{x+\sqrt{x-\cdots}}}_{A}}} = A$

$\sqrt{x+\sqrt{x-A}} = A \Rightarrow \sqrt{x+\sqrt{x-2}} = 2$ — (1)

$x + \sqrt{x-2} = 4 \Rightarrow \sqrt{x-2} = (4-x) \Rightarrow (x-2) = (4-x)^2$ — (2)

$x - 2 = 16 + x^2 - 8x \Rightarrow x^2 - 9x + 18 = 0$

$x^2 - 6x - 3x + 18 = 0$

$x(x-6) - 3(x-6) = 0$

$x_{1,2} = 3, 6$

Discard Cond^n:

* $\sqrt{x+\sqrt{x-2}} = 2 \longrightarrow \sqrt{x-2}$: well defined $\Downarrow$ $x \geq 2$

* $\sqrt{x-2} = 4-x \geq 0 \Rightarrow -x \geq -4 \Rightarrow x \leq 4$

$\boxed{2 \leq x \leq 4} \Rightarrow x \neq 6$, $x = 3$

$2 < 3$
$-2 > -3$

* $\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5-\cdots}}}}}}}$ $\quad (+\,+\,-\,+)$

$= \dfrac{2+\sqrt{5}+\sqrt{15-6\sqrt{5}}}{2}$

'Ramanujan's lost Notebook'

* $\sqrt[3]{\sqrt[3]{2}-1} = \sqrt[3]{\frac{1}{9}} - \sqrt[3]{\frac{2}{9}} + \sqrt[3]{\frac{4}{9}}$

2. Continued Fraction

* $x = \dfrac{1}{1+1} = \dfrac{1}{2}$

* $x = \dfrac{1}{1+\frac{1}{1+1}} = \dfrac{1}{1+\frac{1}{2}} = \dfrac{1}{\frac{3}{2}} = \dfrac{2}{3}$

$x = \dfrac{1}{1+\frac{1}{1+\frac{1}{1+1}}} = \dfrac{1}{1+\frac{1}{1+\frac{1}{2}}} = \dfrac{1}{1+\frac{1}{\frac{3}{2}}} = \dfrac{1}{1+\frac{2}{3}} = \dfrac{1}{\frac{5}{3}} = \dfrac{3}{5}$

Fraction $x \in \mathbb{Q}$

{Continued Fraction}
- Terminating $\to x \in \mathbb{Q}$
- Non terminating $\to x \in \mathbb{Q}'$ (Irrationals)

Lecture-36 (21/2/2022)

2 Contd. fraction:

Divisor) Dividend (Q
R

* $x = \frac{m}{n} = a_1 + \frac{p}{n} = a_1 + \frac{1}{\frac{n}{p}}$, a_1 = Quotient, p = Rem.

$\frac{n}{p} = a_2 + \frac{q}{p} = a_2 + \frac{1}{\left(\frac{p}{q}\right)}$, $a_2 = Q$, $q = R$.

* $x = a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \dots}} \equiv a_1 + \frac{1}{a_2+}\,\frac{1}{a_3+}\dots$ gauss' Notation (Condensed)

* $m < n \rightarrow a_1 = 0$
$\downarrow$
$\frac{m}{n} = \frac{1}{\frac{n}{m}}$

* $x = \frac{3}{5} = \frac{1}{\frac{5}{3}} = \frac{1}{1+\left(\frac{2}{3}\right)}$ $3)\overline{5}(1 \leftarrow a_2$, 3, $2 \rightarrow R$

$\frac{2}{3} = \frac{1}{\left(\frac{3}{2}\right)} = \frac{1}{1+\left(\frac{1}{2}\right)}$ $2)\overline{3}(1$, 2, 1

$\left(\frac{1}{2}\right) = \frac{1}{1+1}$

$\frac{3}{5} = \cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+1}}}$ → Terminating
$\downarrow$
$x \in \mathbb{Q}$

* $x = 0.\bar{2}$
$10x = 2.\bar{2}$ $\Big\} \Rightarrow 9x = 2 \Rightarrow x = \frac{2}{9}$

* $x = \frac{2}{9} = \frac{1}{\frac{9}{2}} = \frac{1}{4+\left(\frac{1}{2}\right)} = \cfrac{1}{4+\cfrac{1}{1+1}}$ $2)\overline{9}(4$, 8, 1

* $x = \frac{5}{8} = \frac{1}{\frac{8}{5}} = \frac{1}{1+\left(\frac{3}{5}\right)} = \cfrac{1}{1+\cfrac{1}{1+\left(\frac{2}{3}\right)}}$ $5)\overline{8}(1$, 5, 3; $3)\overline{5}(1$, 3, 2

$= \cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+1}}}}$

b) Non terminating / Approx cont. fraction / Recurring C.F.

