Lecture - 10 (25/9/2022)

Practice - 2 $\theta \in \mathbb{R}$ *T-functions manipulation/jugglery*

* $\cos(\alpha-\beta)+\cos(\beta-\gamma)+\cos(\gamma-\alpha) = -\frac{3}{2}$ (given) ; $\sin\alpha+\sin\beta+\sin\gamma = ?$ =
$\cos\alpha+\cos\beta+\cos\gamma = ?$ =

$\cos\alpha\cos\beta+\sin\alpha\sin\beta+\cos\beta\cos\gamma+\sin\beta\sin\gamma+\cos\gamma\cos\alpha+\sin\gamma\sin\alpha = -\frac{3}{2}$

$\underbrace{(\sin\alpha+\sin\beta+\sin\gamma)^2}_{0} + \underbrace{(\cos\alpha+\cos\beta+\cos\gamma)^2}_{0} = 0$

$(\sin\alpha+\sin\beta+\sin\gamma)^2 = \sin^2\alpha+\sin^2\beta+\sin^2\gamma + 2\sin\alpha\sin\beta + 2\sin\beta\sin\gamma + 2\sin\gamma\sin\alpha$

$(\cos\alpha+\cos\beta+\cos\gamma)^2 = \cos^2\alpha+\cos^2\beta+\cos^2\gamma + 2\cos\alpha\cos\beta + 2\cos\beta\cos\gamma + 2\cos\gamma\cos\alpha$

Comment on Complex Number:

$A^2+B^2=0 \Rightarrow A^2=-B^2 \Rightarrow A=\pm\sqrt{-B^2}=\pm iB$, $A^2=(\pm iB)^2=-B^2$

$\Downarrow$ if $A,B \in \mathbb{R}$

$A=0$
$B=0$

$A^2+B^2 = (A+B)^2-2AB$

* $\tan\beta = \dfrac{n\sin\alpha\cos\alpha}{1-n\sin^2\alpha}$ (given) ; $\tan(\alpha-\beta) \overset{\text{P.T.}}{=} (1-n)\tan\alpha$

Trick: { In Formula Short }

$\tan(\alpha-\beta) = \dfrac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)} = \dfrac{\sin\alpha\cos\beta-\cos\alpha\sin\beta}{\cos\alpha\cos\beta+\sin\alpha\sin\beta}$

$= \dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

$\tan(\alpha-\beta) = \dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta} = \dfrac{\tan\alpha-\left(\dfrac{n\sin\alpha\cos\alpha}{1-n\sin^2\alpha}\right)}{1+\tan\alpha\left(\dfrac{n\sin\alpha\cos\alpha}{1-n\sin^2\alpha}\right)} = \dfrac{\dfrac{\sin\alpha}{\cos\alpha}-\dfrac{n\sin\alpha\cos\alpha}{1-n\sin^2\alpha}}{1+\dfrac{\sin\alpha}{\cos\alpha}\left(\dfrac{n\sin\alpha\cancel{\cos\alpha}}{1-n\sin^2\alpha}\right)}$

$= \dfrac{\dfrac{\sin\alpha-n\sin^3\alpha-n\sin\alpha\cos^2\alpha}{\cos\alpha(1-n\sin^2\alpha)}}{\dfrac{1-n\cancel{\sin^2\alpha}+n\cancel{\sin^2\alpha}}{(1-n\sin^2\alpha)}} = \dfrac{\dfrac{\sin\alpha-n\sin^3\alpha-n\sin\alpha\cos^2\alpha}{\cos\alpha-n\cos\alpha\sin^2\alpha}}{\dfrac{1}{1-n\sin^2\alpha}} = \dfrac{\sin\alpha(1-n\sin^2\alpha)-n\sin\alpha\cos^2\alpha}{\cos\alpha(\cancel{1-n\sin^2\alpha})}\times(\cancel{1-n\sin^2\alpha})$

$= \dfrac{\sin\alpha}{\cos\alpha}(1-n\sin^2\alpha-n\cos^2\alpha)$

$= (1-n)\tan\alpha$

{In Form. Short}
Useful Results :

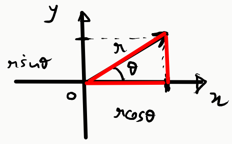
* $\underbrace{\sin(A+B)}_{\sin A\cos B+\cos A\sin B}\overbrace{\sin(A-B)}^{\sin A\cos B-\cos A\sin B} = \sin^2A\cos^2B-\cos^2A\sin^2B = \sin^2A-\sin^2B = \cos^2B-\cos^2A$

* $\underbrace{\cos(A+B)}_{\cos A\cos B-\sin A\sin B}\overbrace{\cos(A-B)}^{\cos A\cos B+\sin A\sin B} = \cos^2A\underbrace{\cos^2B}_{(1-\sin^2B)}-\overbrace{\sin^2A}^{1-\cos^2A}\sin^2B = \cos^2A-\sin^2B\cancel{\cos^2A} - (\sin^2B-\cancel{\cos^2A\sin^2B})$

$= \cos^2A-\sin^2B = \cos^2B-\sin^2A$

Lecture-11 (7/10/2022) 2

* 2D-shape $\longrightarrow \sum(\Delta) \longrightarrow \sum \sin\theta + \sum \cos\theta$
$f(x)$ "Atom"



$\{x, y\} \longrightarrow \{r, \theta\}$ radial

$f(x) = \sum_{i=0}^{n} a_i x^i = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots + a_n x^n$ (Power) n^{th} degree poly machine

$f(x) = \sum_{i=0}^{\infty} a_i x^i = a_0 + a_1 x' + \dots$ $i \in \mathbb{Z}^+$

Infinite Series $\sim$ SWE (QM)
Schrödinger wave Eq^n

$\downarrow$ truncate the Series to low order

E.val Eq^n

$A\psi = \lambda\psi \Rightarrow (A - \lambda I)\psi = 0$
Matrix (operator), #
to calc. λ: $\boxed{|A - \lambda I| = 0}$

$A = \sigma' = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ E.val of σ' : ?

$\left| \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right| = 0 \Rightarrow \begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = 0$

$\lambda^2 - 1 = 0 \Rightarrow \boxed{\lambda = \pm 1}$

$\lambda = -1$: $\left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} - (-1) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right) \psi = 0$

$\boxed{\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}_{2\times2} \psi = 0}$ $\psi = \begin{pmatrix} a \\ b \end{pmatrix}_{2\times1}$

$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}_{2\times2} \begin{pmatrix} a \\ b \end{pmatrix}_{2\times1} = 0 \Rightarrow \begin{pmatrix} a+b \\ a+b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow a + b = 0$ $\boxed{a = -b}$

$b = k \Rightarrow a = -k \Rightarrow \psi = \begin{pmatrix} -k \\ k \end{pmatrix} = k \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

* $s^2 = g_{ab} x^a x^b \longrightarrow s^2 = x^2 + y^2 \longrightarrow \sin^2\theta + \cos^2\theta = 1$
$g_{ab} = \delta_{ab}$

11th grade {In Formul.}

* $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
* $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
* $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

* $\sin 2\theta = 2\sin\theta\cos\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$
* $\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$
$= 2\cos^2\theta - 1 = \dfrac{1 - \tan^2\theta}{1 + \tan^2\theta}$
* $\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

* $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$
* $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$ } ✯

* if $\sin B = 3\sin(2A+B)$; $\longrightarrow$ Prove that $2\tan A + \tan(A+B) = 0$
Pre-given value

$\{A, B\}$ Angles $\longrightarrow$ Relation b/w A & B (given) $\longrightarrow$ diff. Relationship

$\dfrac{a}{b} = \dfrac{c}{d} \Rightarrow \dfrac{a+b}{a-b} = \dfrac{c+d}{c-d}$

Rev. Eng. Trick

* $\dfrac{\sin(2A+B)}{\sin B} = \dfrac{1}{3} \Rightarrow \dfrac{\sin(A + \overbrace{A+B})}{\sin B} = \dfrac{1}{3} \Rightarrow \dfrac{\sin A\cos(A+B) + \cos A \sin(A+B)}{\underset{(A+B-A)}{\sin B}} = \dfrac{1}{3}$

$\dfrac{\overbrace{\sin A\cos(A+B) + \cos A \sin(A+B)}^{a}}{\underbrace{\sin(A+B)\cos A - \cos(A+B)\sin A}_{b}} = \dfrac{1}{3}$ (c / d)

$\boxed{C+D}$

$\dfrac{\{\sin A\cos(A+B) + \cos A\sin(A+B)\} + \{\sin(A+B)\cos A - \cos(A+B)\sin A\}}{\{\sin(A+B)\cos A + \cos(A+B)\sin A\} - \{\sin(A+B)\cos A - \cos(A+B)\sin A\}} = \dfrac{1+3}{1-3}$

$\dfrac{4}{-2} = -2$

$2\tan A + \tan(A+B) = 0$ ✓

$\tan(A+B) = -2\tan A$

$\dfrac{\sin(A+B)}{\cos(A+B)} = \dfrac{-2\sin A}{\cos A}$

$\dfrac{\sin(A+B)\cos A}{\cos(A+B)\sin A} = -2$

HW Booster Shots

* if $2\tan\beta + \cot\beta = \tan\alpha$, P.T : $\cot\beta = 2\tan(\alpha - \beta)$ 4

* $\dfrac{\sin(x+\theta)}{\sin(x+\phi)} = \cos(\theta - \phi) + \cot(x+\phi)\sin(\theta - \phi)$ 1.5

* $\dfrac{\tan(\frac{\pi}{4} + x)}{\tan(\frac{\pi}{4} - x)} = \left(\dfrac{1 + \tan x}{1 - \tan x}\right)^2$ 1

Lecture-12 (8/Oct/2023) 2

HW discussions

* $\frac{\sin(x+\theta)}{\sin(x+\varphi)} = \cdot$ RHS

$$\cos(\theta-\varphi) + \underbrace{\cot(x+\varphi)}_{\frac{\cos(x+\varphi)}{\sin(x+\varphi)}}\sin(\theta-\varphi) = \frac{\sin(x+\varphi)\cos(\theta-\varphi)+\cos(x+\varphi)\sin(\theta-\varphi)}{\sin(x+\varphi)} = \frac{\sin(x+\cancel{\varphi}+\theta-\cancel{\varphi})}{\sin(x+\varphi)} = \frac{\sin(x+\theta)}{\sin(x+\varphi)} = \text{LHS}$$

* $2\tan\beta + \cot\beta = \tan\alpha$ (given) P.T : $\cot\beta = \underbrace{2\tan(\alpha-\beta)}$

$$2\tan(\alpha-\beta) = \frac{2(\tan\alpha-\tan\beta)}{1+\tan\alpha\tan\beta} = \frac{2(2\tan\beta+\cot\beta-\tan\beta)}{1+\tan\alpha\tan\beta} = \frac{2(\tan\beta+\cot\beta)}{1+(2\tan\beta+\cot\beta)\tan\beta} = \frac{2\left(\tan\beta+\frac{1}{\tan\beta}\right)}{1+2\tan^2\beta+1} = \frac{\cancel{2}(\cancel{\tan^2\beta+1})}{\cancel{2}(\cancel{1+\tan^2\beta})}\frac{1}{\tan\beta}$$

$$= \cot\beta$$

* $\tan 3\theta \tan 2\theta \tan\theta = \tan 3\theta - \tan 2\theta - \tan\theta$

Product Sum/Difference

$3\theta = 2\theta+\theta \Rightarrow \tan 3\theta = \tan(2\theta+\theta) = \frac{\tan 2\theta + \tan\theta}{1-\tan 2\theta\tan\theta} \Rightarrow \tan 3\theta - \tan 3\theta\tan 2\theta\tan\theta = \tan 2\theta + \tan\theta \Rightarrow \square$

* $\{\alpha, \beta\}$: $0 \le \alpha, \beta \le 90°$: $\tan\alpha = \frac{m}{m+1}$, $\tan\beta = \frac{1}{2m+1}$ $\longrightarrow$ $\alpha+\beta = ?$

A cute

$$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} = \frac{\pi}{4}$$

* $\alpha+\beta = \frac{\pi}{4}$ $\longrightarrow$ $\underbrace{(1+\tan\alpha)(1+\tan\beta)} = ?$

$$1+\tan\alpha+\tan\beta+\tan\alpha\tan\beta = \underbrace{\left(\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}\right)}_{\tan(\alpha+\beta)=1}(1-\tan\alpha\tan\beta)+(1+\tan\alpha\tan\beta)$$

$$= 1-\cancel{\tan\alpha\tan\beta}+1+\cancel{\tan\alpha\tan\beta} = 2$$

HW

* $\sin^2 6\alpha - \sin^2 4\alpha = \sin 2\alpha \sin 10\alpha$ ✓

diff/Sum Product

* $\frac{\cos^2 33 - \cos^2 57}{\sin^2\frac{21}{2} - \sin^2\frac{69}{2}} = -\sqrt{2}$

* $\sin^2\left(\frac{\pi}{8}+\frac{\alpha}{2}\right) - \sin^2\left(\frac{\pi}{8}-\frac{\alpha}{2}\right) = ?$

* $\cos 2\alpha\cos 2\beta + \sin^2(\alpha-\beta) - \sin^2(\alpha+\beta) = \cos 2(\alpha+\beta)$

* $\tan\alpha - \tan\beta = x$, $\cot\beta - \cot\alpha = y$, P.T $\cot(\alpha-\beta) = ?$ $= \frac{1}{x}+\frac{1}{y}$ ★

Lecture-13 (9/10/2022) 1.5

* $\sin^2 A - \sin^2 B = \sin(A+B)\sin(A-B)$

RHS : $(\sin A\cos B + \cos A\sin B)(\sin A\cos B - \cos A\sin B) = \sin^2 A\underbrace{\cos^2 B}_{(1-\sin^2 B)} - \underbrace{\cos^2 A}_{(1-\sin^2 A)}\sin^2 B = \sin^2 A(1-\sin^2 B) - (1-\sin^2 A)\sin^2 B = \sin^2 A - \sin^2 A\sin^2 B - \sin^2 B + \sin^2 A\sin^2 B$

* $\sin^2 A = \cos^2(A-B) + \cos^2 B - 2\cos(A-B)\cos A\cos B$

RHS = $\cos^2(A-B) + \underbrace{\cos^2 B}_{1-\sin^2 B} - 2\cos(A-B)\cos A\cos B + \cos^2 A - \cos^2 A = \underbrace{(\cos^2 A - \sin^2 B)}_{\cos(A+B)\cos(A-B)} + \cos^2(A-B) - 2\cos(A-B)\cos A\cos B + (1-\cos^2 A)$

$= \cos(A-B)\{\underbrace{\cos(A+B) + \cos(A-B)}_{=2\cos A\cos B} - 2\cos A\cos B\} + (1-\cos^2 A) = \sin^2 A$

Lecture-14 (10/10/2022) 2

HW

* if $\tan(\alpha+\theta) = n\tan(\alpha-\theta)$ P.T : $(n+1)\sin 2\theta = (n-1)\sin 2\alpha$

* if $\tan\alpha = \dfrac{1}{\sqrt{x(x^2+x+1)}}$, $\tan\beta = \dfrac{\sqrt{x}}{\sqrt{x^2+x+1}}$, $\tan\gamma = \sqrt{x^{-3}+x^{-2}+x^{-1}}$ ⟶ P.T $\alpha+\beta=\gamma$

C+D

* $\dfrac{\tan(\alpha+\theta)}{\tan(\alpha-\theta)} = \dfrac{n}{1} \Rightarrow \dfrac{\tan(\alpha+\theta)+\tan(\alpha-\theta)}{\tan(\alpha+\theta)-\tan(\alpha-\theta)} = \dfrac{n+1}{n-1}$

$\tan(\alpha+\beta) = \tan\gamma \Rightarrow \alpha+\beta=\gamma$

* $\cos(\alpha+\beta) = \dfrac{4}{5}$, $\sin(\alpha-\beta) = \dfrac{5}{13}$; $0 \le \alpha,\beta \le \dfrac{\pi}{4}$ ⟶ $\tan 2\alpha = ?$

$\tan 2\alpha = \tan(\alpha+\beta+\alpha-\beta) = \dfrac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)} = \dfrac{56}{33}$

HW 4/5 Dark Side Part 1

* $\cot\alpha\cot 2\alpha + \cot 2\alpha\cot 3\alpha + 2 = \cot\alpha(\cot\alpha - \cot 3\alpha)$ ✓ 6.5

* $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$ ⟶ $\cos(\theta-\frac{\pi}{4}) = ?$ ✓

* $a\tan\alpha + b\tan\beta = (a+b)\tan\left(\dfrac{\alpha+\beta}{2}\right)$ ⟶ $a\cos\beta = b\cos\alpha$ (P.T)

* if $\sin\alpha+\sin\beta = a$, $\cos\alpha+\cos\beta = b$ ⟶ $\cos(\alpha+\beta) = \dfrac{b^2-a^2}{b^2+a^2}$

HW

if α & β are solutions of the Eqn

* $a\tan\theta + b\sec\theta = c$ ⟶ $\tan(\alpha+\beta) = \dfrac{2ac}{a^2-c^2}$ ✓

1 var: θ

↓

Quadratic Eqn

Relationship b/w soln

$\tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

Lecture-15 (16/10/2022) 2

* $\cot\alpha\cot 2\alpha + \cot 2\alpha\cot 3\alpha + \underbrace{2}_{1+1} = \cot\alpha\,(\underbrace{\cot\alpha - \cot 3\alpha}_{})$ $\cot 2\alpha$?

Imp:

$\cot A\cot B + 1 = \dfrac{\cos(A-B)}{\sin A\sin B}$

$\boxed{\text{Not an Id., just a Pattern}}$

* $\underbrace{(\cot\alpha\cot 2\alpha + 1)}_{\frac{\cos(2\alpha-\alpha)}{\sin\alpha\sin 2\alpha}} + \underbrace{(\cot 2\alpha\cot 3\alpha + 1)}_{\frac{\cos(3\alpha-2\alpha)}{\sin 2\alpha\sin 3\alpha}} = \dfrac{\cos\alpha}{\sin\alpha}\left\{\dfrac{\sin\alpha}{\sin\alpha\sin 2\alpha} + \dfrac{\sin\alpha}{\sin 2\alpha\sin 3\alpha}\right\}$

$= \cot\alpha\left\{\dfrac{\sin(2\alpha-\alpha)}{\sin\alpha\sin 2\alpha} + \dfrac{\sin(3\alpha-2\alpha)}{\sin 2\alpha\sin 3\alpha}\right\}$

$= \cot\alpha\left\{\cot\alpha - \cancel{\cot 2\alpha} + \cancel{\cot 2\alpha} - \cot 3\alpha\right\} = RHS$

{In Formula Sheet}

Trick:

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\dfrac{\cos(A-B)}{\sin A\sin B} = \underbrace{\cot A\cot B + 1}$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\dfrac{\sin(A-B)}{\sin A\sin B} = \cot B - \cot A$ (1)

$\dfrac{\sin(A-B)}{\cos B\cos A} = \tan A - \tan B$ (2)

* $a\tan\theta + b\sec\theta = c \Rightarrow c - a\tan\theta = b\sec\theta \Rightarrow c^2 + a^2\underbrace{\tan^2\theta}_{x^2} - 2ca\underbrace{\tan\theta}_{x} = b^2\underbrace{\sec^2\theta}_{(1+\tan^2\theta)}$

Quadratic Eqn

x^2

$\Downarrow$

$\overbrace{(a^2-b^2)}^{a}\tan^2\theta - \overbrace{2ac}^{b}\tan\theta + \overbrace{(c^2-b^2)}^{c} = 0$

$\boxed{\tan\theta = x}$

$\tan\theta_1 = \tan\alpha \qquad x_1 = \alpha$

$\tan\theta_2 = \tan\beta \qquad x_2 = \beta$

$\Downarrow$

$\tan\alpha + \tan\beta = \dfrac{2ac}{a^2-b^2}$

$\tan\alpha\cdot\tan\beta = \dfrac{c^2-b^2}{a^2-b^2}$

$|||$

$ax^2 + bx + c = 0 \Rightarrow x^2 + \dfrac{b}{a}x + \dfrac{c}{a} = 0$

$(x-\alpha)(x-\beta) = 0 \Rightarrow \alpha+\beta = -\dfrac{b}{a}$

$\alpha\beta = \dfrac{c}{a}$

$\Downarrow$

✓ $x^2 - \underbrace{(\alpha+\beta)}x + \alpha\beta = 0$

$\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$1 + \tan^2\theta = \sec^2\theta$

* $\tan(\alpha+\beta) = \dfrac{\frac{2ac}{a^2-b^2}}{1 - \frac{c^2-b^2}{a^2-b^2}} = \dfrac{2ac}{a^2 - \cancel{b^2} - (c^2 - \cancel{b^2})} = \dfrac{2ac}{a^2-c^2}$

* $\tan(\pi\cos\theta) = \cot(\pi\sin\theta) \longrightarrow \cos\left(\theta - \dfrac{\pi}{4}\right) = ?$

$\overbrace{\cos\theta\sin\frac{\pi}{4} + \sin\theta\cos\frac{\pi}{4}}$

$\Downarrow$

$\dfrac{\sin(\pi\cos\theta)}{\cos(\pi\cos\theta)} = \dfrac{\cos(\pi\sin\theta)}{\sin(\pi\sin\theta)} \Rightarrow \underbrace{\cos(\pi\sin\theta)\cos(\pi\cos\theta) - \sin(\pi\sin\theta)\sin(\pi\cos\theta)}_{} = 0$

$\cancel{\cos}(\pi\sin\theta + \pi\cos\theta) = 0 = \cancel{\cos}(2n+1)\dfrac{\pi}{2}$.

$\Downarrow$

* $\cancel{\pi}\sin\theta + \cancel{\pi}\cos\theta = (2n+1)\dfrac{\cancel{\pi}}{2} \Rightarrow \boxed{\sin\theta + \cos\theta = \dfrac{2n+1}{2}} \Rightarrow \underbrace{\underbrace{\dfrac{1}{\sqrt2}}_{\cos\frac{\pi}{4}}\sin\theta + \underbrace{\dfrac{1}{\sqrt2}}_{\sin\frac{\pi}{4}}\cos\theta}_{\cos(\theta-\frac{\pi}{4})} = \dfrac{(2n+1)}{2\sqrt2}$

Comment on Units/dim.

$[\sin\theta] = \#$

$[\theta] = \text{Angle}(°, ^c)$

$[\sin(\underbrace{\theta\cos\theta}_{\text{Angle}})] = \#$

$\#$

Math ≠ Arithmetic

Lecture-16 (20/oct/2022) 1.5+15

comment on periodicity of sin and cos:

Periodic-machine

* $y = f(\theta) = 0 \Rightarrow y = \cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}, \frac{3}{2}\pi, \frac{5}{2}\pi, \ldots (2n+1)\frac{\pi}{2}$ $n \in \mathbb{Z}$

$0 = \cos\frac{\pi}{2}$.

$0 = \cos\frac{3\pi}{2}$

Becoming-zero of the machine

Transcendental
=
Beyond Algebra

* $y = f(-\theta) = \begin{cases} f(\theta) & \text{Even function} \\ -f(\theta) & \text{Odd function} \end{cases}$ $\longrightarrow \cos(-\theta) = \cos\theta$ ✓

* $\boxed{\cos(2n+1)\frac{\pi}{2} = 0, \quad \sin n\pi = 0}$ ✓ In form.

* $\sin\theta = 0 \Rightarrow \theta = 0, \pi, 2\pi, 3\pi, \ldots \ldots n\pi$ $n \in \mathbb{Z}$

HW

* $a\tan\alpha + b\tan\beta \stackrel{\text{given}}{=} (a+b)\tan\left(\frac{\alpha+\beta}{2}\right)$ PT: $\overline{a\cos\beta = b\cos\alpha}$ $\alpha \neq \beta$

* if α & β are sol^n of $a\cos\theta + b\sin\theta = c$, $\cos(\alpha-\beta) = \frac{2c^2 - (a^2+b^2)}{a^2+b^2}$

Lecture-17 (22/oct/2022) 2

* $a\tan\alpha + b\tan\beta = (a+b)\tan\left(\frac{\alpha+\beta}{2}\right) = a\left\{\tan\alpha - \tan\left(\frac{\alpha+\beta}{2}\right)\right\} = b\left\{\tan\left(\frac{\alpha+\beta}{2}\right) - \tan\beta\right\}$

$\underbrace{a\tan\left(\frac{\alpha+\beta}{2}\right) + b\tan\left(\frac{\alpha+\beta}{2}\right)}$, $\frac{\alpha}{2}+\frac{\beta}{2}$

$\frac{\sin\left(\cancel{\frac{\alpha}{2}-\frac{\beta}{2}}\right)}{\cos\alpha\cos\cancel{\frac{\alpha+\beta}{2}}}$ $\frac{\sin\left(\cancel{\frac{\alpha}{2}-\frac{\beta}{2}}\right)}{\cos\cancel{\frac{\alpha+\beta}{2}}\cos\beta}$

$\tan A - \tan B = \frac{\sin(A-B)}{\cos A\cos B}$

$\Rightarrow a\cos\beta = b\cos\alpha$

yrs = 1-2

8 months

Trig. vol. 2 – full blown Identities / graphs (Till High School) $\xrightarrow[\text{of flight}]{2^{nd}\text{ line}}$ Coord. – slope / Ratios $\longrightarrow$ Difference, Limits, Continuity, Derivative, Diff./Int. Calculus

└ T. Equation

1st line of flight

Coord. Geometry (11-12)

Section formula

└→ Conic Sections: St. Lines, Circle, Para, Ellipse, Hyperbola

$\xrightarrow{3^{rd}\text{ line of flight}}$ Set, Relation, Function, Binary op, Group theory → Discrete; → Continuous SO(n), SU(n)

Cantor, Gödel, Turing } GEB Book

Probability theory → Complexity: P, NP: Hard, Easy

Counting ^nP_r ^nC_r

Statistics

Puzzle

* $\text{Max}(7\cos\theta + 24\sin\theta) = ?$ ✓ → Pre-Calc. ✓

$\text{Min}(7\cos\theta + 24\sin\theta) = ?$ → Post-Calc. (Later) ✗

Rule: Min # of hrs = 1

* $f(\theta) = \sqrt{3}\sin\theta + 6\cos\theta$ ⟶ Simplify it! (Reduction)

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

Lecture-18 (23/oct/2022) 2

* ∃ 2 kinds of phenomena in the world

Computable ✓ | Uncomputable ✓

* $\underset{\sim}{F} = \dfrac{dp}{dt} = m\underbrace{\dfrac{d^2x}{dt^2}}_{\underset{\sim}{a}}$ NLM II (Near Equi^m)

2nd order in t-val Diff. Equation ??

Leibniz

* Will it Rain tommorow?

* Feeling / Duration / States of 'Consciousness'

Bergson / Nietzsche

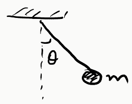
" Laws of Nature are unprovable, but you taste them / you experience them / you Palpate them. "

– A & D

Pendulum (SHM ~ Harmonic Oscillator)

* $\underset{\sim}{F} = -kx$ spring force

$m\dfrac{d^2x}{dt^2}$



$\dfrac{d^2x}{dt^2} + \underbrace{\dfrac{k}{m}}_{=\omega^2} x = 0 \Rightarrow \boxed{\dfrac{d^2x}{dt^2} + \omega^2 x = 0}$ ω : freq. of H-O

H-O D.E (2nd order)

* $x(t) = A\sin(\omega t + \phi)$ Exact Analytic Solution

$\boxed{\text{Ansatz "guess Sol}^n\text{"}}$

$\dfrac{dx}{dt} = A\cos(\omega t+\phi).\omega$

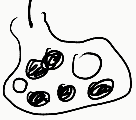
$\dfrac{d^2x}{dt^2} = -A\omega^2\sin(\omega t+\phi) = -\omega^2 x \Rightarrow \dfrac{d^2x}{dt^2} + \omega^2 x = 0$

H-O Eq^n

* $x(t) = A\underbrace{\sin(\omega t+\phi)}_{\text{Predictable}}$ Final trajectory

FAPP / Probabilistic / Statistic / Averages

Typicality = Law of Large # = Belief



Black is picked

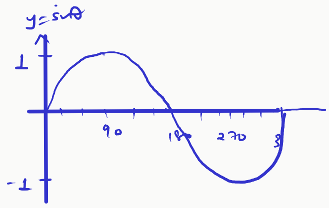
400 B

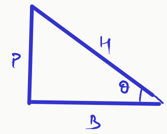
30 W

Lecture-19 (24/oct/2022) 1.5

c. Taste of Calculus Part 2 (Maximum / Minimum) check Alg. vol-2

upper bound " lower bound "





$\sin\theta = \frac{P}{H}$

* $f(\theta) = a\cos\theta + b\sin\theta$

Inequality Relⁿ (Range)

$y = \sin\theta$: $0 \quad \frac{1}{2} \quad \frac{1}{\sqrt{2}} \quad \frac{\sqrt{3}}{2} \quad 1 \quad \frac{\sqrt{3}}{2} \quad \frac{1}{\sqrt{2}} \quad \frac{1}{2} \quad 0 \quad \frac{-1}{2} \quad \frac{-1}{\sqrt{2}} \quad \frac{-\sqrt{3}}{2} \quad -1$

$\frac{-\sqrt{3}}{2} \quad \frac{-1}{\sqrt{2}} \quad \frac{-1}{2} \quad 0$

Range

$$\boxed{\begin{array}{lll} -1 \le \sin\theta \le 1 & -\infty < \tan\theta < \infty & -\infty < \cot\theta < \infty \\ -1 \le \cos\theta \le 1 & \operatorname{cosec}\theta \in (-\infty,-1] \cup [1,\infty) & \\ & \sec\theta \in (-\infty,-1] \cup [1,\infty) & \end{array}}$$

{In Form.} Later: ITF (Trig. Adv.)

Simplify / Analysis

Trick

* $f(\theta) = a\cos\theta + b\sin\theta$ $\quad a, b \in \mathbb{R}$ $\quad \boxed{\begin{array}{l} a = r\sin\alpha \\ b = r\cos\alpha \end{array}}$

$= r\{\sin\alpha\cos\theta + \cos\alpha\sin\theta\}$

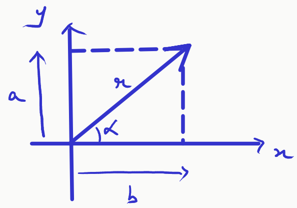
$\boxed{f(\theta) = r\sin(\alpha+\theta)}$

$\Downarrow$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$-1 \le \sin(\alpha+\theta) \le 1 \Rightarrow -r \le \underbrace{r\sin(\alpha+\theta)}_{f(\theta)} \le r \Rightarrow \boxed{-r \le f(\theta) \le r}$



givens : a, b, θ

Ad. hoc var. : r, α

$\Big\} \longrightarrow$ $a^2+b^2 = r^2(\underbrace{\sin^2\alpha+\cos^2\alpha}_{1}) \Rightarrow r = \sqrt{a^2+b^2}$

$\frac{a}{b} = \frac{r\sin\alpha}{r\cos\alpha} \Rightarrow \tan\alpha = \frac{a}{b}$

$b = r\cos\alpha$

$a = r\sin\alpha$

$$\boxed{-\sqrt{a^2+b^2} \le a\cos\theta + b\sin\theta \le \sqrt{a^2+b^2} \quad \forall\,\theta}$$

{In Form.}

$\underbrace{-\sqrt{a^2+b^2}}_{\text{Minimum val}}$ $\qquad \underbrace{\sqrt{a^2+b^2}}_{\text{Maximum val}}$

$10 = 2 \times 5$

$12 = 2 \times 6$

* $f(\theta) = 3\cos\theta + 5\sin\left(\theta - \frac{\pi}{6}\right) \longrightarrow -\sqrt{19} \le f(\theta) \le \sqrt{19}$

* $g(\theta) = 4\sin\theta - 3\cos\theta + 7 \longrightarrow 2 \le f(\theta) \le 12$

* a, b, θ : $a \le 3\cos\theta + 5\sin\left(\theta - \frac{\pi}{6}\right) \le b \qquad a, b = ?$

* $\alpha(\theta) = (2\sqrt{3}+3)\sin\theta + 2\sqrt{3}\cos\theta$

$\sqrt{(2\sqrt{3}+3)^2 + (2\sqrt{3})^2} = \sqrt{(2\sqrt{3})^2 + 9 + 6\cdot2\sqrt{3} + (2\sqrt{3})^2} = \sqrt{12\sqrt{3} + 33}$ ✓

Lecture-20 (25/oct) 1.5

d. Addition of T-functions

* $f(\theta) = a\cos\theta + b\sin\theta = r\{\underbrace{\sin\alpha\cos\theta + \cos\alpha\sin\theta}_{\sin(\alpha+\theta)}\} = r\sin(\alpha+\theta) = \sqrt{a^2+b^2}\,\sin(\alpha+\theta)$ ✓

$\downarrow$

$r\{\cos\alpha\cos\theta + \sin\alpha\sin\theta\} = r\cos(\alpha-\theta) = r\cos(\theta-\alpha) = \sqrt{a^2+b^2}\cos(\alpha-\theta)$

1st subs :

$a = r\sin\alpha$

$b = r\cos\alpha$

$\tan\alpha = \frac{a}{b}$

$r = \sqrt{a^2+b^2}$

2nd subs.

$a = r\cos\alpha$

$b = r\sin\alpha$

* $f(\theta) = \sqrt{3}\sin\theta + \cos\theta \begin{cases} \to r\sin(\theta+\alpha) \\ \to r\cos(\theta-\alpha) \end{cases}$ $r = 2$

$$f(\theta) = \sqrt{a^2+b^2}\left\{ \underbrace{\frac{a}{\sqrt{a^2+b^2}}}_{\sin\alpha}\sin\theta + \underbrace{\frac{b}{\sqrt{a^2+b^2}}}_{\cos\alpha}\cos\theta \right\} = 2\left\{ \underbrace{\frac{\sqrt{3}}{2}}_{\sin\frac{\pi}{3}}\sin\theta + \underbrace{\frac{1}{2}}_{\cos\frac{\pi}{3}}\cos\theta \right\} = 2\cos\left(\theta - \frac{\pi}{3}\right)$$

$$2\left\{ \underbrace{\frac{\sqrt{3}}{2}}_{\cos\frac{\pi}{6}}\sin\theta + \underbrace{\frac{1}{2}}_{\sin\frac{\pi}{6}}\cos\theta \right\} = 2\sin\left(\theta + \frac{\pi}{6}\right)$$

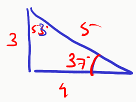
* $f(\theta) = 3\cos\theta - 4\sin\theta$ $r = 5$

$= 5\left\{ \underbrace{\frac{3}{5}}_{\sin\alpha}\cos\theta - \underbrace{\frac{4}{5}}_{\cos\alpha}\sin\theta \right\} = \underline{5\sin(\alpha-\theta)}$; $\sin\alpha = \frac{3}{4}$, $\cos\alpha = \frac{4}{5} \Rightarrow \tan\alpha = \frac{3}{4} \Rightarrow \alpha = 37°$

$= 5\{\cos\alpha\cos\theta - \sin\alpha\sin\theta\} = 5\cos(\theta+\alpha)$; $\sin\alpha = \frac{4}{5}$ $\cos\alpha = \frac{3}{5} \Rightarrow \tan\alpha = \frac{4}{3} \Rightarrow \alpha = 53°$

$\parallel$

$5\cos\left(\theta + \tan^{-1}\left(\frac{4}{3}\right)\right)$



Physical triangle

* $h(\theta) = \sin 100 + \cos 100 \begin{cases} \to >0 \\ \to <0 \\ \to =0 \\ \to \text{Imaginary} \end{cases}$

$= \sqrt{2}\sin 145 > 0$

$= \sqrt{2}\cos 55 > 0$

* $y = \underset{\uparrow}{y_1} + \underset{\uparrow}{y_2} = \overset{\text{Amplitude}}{A_1}\underbrace{\sin\omega t} + A_2\sin(\omega \overset{\text{time}}{t} + \phi) = ?$ Add the SHMs

1^{st} H.O 2^{nd} H.O phase "Phi"

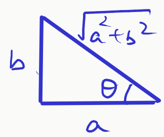
Lecture-21 (28/oct)

Comment on SHM

* $y = \overset{\text{given}}{A_1}\sin\omega t + A_2\sin(\overset{\text{given 2}}{\omega}\overset{\text{Time}}{t} + \overset{\text{given}}{\phi}) = A_1\sin\omega t + A_2\sin\omega t\cos\phi + A_2\cos\omega t\sin\phi$

$= \underbrace{(A_1 + A_2\cos\phi)}_{=a}\sin\omega t + \underbrace{(A_2\sin\phi)}_{=b}\cos\omega t = a\sin\omega t + b\cos\omega t$

$= \sqrt{a^2+b^2}\left\{ \underbrace{\frac{a}{\sqrt{a^2+b^2}}}_{=\cos\theta}\sin\omega t + \underbrace{\frac{b}{\sqrt{a^2+b^2}}}_{\sin\theta}\cos\omega t \right\}$

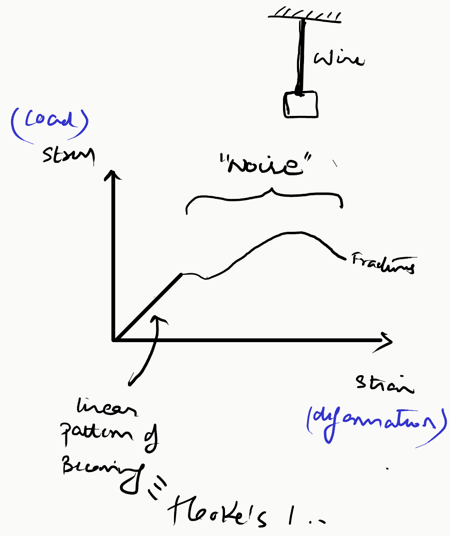


$\sin\theta = \frac{b}{\sqrt{a^2+b^2}}$ $\tan\theta = \frac{b}{a}$

$\cos\theta = \frac{a}{\sqrt{a^2+b^2}}$

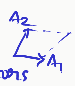
$= \underbrace{\sqrt{a^2+b^2}}\left\{ \sin(\omega t + \theta) \right\}$

$\sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$



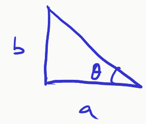
M. Imp. Formula {In formula sheet}

$$\boxed{y = A_1\sin\omega t + A_2\sin(\omega t+\phi) = \underbrace{\sqrt{A_1^2+A_2^2+2A_1A_2\cos\phi}}_{=A_{net}}\sin(\omega t+\theta) = A_{net}\sin(\omega t+\theta) \; ; \; \tan\theta = \frac{\overset{\text{Const.}}{A_2\sin\phi}}{A_1 + A_2\cos\phi}}$$



- vectors
- kinematics
- SHM
- Waves
- EMI
- A.C
- Wave optics

* $y = \overset{A_1}{4\sin\omega t} + \overset{A_2}{3}\underbrace{\sin(\omega t + \overset{\varphi}{30})}_{3\left\{\sin\omega t \frac{\sqrt{3}}{2} + \cos\omega t \frac{1}{2}\right\}} = \underbrace{\left(4 + \frac{3\sqrt{3}}{2}\right)}_{a}\sin\omega t + \underbrace{\frac{3}{2}}_{b}\cos\omega t = a\sin\omega t + b\cos\omega t$



$\cos\theta = \frac{a}{\sqrt{a^2+b^2}}$

$\sin\theta = \frac{b}{\sqrt{a^2+b^2}}$

$\tan\theta = \frac{b}{a} = \frac{3/2}{4+\frac{3\sqrt{3}}{2}}$

$\tan\theta = \frac{3}{8+3\sqrt{3}}$

$= \sqrt{a^2+b^2}\left\{\underbrace{\frac{a}{\sqrt{a^2+b^2}}}_{\cos\theta}\sin\omega t + \underbrace{\frac{b}{\sqrt{a^2+b^2}}}_{\sin\theta}\cos\omega t\right\} = \sqrt{\left(4+\frac{3\sqrt{3}}{2}\right)^2 + \left(\frac{3}{2}\right)^2}\sin(\omega t + \theta)$

$= \sqrt{16 + \frac{27}{4} + 12\sqrt{3} + \frac{9}{4}}\sin(\omega t + \theta) = \sqrt{12\sqrt{3} + 25}\sin\left(\omega t + \tan^{-1}\left(\frac{3}{8+3\sqrt{3}}\right)\right)$

$\parallel$

direct

* $y = \sqrt{16 + 9 + 24\frac{\sqrt{3}}{2}}\sin(\omega t + \theta)$

HW

* $y_1 = 3\sin\omega t$, $y_2 = 4\cos\omega t \longrightarrow y = y_1 + y_2$ → 1. Amp. of resultant → 2. Eqn
* $y_1 = 5\sin(\omega t + 30)$, $y_2 = 10\cos\omega t$
* $y_1 = A_1\sin\omega t$, $y_2 = A_2\sin(\omega t + \pi/3)$ } $y = y_1 + y_2$ HW
* $x = A_1\sin\omega t$, $y = A_2\sin(\omega t + \varphi)$ ☆

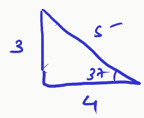
M	T	W	T	F	Sat	Sun
Math	Bio	Piano	Chem.	Bio	Phy	Phy
					Drum	german
					German	Math
					Math	Chem.

school

Not Fun!!

Lecture-22 (29/oct) 2

* $y = 3\sin\omega t + 4\underbrace{\cos\omega t}_{\sin(\omega t + \frac{\pi}{2})} = \overset{A_1}{3}\sin\omega t + \overset{A_2}{4}\sin(\omega t + \underset{\varphi}{\tfrac{\pi}{2}}) = 5\sin(\omega t + \theta) = 5\sin(\omega t + 53^\circ)$



2nd App

$= 5\left\{\underbrace{\frac{3}{5}}_{\cos\theta}\sin\omega t + \underbrace{\frac{4}{5}}_{\sin\theta}\cos\omega t\right\} = 5\sin(\omega t + \theta)$, $\tan\theta = \frac{4}{3} \Rightarrow \theta = 53^\circ$

* $y = 5\sin(\omega t + 30^\circ) + 10\cos\omega t = \sqrt{175}\sin\left(\omega t + \tan^{-1}\frac{5}{\sqrt{3}}\right)$ $\qquad \tan\theta = \frac{\frac{25}{2}}{\frac{5\sqrt{3}}{2}}$

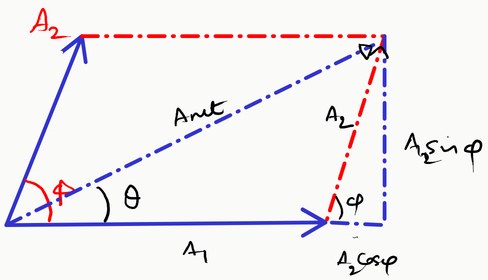
Vector Trick

Angle diff $= \omega t + \varphi - \omega t = \varphi$

* $y = \overset{\#}{A_1}\sin\omega t + \overset{\#}{A_2}\sin(\omega t + \varphi) = A_{net}\sin(\omega t + \theta)$ → Angle b/w net & A_1

✓ $A_{net} = \sqrt{A_1^2 + A_2^2 + 2\underset{\sim}{A_1}\cdot\underset{\sim}{A_2}}$ $\qquad \underset{\sim}{A_1}\cdot\underset{\sim}{A_2} = |A_1||A_2|\cos\varphi$

$\theta = \tan^{-1}\left(\frac{A_2\sin\varphi}{A_1 + A_2\cos\varphi}\right)$



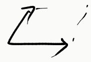
$\parallel^{gm}$ law of vector

Imp pt: (check-points)

1. Convert every T-fun into sine with 'ωt' term as +ve.
2. +ve sign in b/w the T-funn
3. A_1 = Number with smaller Angle
4. $y_{net} = A_{net}\sin(\text{Angle of } A_1 + \theta)$

HW (using vector trick)

* $y = 4\sin(\omega t + 120^\circ) + 3\sin(\omega t + 30^\circ)$
* $g = 3\sin(\omega t + 30^\circ) - 4\sin(\omega t + 60^\circ)$
* $z = 3\sin(\omega t + 30^\circ) + 4\cos(\omega t + 60^\circ)$

 with vector diag

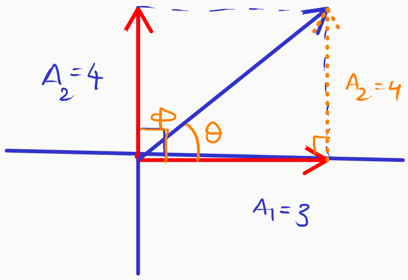
Lecture-23 (30/oct) 1.5

* $y = 4\sin(\omega t + 120) + \underset{\uparrow A_1}{3}\sin(\underbrace{\omega t + 30}_{=x})$, $x = \omega t + 30^\circ$ transformation ($\sim$ Rot^n)

$= \underset{A_1}{3}\sin x + \underset{A_2}{4}\sin(x + \underbrace{90}_{\phi})$

$= A_{net}\sin(x + \theta)$ $\quad A_{net} = \sqrt{3^2 + 4^2 + 2.3.4\cos 90}$

$\theta = \tan^{-1}\frac{4}{3} = 53$



$\tan\theta = \frac{4}{3} \Rightarrow \theta = 53^\circ$

$y = 5\sin(x + 53) = 5\sin(\omega t + 30 + 53)$ ✓

* $z = 3\sin(\omega t + 30) + 4\underbrace{\cos(\omega t + 60^\circ)}_{\underbrace{\sin(90 - \omega t - 60)}_{-\sin(\omega t - 30)}} = 3\sin(\omega t + 30) - \underbrace{4\sin(\omega t - 30)}_{4\sin(180 + \omega t - 30)} = 3\sin(\underbrace{\omega t + 30}_{=x}) + 4\sin(\omega t + 150)$

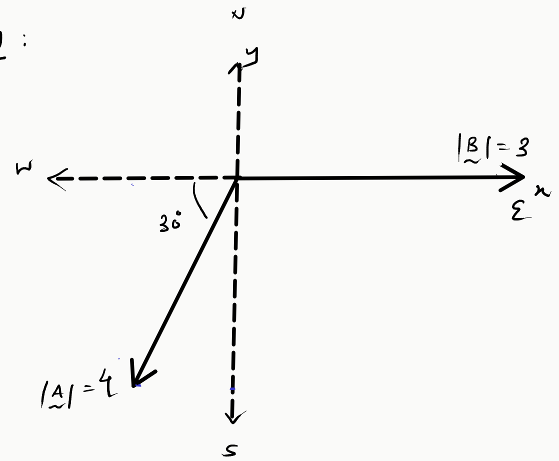
$= \underset{A_1}{3}\sin x + \underset{A_2}{4}\sin(x + \underbrace{120}_{\phi}) = A_{net}\sin(x + \theta)$; $A_{net} = \sqrt{3^2 + 4^2 + 2.3.4\cos 120}$

$\tan\theta = \dfrac{4\sin 120}{3 + 4\cos 120} = \dfrac{\frac{\cancel{4}^{2}\sqrt{3}}{\cancel{2}}}{3 - 4\frac{1}{2}} = -2\sqrt{3}$

$= \sqrt{13}\sin(x + \tan^{-1}2\sqrt{3}) = \sqrt{13}\sin(\omega t + 30 + \tan^{-1}2\sqrt{3})$

HW :

*



	$\underset{\sim}{A}$	$\underset{\sim}{B}$	$\underset{\sim}{C} = \underset{\sim}{A} + \underset{\sim}{B}$
Mag	4	3	?
Dir^n	30° S of W	E	?