* $x = \sqrt{19} = 4 + (\sqrt{19} - 4)$ 'step 1: add/subt. # closest to it'

$\sqrt{19} - 4 = \frac{(\sqrt{19}-4)(\sqrt{19}+4)}{\sqrt{19}+4} = \frac{3}{\sqrt{19}+4}$

* $x = 4 + \frac{3}{\sqrt{19}+4} = 4 + \frac{1}{\left(\frac{\sqrt{19}+4}{3}\right)}$

$\frac{\sqrt{19}+4}{3} = 2 + \underbrace{\frac{\sqrt{19}-2}{3}} = 2 + \frac{5}{\sqrt{19}+2}$

$\frac{(\sqrt{19}-2)}{3}\frac{(\sqrt{19}+2)}{\sqrt{19}+2} = \frac{\cancel{15}\,5}{\cancel{3}(\sqrt{19}+2)}$

$= 2 + \frac{1}{\left(\frac{\sqrt{19}+2}{5}\right)}$

logic (step 2)

* greatest int of $\frac{\sqrt{19}+4}{3} = 2$

Subtract great int from $\frac{\sqrt{19}+4}{3} \Rightarrow \frac{\sqrt{19}+4}{3} - 2$
$= \frac{\sqrt{19}-2}{3}$

* $\frac{\sqrt{19}+2}{5} = 1 + \underbrace{\frac{\sqrt{19}-3}{5}} = 1 + \frac{2}{(\sqrt{19}+3)}$

$\frac{\sqrt{19}-3}{5} \times \frac{\sqrt{19}+3}{\sqrt{19}+3} = \frac{2}{\sqrt{19}+3}$

* $x = \sqrt{19} = 4 + \cfrac{3}{\underbrace{\sqrt{19}+4}}$
$\cfrac{1}{2+\cfrac{5}{\underbrace{\sqrt{19}+2}}}$
$\cfrac{1}{1+\cfrac{2}{\underbrace{\sqrt{19}+3}}}$

$= 4 + \cfrac{1}{2+\cfrac{1}{1+\dots}}$

c. Contd. Frac → Fraction

* $x = \cfrac{1}{1+\left(\cfrac{1}{1+\cfrac{1}{1+\dots}}\right)} = \frac{1}{1+x}$

$x^2 + x - 1 = 0 \Rightarrow x = \frac{-1 \pm \sqrt{5}}{2}$

$x = \frac{\sqrt{5}-1}{2} = \varphi$ 'Golden Ratio' Aristotle
$= 0.61803\dots\dots$ irrational #

* $x = \cfrac{1}{2+\left(\cfrac{1}{2+\cfrac{1}{2+\dots}}\right)} = \frac{1}{2+x}$

$x^2 + 2x - 1 = 0 \Rightarrow x = -1 \pm \sqrt{2}$

$x = \sqrt{2} - 1 = \delta_s$ 'Silver ratio'
$= 0.414213\dots\dots$

Lecture-37 (26/2/2022)

Practice-3

$$a\frac{b}{c} = \frac{ac+b}{c} = a + \frac{b}{c}$$

* $x + \frac{1}{x} = 25\frac{1}{25} = 25 + \frac{1}{25}$

Method 1

* $x^2 + 1 = \left(25 + \frac{1}{25}\right)x \Rightarrow x^2 - \left(25 + \frac{1}{25}\right)x + \underbrace{1}_{25 \times \frac{1}{25}} = 0$

$$\underbrace{x^2 - 25x}_{x(x-25)} \underbrace{- \frac{1}{25}x + 25 \cdot \frac{1}{25}}_{-\frac{1}{25}(x-25)} = 0 \Rightarrow \left(x - \frac{1}{25}\right)(x-25) = 0$$

$$\Downarrow$$

$$x_{1,2} = 25, \frac{1}{25}$$

Method 2

* $x + \frac{1}{x} = 25 + \frac{1}{25} \Rightarrow x = 25, \frac{1}{25}$

* $x^2 - (\sqrt{3}+1)x + \sqrt{3} = 0$

$$x_{1,2} = \sqrt{3}, 1$$

*

$$\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}, \quad x \neq -1, -2, -4$$