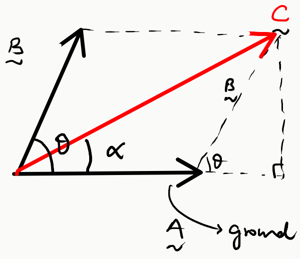
mit vector

$\underset{\sim}{A} = a\hat{i} + b\hat{j}$

mag.

Lecture-24 (31/oct) 1.5

e. Addition of Vectors (Detour : Check vectors$^{\text{Later}}$) : Manipulation of Vectors



$*\ \boxed{\underset{\sim}{C} = \underset{\sim}{A} + \underset{\sim}{B}} \Rightarrow |\underset{\sim}{C}|^2 = \underset{\sim}{C}\cdot\underset{\sim}{C} = (\underset{\sim}{A}+\underset{\sim}{B})\cdot(\underset{\sim}{A}+\underset{\sim}{B}) = \underset{\sim}{A}\cdot\underset{\sim}{A} + \underset{\sim}{B}\cdot\underset{\sim}{B} + \underbrace{\underset{\sim}{A}\cdot\underset{\sim}{B} + \underset{\sim}{B}\cdot\underset{\sim}{A}}_{= 2\underset{\sim}{A}\cdot\underset{\sim}{B}}$

M D M D M D

Master Equation

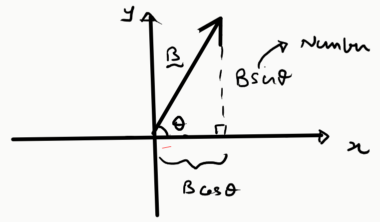
only for (vectors : Commutativity)

Extra structure

$$\boxed{|\underset{\sim}{C}|^2 = |\underset{\sim}{A}|^2 + |\underset{\sim}{B}|^2 + 2\underset{\sim}{A}\cdot\underset{\sim}{B}}\ ,\quad \underset{\sim}{A}\cdot\underset{\sim}{B} = |\underset{\sim}{A}||\underset{\sim}{B}|\cos\theta$$

'11^{gm} law of vectors'

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$



Basis vectors $\hat{i}\cdot\hat{i} = 1$

$\underset{\sim}{B} = (|\underset{\sim}{B}|\cos\theta)\hat{i} + (|\underset{\sim}{B}|\sin\theta)\hat{j}$

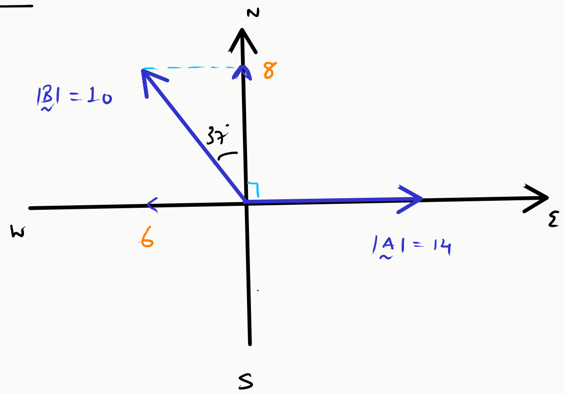
$|\underset{\sim}{B}|^2 = (|\underset{\sim}{B}|\cos\theta)^2 + (|\underset{\sim}{B}|\sin\theta)^2$

$\tan\theta = \dfrac{B\sin\theta}{B\cos\theta}$

e.1 Different formats

Type 1

Q1



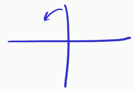
$\underset{\sim}{B}$: 37 W of N

Resultant of $\underset{\sim}{A}, \underset{\sim}{B}$

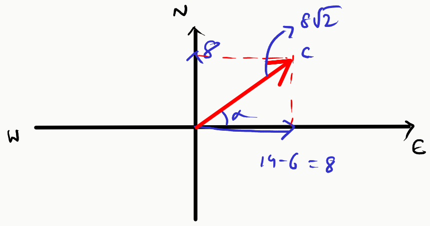
$\underset{\sim}{C} \equiv \underset{\sim}{A} + \underset{\sim}{B} = ?$ → M = $8\sqrt{2}$; → D : $\alpha = 45°$

$|C|^2 = (10)^2 + (14)^2 + \underbrace{2(10)\cdot(14)\cos(90+37)}_{-2\cdot 10\cdot 14\cdot \frac{3}{5}}$

$= 128$



$\cos(90+\theta) = -\sin\theta$



$\tan\alpha = \dfrac{8}{8} = 1 \Rightarrow \alpha = 45°$

Type 2

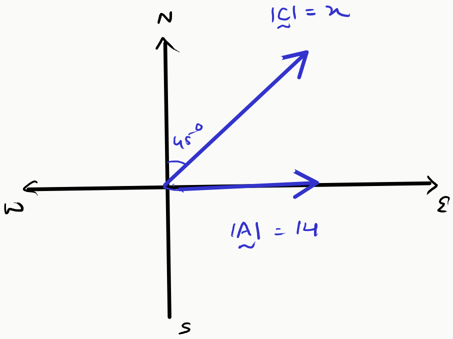
Q2

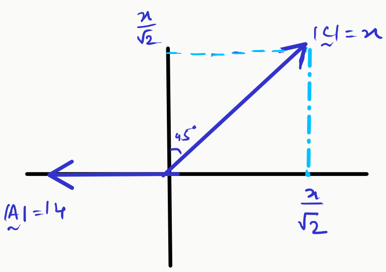
	M	D
$\underset{\sim}{A}$	14	E
$\underset{\sim}{B}$	10	?
$\underset{\sim}{C}$	?	N.E

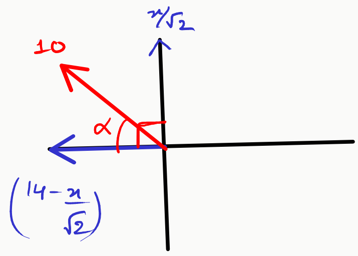
$\underset{\sim}{C} \equiv \underset{\sim}{A} + \underset{\sim}{B}$ (not given)

$\underset{\sim}{B} = \underset{\sim}{C} + (-\underset{\sim}{A})$ (given)

$|\underset{\sim}{B}| = 10$







HW :

$10^2 = \left(\dfrac{x}{\sqrt{2}}\right)^2 + \left(14 - \dfrac{x}{\sqrt{2}}\right)^2 \Rightarrow x = ?$

$\tan\alpha = \dfrac{\frac{x}{\sqrt{2}}}{\left(14 - \frac{x}{\sqrt{2}}\right)} = ?$

Sat : Test - Trigo. Vol. 2 } S-10 No new questions

$*\ 100 = \dfrac{x^2}{2} + \underset{196}{14^2} + \dfrac{x^2}{2} - 2\cdot 14\cdot\dfrac{x}{\sqrt{2}} \longrightarrow x^2 - 14\sqrt{2}x + 96 = 0 \Rightarrow x = \dfrac{14\sqrt{2} \pm \sqrt{392 - 384}}{2} = \dfrac{14\sqrt{2} \pm \overset{2\sqrt{2}}{\sqrt{8}}}{2}$

→ $8\sqrt{2}$ → $\tan\alpha = \frac{4}{3}$, $\alpha = 53°$

→ $6\sqrt{2}$ → $\tan\alpha = \frac{3}{4}$, $\alpha = 37°$

Lecture-25 (5/Nov.) 2

Test 1 (Total = 50)

1. $\frac{\cos^4\alpha}{\cos^2\beta} + \frac{\sin^4\alpha}{\sin^2\beta} = 1$ (given), $\sin^4\alpha + \sin^4\beta = ?$

2. $\frac{\sin^4\theta}{\lambda} + \frac{\cos^4\theta}{\mu} = \frac{1}{\lambda+\mu}$ (given), $\frac{\sin^{4n}\theta}{\lambda^{2n-1}} + \frac{\cos^{4n}\theta}{\mu^{2n-1}} = ?$

3. $\tan\beta = \frac{n\sin\alpha\cos\alpha}{1-n\sin^2\alpha}$ (given), $\tan(\alpha-\beta) = (1-n)\tan\alpha$ P.T

4. $\sin B = 3\sin(2A+B)$ (given), $2\tan A + \tan(A+B) = 0$ P.T

5. $\cot\alpha\cot2\alpha + \cot2\alpha\cot3\alpha + 2 = \cot\alpha(\cot\alpha - \cot3\alpha)$ P.T

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

$\sin(A\pm B) = \sin A\cos B \pm \cos A\sin B$

$\cos(A\pm B) = \cos A\cos B \mp \sin A\sin B$

$\tan(A\pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

$\frac{\sin(A\pm B)}{\sin A\sin B} = \cot B \pm \cot A$

$\frac{\sin(A\pm B)}{\cos A\cos B} = \tan A \pm \tan B$

$\frac{\cos(A\pm B)}{\cos A\cos B} = 1 \mp \tan A\tan B$

$\frac{\cos(A\pm B)}{\sin A\sin B} = \cot A\cot B \mp 1$

$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$

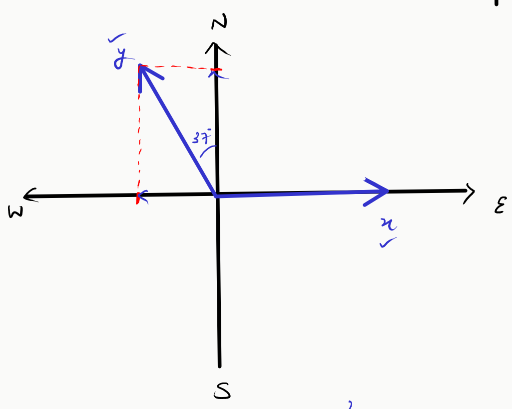
Type III

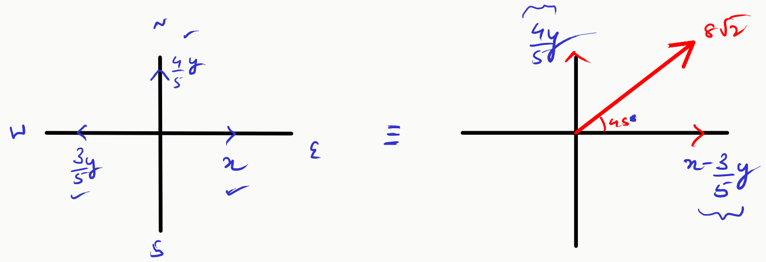
Q3

	M	D
$\underset{\sim}{A}$	x	E ✓
$\underset{\sim}{B}$	y	$37°$ W of N ✓
$\underset{\sim}{C}$	$8\sqrt2$	N-E ✓ given

$x = ?$
y

$\underset{\sim}{C} = \underset{\sim}{A} + \underset{\sim}{B}$ resultant vector





$8\sqrt2\cos45 = \frac{4}{5}y \Rightarrow \boxed{y=10}$

$\tan45 = 1 \Rightarrow \boxed{x - \frac{3}{5}y = \frac{4}{5}y} \Rightarrow x = \frac{7}{5}y \Rightarrow \boxed{x=14}$

Type-IV HW

Q.4 *

	M	D
$\underset{\sim}{A}$	14	E
$\underset{\sim}{B}$	x	y
$\underset{\sim}{C}$	$8\sqrt2$	N-E

$x, y = ?$

e.2 Application to Relative Motion (Kinematics)

* $\boxed{\underset{\sim}{v}_{AB} = \underset{\sim}{v}_A - \underset{\sim}{v}_B} \longleftrightarrow \boxed{\underset{\sim}{C} = \underset{\sim}{A} + \underset{\sim}{B}}$

Relative vel of A WRT B

WRT ground

'Relative motion'

$\underset{\sim}{v}_A = \underset{\sim}{v}_{AB} + \underset{\sim}{v}_B$

$\underset{\sim}{v}_{AB}$ — M, D

Q5 A car 'A' moving with 7m/s in N, $\underbrace{\text{for driver B}}^{\text{WRT B}}$, A appears N-W.
B moving in 37° N of E

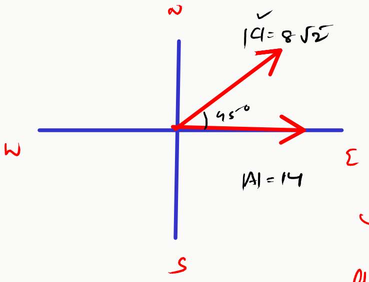
$\Big\} \longrightarrow$ vel. of B = ?
vel. of A wrt B = ?

Lecture-26 (6/Nov) 2

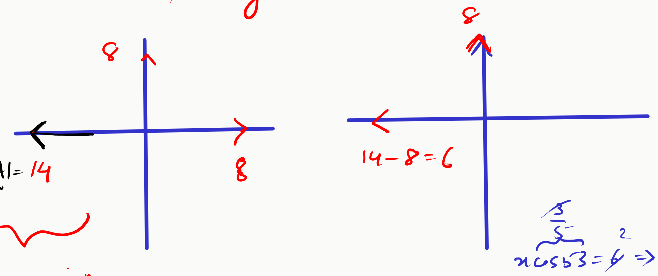
HW discussion

1.

	M	D
$\underset{\sim}{A}$	14	E ✓
$\underset{\sim}{B}$	x	y
$\underset{\sim}{C}$	$8\sqrt{2}$	N-E ✓



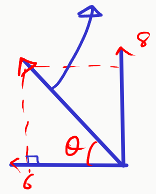
Most Imp. diag.



flip in the dirⁿ of $\underset{\sim}{A}$ so that

$\underset{\sim}{B} = \underset{\sim}{C} + (-\underset{\sim}{A})$

$x\cos 53 = 6 \Rightarrow x = 10$



$\underset{\sim}{C} = \underset{\sim}{A} + \underset{\sim}{B} \Rightarrow \underset{\sim}{B} = \underset{\sim}{C} - \underset{\sim}{A}$

$\tan\theta = \frac{8}{6} = \frac{4}{3}$

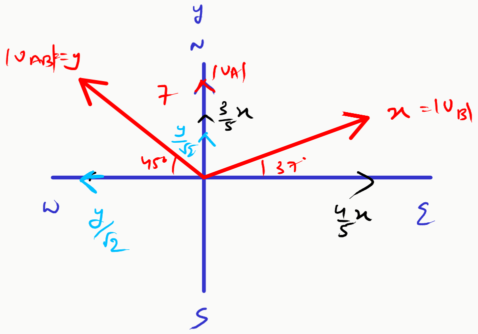
$\boxed{\theta = 53°}$

53° N of W

2.

	M	D
v_A	7	N
v_B	x	37° N of E
v_{AB}	y	N-W

$\underset{\sim}{v}_{AB} = \underset{\sim}{v}_A - \underset{\sim}{v}_B \Rightarrow \underset{\sim}{v}_A = \underset{\sim}{v}_B + \underset{\sim}{v}_{AB}$

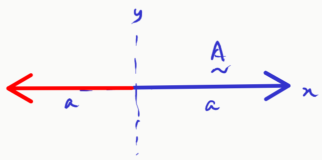


logical Equality / physical Equalⁿ

$\hat{x}: \quad \frac{y}{\sqrt{2}} = \frac{4}{5}x$

Set of Linear Eqⁿ in 2 var.

$\hat{y}: \quad \frac{3}{5}x + \frac{1}{\sqrt{2}}y = 7$



$\underset{\sim}{A} = a\hat{i}$

$|A|^2 = \underset{\sim}{A}\cdot\underset{\sim}{A} = a^2$

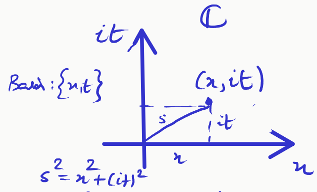
direction duality

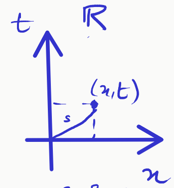
$\underset{\sim}{A} = a(-\hat{i}) = -a\hat{i}$

$|A|^2 = a^2$

Complex Space $\mathbb{C}$

Real space $\mathbb{R}$





$s^2 = x^2 + (it)^2$

$s^2 = x^2 - t^2$

$= \eta_{ab} x^a x^b$

$s^2 = g_{ab} x^a x^b$

$s^2 = x^2 + t^2$

$\mathbb{R}^2$

$g_{ab} = \eta_{ab} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

basis: $\{x, y\}$

$s^2 = x^2 - y^2$

$z = a + ib \leftrightarrow \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$

$g_{ab} = \delta_{ab} = \mathbb{I}$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

basis: $\{x, y\}$

$s^2 = x^2 + y^2$

Comment on Complex Space and Real Space

$\mathbb{C}$ $\mathbb{R}$

$\mathbb{R} \subset \mathbb{C}$ 'Container-Contained Relation'

* "Space, is the material with which 'the mind' builds up number the medium in which the mind places it."

" Science confines itself, however, to drawing our attention to this material."

– Intro to metaphy. – Bergson

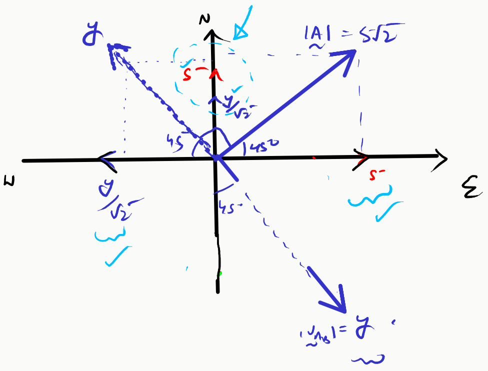
Lecture-27 (20/Nov) 2.5

Leibniz ? Infinitely small

$\exists$ Comparison of the points

Q6 A moving N-E $5\sqrt{2}$ m/s, B moving with 10 m/s. but for driver of B, A is moving S-E dirⁿ of B, $|v_{AB}| = ?$

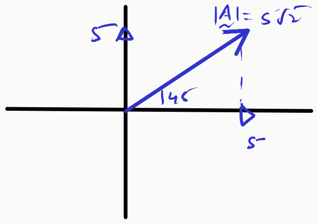
	M	D
v_A	$5\sqrt{2}$	N-E
v_B	10	?
v_{AB}	y	S-E

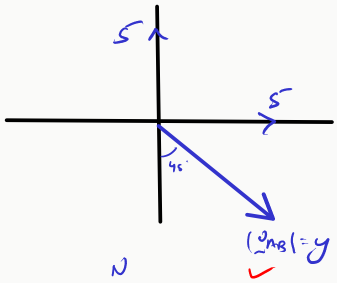


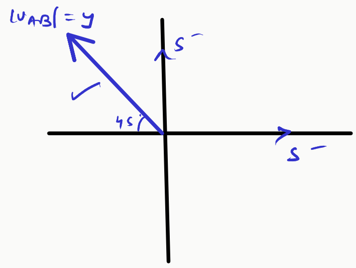
$\hat{x}: \quad \frac{y}{\sqrt{2}} = 5$ ✓

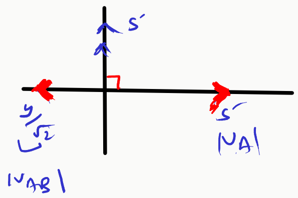
$\hat{y}: \quad y^2 + (5\sqrt{2})^2 = 10^2$

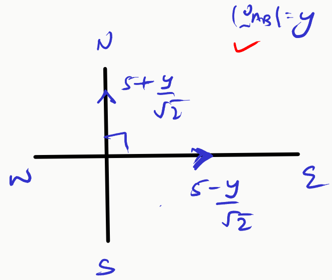
$\underbrace{v_{AB}}_{\text{Master}} = v_A - v_B \Rightarrow v_B = \underbrace{v_A - v_{AB}}_{\text{known}} = v_A + (-v_{AB})$











$v_B = v_A - v_{AB}$

$|v_B|^2 = |v_A - v_{AB}|^2$

$= |v_A|^2 + |v_{AB}|^2$

$10^2 = \underbrace{\left(5 + \frac{y}{\sqrt{2}}\right)^2 + \left(5 - \frac{y}{\sqrt{2}}\right)^2}$ $\Rightarrow$

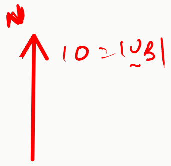
$25 + \frac{y^2}{2} + \cancel{\frac{10y}{\sqrt{2}}} + 25 + \frac{y^2}{2} - \cancel{\frac{10y}{\sqrt{2}}} = 100$

$y^2 = 50 \Rightarrow \boxed{y = 5\sqrt{2}}$

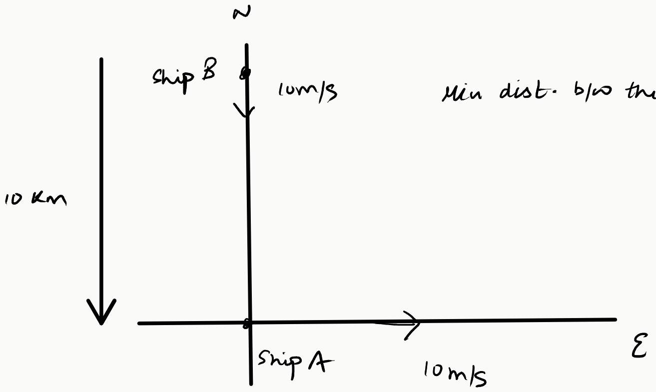
'Magnitude of v_{AB}'

$5 + \frac{(5\sqrt{2})}{\sqrt{2}} = 10$

$5 - \frac{(5\sqrt{2})}{\sqrt{2}} = 0$



Q7



Min dist. b/w the 2 ships?

$\left\{\begin{matrix}\sin\theta \\ \cos\theta\end{matrix}\right.$ i

$\underbrace{\log}$ $\begin{cases} 10 \to \in N \\ e = 2.15.. \end{cases}$ $\longrightarrow$ $e^{i\theta}$

Mathematician / Poet / Author

{ Lewis Carroll 'Alice in the Wonderland'

$\downarrow$

$\boxed{\text{Kid's Story}}$

Topology / Becomings / Progression)

Duration / transformation

Deleuze, $\underbrace{\text{Logic of Sense}}$

20th c. French Phil.

'Lewis Carroll'

Manuel DeLanda

* Singular
  - one / unit
  - Unique / Irreplaceable

Lecture-28 (12/Nov) 2.5

2. T-Identities for Multiple / Submultiple Angle

2.a. Double Angle Formulas

{Note: derivation are sooo Easy but spotting the identity seems difficult}

Space = Countable  θ: dir = uncountable

* $\sin(A+B) = \sin A\cos B + \cos A \sin B$ (the most fundamental)

$\cos(A+B) = \cos A\cos B - \sin A \sin B$

A = A (law of self identity)

$$\sin^2(A+B) + \cos^2(A+B) = \sin^2 A\cos^2 B + \cos^2 A\sin^2 B + \cos^2 A\cos^2 B + \sin^2 A \sin^2 B + \cancel{2\sin A\sin B\cos A\cos B} - \cancel{2\sin A\sin B\cos A\cos B}$$

$$= \sin^2 A(\sin^2 B + \cos^2 B) + \cos^2 A(\sin^2 B + \cos^2 B)$$

$$= (\sin^2 B + \cos^2 B)\underbrace{(\sin^2 A + \cos^2 A)}_{=1} \overset{?}{=} 1$$

{Pythagoras assumed}

* A = B $\Rightarrow$ $\sin 2\theta = 2\sin\theta\cos\theta \xrightarrow[\theta \to \theta/2]{} \sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$ } wonders!

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta \longrightarrow \sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}} \xrightarrow{\theta \to \frac{\theta}{2}} \boxed{\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}} \quad \text{I.}$$

$$= 2\cos^2\theta - 1 \longrightarrow \cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}} \xrightarrow{\theta \to \theta/2} \boxed{\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}}} \quad \text{II.}$$

* $\sin 45^\circ = \frac{1}{\sqrt{2}}$, $\cos 45 = \frac{1}{\sqrt{2}}$, $\sin 22.5 = ?$ $\cos 22.5^\circ = ?$ $\sin\frac{15}{2} = ?$

* $$\sin 22.5 = \sqrt{\frac{1-\cos 45}{2}} = \sqrt{\frac{1-\frac{1}{\sqrt{2}}}{2}} = \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}} = \sqrt{\frac{2-\sqrt{2}}{4}}$$

$$\cos 22.5 = \sqrt{\frac{1+\cos 45}{2}} = \sqrt{\frac{1+\frac{1}{\sqrt{2}}}{2}} = \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}} = \sqrt{\frac{2+\sqrt{2}}{4}}$$

$\tan\frac{\theta}{2} = \frac{2\tan\theta/4}{1-\tan^2\theta/4}$ wonders!

* $$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} = \frac{2\sin\theta\cancel{\cos\theta}}{\cos^{\cancel{2}}\theta(1-\tan^2\theta)} = \frac{2\tan\theta}{1-\tan^2\theta}$$

* $$\boxed{\tan\theta/2 = \frac{\sin\theta/2}{\cos\theta/2} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}} \quad \text{III}$$

$$\tan\underbrace{22.5}_{\frac{45}{2}} = \frac{\sin 22.5}{\cos 22.5} = \sqrt{\frac{1-\cos 45}{1+\cos 45}} = \sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}}$$

Q1 $$y = \sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}}$$ trig Expression!

$2(1+\cos 8\theta) = 2\cdot 2\cos^2 4\theta$

$\cos 2\theta = 2\cos^2\theta - 1$

$2\cos^2\theta = 1+\cos 2\theta$

$2\cos^2\theta/2 = 1+\cos\theta$

$$= \sqrt{2+\sqrt{2+2\cos 4\theta}}$$

$$= \sqrt{2+\sqrt{2(1+\cos 4\theta)}}$$

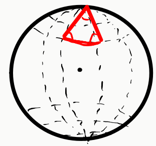
$$= \sqrt{2+\sqrt{2\cdot 2\cos^2 2\theta}} = \sqrt{2+2\cos 2\theta} = \sqrt{2(1+\cos 2\theta)}$$

$$= \sqrt{2\cdot 2\cos^2\theta} = 2\cos\theta$$

HW (~JEE type)

* $\sin 18 = ?$, $\sin 36^\circ = ?$

Riemannian



sphere (3D)

Deviation from Euclidean (~ NLM-2)

I am Rohan (static)

= to Be = to Endure

= to Be in duration

tautology

e⁻ is, EM wave is, thought is; idea is, field is, I is; Dinesh is, ⋮

How is "isness" of one being diff. from another?

Same: {all exists / actualized in this world}

Different ⇓ ∃ diff. kinds of Existences {Exist in a completely diff. world}

It is a trap

Postulate: Another world (universe)

Univocity of Being 'Duns Scotus'

‖

Universe – univocal

Bhagwat Gita ‖ } Vyasa

existence

"Being is said of everything in the single & the same sense but that of which it is said differs. It is said of difference-in-itself"

Le Monde — DR, Deleuze

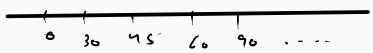
1. Descartes = coord. sys.

2. Spinoza : God = Nature

3. Leibniz = Calculus → Cartesian Interpretation of Calculus

Lecture-29 (13/Nov) 1.5

Needs development!!



Q2 $\sin 18 = \sqrt{\frac{1-\cos 36}{2}} = \sqrt{\frac{1-\sqrt{\frac{(1+\cos 72)}{2}}}{2}}$ .... ✗

$\parallel$

$\cos 72$

* $\underbrace{18^\circ}_{=\theta} \times 5 = 90^\circ \Rightarrow \boxed{5\theta = 90^\circ} \Rightarrow 2\theta + 3\theta = 90^\circ \Rightarrow 2\theta = 90^\circ - 3\theta$

$$\boxed{\underbrace{\sin 2\theta}_{2\sin\theta\cos\theta} = \sin(90-3\theta) = \underbrace{\cos 3\theta}_{??}}$$

$$\sin\theta = \frac{\cos 3\theta}{2\cos\theta} = \frac{\cos(\theta+2\theta)}{2\cos\theta} = \frac{\cos\theta\cos 2\theta - \sin\theta\overbrace{\sin 2\theta}^{2\sin\theta\cos\theta}}{2\cos\theta} = \frac{\cos 2\theta - 2\sin^2\theta}{2} = \frac{1-2\sin^2\theta - 2\sin^2\theta}{2} = \frac{1-4\sin^2\theta}{2}$$

$$\boxed{4\sin^2\theta + 2\sin\theta - 1 = 0} \longrightarrow \sin\theta = \frac{-2 \pm \sqrt{4+16}}{8} = \frac{-2\pm\sqrt{20}}{8} = \frac{-2\pm 2\sqrt{5}}{8}$$

→ $-$ < 0 $\notin$ I^{st} quad.

→ $\frac{\sqrt{5}-1}{4}$

* $+$: $a + b = c$; $\underbrace{a, b, c}$ $\in \mathbb{Z}$

Binary operator {usual Addition}

being endowed with all the Properties of that Set

$a+b = b+a$ Comm. UNDER '+'

* $+'$ (Enforced Interaction) : $a +' b = \lambda$, $a \in \mathbb{Q}$, $b \in \mathbb{R}$, $\lambda \in \mathbb{R}$

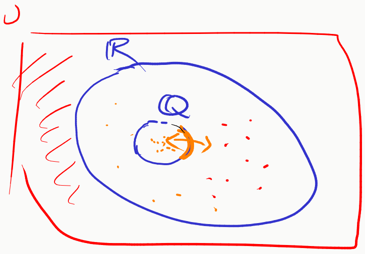
$1 +' \sqrt{3} = \lambda$

they don't interact with each other

$\frac{7}{2}$ ⋮ $\sqrt{3}$

Boundary

Note : ∃ $\sqrt{}$ operation (postulate)



$\mathbb{R} \cap \mathbb{Z} = \mathbb{Z}$

$$\boxed{\mathbb{R} \cap \mathbb{Z}' \neq \phi}$$

* $\hat{+}$ (Enforced Interaction) : $a \hat{+} i b = z$; $a, b \in \mathbb{R}$, $i \in \mathbb{C}$

$\underbrace{\exists \text{ diff of Kind}}$

$\parallel$

∄ any transformation that take us from '1' to 'another'

* $a^n \equiv a \times a \times \cdots \times a$, $\boxed{n \in \mathbb{N}}$ (Constraint)

new creature

to facilitate 'Method of division (split)'

$$\boxed{\sqrt{a} \equiv a^{1/2}} \longrightarrow a^1 = \sqrt{a} \times \sqrt{a}$$

$\frac{-1}{12}$ $\frac{\pi}{12}$ $\sqrt{3}$ i $\frac{i}{\sqrt{2}}$ $2i$

Boundary

Note : ∃ $\sqrt{-1} = i$; ("postulate") usual multip. $2 \times i = 2i$

* $x = a^n$ → $n \in \mathbb{Z}$
→ $n \in \mathbb{Q}$
→ $n \in \mathbb{R}$ → Computable
→ Uncomputable
→ $n \in \mathbb{C}$ i (New Creature)
→ $n \in$ Quaternions $\{i, j, k\}$

$\{i, j\} = ij + ji = k$

$i^2 = j^2 = k^2 = -1$

Removing structure At the Boundary (Spatially speaking)

Lecture-30 (17/Nov) 2

Q3 $\sin 36 = ?$

$$\boxed{\theta = 36 \longrightarrow 5\theta = 180^\circ \Rightarrow 2\theta + \underbrace{3\theta}_{} = 180^\circ \Rightarrow 2\theta = 180 - 3\theta}$$ Most Imp. Trick

$\sin2\theta = \sin^2\theta = 2\cos^2\theta - 1$ ...

* $\underbrace{\sin 2\theta}_{2\cancel{\sin\theta}\cos\theta} = \sin(180 - 3\theta) = \underbrace{\sin 3\theta}_{} \Rightarrow 2\cos\theta = 2\cos^2\theta - 1 + 2\cos^2\theta \Rightarrow \boxed{4\cos^2\theta - 2\cos\theta - 1 = 0}$

$\sin(\theta + 2\theta) = \cancel{\sin\theta}\underbrace{\cos 2\theta}_{(2\cos^2\theta - 1)} + \cos\theta\underbrace{\sin 2\theta}_{2\cancel{\sin\theta}\cos\theta}$

$\Downarrow$

$\cos\theta = \dfrac{2 \pm \sqrt{4+16}}{8} = \dfrac{2 \pm \sqrt{20}}{8}$

$= \dfrac{1 \pm \sqrt{5}}{4} \Rightarrow \boxed{\cos\theta = \dfrac{1+\sqrt5}{4}} > 0$

$\because$ 1st quad.

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 1 - 2\sin^2\theta$

$= 2\cos^2\theta - 1$

$\Downarrow$

$\sin\theta = \sqrt{\dfrac{1 - \cos 2\theta}{2}}$

S	A
T	C

$\sqrt4 = 2 < \sqrt5 < \sqrt9 = 3$

* $\cos 36 = \dfrac{1+\sqrt5}{4} \longrightarrow \sin^2 36 = 1 - \underbrace{\cos^2 36}_{\left\{\frac{1+\sqrt5}{4}\right\}^2} \Rightarrow \sin^2 36 = \dfrac{16 - (1 + 5 + 2\sqrt5)}{16} = \dfrac{10 - 2\sqrt5}{16}$

$$\sin 36 = \frac{\sqrt{10 - 2\sqrt5}}{4}$$

Table of special angles

	$18^\circ\left(\frac{\pi}{10}\right)$	$36^\circ\left(\frac{\pi}{5}\right)$	$54^\circ\left(\frac{3\pi}{10}\right)$	$72^\circ\left(\frac{2}{5}\pi\right)$
$\sin\theta$	$\frac{\sqrt5 - 1}{4}$	$\frac{\sqrt{10-2\sqrt5}}{4}$	$\frac{\sqrt5+1}{4}$	$\frac{\sqrt{10+2\sqrt5}}{4}$
$\cos\theta$	$\frac{\sqrt{10+2\sqrt5}}{4}$	$\frac{\sqrt5+1}{4}$	$\frac{\sqrt{10-2\sqrt5}}{4}$	$\frac{\sqrt5-1}{4}$

JEE-2

Q4. $\sin\frac{\pi}{10} + \underbrace{\sin\frac{13\pi}{10}}_{\sin\left(\pi + \frac{3}{10}\pi\right) = -\sin\frac{3}{10}\pi} = \sin\overset{18}{\frac{\pi}{10}} - \sin\overset{54}{\frac{3}{10}\pi} = \sin 18 - \sin 54 = \frac{\sqrt5 - 1}{4} - \left(\frac{\sqrt5+1}{4}\right) = -\frac12$

Q5. $\underbrace{\sin^2 72}_{\cos 18} - \sin^2 60 = \left(\dfrac{\sqrt{10+2\sqrt5}}{4}\right)^2 - \dfrac34 = \dfrac{10+2\sqrt5}{4^2} - \dfrac34$

$= \dfrac14\left\{\dfrac{10 + 2\sqrt5 - 12}{4}\right\} = \dfrac{2\sqrt5 - 2}{16} = \dfrac{\sqrt5 - 1}{8}$

Leibnizian Calculus for Moron

Aristotelian logic

$\downarrow$

1st Calculator

Formal logic, Leibniz

$\downarrow$

(1847)

The mathematical Analysis of logic, George Boole

HW

* $\sin^2 24 - \sin^2 6 = ? = \sin^2 4\theta - \sin^2\theta$
* $\sin\frac{\pi}{10}\sin\frac{13\pi}{10} = ? = -\frac14$
* $\sin\frac{\pi}{5}\sin\frac{2\pi}{5}\sin\frac{3\pi}{5}\sin\frac{4\pi}{5} = ?$
* $16\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{14\pi}{15} = ?$

irreducible

$\sum F_{net} \neq 0$

"difference of the force is the essence of Force"

– Nietzsche & Phil.

$E = h\nu$

Einstein

|||

Full wealth unexplored

$\{dx\}$ → Cartesian Calculus

||

Number Powered Calculus

$\frac{dy}{dx}$

Trigo + Calculus

Fourier series

$\downarrow$

AP
GP
HP
AGP
AHP..

→ Convergent
→ divergent

"special cases"

Comment on Boolean Algebra (CS+Math)

SOP ⟷ POS

$(x\cdot y) + (z\cdot k)$ $\quad (x+y)\cdot(z+k)$

AND OR

* Decimal : $\mathbb{R}$ (Computable)

$1 + 1 = 2$

$\{0,1,2,3,4,5,6,7,8,9\}$ Russell Turing Church

Symbols

3 = −*?/ ज ।@A

* Binary = $\{0,1\}$ $\quad 1+1 \neq 2$

$1+0=1$
$1\cdot0=0$ > operation : Complementarity

Lecture-31 (19/Nov) 2 $\quad 18^\circ = \frac{\pi}{10} \rightarrow$

HW discuss.

* $\sin\frac{\pi}{5}\sin\frac{2\pi}{5}\sin\frac{3\pi}{5}\sin\frac{4\pi}{5} = \left(\sin\frac{\pi}{5}\sin\frac{2\pi}{5}\right)^2 = \left\{\frac{\sqrt{10-2\sqrt5}}{4}\cdot\frac{\sqrt{10+2\sqrt5}}{4}\right\}^2$

$\left.\begin{array}{l}2+3=5\\1+4=5\end{array}\right\}$ $\quad \frac{2\pi}{5}+\frac{3}{5}\pi = \pi$, $\quad \frac{\pi}{5}+\frac{4}{5}\pi = \pi \longrightarrow$ $\sin(\pi-\theta)=\sin\theta$

$\underbrace{\sin\left(\pi-\frac{3\pi}{5}\right)}_{\sin\frac{2\pi}{5}} = \sin\frac{3\pi}{5}$

$\longrightarrow \frac{(10-2\sqrt5)(10+2\sqrt5)}{16^2} = \frac{100-20}{256} = \frac{80}{256}$

* $16\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{14\pi}{15}$

$\frac{2}{15}+\frac{8}{15} = \frac{10}{15} = \frac{2\pi}{3} \longrightarrow 120^\circ$

$\frac{8}{18}-\frac{2}{15} = \frac{6}{15} = \frac{2\pi}{5}$

$\left.\begin{array}{l}\frac{4}{15}+\frac{14}{15} = \frac{18}{15}\pi = \frac{6\pi}{5}\end{array}\right\} = \frac{5\pi+\pi}{5} = \pi+\frac{\pi}{5}$

$\frac{14}{15}-\frac{4}{15} = \frac{10}{15} = \frac{2\pi}{3}$

$\rightarrow 4\left\{\left(2\cos\overset{A}{\frac{2\pi}{15}}\cos\overset{B}{\frac{8\pi}{15}}\right)\cdot\left(2\cos\overset{A}{\frac{4\pi}{15}}\cos\overset{B}{\frac{14\pi}{15}}\right)\right\} = 4\left\{\left(\underbrace{\cos\frac{2\pi}{3}}+\cos\frac{2\pi}{5}\right)\left(\underbrace{\cos\frac{6\pi}{5}}+\overbrace{\cos\frac{2\pi}{3}}^{-\cos\pi/3}\right)\right\}$

$\cos\left(\frac{3\pi-\pi}{3}\right) = \cos\left(\pi-\frac{\pi}{3}\right) = -\cos\frac{\pi}{3}$ $\qquad \cos\left(\pi+\frac{\pi}{5}\right) = -\cos\frac{\pi}{5}$

$\rightarrow 4\left\{\left(-\cos\frac{\pi}{3}+\cos\frac{2\pi}{3}\right)\cdot\left(-\cos\frac{\pi}{5}+\underbrace{\cos\frac{2\pi}{3}}_{-\frac{1}{2}}\right)\right\} = 4\left\{\left(-\frac{2}{2\times2}+\frac{\sqrt5-1}{4}\right)\left(-\frac{(\sqrt5+1)}{4}-\frac{2}{2\times2}\right)\right\}$

$= \frac{4}{4\cdot4}\left\{\underbrace{(-2+\sqrt5-1)}_{\sqrt5-3}\underbrace{(-\sqrt5-1-2)}_{-\sqrt5-3}\right\} = \frac{1}{4}\{9-5\} = 1$

D		B
0	=	0
1	=	1
2	=	10
3	=	11

$\left\{\begin{array}{l}+\leftrightarrow\cdot\\1\leftrightarrow0\end{array}\right\}$

$\bar{1} = 0$

$\bar{0} = \bar{\bar{1}} = 1$

$\begin{array}{r}0\\+0\\\hline00\end{array}\quad\begin{array}{r}0\\+1\\\hline01\end{array}\quad\begin{array}{r}1\\+1\\\hline10\end{array}\quad\begin{array}{r}11\\+1\\\hline100\end{array}$

$a^m\cdot a^n = a^{m+n}$

$\sqrt5 = 5^{1/2}$

$\sqrt5\times\sqrt5 = 5^{1/2}\cdot5^{1/2} = 5^{\frac12+\frac12} = 5^{\frac{1+1}{2}} = 5^{\frac22} = 5^1$

$\sin n\pi = 0$, $\cos n\pi = (-1)^n$

'Transcendental'

Transcendence = Beyond

$\cos(A+B)+\cos(A-B) = 2\cos A\cos B$

$\cos(A+B)-\cos(A-B) = 2\sin A\sin B$

$\cos(-\theta) = \cos\theta$

S | A

T | C

A CA'

Lecture-32 (20/Nov) 2

2.b. Triple Angle formula

$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$\sin 2\theta = 2\sin\theta\cos\theta$
$\sin 3\theta = \dots$ (Later!)