$$\frac{1}{x+1} + \frac{2}{x+2} = \frac{1}{x+4} + \frac{3}{x+4}$$

$$\frac{1}{x+1} - \frac{1}{x+4} = \frac{3}{x+4} - \frac{2}{x+2}$$

$$\frac{\cancel{x}+4-\cancel{x}-1}{(x+1)\cancel{(x+4)}} = \frac{3x+6-2x-8}{\cancel{(x+4)}(x+2)}$$

$$\frac{3}{x+1} = \frac{x-2}{x+2} \Rightarrow 3x+6 = x^2 \overbrace{-2x+x}^{-x} - 2$$

$$\Downarrow$$

$$\boxed{x^2 - 4x - 8 = 0}$$

*

$$x_{1,2} = \frac{4 \pm \sqrt{16+32}}{2} = \frac{4 \pm \sqrt{48}}{2} = 2(1 \pm \sqrt{3})$$

* $abx^2 + (b^2 - ac)x - bc = 0 \longrightarrow x_{1,2} = -\frac{b}{a}, \frac{c}{b}$

Perfect Square:

* $x^2 + \underbrace{\frac{(b^2-ac)}{ab}}x - \frac{c}{a} = 0$

$$x^2 + \frac{(b^2-ac)}{ab}x - \frac{c}{a} + \left(\frac{b^2-ac}{2ab}\right)^2 - \left(\frac{b^2-ac}{2ab}\right)^2 = 0$$

$$\left\{x + \underbrace{\left(\frac{b^2-ac}{2ab}\right)}_{K}\right\}^2 - \left\{\frac{c}{a} + \underbrace{\left(\frac{b^2-ac}{2ab}\right)^2}_{K}\right\} = 0$$

$$(x+K)^2 - \left(\sqrt{\frac{c}{a} + K^2}\right)^2 = 0$$

* $\left(x + K - \sqrt{\frac{c}{a} + K^2}\right)\left(x + K + \sqrt{\frac{c}{a} + K^2}\right) = 0$

Inner Calculation:

* $K = \frac{b^2 - ac}{2ab} = \frac{b}{2a} - \frac{c}{2b}$

$$\frac{c}{a} + K^2 = \frac{c}{a} + \frac{b^2}{4a^2} + \frac{c^2}{4b^2} - \cancel{2}\left(\frac{b}{\cancel{2}a}\right)\left(\frac{c}{2b}\right)$$

$$= \frac{c}{2a} + \frac{b^2}{4a^2} + \frac{c^2}{4b^2} = \left(\frac{b}{2a} + \frac{c}{2b}\right)^2$$

* $x + K - \underbrace{\sqrt{\frac{c}{a} + K^2}} = 0 \Rightarrow \left(x + \cancel{\frac{b}{2a}} - \frac{c}{2b} - \cancel{\frac{b}{2a}} - \frac{c}{2b}\right) = 0$

$$\Downarrow$$

$$\boxed{x_1 = \frac{c}{b}}$$

$x + K + \underbrace{\sqrt{\frac{c}{a} + K^2}} = 0 \Rightarrow \left(x + \frac{b}{2a} - \cancel{\frac{c}{2b}} + \frac{b}{2a} + \cancel{\frac{c}{2b}}\right) = 0$

$$\Downarrow$$

$$\boxed{x_2 = -\frac{b}{a}}$$

Shridharacharya's formula:

* $x = \frac{-(b^2-ac) \pm \sqrt{(b^2-ac)^2 - 4(ab)(-bc)}}{2(ab)}$

$$= \frac{-(b^2-ac) \pm \sqrt{(b^2-ac)^2 + 4ab^2c}}{2ab}$$

$$= \frac{-(b^2-ac) \pm \sqrt{b^4 + a^2c^2 - 2b^2ac + 4ab^2c}}{2ab}$$

$$= \frac{-(b^2-ac) \pm \sqrt{b^4 + a^2c^2 + 2acb^2}}{2ab}$$

$$= \frac{-(b^2-ac) \pm \sqrt{(b^2+ac)^2}}{2ab}$$

$$= \frac{-(b^2-ac) \pm (b^2+ac)}{2ab}$$

$$x_1 = \frac{-\cancel{b^2} + ac + \cancel{b^2} + ac}{2ab} = \frac{c}{b}$$

$$x_2 = \frac{-b^2 + ac - b^2 - ac}{2ab} = -\frac{b}{a}$$