$f(\sin A)$

* $\sin(A+B) = \sin A\cos B + \cos A\sin B \xrightarrow[B=2A]{} \sin 3A = \sin(A+2A) = \sin A\underbrace{\cos 2A}_{(1-2\sin^2 A)} + \cos A\underbrace{\sin 2A}_{2\sin A\cos A} = \sin A - 2\sin^3 A + 2\sin A\underbrace{(\cos^2 A)}_{(1-\sin^2 A)}$

$= \sin A - 2\sin^3 A + 2\sin A - 2\sin^3 A$

$\boxed{\sin 3\theta = 3\sin\theta - 4\sin^3\theta}$ → cubic in $\sin\theta$

* $\cos 3A = \cos(A+2A) = \cos A\underbrace{\cos 2A}_{(2\cos^2 A-1)} - \sin A\underbrace{\sin 2A}_{2\sin A\cos A} = 2\cos^3 A - \cos A - (2\underbrace{\sin^2 A}_{2(1-\cos^2 A)}\cos A) = 2\cos^3 A - \cos A - 2\cos A + 2\cos^3 A$

$\boxed{\cos 3\theta = 4\cos^3\theta - 3\cos\theta}$ → cubic in $\cos\theta$

* $\tan(A+B) = \dfrac{\tan A + \tan B}{1-\tan A\tan B} \xrightarrow{B=2A} \tan(3A) = \dfrac{\sin 3A}{\cos 3A} = \dfrac{3\sin A - 4\sin^3 A}{\underbrace{4\cos^3 A - 3\cos A}_{\tan A\left(\frac{3-4\sin^2 A}{4\cos^2 A - 3}\right)}} = f(\tan A)$

$\tan(3A) = f(\tan\theta)$

'H.W. Activity'

$\tan(A+2A) = \dfrac{\tan A + \tan 2A}{1-\tan A\tan 2A} = \dfrac{\tan A + \left(\frac{2\tan A}{1-\tan^2 A}\right)}{1-\tan A\left(\frac{2\tan A}{1-\tan^2 A}\right)} = \dfrac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

$$\boxed{\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}}$$

* $\sin(A+B+C) = \underbrace{\sin(A+B)}_{\sin A\cos B+\cos A\sin B}\cos C + \underbrace{\cos(A+B)}_{\cos A\cos B-\sin A\sin B}\sin C = \sin A\cos B\cos C + \cos A\sin B\cos C + \cos A\cos B\sin C - \sin A\sin B\sin C$ □

* $\cos(A+B+C) = \underbrace{\cos(A+B)}_{\cos A\cos B-\sin A\sin B}\cos C - \underbrace{\sin(A+B)}_{\sin A\cos B+\cos A\sin B}\sin C = \cos A\cos B\cos C - \sin A\sin B\cos C - \sin A\cos B\sin C - \cos A\sin B\sin C$ □

* $\tan(A+B+C) = \dfrac{\overbrace{\tan(A+B)}^{\frac{\tan A+\tan B}{1-\tan A\tan B}} + \tan C}{1-\underbrace{\tan(A+B)}_{\frac{\tan A+\tan B}{1-\tan A\tan B}}\tan C} = \dfrac{\tan A+\tan B+\tan C-\tan A\tan B\tan C}{1-\tan A\tan B-\tan B\tan C-\tan C\tan A}$ □

Lecture-33 (24/Nov) 1.5

"Ruminate every Aphorism." — Nietzsche

"Every word has to be Commented upon" — Deleuze

$\boxed{\text{Read Slow}}$

$\cos^2 A = \cos B \Rightarrow B = \cos^{-1}(\cos^2 A)$

I.T.F

* $\sin 2\theta = 2\sin\theta\cos\theta$; $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$, $\sin^2\theta + \cos^2\theta = 1$

* $\sin 4\theta = \sin 2(2\theta) = 2\overbrace{\sin 2\theta}^{2\sin\theta\cos\theta}\underbrace{\cos 2\theta}_{1-2\sin^2\theta} = 4\sin\theta(1-2\sin^2\theta)\sqrt{1-\sin^2\theta}$ ✓

* $\overset{f(\cos\theta)}{\cos 5\theta} = \cos(2\theta+3\theta) = \underbrace{\overbrace{\cos 2\theta}^{2\cos^2\theta-1}\underbrace{\cos 3\theta}_{4\cos^3\theta-3\cos\theta}}_{\mathrm{I}} - \underbrace{\overbrace{\sin 2\theta}^{2\sin\theta\cos\theta}\underbrace{\sin 3\theta}_{3\sin\theta-4\sin^3\theta}}_{\mathrm{II}}$

$\mathrm{I} \equiv (2\cos^2\theta - 1)(4\cos^3\theta - 3\cos\theta) = 8\cos^5\theta - \underbrace{6\cos^3\theta - 4\cos^3\theta}\, + 3\cos\theta = 8\cos^5\theta - 10\cos^3\theta + 3\cos\theta$ ✓

$\mathrm{II} \equiv (2\sin\theta\cos\theta)(3\sin\theta - 4\sin^3\theta) = (2\underbrace{\sin^2\theta}_{(1-\cos^2\theta)}\cos\theta)(3-4\underbrace{\sin^2\theta}_{(1-\cos^2\theta)}) = (2\cos\theta - 2\cos^3\theta)\underbrace{(3-4+4\cos^2\theta)}_{(4\cos^2\theta-1)}$

$= 8\cos^3\theta - 2\cos\theta - 8\cos^5\theta + 2\cos^3\theta = 10\cos^3\theta - 8\cos^5\theta - 2\cos\theta$

* $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$

$\underbrace{\sin\theta + \cos\theta} = f(\theta)$

$\cancel{x + y} =$

Trig. Machines

$\underbrace{x}\underbrace{+x^2} \neq 0$

$x(x+1) = 0$

Lecture-34 (26/Nov.) 2

* Math ⊂ Platonic heaven/Eternal
* space ⇒ Curving
* $g_{ab} = g_{ab}(x)$
  - $g_{ab} = \delta_{ab} = \mathbb{1} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}_{2\times2}$ Kronecker delta
  - $g_{ab} = \eta_{ab} = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}_{2\times2}$ minkowski metric 2D
  - $g_{ab} = \begin{pmatrix}r & 0\\ 0 & r\sin\theta\end{pmatrix}$

  ⋮

$x_2 + x_1 = x_2 - x$.

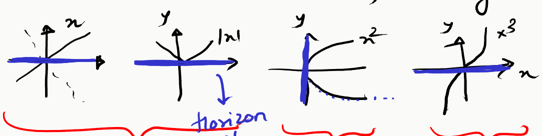
$\cancel{\int dx = \int dx}$?

$\frac{dx}{dx} = 1$

Matrix = Juxtaposition (Number)

Preservation of Entity

* Algebra ⇒ ∃ x (var.) , Rules of manipulations
  - → Like term ⇐ self-identity ($x = x$) Aristotle; $x + x = 2x$
  - → Unlike term ⇐ $x \neq y$, $y = y$ (Aristotelian); $x + y$



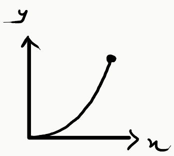
Linear Quad Cubic . . . .

$f(\theta) = \sum_{i=0}^{n} a_i x^i = a_0 x^0 + \overbrace{a_1 x + a_2 x^2} + a_3 x^3 + \ldots a_n x^n$, $n \in \mathbb{N}$

Progression

Macroscopic evaluations

$\boxed{\text{Geometry (Cartesian) prohibits } \infty. \text{ Cannot "feel" it.}}$



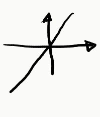
∄ Closed/Bounded Machines

"∞" precision to Analysis

Geometry = Patchwork

Algebra = Relationships b/w entities
- → infinitely big
- → infinitely small

* UnBounded map/Machines :



$y = ax + b$ →

Algebra

$\lim_{x\to\infty} y = \lim_{x\to\infty}(ax + b(x^0)) \to \infty$ upper bound

$\lim_{x\to-\infty} y = \lim_{x\to-\infty}(ax + bx^0) \to -\infty$ lower bound

$\infty = \underbrace{\infty_1} + \infty_2 + \infty_3 + \cdots \cdots \infty_n$

Every part is infinite still

$: \quad 1 = \overbrace{0 + 0.1 + \ldots}^{\text{Method of Division}} \Rightarrow$ Infinite → $\boxed{\text{Intuition}}$

$\boxed{\text{finiteness}}$ Conventional ↳ 'Analysis'

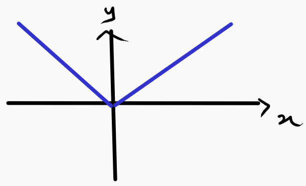
Think!

$\boxed{\text{How does } -\infty \text{ exists in Number theory / Calculus ?}}$

Modulus Machine

$$* \quad y = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

Point of transformation



$$\lim_{x\to\infty} y = \lim_{x\to\infty} |x| = \lim_{x\to\infty} x \xrightarrow{\text{direct subst.}} \infty \quad \text{upper bound}$$

$$\lim_{x\to-\infty} y = \lim_{x\to-\infty} |x| = \lim_{x\to-\infty} (-x) \to \underbrace{-(-\infty)}_{\infty} \quad \text{lower bound}$$

"normal"

$1 \times \infty = 1\infty$

weird ? $\begin{cases} 2\times\infty = 2\infty \\ -3\times\infty = -3\infty \end{cases}$

$-1 \times \infty = -\infty$ "Normal"

? weird $\boxed{0 \times \infty = 0}$

Lecture-35 (1/dec) 1.5'

3. Transformation of Sum ↔ Product

$*$ $\sin(A+B) = \sin A\cos B + \cos A \sin B$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A\cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

SOP

Set 1 :

$*$ $\sin(A+B) + \sin(A-B) = 2\sin A \cos B$

$*$ $\sin(A+B) - \sin(A-B) = 2\cos A \sin B$

$*$ $\cos(A+B) + \cos(A-B) = 2\cos A \cos B$

$*$ $\cos(A-B) - \cos(A+B) = 2\sin A \sin B$

Sleight of hand $\boxed{\begin{matrix} A+B=C \\ A-B=D \end{matrix}} \Rightarrow A = \frac{C+D}{2}, \; B = \frac{C-D}{2}$

Set 2

$$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$\cos C - \cos D = 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right)$$

Trick : set 2

C D $\frac{C+D}{2}$ $\frac{C-D}{2}$

S + S = 2SC

S − S = 2CS

C + C = 2CC

C − C = 2SS* $\frac{D-C}{2}$

A+B A−B A B

$*$ −ve sign

set 1

vol-1 (Algebra)

Machine

Rational Irrational Transcendental

$$* \quad \underbrace{\cos 53}_{3/5} + \underbrace{\cos 37}_{4/5} = 2\cos\left(\frac{90}{2}\right)\cos\left(\frac{16}{2}\right) = 2\cdot\frac{1}{\sqrt{2}}\cdot\cos 8 = \sqrt{2}\cos 8 \Rightarrow \cos 8 = \frac{7}{5\sqrt{2}}$$

$$* \quad \sin^2 75 = \sin 75\cos 15 = \frac{1}{2}\left(2\sin 75 \cos 15\right) = \frac{1}{2}\left\{\overbrace{\sin 90}^{1} + \overbrace{\sin 60}^{\sqrt{3}/2}\right\} = \frac{2+\sqrt{3}}{4}$$

$$* \quad \cos 20 \cos 40 \cos 60 \cos 80 = \frac{1}{2}\left\{(2\cos 20\cos 40)\cos 60 \cos 80\right\}$$

A B

S+S = 2SC

S−S = 2CS

C+C = 2CC

C−C = 2SS*

$= \frac{1}{2}\{(\cos 60 + \cos 20)\cos 60 \cos 80\}$

$= \frac{1}{2}\{\frac{1}{4}\cos 80 + \frac{1}{2}\underbrace{\cos 20 \cos 80}_{\frac{1}{4}(\cos 100 + \cos 60)}\} = \frac{1}{2}\cdot\frac{1}{4}\{\cancel{\cos 80} + \underbrace{\cos 100}_{\cos(180-80) = -\cancel{\cos 80}} + \cos 60\} = \frac{1}{16}$ ✓

HW

* $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13} =$
* $\sin 10^\circ \sin 30 \sin 50 \sin 70 =$
* $\sin 20^\circ \sin 40 \sin 60 \sin 80 =$
* $4\cos 12^\circ \cos 48^\circ \cos 72^\circ = ?$ simplify } HW
* $\cos 10 \cos 30 \cos 50 \cos 70 = ? = \frac{3}{16}$

Lecture-36 (3 dec) 15

C	D	$\frac{C+D}{2}$	$\frac{C-D}{2}$

$S + S = 2SC$

$S - S = 2CS$

$C + C = 2CC$ ✓

$C - C = 2SS^*$

A+B A-B A B

$\cos(-\theta) = \cos\theta$

* $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13} = \underbrace{\cos\left(\frac{10\pi}{13}\right)}_{\cos\left(\frac{13\pi - 3\pi}{13}\right) = \cos\left(\pi - \frac{3\pi}{13}\right) = -\cancel{\cos\frac{3\pi}{13}}} + \underbrace{\cos\left(\frac{8\pi}{13}\right)}_{-\cancel{\cos\frac{5\pi}{13}}} + \cancel{\cos\frac{3\pi}{13}} + \cancel{\cos\frac{5\pi}{13}} = 0$

S A
T C

* $\sin 20 \sin 40 \sin 60 \sin 80 = \frac{1}{4}\{\underbrace{2\sin 20 \sin 40}_{(\cos(20) - \cos 60)}\, 2\sin 60 \sin 80\} = \frac{1}{8}\{\underbrace{2\cos 20 \cos 20}_{\cancel{\cos 40} + \cos 0 \to 1} + \underbrace{2\cos 20 \cos 40}_{\cos 60 + \cancel{\cos 20}} - \underbrace{2\cos 60 \cos 20}_{\cos\cancel{40} + \cancel{\cos 40}} - \underbrace{2\cos 60 \cos 40}_{\cos 100 + \cancel{\cos 20}}\} = \frac{3}{16}$

$(\cos(20) - \cos(140))$, $-\cos 40$, $\cos 60 \to \frac{1}{2}$, $-\cancel{\cos 80}$

* $\sin 10 \sin 30 \sin 50 \sin 70 = \frac{1}{4}\{(\underbrace{2\sin 10 \sin 30}_{\cos(20) - \cos 40})(\underbrace{2\sin 50 \sin 70}_{\cos 20 - \cos 120})\} = \frac{1}{8}\{\underbrace{2\cos 20 \cos 20}_{\cos 40 + \cos 0} + \underbrace{2\cos 20 \cos 60}_{\cos 80 + \cos 40} - \underbrace{2\cos 40 \cos 20}_{\cos 60 + \cos 20} - \underbrace{2\cos 40 \cos 60}_{\cos 100 + \cos 20}\}$

$\cos 120 = (-\cos(180-60)) = -\cos 60$, $\cos 0 \to 1$, $\cos 100 \to -\cos 80$

$= \frac{1\cdot 2}{8}\{\underbrace{\cos 40 + \cos 80}_{(2\cos 60 \cdot \cos 20)} + \frac{\cos 0}{2} - \frac{\cos 60}{2} - \frac{2\cos 20}{2}\} = \frac{1}{4}\{\cancel{2}\cdot\frac{1}{\cancel{2}}\cancel{\cos 20} + \underbrace{\frac{\cos 0 - \cos 60}{2}}_{\frac{1 - \frac{1}{2}}{2}} - \frac{\cancel{2}\cancel{\cos 20}}{\cancel{2}}\} = \frac{1}{4}\cdot\frac{1}{4} = \frac{1}{16}$

$\sin 10 \sin 30 \sin 50 \sin 70 = \cos 20 \cos 40 \cos 60 \cos 80 = \frac{1}{16}$

HW

* $\tan 20 \tan 40 \tan 80 = ?$ ✓
* $\sin\alpha \sin(60-\alpha)\sin(60+\alpha) = \frac{1}{4}\sin 3\alpha$ P.T. } HW

C	D	$\frac{C+D}{2}$	$\frac{C-D}{2}$

$S + S = 2SC$

$S - S = 2CS$

$C + C = 2CC$

✓ $C - C = 2SS^*$

A+B A-B A B

Lecture-37 (5/dec) 1.5

* $4\sin\phi\,\sin\left(\frac{\pi}{3}+\phi\right)\sin\left(\frac{2\pi}{3}+\phi\right) = 2\sin\phi\left\{\underbrace{2\sin\left(\frac{\pi}{3}+\phi\right)\cdot\sin\left(\frac{2\pi}{3}+\phi\right)}_{\cos\left(\frac{\pi}{3}\right)-\cos\underbrace{(\pi+2\phi)}_{(-\cos 2\phi)}}\right\} = 2\sin\phi\cdot\left\{\underbrace{\cos\frac{\pi}{3}}_{\frac{1}{2}}+\cos 2\phi\right\} = 2\sin\phi\left\{\frac{1}{2}+\cos 2\phi\right\}$

$= \cancel{\sin\phi} + \underbrace{2\sin\phi\cos 2\phi}_{\sin 3\phi + \underbrace{\sin(-\phi)}_{-\cancel{\sin\phi}}} = \sin 3\phi$

'Machine'

* $\underbrace{f(\alpha,\beta)}_{} = \sin\alpha\sin\beta$, $\alpha+\beta = 90^\circ$; $f(\alpha,\beta)$ Min/Max. value

Multivariable function

| functions → Single var. Calculus $\frac{d}{dx}$
| Multivar. fun → Multivar Calculus
| $\underbrace{\partial_x = \frac{\partial}{\partial x}\Big|_{y,z}}$
| Naive interpretation of Calculus

| Accuracy over speed

* $f(\alpha,\beta) = \sin\alpha\sin\beta = \frac{1}{2}(2\sin\alpha\sin\beta) = \frac{1}{2}\left\{\cos(\alpha-\beta)-\cancel{\cos\underbrace{(\alpha+\beta)}_{90}}^{0}\right\}$

* $\boxed{-1 \le \cos\theta \le 1 \qquad \forall\,\theta}$

* $-\frac{1}{2} \le \underbrace{\frac{\cos(\alpha-\beta)}{2}}_{f(\alpha,\beta)} \le \frac{1}{2}$

HW

* $\tan(60+\theta)\tan(60-\theta) = \frac{2\cos 2\theta+1}{2\cos 2\theta-1}$ P.T

$C \qquad D \qquad \frac{C+D}{2} \quad \frac{C-D}{2}$

$S + S = 2SC$

$S - S = 2CS$

$C + C = 2CC$

$C - C = 2SS^*$

$A+B \qquad A-B \qquad A \quad B$

* $\sin 50\cos 85 = \frac{1-\sqrt{2}\sin 35}{2\sqrt{2}}$

Type 2 : Change of flavor

* $\frac{\cos 9\alpha - \cos 5\alpha}{\sin 17\alpha - \sin 3\alpha} = -\frac{\sin 2\alpha}{\cos 10\alpha}$

* $\frac{\cos 4\beta + \cos 3\beta + \cos 2\beta}{\sin 4\beta + \sin 3\beta + \sin 2\beta} = ?$

* $\left(\frac{\cos\alpha+\cos\beta}{\sin\alpha-\sin\beta}\right)^n + \left(\frac{\sin\alpha+\sin\beta}{\cos\alpha-\cos\beta}\right)^n = \begin{cases} ? & n \text{ even} \\ ? & n \text{ odd} \end{cases}$

* $\sin 4\theta + \sin 2\theta = 2\sin 3\theta\cos\theta$

$(x+y)^n = \sum_{r=0}^{n} {}^nC_r\, x^r y^{n-r}$

$= {}^nC_0\, x^0 y^n + {}^nC_1\, x^1 y^{n-1} + \dots$

${}^nC_n\, x^n y^0$

${}^nC_r = \binom{n}{r} = \frac{n!}{(n-r)!\,r!}$

Lecture-38 (17/dec) 2

* $4\cos 12 \cos 48 \cos 72 = 2\underbrace{(2\cos 12\cos 48)}_{(\cos 60 + \cos 36)}\cos 72 = 2\cancel{\cos 60}\cancel{\cos 72} + \underbrace{2\cos 36\cos 72}_{\cos 108 + \cos 36} = \cos 36 = \sin 54$

$\cos(180-72) = -\cancel{\cos 72}$

C	D	$\frac{C+D}{2}$	$\frac{C-D}{2}$

$S + S = 2SC$
$S - S = 2CS$
$C + C = 2CC$
$C - C = 2SS^*$

$A+B \quad A-B \quad A \quad B$

* $\tan 20 \tan 40 \tan 80 = \dfrac{2\sin 20 \sin 40 \sin 80}{2\cos 20 \cos 40 \cos 80} = \dfrac{(\cos 40 - \cos 120)\sin 20}{(\cos 40 + \cos 120)\cos 20} = \dfrac{\overbrace{2\cos 40 \sin 20}^{\sin 60 - \sin 20} - 2\overbrace{\cos 120}^{-1}\sin 20}{\underbrace{2\cos 40\cos 20}_{\cos 60 + \cos 20} + 2\underbrace{\cos 120}_{-1}\cos 20}$

$= \dfrac{\sin 60 - \cancel{\sin 20} + \cancel{\sin 20}}{\cos 60 + \cancel{\cos 20} - \cancel{\cos 20}} = \tan 60 = \sqrt{3}$

* $f(\beta) = \dfrac{\cos 4\beta + \cos 3\beta + \cos 2\beta}{\sin 4\beta + \sin 3\beta + \sin 2\beta} = \dfrac{(2\cos 3\beta\cos\beta) + \cos 3\beta}{(2\sin 3\beta\cos\beta) + \sin 3\beta} = \dfrac{\cos 3\beta\,\cancel{(2\cos\beta+1)}}{\sin 3\beta\,\cancel{(2\cos\beta+1)}} = \cot 3\beta$

HW

* $(\sin 3\alpha + \sin\alpha)\sin\alpha + (\cos 3\alpha - \cos\alpha)\cos\alpha = ? = 0$ ✓
* $\cos 2\theta \cos\frac{\theta}{2} - \cos 3\theta\cos\frac{9\theta}{2} = ? = \sin 5\theta \sin\frac{5\theta}{2}$ ✓
* $\sin\psi + \sin\left(\psi + \frac{2\pi}{3}\right) + \sin\left(\psi + \frac{4\pi}{3}\right) = ? = 0$ ✓
* $(\cos\alpha \pm \cos\beta)^2 + (\sin\alpha \pm \sin\beta)^2 = \begin{cases} ? = 4\cos^2\left(\frac{\alpha-\beta}{2}\right) & + \\ ? = 4\sin^2\left(\frac{\alpha-\beta}{2}\right) & - \end{cases}$

Sum

$1 = 1$
$\sin\alpha = \sin\alpha$
$\sin\alpha \neq \sin\beta$
$\sin\alpha - \sin\beta =$

C	D	$\frac{C+D}{2}$	$\frac{C-D}{2}$

$S + S = 2SC$
$S - S = 2CS$
$C + C = 2CC$
$C - C = 2SS^*$

$A+B \quad A-B \quad A \quad B$

* $\dfrac{\cos 2\alpha\cos 3\alpha - \cos 2\alpha\cos 7\alpha + \cos\alpha\cos 10\alpha}{\sin 4\alpha\sin 3\alpha - \sin 2\alpha\sin 5\alpha + \sin 4\alpha\sin 7\alpha} = ? = \dfrac{\cos\alpha + \cos 11\alpha}{\cos\alpha - \cos 11\alpha} = \cot 6\alpha\cot 5\alpha$ ✓

* if $\sin\theta + \sin\phi = \sqrt{3}(\cos\phi - \cos\theta) \longrightarrow \sin 3\theta + \sin 3\phi = ? =$

Constraint Eq^n (θ/ϕ)

* $\left(\dfrac{\cos\alpha + \cos\beta}{\sin\alpha - \sin\beta}\right)^n + \left(\dfrac{\sin\alpha + \sin\beta}{\cos\alpha - \cos\beta}\right)^n = \left(\dfrac{2\cancel{\cos\frac{\alpha+\beta}{2}}\cos\frac{\alpha-\beta}{2}}{2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}}\right)^n + \left(\dfrac{2\cancel{\sin\frac{\alpha+\beta}{2}}\cos\frac{\alpha-\beta}{2}}{-2\cancel{\sin\frac{\alpha+\beta}{2}}\sin\frac{\alpha-\beta}{2}}\right)^n$

$= \left[1 + (-1)^n\right]\cot^n\frac{\alpha-\beta}{2} = \begin{cases} 0 & \text{odd} \\ 2\cot^n\frac{\alpha-\beta}{2} & \text{Even} \end{cases}$

C	D	$\frac{C+D}{2}$	$\frac{C-D}{2}$

$S + S = 2SC$
$S - S = 2CS$
$C + C = 2CC$
$C - C = 2SS^*$

Lecture-39 (18/dec) 1:50 min

* $\underbrace{\sin\theta + \sin\phi}_{\cancel{2}\sin\frac{\theta+\phi}{2}\cos\frac{\theta-\phi}{2}} = \sqrt{3}\,(\underbrace{\cos\phi - \cos\theta}_{-\cancel{2}\sin\frac{\phi+\theta}{2}\sin\frac{\phi-\theta}{2}})$ given ; $\sin 3\theta + \sin 3\phi \to 2\sin\frac{3}{2}(\theta+\phi)\cos\frac{3}{2}(\theta-\phi) \to ?$

$$\sin\frac{\theta+\phi}{2}\left\{\cos\frac{\theta-\phi}{2} + \sqrt{3}\sin\frac{\phi-\theta}{2}\right\} = 0$$

Case I

$\sin\frac{\theta+\phi}{2} = 0 = \sin 0$

$\Downarrow$

$\boxed{\frac{\theta+\phi}{2} = 0} \Rightarrow \theta = -\phi$

$\sin 3\theta + \sin 3\phi = 0$

$\cos\frac{\theta-\phi}{2} = \sqrt{3}\sin\frac{\theta-\phi}{2}$

$\Downarrow$

$\tan\frac{\theta-\phi}{2} = \frac{1}{\sqrt{3}} = \tan\frac{\pi}{6}$

$\Downarrow$

$\boxed{\frac{\theta-\phi}{2} = \frac{\pi}{6}} \Rightarrow \theta = \frac{\pi}{3} + \phi$

$\sin 3\theta + \sin 3\phi = 0$

C D $\frac{C+D}{2}$ $\frac{C-D}{2}$

S + S = 2 S C
S − S = 2 C S
C + C = 2 C C
C − C = 2 S S*

$x = xy$

$\Downarrow$

$x - xy = 0 \Rightarrow x(1-y) = 0$

$\Downarrow$

$x = 0,\ y = 1$

$A \cdot B = 0$

$A = 0$ or $B = 0$

$a^0 = 1 : a \neq 0$

$\boxed{a=0}$ Indeterminate

new Type 3 —: T + ratio

* if $\frac{\sin(\theta+\alpha)}{\cos(\theta-\alpha)} = \frac{1-m}{1+m} \longrightarrow \tan\left(\frac{\pi}{4}-\theta\right)\tan\left(\frac{\pi}{4}-\alpha\right) = ? = m$

C + D

* if $a\sin\theta = b\sin\left(\theta+\frac{2\pi}{3}\right) = c\sin\left(\theta+\frac{4\pi}{3}\right)$; $ab + bc + ca = ?$

* if $\frac{\tan(\theta+\alpha)}{a} = \frac{\tan(\theta+\beta)}{b} = \frac{\tan(\theta+\gamma)}{c} \longrightarrow \frac{a+b}{a-b}\sin^2(\alpha-\beta) + \frac{b+c}{b-c}\sin^2(\beta-\gamma) + \frac{c+a}{c-a}\sin^2(\gamma-\alpha) = ?$

D A

True 1

F ②

S
S ③

Lecture-40 (20/12) 2

* $\frac{\sin(\theta+\alpha)}{\cos(\theta-\alpha)} = \frac{1-m}{1+m} \Rightarrow \frac{\sin(\theta+\alpha) + \overbrace{\cos(\theta-\alpha)}^{\sin\left(\frac{\pi}{2}-\theta+\alpha\right)}}{\sin(\theta+\alpha) - \underbrace{\cos(\theta-\alpha)}_{\sin\left(\frac{\pi}{2}-\theta+\alpha\right)}} = \frac{\cancel{2}}{-\cancel{2}m} = -\frac{1}{m}$

$$\frac{\sin\left(\frac{2\alpha+\frac{\pi}{2}}{2}\right)\cos\left(\frac{\frac{\pi}{2}-2\theta}{2}\right)}{\cos\left(\frac{2\alpha+\frac{\pi}{2}}{2}\right)\cdot\sin\left(\frac{-\frac{\pi}{2}+2\theta}{2}\right)} = -\frac{1}{m} \Rightarrow -\underbrace{\tan\left(\frac{\pi}{4}+\alpha\right)}_{\cot\left(\frac{\pi}{2}-\frac{\pi}{4}-\alpha\right) = \cot\left(\frac{\pi}{4}-\alpha\right)} \cdot \cot\left(\frac{\pi}{4}-\theta\right) = -\frac{1}{m} \Rightarrow \tan\left(\frac{\pi}{4}-\alpha\right)\tan\left(\frac{\pi}{4}+\theta\right) = m$$

C + D

$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$

C D $\frac{C+D}{2}$ $\frac{C-D}{2}$

S + S = 2 S C
S − S = 2 C S
C + C = 2 C C
C − C = 2 S S*

$\frac{a\cdot\cancel{c}}{b\cancel{c}} = \frac{a}{b}$

$\frac{a+c}{b+c} = \frac{a}{b+c} + \underbrace{\frac{c}{b+c}}_{\underbrace{\frac{1}{\frac{b+c}{c}}}_{\frac{1}{\frac{b}{c}+1}}}$

* $a\sin\theta = b\sin\left(\theta+\frac{2\pi}{3}\right) = c\sin\left(\theta+\frac{4\pi}{3}\right) \longrightarrow ab+bc+ca = ?$ Ratio

$a\sin\theta = b\sin\left(\theta+\frac{2\pi}{3}\right) = c\sin\left(\theta+\frac{4\pi}{3}\right) = \lambda$ (Trick) $\Rightarrow \frac{\lambda}{a} = \sin\theta,\ \frac{\lambda}{b} = \sin\left(\theta+\frac{2\pi}{3}\right),\ \frac{\lambda}{c} = \sin\left(\theta+\frac{4\pi}{3}\right)$

$C \quad D \quad \frac{C+D}{2} \quad \frac{C-D}{2}$

$S+S = 2SC$

$$\frac{\lambda}{a}+\frac{\lambda}{b}+\frac{\lambda}{c} = \sin\theta + \sin\left(\theta+\frac{2\pi}{3}\right) + \sin\left(\theta+\frac{4\pi}{3}\right) = \sin\left(\theta+\frac{2\pi}{3}\right) + \cancel{2}\underbrace{\sin\left(\frac{2\theta+\frac{4\pi}{3}}{2}\right)}_{\sin\left(\theta+\frac{2\pi}{3}\right)}\cdot\overbrace{\cos\left(\frac{\frac{4\pi}{3}}{\cancel{2}}\right)}^{-\cos\frac{\pi}{3}} = 0$$

$$\Downarrow$$

$$\lambda\left(\frac{bc+ca+ab}{abc}\right) = 0$$

$\to \lambda = 0$ (Not possible)

$\to \frac{ab+bc+ca}{abc} = 0 \quad \because abc \neq 0 \Rightarrow \boxed{ab+bc+ca=0}$ ✓

$$\frac{\tan A+\tan B}{\tan A-\tan B} = \frac{\frac{SA}{CA}+\frac{SB}{CB}}{\frac{SA}{CA}-\frac{SB}{CB}} = \frac{\sin(A+B)}{\sin(A-B)}$$

* $\frac{\tan(\theta+\alpha)}{a} = \frac{\tan(\theta+\beta)}{b} = \frac{\tan(\theta+\gamma)}{c} \to \overbrace{\frac{a+b}{a-b}\sin^2(\alpha-\beta)}^{1} + \overbrace{\frac{b+c}{b-c}\sin^2(\beta-\gamma)}^{2} + \overbrace{\frac{c+a}{c-a}\sin^2(\gamma-\alpha)}^{3} = ?$

$$\frac{\tan(\theta+\alpha)}{a} = \frac{\tan(\theta+\beta)}{b}$$

$$\frac{a}{b} = \frac{\tan(\theta+\alpha)}{\tan(\theta+\beta)}$$

$$\frac{a+b}{a-b} = \underbrace{\frac{\tan(\theta+\alpha)+\tan(\theta+\beta)}{\tan(\theta+\alpha)-\tan(\theta+\beta)}}_{\frac{\sin(2\theta+\alpha+\beta)}{\sin(\alpha-\beta)}}$$

$$\frac{\tan(\theta+\beta)}{b} = \frac{\tan(\theta+\gamma)}{c}$$

$$\frac{b}{c} = \frac{\tan(\theta+\beta)}{\tan(\theta+\gamma)} \Rightarrow \frac{b+c}{b-c} = \underbrace{\frac{\tan(\theta+\beta)+\tan(\theta+\gamma)}{\tan(\theta+\beta)-\tan(\theta+\gamma)}}_{\frac{\sin(2\theta+\beta+\gamma)}{\sin(\beta-\gamma)}}$$

$$\frac{\tan(\theta+\alpha)}{a} = \frac{\tan(\theta+\gamma)}{c}$$

$$\Downarrow$$

$$\frac{a+c}{a-c} = \underbrace{\frac{\tan(\theta+\alpha)+\tan(\theta+\gamma)}{\tan(\theta+\alpha)-\tan(\theta+\gamma)}}_{\frac{\sin(2\theta+\alpha+\gamma)}{\sin(\alpha-\gamma)}}$$

* ① $= \frac{a+b}{a-b}\sin^2(\alpha-\beta) = \sin(\overset{A}{2\theta+\alpha+\beta})\cdot\sin(\overset{B}{\alpha-\beta}) = \frac{1}{2}\left\{\cos(\cancel{2\theta+2\beta}) - \cos(\cancel{2\theta+2\alpha})\right\}$

② $= \frac{b+c}{b-c}\sin^2(\beta-\gamma) = \sin(2\theta+\gamma+\beta)\cdot\sin(\beta-\gamma) = \frac{1}{2}\left\{\cos(\cancel{2\theta+2\gamma}) - \cos(\cancel{2\theta+2\beta})\right\}$

③ $= \frac{c+a}{c-a}\sin^2(\gamma-\alpha) = \sin(2\theta+\gamma+\alpha)\cdot\sin(\gamma-\alpha) = \frac{1}{2}\left\{\cos(\cancel{2\theta+2\alpha}) - \cos(\cancel{2\theta+2\gamma})\right\}$

$+$

$\boxed{①+②+③=0}$

$C \quad D \quad \frac{C+D}{2} \quad \frac{C-D}{2}$

$S+S = 2SC$

$S-S = 2CS$

$C+C = 2CC$

$C-C = 2SS$*

$A+B \quad A-B \quad A-B$

Lecture – 41 (23/12) 2

* $$\frac{\cos 6\alpha + 6\cos 4\alpha + 15\cos 2\alpha + 10}{\cos 5\alpha + 5\cos 3\alpha + 10\cos\alpha} = f(\cos\alpha) = ?$$

$$N \equiv \underbrace{\cos 6\alpha + \cos 4\alpha}_{} + 5\cos 4\alpha + 15\cos 2\alpha + 10 = (2\cos 5\alpha\cos\alpha) + 5\underbrace{(\cos 4\alpha + \cos 2\alpha)}_{(2\cos 3\alpha\cos\alpha)} + \underbrace{10\cos 2\alpha + 10}_{10(\cos 2\alpha + 1)}$$

$\cos 6\alpha + 6\cos 4\alpha$ → $\cos 6\alpha + \cos 4\alpha + 5\cos 4\alpha$ → $(2\cos 5\alpha\cos\alpha)$

$10(\cos 2\alpha + 1)$, $1 = \cos 0\alpha$ (Trick) → $2\cos\alpha\cdot\cos\alpha$

$$= 2\cos\alpha\,(\cos 5\alpha + 5\cos 3\alpha + 10\cos\alpha)$$

* $$\frac{N}{D} = 2\cos\alpha$$

$C + D$, $\frac{C+D}{2}$, $\frac{C-D}{2}$

$S + S = 2SC$

$S - S = 2CS$

$C + C = 2CC$

$C - C = 2SS^*$

$A+B$ $A-B$ A B

* $$\frac{\sin(A-C) + 2\sin A + \sin(A+C)}{\sin(B-C) + 2\sin B + \sin(B+C)} = \frac{\sin A}{\sin B}$$

* a, b, c : $b - a = c - b = k$ "common" "difference" $\sim 10^{15}$ ⇒ a, b, c is a Arithmetic progression (AP)

Eng. 1st Std. (50-60 yr.)

French. 1st Std. 1635 – WWII (250 yrs)

Nietzsche

Sutra = Aphorism

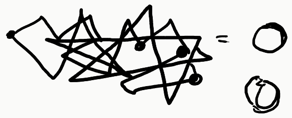
$\sim 10{,}000$ → 2022

Sanskrit Canon → Upanisads, → Brahma Sutra, → Gita

Mahabharat

Flow of River → Wiki

Commentary



Rhizome

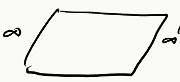
Milieu

* if $\sin(y+z-x)$, $\sin(z+x-y)$, $\sin(x+y-z)$: they form AP

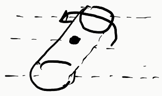
⇓ P.T.

$\tan x$, $\tan y$, $\tan z$ also form an AP.

$S = 1 + 2 + 3 + \ldots$ weird progression → $\infty \in \mathbb{R}$; → $-\frac{1}{12} \in \mathbb{C}$ "sic" infinity 'Ramanujan'



$\ldots\infty$ $\infty\ldots$



Analytical Continuation

possible on $\mathbb{C} \sim \mathbb{R}^2$

* $\sin(x-y)\cos z = \sin(y-z)\cos x \;\ldots$

type 4: change of flavour (warm-up to next section)

HW

* $$\frac{\sec 8\theta - 1}{\sec 4\theta - 1} = \frac{\tan 8\theta}{\tan 2\theta}$$

* $$\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8} = ?$$

$$\sin^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \sin^4\frac{5\pi}{8} + \sin^4\frac{7\pi}{8} = ?$$

* $$\cos^2\alpha + \cos^2\left(\alpha + \frac{2\pi}{3}\right) + \cos^2\left(\alpha - \frac{2\pi}{3}\right) = ?$$

'first principles'

$$\sin 2\theta = 2\sin\theta\cos\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$
$$= 2\cos^2\theta - 1$$

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{\sin 2\theta}{\cos 2\theta}$$

$$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

$$\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}}$$

Lecture-42 (24/dec) 1.5

4. Full Blown Identities (All bets are off.) : Broadening the horizon (Dark side T-vol 2)

$$* \quad \frac{\sec 8\theta - 1}{\sec 4\theta - 1} = \frac{\tan 8\theta}{\tan 2\theta}$$

$$\frac{\tan 8\theta}{\tan 2\theta} = \frac{\frac{\sin 8\theta}{\cos 8\theta}}{\frac{\sin 2\theta}{\cos 2\theta}} = \frac{\sin 8\theta \cdot \cos 2\theta}{\cos 8\theta \cdot \sin 2\theta}$$

$$* \quad LHS = \frac{\frac{1}{\cos 8\theta} - 1}{\frac{1}{\cos 4\theta} - 1} = \frac{\frac{1-\cos 8\theta}{\cos 8\theta}}{\frac{1-\cos 4\theta}{\cos 4\theta}} = \frac{1-\cos 8\theta}{1-\cos 4\theta} \cdot \frac{\cos 4\theta}{\cos 8\theta}$$

trick $\cos\theta$ ... $\cos\theta$

1st way

$$\frac{2\sin 4\theta \cdot \sin 4\theta}{2\sin 2\theta \cdot \sin 2\theta} \cdot \frac{\cos 4\theta}{\cos 8\theta}$$

"

$$\frac{\sin 8\theta \cdot \sin 4\theta}{\cancel{2}\sin 2\theta \cdot \cos 2\theta \cdot \sin 2\theta \cdot \cos 8\theta} = \frac{\tan 8\theta}{\tan 2\theta} \cdot \frac{\cancel{\sin 4\theta}}{\cancel{\sin 4\theta}} = RHS$$

$$\frac{\sin 2\theta}{\cos 2\theta} = \tan 2\theta$$

2nd

$$\frac{1-\cos 8\theta}{1-\cos 4\theta} \cdot \frac{\cos 4\theta}{\cos 8\theta} = \frac{2\sin 4\theta \sin 4\theta \cdot \cos 4\theta}{2\sin 2\theta \cdot \sin 2\theta \cdot \cos 8\theta}$$

$$\frac{1-\cos 8\theta}{1-\cos 4\theta} = \frac{2\sin^2 4\theta}{2\sin^2 2\theta}$$

"

$$\frac{\sin 8\theta}{\cos 8\theta} \cdot \frac{\sin 4\theta}{2\sin 2\theta \cdot \frac{\cos 2\theta}{\cos 2\theta} \cdot \sin 2\theta}$$

"

$$\frac{\tan 8\theta \cdot \sin 4\theta}{\cancel{\sin 4\theta} \cdot \tan 2\theta} = \frac{\tan 8\theta}{\tan 2\theta}$$

$2\sin^2\theta = 1 - \cos 2\theta$

$\cos 2\theta = 1 - 2\sin^2\theta$

$= 2\cos^2\theta - 1$

$(a+b)c \rightarrow ac + bc$

$\downarrow$

$d \cdot c$

$C \quad D \qquad \frac{C+D}{2} \quad \frac{C-D}{2}$

$S + S = 2SC$

$S - S = 2CS$

$C + C = 2CC$

$C - C = 2SS^*$

$\sin 2\theta = 2\sin\theta\cos\theta$

Reduction

$$* \quad \cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8} = 2\left\{\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8}\right\}$$

$$\cos^4\frac{5\pi}{8} = \left\{\cos\left(\pi - \frac{3\pi}{8}\right)\right\}^4 = \cos^4\frac{3\pi}{8}$$

$$\cos^4\frac{7\pi}{8} = \left\{\cos\left(\pi - \frac{\pi}{8}\right)\right\}^4 = \cos^4\frac{\pi}{8}$$

$$\cos^4\frac{\pi}{8} = \left(\frac{1+\cos\frac{\pi}{4}}{2}\right)^2 \qquad \cos^4\frac{3\pi}{8} = \left(\frac{1+\cos\frac{3\pi}{4}}{2}\right)^2$$

$$= \frac{2}{4}\left\{\left(1+\frac{1}{\sqrt{2}}\right)^2 + \left(1-\frac{1}{\sqrt{2}}\right)^2\right\}$$

$$= \frac{1}{2\times 2}\left\{(\sqrt{2}+1)^2 + (\sqrt{2}-1)^2\right\} = \frac{3+\cancel{2\sqrt{2}}+3-\cancel{2\sqrt{2}}}{2\times 2} = \frac{3}{2}$$

S | A

T | C

$\frac{7\pi}{8} = \frac{8\pi - \pi}{8}$

$\frac{5\pi}{8} = \frac{8\pi - 3\pi}{8}$

$\cos 2\theta = 2\cos^2\theta - 1$

$2\cos^2\theta = 1 + \cos 2\theta$

$\frac{3\pi}{4} = \frac{4\pi - \pi}{4} = \pi - \frac{\pi}{4}$

$\cos\frac{3\pi}{4} = \cos\left(\pi - \frac{\pi}{4}\right) = -\cos\frac{\pi}{4}$

$= -\frac{1}{\sqrt{2}}$

Lecture-43 (29/12) 1.5

* $\sin^4\frac{\pi}{8}+\sin^4\frac{3\pi}{8}+\sin^4\frac{5\pi}{8}+\sin^4\frac{7\pi}{8} = 2\left\{\sin^4\frac{\pi}{8}+\sin^4\frac{3\pi}{8}\right\} = 2\left\{\sin^4\frac{\pi}{8}+\cos^4\frac{\pi}{8}\right\}$

$\sin^4\frac{7\pi}{8} = \left(\sin\left(\pi-\frac{\pi}{8}\right)\right)^4 = \left(\sin\frac{\pi}{8}\right)^4$

$\sin^4\frac{3\pi}{8} = \sin^4\left(\frac{\pi}{2}-\frac{\pi}{8}\right) = \cos^4\frac{\pi}{8}$

$= 2\left\{1-\frac{1}{2}\left(\sin\frac{\pi}{4}\right)^2\right\} = 2\left\{1-\frac{1}{4}\right\} = 2\cdot\frac{3}{4} = \frac{3}{2}$

S | A
T | C

$\frac{7\pi}{8} = \frac{8\pi-\pi}{8} = \pi-\frac{\pi}{8}$

$\cos 2\theta = 2\cos^2\theta-1$
$= 1-2\sin^2\theta$

$(\sin^2\theta+\cos^2\theta)^2 = 1^2$

$\sin^4\theta+\cos^4\theta = -2\sin^2\theta\cos^2\theta+1$

$= 1-\frac{1}{2}(\sin 2\theta)^2$

* $\cos^2 A+\cos^2\left(A+\frac{2\pi}{3}\right)+\cos^2\left(A-\frac{2\pi}{3}\right) = \frac{1}{2}\left\{3+\cos 2A+\cos\left(2A+\frac{4\pi}{3}\right)+\cos\left(2A-\frac{4\pi}{3}\right)\right\}$

$= \frac{1}{2}\left\{3+\cancel{\cos 2A}+\cancel{2\cos(2A)}\underbrace{\cos\left(\frac{4}{3}\pi\right)}_{\cos\left(\frac{3\pi+\pi}{3}\right) = -\cos\frac{\pi}{3}}\right\} = \frac{3}{2}$

S | A
T | C

$\cos 2\theta = 2\cos^2\theta-1$
$\Downarrow$
$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$

C D
C-C

Trick question:

* $\frac{\sin\alpha}{\cos 3\alpha}+\frac{\sin 3\alpha}{\cos 9\alpha}+\frac{\sin 9\alpha}{\cos 27\alpha} \overset{P.T}{=} \frac{1}{2}(\tan 27\alpha-\tan\alpha)$

$\frac{\sin 27\alpha}{\cos 27\alpha}-\frac{\sin\alpha}{\cos\alpha}$

$\Downarrow$

$\underbrace{\frac{1}{2}\left\{\frac{2\sin\alpha}{\cos 3\alpha}+\frac{2\sin 3\alpha}{\cos 9\alpha}+\frac{2\sin 9\alpha}{\cos 27\alpha}\right\}}_{f(\tan\alpha)} = \frac{1}{2}\left\{\frac{\overbrace{2\sin\alpha\cos\alpha}^{\sin 2\alpha}}{\cos 3\alpha\cos\alpha}+\frac{\overbrace{2\sin 3\alpha\cos 3\alpha}^{\frac{\sin 6\alpha}{\cos 9\alpha\cos 3\alpha}}}{\cos 9\alpha\cos 3\alpha}+\frac{\overbrace{2\sin 9\alpha\cos 9\alpha}^{\frac{\sin 18\alpha}{\cos 27\alpha\cos 9\alpha}}}{\cos 27\alpha\cos 9\alpha}\right\}$

$\frac{\sin(3\alpha-\alpha)}{\cos 3\alpha\cos\alpha} = \frac{\sin 3\alpha\cos\alpha-\cos 3\alpha\sin\alpha}{\cos 3\alpha\cos\alpha}$

$= \tan 3\alpha-\tan\alpha$

$\sin(A+B) = \sin A\cos B+\cos A\sin B$

$= \frac{1}{2}\left\{\frac{\sin(3\alpha-\alpha)}{\cos 3\alpha\cos\alpha}+\frac{\sin(9\alpha-3\alpha)}{\cos 9\alpha\cos 3\alpha}+\frac{\sin(27\alpha-9\alpha)}{\cos 27\alpha\cos 9\alpha}\right\}$

$= \frac{1}{2}\left\{\cancel{\tan 3\alpha}-\tan\alpha+\cancel{\tan 9\alpha}-\cancel{\tan 3\alpha}+\tan 27\alpha-\cancel{\tan 9\alpha}\right\} = \frac{1}{2}\{\tan 27\alpha-\tan\alpha\}$

HW

$\sin 3\alpha+\sin 3\alpha$ (Trick : $2=1+1$)

* $\frac{\sin 5\alpha-2\sin 3\alpha+\sin\alpha}{\cos 5\alpha-\cos\alpha} = \tan\alpha$

* $\sin 2\beta+2\sin 4\beta+\sin 6\beta = 4\cos^2\alpha\sin 4\alpha$

* $\frac{\sin\gamma-\sin 3\gamma}{\sin^2\gamma-\cos^2\gamma} = 2\sin\gamma$

* $\cos\theta\cos 2\theta\cos 2^2\theta\cos 2^3\theta\ldots\ldots$ $\quad \cos 2^{n-1}\theta = \frac{\sin 2^n\theta}{2^n\sin\theta}$

Transcendental series

* $\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7} = ?$

(60 CE)
Hero of Alexandria
Engineer / Mathematician

$f(x) = \sum_{i=1}^{n} a_i x^i$

upper limit
Lower limit
Finite (Truncates) $\to n=m$ Polynomial of deg. m
$\to n\to\infty$ Not a poly.

Lecture-44 (30/dec) 1.5

$$* \quad x+1 \,\overline{\smash{)}\, x^3+x^2+2x+2}\,\big(\quad x^2+2$$

$$\cancel{x^3+x^2}$$

$$2x+2$$
$$2x+2$$
$$0$$

$$x+1 \,\overline{\smash{)}\, x+x^2+x^3+ \ldots\ldots}\,\big($$

$$\frac{x^3+x^2+2x+2}{x+1} = (x^2+2) + 0$$

division of a polynomial

$$x+1 \,\overline{\smash{)}\, 2+2x+x^2+x^3}\,\big($$

Starting pt of the division = Highest degree (Threshold / cut off)

α. Comparing the sizes of sets (Ref : set theory : Alg. vol. 2)

* Cantor = theologian (Study of God) ⟶ SET theory
Infinity

Number line = space "1-D"

$0 \quad 1 \quad 2 \quad 3 \ldots\ldots \longrightarrow \infty$

Eye = frames = observers = Judgment ground

$\boxed{\text{Continuum} = \text{flow}}$ = ∞ → Identity, ↑ Representation

River

$\boxed{\text{space} \Rightarrow \text{Counting}}$

Bergson

* 1 : unit ⟶ chosen / arbitrary / utilitarian
a priori

↓

Representation = Reproduction
Math. dynamic

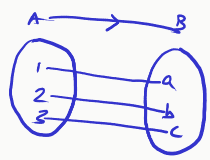
1. Density Argument

* $\mathbb{Z} = \{\ldots 0, 1, 2 \ldots\}$: $n(\mathbb{Z})$ = invariant , $C = \{\Delta, 木, 大\}$
symbols; $0 \to A$, $1 \to B$, $2 \to C$ Representation

translation = transform

1:39 ↑ I = Rehan, 1:41 ↑ I = Rehan

* $\mathbb{N} = \{1, 2, 3 \ldots\}$, $A = \{1, 2, 3\}$; $B = \{a, b, c\}$



$1 \xrightarrow{\text{transfⁿ}} a$
$2 \longrightarrow b$
$3 \longrightarrow c$

⇒ ∃ 1-1 correspondence ⇒ $\boxed{n(A) = n(B) = \text{Cardinality} \quad \text{Invariant of the transfⁿ}}$ = Identity

Gödel

Id = Rehan = Constant

* $1, 2, 3 \ldots\ldots \omega, \omega+1, \omega+2, \ldots\ldots 2\omega, \ldots 3\omega \ldots 4\omega \ldots\ldots \omega^2 \ldots \omega^3 \ldots \omega^\omega$

something > every Nat. #

Smallest uncountable ordinal number

(Cantor, 1883)
Ordinal Numbers
↓
Well ordered
∃ 1st ordinal #

(choice)
unit = 1

* $0, 1, 2, 3 \ldots n, \ldots \aleph_0 \;\; \aleph_1 \;\; \aleph_2 \ldots \aleph_\alpha \ldots$

Smallest Cardinal #
(1st infinity)

* X any set
  - $n(X) < n(\mathbb{N}) \longrightarrow$ X : finite set
  - $n(X) = n(\mathbb{N}) = \aleph_0 \longrightarrow$ $\boxed{\text{X : Countably infinite}}$
  - $n(X) > n(\mathbb{N}) \longrightarrow$ X : Uncountable

fun^n
Power set
Set th..

Lecture-45 (31/dec) 2

* Peano's Axioms : (Formal system)
Giuseppe Peano (~1890)

1. Postulate the existence of 0 (judgment ground = origin)
2. Succession operator (+, S)

} $0, 1, 2, 3 \ldots \Rightarrow$ Natural Numbers.

→ Transformation ∃ Invariant

∃ unity
||
Identity = $\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

'Preference/Arbitrary'

$0 \rightarrow 0$
$S(0) = S0 \rightarrow 1$
$S(S(0)) = SS0 \rightarrow 2$
$S(S(S(0))) = SSS0 \rightarrow 3$

S' : $S' = 0 + \frac{1}{2}$ $\longrightarrow 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \frac{7}{2} \ldots\ldots \supset \mathbb{N}$
New transformation

$\boxed{\text{Method of division}}$

- → Platonic ~ 10^{15} : Laws of self Identity $I = I$
  1, 2 (fixed) → $\frac{1}{2}$
- → Bergsonian/Deleuzian ~ 15 : Laws of self differentiation

Composition of mapping (later)

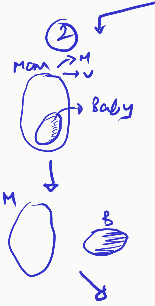
* $x \xrightarrow{i/p} \boxed{f(x)} \xrightarrow{a} \boxed{g(x)} \xrightarrow{b} \boxed{h(x)} \rightarrow c$

$a = f(x)$ $\quad b = g(a)$ $\quad c = h(b)$

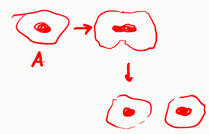
$$\underbrace{\qquad\qquad\qquad}_{c = h(g(f(x))) = h \circ g \circ f(n)}$$

composition operator

(2) (1)



M ≠ B



$A_1 \neq A_2$

'difference is replicated'

Bergsonian

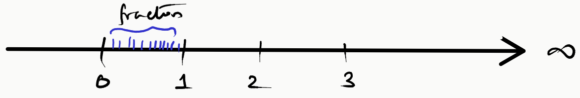
∃ diff of degree → Extensive
∃ diff in kind → Intensive
} → INTENSITY
||
Mixture of "Extensive" + "Intensive"

usual Thermodynamics term

$P = \frac{F}{A}$

Area : Vector

(Normal)

* 

* $\mathbb{Z} = \{\cdots 0, 1, 2, \cdots\}$ Wholes / Integers : $n(\mathbb{Z}) = \infty$ less dense

  $\mathbb{Q} = \{\frac{p}{q} : p, q \in \mathbb{Z}\}$ Part in terms of wholes : $n(\mathbb{Q}) = \underbrace{\infty \times \infty}_{}$ more dense

  Broken / fractional / fractal

  } Problem in set-up.

* Arrangement of $\mathbb{Z}$: .... $-2, -1, 0, 1, 2, \ldots$ 'logically not missing out any'

  " " $\mathbb{Q}$: .... $\frac{1}{2}, \frac{1}{3}$ .... missing! (problem)

  Density Argument won't work

Lecture-46 (7/1/2023) 1

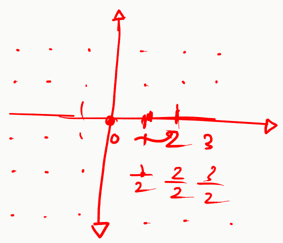
2. Diagonal Argument (2 dim. argument) : Intro. another "dim."

Numerator change (x axis) →

dino. change (y axis) ↓

$\frac{1}{1}$	$\frac{2}{1}$	$\frac{3}{1}$	$\frac{4}{1}$	$\frac{5}{1}$	....
$\frac{1}{2}$	$\frac{2}{2}$	$\frac{3}{2}$	$\frac{4}{2}$	$\frac{5}{2}$	....
$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{4}{3}$	$\frac{5}{3}$	...
⋮					

Linear	Rotation
$\underset{\sim}{p} = m\underset{\sim}{v}$	$\underset{\sim}{L} = \underset{\sim}{r} \times \underset{\sim}{p}$
$\underset{\sim}{F} = \frac{d\underset{\sim}{p}}{dt}$	$\tau = \frac{d\underset{\sim}{L}}{dt} = \underset{\sim}{r} \times \frac{d\underset{\sim}{p}}{dt} = \underset{\sim}{r} \times \underset{\sim}{F}$



$A \times B = \{(a, b) : a \in A, b \in B\}$

* Move on $\hat{x}$: 1 2 3 4 5 .... (Missed!)

  $\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3 \ldots$ "

* Move on $\hat{y}$: $\frac{1}{1}$ $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$ .... $\frac{1}{n}$ (Still not capturing all)

* Move diagonally : $\boxed{n(\mathbb{Z}) = n(\mathbb{Q})}$ all integers matches (pairs up) to all Rational

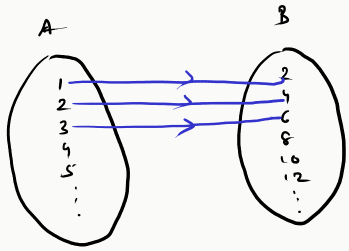
Ex:

* $A = \mathbb{N} = \{1, 2, \ldots\}$
  $B = \{\text{Even } \#\}$

  1 matches with its double 2
  2 matches with its double 4
  3 matches with its double 6
  } $\exists$ a machine to do the transformation (1-1)

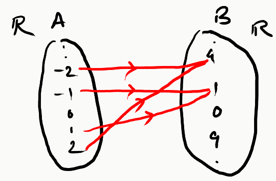
  ⇓

  No gaps ⇒ $\underbrace{n(\mathbb{N})}_{} = n(\text{Even } \#)$
  $\equiv \aleph_0$ (Aleph-Null)



$f(x) = 2x$ (1-1 map)

$f : A \to B = \{(x, 2x), x \in \mathbb{N}\}$



$f(x) = x^2$

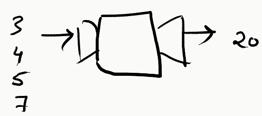
$f : A \to B = \{(x, x^2) : x \in A\}$

Lecture-47 (8/Jan/2023) 1.5

'Real' problem

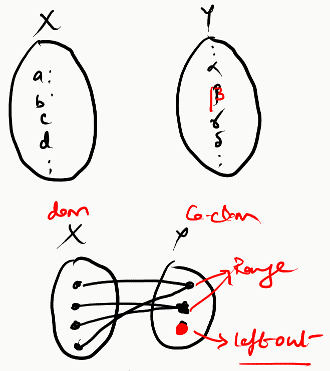
* 1-1 map / Machine ≡ Injection / Injective map
* $\forall$ ⇒ Exhaust all the elements

* $\boxed{f : X \to Y \longrightarrow \forall y \in Y \; \exists x \in X : f(x) = y}$ ≡ onto machine / Surjection "over/above"



" Every Element y can be mapped from element x : f(x) = y "

" Every element of function's Co-dom. is the image of at least one element of it's domain



✓ function

not a surjective fun^n

(Non-surjective fun^n)

* $f(x) = x$ Identity transf^n

* $\underbrace{f(x)}_{y} = 2x+1$ $f: \mathbb{R} \to \mathbb{R}$

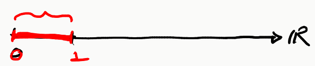
$\forall y \in \mathbb{R}$ $\underbrace{\exists x \in \mathbb{R}}$ $\longrightarrow$ $y = 2x+1 \Rightarrow \underbrace{x = \frac{y-1}{2}}$ $\Rightarrow \forall y \in \mathbb{R} \ \exists x \in \mathbb{R}$

Existent / Nice / not wild anywhere (Not Blowing up) ✓ :)

* Injective + Surjective = Bijection / Invertible function
(1-1) (onto)

'Real' #

scaled down Number line



∈ [0,1]

* Any 4 digits ; # b/w 0 & 1 ; 12 friends wrote

0 . 2 7 2 1 2 1 5 6 . . . .

1 ⟷ 0 . 2 7 2 0 1 9 2 6 1 5 add 1 to 2

2 ⟷ 0 . 3 7 2 0 1 9 2 6 1 5 . . . . add 1 to 7

3 ⟷ 0 . 3 8 2 0 1 9 2 6 1 5 . . . . add 1 to 2

4 ⟷ 0 . 3 8 1 1 1 9 2 6 1 5 . . . . add 1 to 0

5 ⟷ 0 . 3 8 1 1 2 9 2 6 1 5 . . . .

6 ⟷ 0 . 3 8 1 1 2 0 2 6 1 5 . . . .

7 ⟷ 0 . 3 8 1 1 2 1 2 6 1 5 . . . .

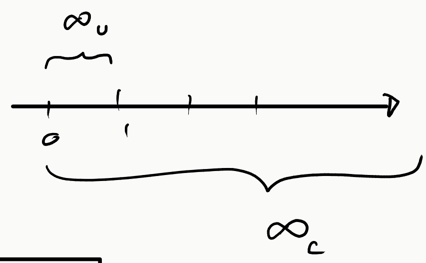
8 ⟷ 0 . 3 8 1 1 2 2 2 6 1 5 . . . . add 1 to diagonal

9 ⟷ 0 . 2 8 1 1 2 2 2 6 1 5 6 . . . Subtract 1

Counting More

* # ∈ [0,1] ⟶ ℝ matches to ℤ

(Move ŷ axis) $\boxed{n(\mathbb{Z}) = n(\mathbb{R})}$ ✗ wrong



List goes on, still will miss a #

Make a real # that isn't on the list !

$\Rightarrow$ $\boxed{\overset{\aleph_0}{n(\mathbb{Z})} < n(\mathbb{R})}$

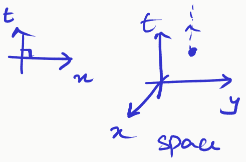
Comes from Counting (Countable ∞) ∞_c

uncountable ∞_u

To Prove :

$\boxed{\aleph_1 = 2^{\aleph_0}}$

Lecture-48 (12/Jan) 1.5



* $\mathbb{N} = \{1, 2, \dots\} \longrightarrow n(\mathbb{N}) = \aleph_0$

$\mathbb{R} \neq \{\pi, \sqrt{13}, -\frac{1}{12}, \sqrt[5]{2} \dots\}$

?? (Relation)

$\boxed{\mathbb{R} = \text{continuum}} \longrightarrow n(\mathbb{R}) = c$ (Cardinality of the continuum)

$A = \{1, 2, 3\}$ $n(A) = 3$

$P(A) = \{ \phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\} \}$; $n(P(A)) = 2^3$

$\boxed{n(P(A)) = 2^{n(A)}}$

list of all the subsets of $\mathbb{N}$

* $P(\mathbb{N}) = \{ \phi$
$\{1\}, \{2\}, \{3\} \dots \{\omega\} \dots$
$\{1,2\}, \{1,3\}, \{1,4\} \dots \{1,\omega\} \dots$
$\{2,3\}, \{2,4\}, \{2,5\}, \dots \{2,\omega\} \dots$
$\vdots \quad \dots \quad \dots \}$ $\longrightarrow n(P(\mathbb{N})) = 2^{n(\mathbb{N})} = 2^{\aleph_0}$

* $\boxed{\text{Continuum Hypo.} \Rightarrow \nexists\ A : \aleph_0 < n(A) < c \Rightarrow c = 2^{\aleph_0} = \aleph_1}$

$n(P(\mathbb{N})) = n(\mathbb{R})$

Transfinite # Cardinal

$0, 1, 2, 3 \dots n \dots \aleph_0, \aleph_1, \aleph_2, \dots \aleph_\alpha \dots$

smallest Cardinal # (1^{st} Infinity)

(Ref: what is the Cantor's Continuum prob.?, Gödel, 1947)

Famous JEE Identity

HW without Induction

* $\cos A \cos 2A \cos 2^2A \cos 2^3A \dots \cos 2^{n-1}A = \dfrac{\sin 2^nA}{2^n \sin A}$ To Prove

$C + D$, $\frac{C+D}{2}$, $\frac{C-D}{2}$

$S + S = 2SC$
$S - S = 2CS$
$C + C = 2CC$
$C - C = 2SS^*$
$A+B$ $A-B$ A B

* LHS $= \dfrac{1}{2\sin A} \{ \underbrace{2\sin A \cos A}_{\sin 2A} \cos 2A \cos 2^2A \cos 2^3A \dots \cos 2^{n-1}A \}$

$= \dfrac{1}{2^2 \sin A} \{ \underbrace{2\sin 2A \cos 2A}_{\sin 4A} \cos 2^2A \cos 2^3A \dots \cos 2^{n-1}A \}$

$= \dfrac{1}{2^3 \sin A} \{ \underbrace{2\sin 4A \cos 4A}_{\sin 8A = \sin 2^3A} \cos 2^3A \dots \cos 2^{n-1}A \} = \dfrac{1}{2^4 \sin A} \{ \underbrace{2\sin 2^3A \cos 2^3A}_{\sin 2^4A} \cos 2^4A \dots \cos 2^{n-1}A \}$

$= \dfrac{1}{2 \cdot 2^4 \sin A} \{ \underbrace{2\sin 2^4A \,(\cos 2^4A}_{\sin 2^5A} \cos 2^5A \dots \underbrace{2\cos 2^{n-2}A}_{\sin 2^{n-1}A} \cos 2^{n-1}A) \}$

$\vdots$

$= \dfrac{1}{2 \cdot 2^{n-1} \sin A} \{ 2\sin 2^{n-1}A \cos 2^{n-1}A \} = \dfrac{1}{2^n \sin A} \{ \sin 2^nA \}$

I am not a being, I am a Becoming

* $\cos\frac{\pi}{7} \cos\frac{2\pi}{7} \cos\frac{4\pi}{7} =$ ✓

C D $\frac{C+D}{2}$ $\frac{C-D}{2}$

$S + S = 2SC$
$S - S = 2CS$
$C + C = 2CC$ ✓
$C - C = 2SS^*$
$A+B$ $A-B$ A B

hw

* $\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{14\pi}{15} = ?$

* $\cos 20 \cos 40 \cos 60 \cos 80 = ? = \frac{1}{16}$

* $\sin\frac{\pi}{18}\sin\frac{5\pi}{18}\sin\frac{7\pi}{18} = ? = \frac{1}{8}$

hw

* if $\theta = \frac{\pi}{2^n+1} \longrightarrow 2^n\cos\theta\cos 2\theta\cos 2^2\theta \ldots \cos 2^{n-1}\theta = ?$

* if $\tan\alpha = \frac{p}{q}$, $\alpha = 6\beta$: $0 < \alpha < 90$

$$\frac{1}{2}\left\{p\,\mathrm{cosec}\,2\beta - q\sec 2\beta\right\} = ?$$

Lecture-49 (13/Jan) 2

$n = 1,2,3,\dots \in \mathbb{Z}^+$

* $$\boxed{\cos\theta\cos2\theta\cos2^2\theta\dots\cos2^{n-1}\theta = \frac{\sin2^n\theta}{2^n\sin\theta}}$$

"Identity" (special series)

'Cos product series'

HW Easy! (2.5/10)

* $\tan\alpha = \frac{p}{q}$, $\alpha = 6\beta$; $0<\alpha<90^\circ$ $\longrightarrow$ $\sin\alpha = \frac{p}{\sqrt{p^2+q^2}}$, $\cos\alpha = \frac{q}{\sqrt{p^2+q^2}}$

$\sin 6\beta = \sin 3(2\beta)$

$\frac{1}{2}\{p\,\mathrm{cosec}\,2\beta - q\sec2\beta\} = \frac{1}{2}\left\{\sqrt{p^2+q^2}\frac{\sin\alpha}{\sin2\beta} - \sqrt{p^2+q^2}\frac{\cos\alpha}{\cos2\beta}\right\}$

$= \sqrt{p^2+q^2}\left\{\frac{\sin\alpha\cos2\beta - \cos\alpha\sin2\beta}{2\sin2\beta\cos2\beta}\right\} = \sqrt{p^2+q^2}\left\{\frac{\sin(\alpha-2\beta)}{\sin4\beta}\right\} = \sqrt{p^2+q^2}\frac{\sin(\cancel{6\beta-2\beta})}{\cancel{\sin4\beta}} = \sqrt{p^2+q^2}$

JEE:

* $\theta = \frac{\pi}{2^n+1}$ $\rightarrow$ $\underbrace{\cancel{2}\cos\theta\cos2\theta\cos2^2\theta\dots\cos2^{n-1}\theta}$ = ? value

$\frac{\sin2^n\theta}{\cancel{2^n}\sin\theta} = \frac{\sin(\pi-\theta)}{\sin\theta} = \frac{\sin\theta}{\sin\theta} = 1$

Trick

$\theta = \frac{\pi}{2^n+1} \Rightarrow 2^n\theta+\theta = \pi$

$\Downarrow$

$\boxed{2^n\theta = \pi-\theta}$

* if $\tan\beta = \frac{\tan\alpha+\tan\gamma}{1+\tan\alpha\tan\gamma}$ $\xrightarrow{P.T.}$ $\sin2\beta = \frac{\sin2\alpha+\sin2\gamma}{1+\sin2\alpha\sin2\gamma}$

$\sin2\beta = \frac{2\tan\beta}{1+\tan^2\beta}$; $\tan\beta = \frac{\frac{\sin\alpha}{\cos\alpha}+\frac{\sin\gamma}{\cos\gamma}}{1+\frac{\sin\alpha}{\cos\alpha}\frac{\sin\gamma}{\cos\gamma}} = \frac{\frac{\sin\alpha\cos\gamma+\sin\gamma\cos\alpha}{\cancel{\cos\alpha\cos\gamma}}}{\frac{\cos\alpha\cos\gamma+\sin\alpha\sin\gamma}{\cancel{\cos\alpha\cos\gamma}}}$

$\tan\beta = \frac{\sin(\alpha+\gamma)}{\cos(\alpha-\gamma)}$

$\sin2\beta = \frac{\frac{2\sin(\alpha+\gamma)}{\cos(\alpha-\gamma)}}{1+\frac{\sin^2(\alpha+\gamma)}{\cos^2(\alpha-\gamma)}} = \frac{\frac{2\sin(\alpha+\gamma)}{\cancel{\cos(\alpha-\gamma)}}}{\frac{\cos^2(\alpha-\gamma)+\sin^2(\alpha+\gamma)}{\cos^2(\alpha-\gamma)}} = \frac{2\sin(\alpha+\gamma)\cos(\alpha-\gamma)}{\cos^2(\alpha-\gamma)+\sin^2(\alpha+\gamma)}$

$= \frac{\sin2\alpha+\sin2\gamma}{\cos^2(\alpha-\gamma)+\sin^2(\alpha+\gamma)} = \frac{\sin2\alpha+\sin2\gamma}{\frac{1}{2}\left(1+\cos(2\alpha-2\gamma)+1-\cos(2\alpha+2\gamma)\right)}$

$= \frac{\cancel{2}(\sin2\alpha+\sin2\gamma)}{\cancel{2}+\underbrace{\cos(2\alpha-2\gamma)+\cos(2\alpha+2\gamma)}_{\cancel{2}\cos2\alpha\cdot\cos(2\gamma)}} = \frac{\sin2\alpha+\sin2\gamma}{1+\cos2\alpha\cos2\gamma}$

$\tan2\beta = \frac{2\tan\beta}{1-\tan^2\beta}$

$\cos2\beta = \cos^2\beta-\sin^2\beta$

$= \frac{(1-\tan^2\beta)}{\sec^2\beta}$

$= \frac{1-\tan^2\beta}{1+\tan^2\beta}$

$\sin2\beta = \frac{2\tan\beta}{1+\tan^2\beta}$

C D $\frac{C+D}{2}$ $\frac{C-D}{2}$

$S+S = 2SC$

$S-S = 2CS$

$C+C = 2CC$

$C-C = 2SS^*$

A+B A−B A B

$\cos2\theta = 2\cos^2\theta-1$

$= 1-2\sin^2\theta$

$\cos^2\theta = \frac{1+\cos2\theta}{2}$

$\cos^2(\alpha-\gamma) = \frac{1+\cos(2\alpha-2\gamma)}{2}$

HW

* $\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}\cos\frac{4\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15} = ?$

* $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha = \cot\alpha$ P.T

Lecture-50 (14/Jan) 2

Cos prod. Id.

* $\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\underbrace{\cos\frac{14\pi}{15}}_{\cos\left(\frac{15\pi-\pi}{15}\right)=\cos\left(\pi-\frac{\pi}{15}\right)=-\cos\frac{\pi}{15}} = -\overset{\theta}{\cos\frac{\pi}{15}}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15} = -\frac{\sin 2^4\frac{\pi}{15}}{2^4\sin\frac{\pi}{15}} = \frac{-\overset{-\sin\pi/15}{\sin\left(\frac{16\pi}{15}\right)}}{16\sin\frac{\pi}{15}} = \frac{1}{16}$

* $\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}\cos\frac{4\pi}{15}\underbrace{\cos\frac{5\pi}{15}}_{\frac{1}{2}}\cos\frac{6\pi}{15}\overset{-\cos\frac{8\pi}{15}}{\cos\frac{7\pi}{15}} = \frac{+1}{2}\underbrace{\left\{\overset{\theta}{\cos\frac{\pi}{15}}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\right\}}_{\frac{\cancel{\sin 2^4\frac{\pi}{15}}}{2^4\cancel{\sin\frac{\pi}{15}}}}\underbrace{\left\{\overset{\cos\frac{\pi}{5}\ \cos\frac{2\pi}{5}}{\cos\frac{3\pi}{15}\cos\frac{6\pi}{15}}\right\}}_{\frac{\cancel{\sin 2^2\frac{\pi}{5}}}{2^2\cancel{\sin\frac{\pi}{5}}}=\frac{1}{2^2}} = \frac{1}{2^7}$

$\log_2 128 = \log_2 2^7 = 7$

4/10

* $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha = \cot\alpha$

HINT: (Rev. Eng / Don't convert to sinc) Diff. way to prev.

* $\underbrace{(\cot\alpha - \tan\alpha)} - 2\tan 2\alpha - 4\tan 4\alpha - 8\cot 8\alpha = 0$

HW

* $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14} = ?$

* $\boxed{(1+\sec 2\alpha)(1+\sec 4\alpha)(1+\sec 8\alpha)\ldots(1+\sec 2^n\alpha) = \tan 2^n\alpha\cot\alpha}$ ✓ $n\in N$ without Induction

* if $\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right) = \tan^3\left(\frac{\pi}{4}+\frac{\alpha}{2}\right) \xrightarrow{\text{P.T.}} \sin\theta = \frac{3\sin\alpha + \sin^3\alpha}{1+3\sin^2\alpha}$

Lecture-57 (15/Jan) 1.5+15

Tip: whenever you see Only tan/cot → Don't Convert to sin/cos

'diff. than vol-1'

* $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha = \cot\alpha \Rightarrow (\cot\alpha - \tan\alpha) - 2\tan 2\alpha - 4\tan 4\alpha - 8\cot 8\alpha = 0$

Trick Identity

* $\cot\alpha - \tan\alpha = \frac{1}{\tan\alpha} - \tan\alpha = \frac{1-\tan^2\alpha}{\tan\alpha} = 2\underbrace{\left(\frac{1-\tan^2\alpha}{2\tan\alpha}\right)}_{\frac{1}{\tan 2\alpha}} = 2\cot 2\alpha$

* $\underbrace{2\cot 2\alpha - 2\tan 2\alpha}_{4\cot 4\alpha} - 4\tan 4\alpha - 8\cancel{\cot 8\alpha} = 0$

$\underbrace{4\cot 4\alpha - 4\tan 4\alpha}_{\cancel{8\cot 8\alpha}}$

$*\ \sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\underbrace{\sin\frac{7\pi}{14}}_{1}\underbrace{\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}}_{\sin\frac{5\pi}{14}\sin\frac{3\pi}{14}\sin\frac{\pi}{14}} = \left(\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\right)^2 = \left(\frac{\sin 2^3\frac{\pi}{7}}{2^3\sin\frac{\pi}{7}}\right)^2 = \frac{1}{2^6}$

$\cos\frac{6\pi}{14}\quad \cos\frac{4\pi}{14}\quad \cos\frac{2\pi}{14}$

$\frac{2\pi}{7}\quad \frac{\pi}{7}$

$-\cos\frac{4\pi}{7}$

$\cos 2\theta = 2\cos^2\theta - 1$

$2\cos^2\theta = 1+\cos 2\theta$

$2\cos^2 2\theta = 1+\cos 4\theta$

$2\cos^2 4\theta = 1+\cos 8\theta$

$\vdots$

$2\cos^2 2^n\theta = 1+\cos 2^{n+1}\theta$

$\downarrow$

$2\cos^2 2^{n-1}\theta = 1+\cos 2^n\theta$

$*\ (1+\sec 2\theta)(1+\sec 4\theta)(1+\sec 8\theta)\cdots(1+\sec 2^n\theta) = \tan 2^n\theta\cot\theta$

$*\ \frac{(1+\cos 2\theta)(1+\cos 4\theta)(1+\cos 8\theta)\cdots(1+\cos 2^n\theta)}{\cos 2\theta\cos 4\theta\cos 8\theta\cdots\cos 2^n\theta} = \frac{(2\cos^2\theta)(2\cos^2 2\theta)(2\cos^2 4\theta)\cdots(2\cos^2 2^{n-1}\theta)}{\cos 2\theta\cos 4\theta\cos 8\theta\cdots\cos 2^{n-1}\theta\cos 2^n\theta}$

$= \frac{2^n\cos\theta}{\cos 2^n\theta}\underbrace{(\cos\theta\cos 2\theta\cos 4\theta\cdots\cos 2^{n-1}\theta)}_{} = \tan 2^n\theta\cot\theta\ \square$

$= \frac{\sin 2^n\theta}{2^n\sin\theta}$ cos prod id

$*\ \boxed{(1+\sec 2\theta)(1+\sec 4\theta)(1+\sec 8\theta)\cdots(1+\sec 2^n\theta) = \tan 2^n\theta\cot\theta}$ sec product series identity

$(1+\sec 2\theta)(1+\sec 4\theta)(1+\sec 8\theta)(1+\sec 16\theta) = \tan 16\theta\cot\theta$

$*\ \tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right) = \tan^3\left(\frac{\pi}{4}+\frac{\alpha}{2}\right) \longrightarrow \sin\theta = \frac{3\sin\alpha+\sin^3\alpha}{1+3\sin^2\alpha}$

$\Downarrow$

$\frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}} = \left(\frac{1+\tan\frac{\alpha}{2}}{1-\tan\frac{\alpha}{2}}\right)^3 \Rightarrow \overbrace{\frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}}^{A(\theta)} = \left(\overbrace{\frac{\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}}}^{A(\alpha)}\right)^3 \Rightarrow \boxed{A(\theta) = (A(\alpha))^3}$

Simplify the expression

$*\ A = \frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta} \longrightarrow A^2 = \left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)^2 = \frac{1+2\sin\theta\cos\theta}{1-2\sin\theta\cos\theta} = \frac{1+\sin 2\theta}{1-\sin 2\theta}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = 2\cos^2\theta - 1$

$= 1-2\sin^2\theta$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$\boxed{A^2 = \left(\frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}\right)^2 = \frac{1+\sin\theta}{1-\sin\theta}} \rightarrow A^2 = \frac{1+\sin\theta}{1-\sin\theta}$

$(1+\sin\alpha)\quad(1-\sin\alpha)$

$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a-b}{a+b} = \frac{c-d}{c+d}$

$*\ A^2(\theta) = (A^2(\alpha))^3 \Rightarrow \frac{1+\sin\theta}{1-\sin\theta} = \frac{(1+\sin\alpha)^3}{(1-\sin\alpha)^3} \xrightarrow[\text{Devid.}]{\text{Comp}} \frac{2\sin\theta}{2} = \frac{(\overbrace{1+\sin\alpha}^{a})^3-(1-\sin\alpha)^3}{(1+\sin\alpha)^3+(1-\sin\alpha)^3}$

Algebra

$(a+b)^3 = a^3+b^3+3ab(a+b) \Rightarrow a^3+b^3 = (a+b)^3-3ab(a+b) = (a+b)((a+b)^2-3ab) = (a+b)(a^2+b^2-ab)$

$(a-b)^3 = a^3-b^3-3ab(a-b) \Rightarrow a^3-b^3 = (a-b)^3+3ab(a-b) = (a-b)((a-b)^2+3ab) = (a-b)(a^2+b^2+ab)$

$*\ \sin\theta = \frac{\cancel{2}\sin\alpha\,(1+\sin^2\alpha+\cancel{2\sin\alpha}+1+\sin^2\alpha-\cancel{2\sin\alpha}+(1-\sin^2\alpha))}{\cancel{2}\,(2+2\sin^2\alpha+\cancel{2\sin\alpha}-\cancel{2\sin\alpha}+\sin^2\alpha-1)} = \frac{3\sin\alpha+\sin^3\alpha}{1+3\sin^2\alpha}\ \square$

HW

* if $\tan^2\theta = 2\tan^2\phi + 1 \longrightarrow \cos 2\theta + \sin^2\phi = ?$
* $\tan 4\theta = \dfrac{4\tan\theta\,(1-\tan^2\theta)}{1-6\tan^2\theta+\tan^4\theta}$

2/10 ✓

* $(2\cos\theta-1)(2\cos 2\theta-1)(2\cos 2^2\theta-1)\dots(2\cos 2^{n-1}\theta-1) = \dfrac{2\cos 2^n\theta+1}{2\cos\theta+1}$

(7/10) ✓

* $\tan(\alpha+100^\circ) = \tan(\alpha+50^\circ)\tan\alpha\tan(\alpha-50^\circ) \longrightarrow$ smallest +ve value of α : equality held.

trig. eqⁿ

Lecture-52 (19 Jan) 2

* $\tan^2\theta = 2\tan^2\phi + 1 \longrightarrow \underbrace{1+\tan^2\theta}_{\sec^2\theta} = 2(\underbrace{\tan^2\phi+1}_{\sec^2\phi = \frac{1}{\cos^2\phi}}) \Rightarrow 2\cos^2\theta = \cos^2\phi$

* $LHS = (2\cos\theta-1)(2\cos2\theta-1)(2\cos2^2\theta-1)\dots\dots(2\cos2^{n-1}\theta-1)$

$= \dfrac{1}{(2\cos\theta+1)}\left\{(2\cos\theta+1)(2\cos\theta-1)(2\cos2\theta-1)(2\cos2^2\theta-1)\dots(2\cos2^{n-1}\theta-1)\right\}$

$(2\cos\theta+1)(2\cos\theta-1) = (4\cos^2\theta-1) = (2\cos2\theta+1)$

$(2\cos2\theta+1)(2\cos2\theta-1) = 4\cos^2 2\theta-1 = (2\cos4\theta+1)$

$(2\cos4\theta+1)(2\cos4\theta-1) = 4\cos^2 4\theta-1 = (2\cos8\theta+1)\dots$

$(2\cos 2^{n-2}\theta-1)$

$4\cos^2 2^{n-2}\theta-1 = (2\cos2^{n-1}\theta+1)$

$(2\cos2^{n-1}\theta+1)(2\cos2^{n-1}\theta-1) = (2\cos2^n\theta+1)$

$\cos2\theta = 2\cos^2\theta-1$
$2\cos2\theta = 4\cos^2\theta-2$
$4\cos^2\theta = 2\cos2\theta+2$

$2\cos8\theta = 4\cos^2 4\theta-1$

$2\cos4\theta = 4\cos^2 2\theta-2$

* $\tan4\theta = \dfrac{4\tan\theta(1-\tan^2\theta)}{1-6\tan^2\theta+\tan^4\theta}$

$\tan2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

* $LHS = \tan4\theta = \dfrac{2\tan2\theta}{1-\tan^2 2\theta} = \dfrac{2\left(\dfrac{2\tan\theta}{1-\tan^2\theta}\right)}{1-\left(\dfrac{2\tan\theta}{1-\tan^2\theta}\right)^2} = \dfrac{4\tan\theta\,(1-\tan^2\theta)}{(1-\tan^2\theta)^2-4\tan^2\theta} = \dfrac{4\tan\theta\,(1-\tan^2\theta)}{1-6\tan^2\theta+\tan^4\theta}$

* $\tan(\alpha+100) = \underbrace{\tan(\alpha+50)\tan\alpha\tan(\alpha-50)}_{(\text{T. fun})\ (\text{val}^n \text{ of } \alpha)}$ 'T. Equation'

$$\tan(A+B) = \frac{\tan A+\tan B}{1-\tan A\tan B}$$

C D $\frac{C+D}{2}$ $\frac{C-D}{2}$

$S+S = 2SC$

$S-S = 2CS$

$C+C = 2CC$

$C-C = 2SS^*$

A+B A−B A B

* $\dfrac{\tan(\alpha+100)}{\tan(\alpha-50)} = \tan(\alpha+50)\tan\alpha$

$\Downarrow$

* $\dfrac{2\sin(\alpha+100)\cdot\cos(\alpha-50)}{2\cos(\alpha+100)\sin(\alpha-50)} = \dfrac{2\sin(\alpha+50)\sin\alpha}{2\cos(\alpha+50)\cos\alpha} \Rightarrow \dfrac{\sin(2\alpha+50)+\sin(150)}{\sin(2\alpha+50)-\sin(150)} = \dfrac{\cos(50)-\cos(2\alpha+50)}{\cos(2\alpha+50)+\cos(50)}$

$$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$$

$\dfrac{\cancel{2}\sin(2\alpha+50)}{\cancel{2}\sin(150)}$ $= \dfrac{2\cos 50}{-2\cos(2\alpha+50)}$ $\Rightarrow \underbrace{2\sin(2\alpha+50)\cos(2\alpha+50)}_{\sin(4\alpha+100)} = -\cos 50$

$\sin(150) = \frac{1}{2}$

$\sin(270-50)$

S A

T C

$\sin(4\alpha+100) = \sin(270-50) \Rightarrow 4\alpha+100 = 220 \Rightarrow 4\alpha = 120 \Rightarrow \boxed{\alpha = 30^\circ} > 0$

HW

* $\dfrac{\cos 7\alpha - \cos 8\alpha}{1+2\cos 5\alpha} = \cos 2\alpha - \cos 3\alpha$ ✓

* $\dfrac{1+\cos 4\alpha}{\cot\alpha - \tan\alpha} = \dfrac{1}{2}\sin 4\alpha$ ✓

* $\cot\dfrac{\pi}{24} = \sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$ → JEE

* $\tan\dfrac{\pi}{16} = \sqrt{4+2\sqrt{2}} - (\sqrt{2}+1)$

HW

Hint: use special values

* if $\tan\dfrac{\theta}{2} = \sqrt{\dfrac{a-b}{a+b}}\tan\dfrac{\phi}{2} \longrightarrow \cos\theta = \dfrac{a\cos\phi+b}{a+b\cos\phi}$

Lecture-53 (21/Jan) 2

C D $\frac{C+D}{2}$ $\frac{C-D}{2}$

$S+S = 2SC$

$S-S = 2CS$

$C+C = 2CC$

$C-C = 2SS^*$

A+B A−B A B

* LHS $= \dfrac{\cos 7\alpha - \cos 8\alpha}{1+2\cos 5\alpha} = \underbrace{\cos 2\alpha - \cos 3\alpha}_{2\sin\frac{5\alpha}{2}\sin\frac{\alpha}{2}}$

Hints / suggestions :

* $N \equiv \cos 7\alpha - \cos 8\alpha = 2\sin\dfrac{15\alpha}{2}\sin\dfrac{\alpha}{2}$

* LHS $= \dfrac{\cos 7\alpha - \cos 8\alpha}{1+2\cos 5\alpha} \underset{\text{step}}{\overset{1^{st}}{=}} \dfrac{2\sin\frac{5\alpha}{2}(\cos 7\alpha - \cos 8\alpha)}{2\sin\frac{5\alpha}{2}(1+2\cos\alpha)} = \dfrac{2\sin\frac{5\alpha}{2}\cos 7\alpha - 2\sin\frac{5\alpha}{2}\cos 8\alpha}{2\left\{\sin\frac{5\alpha}{2} + \underbrace{2\sin\frac{5\alpha}{2}\cos 5\alpha}_{\sin\frac{15\alpha}{2}-\sin\frac{5\alpha}{2}}\right\}} = \dfrac{\left(\sin\frac{19\alpha}{2}-\sin\frac{9\alpha}{2}\right)-\left(\sin\frac{21\alpha}{2}-\sin\frac{11\alpha}{2}\right)}{2\left\{\cancel{\sin\frac{5\alpha}{2}}+\sin\frac{15\alpha}{2}-\cancel{\sin\frac{5\alpha}{2}}\right\}}$

21 9 12 19

$= \dfrac{2\sin\frac{15\alpha}{2}\cos 2\alpha - \left\{2\sin\frac{15}{2}\alpha\cos(-3\alpha)\right\}}{2\sin\frac{15\alpha}{2}} = \cos 2\alpha - \cos 3\alpha$

* $\dfrac{1+\cos 4\alpha}{\cot\alpha - \tan\alpha} = \dfrac{1}{2}\sin 4\alpha$; RHS $= \dfrac{1}{2}\sin 2(2\alpha) = \dfrac{1}{2}\cdot\cancel{2}\sin 2\alpha\cos 2\alpha = \sin 2\alpha\cos 2\alpha$

$D = \cot\alpha - \tan\alpha = \dfrac{\cos\alpha}{\sin\alpha} - \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{2(\cos^2\alpha - \sin^2\alpha)}{2\sin\alpha\cos\alpha} = \dfrac{2\cos 2\alpha}{\sin 2\alpha} = 2\cot 2\alpha$

LHS $= \dfrac{\sin 2\alpha}{2\cos 2\alpha}\left(1+\cos 4\alpha\right) = \dfrac{\sin 2\alpha}{\cancel{2\cos 2\alpha}}\cdot \cancel{2}\cos^{\cancel{2}} 2\alpha$

$\cos 2\alpha = 2\cos^2\alpha - 1$
$\cos 4\alpha + 1 = 2\cos^2 2\alpha$

* $\{\theta, \varphi\}$, To prove $\cos\theta = \dfrac{a\cos\varphi + b}{a+b\cos\varphi}$ ✓ : given $\tan\dfrac{\theta}{2} = \sqrt{\dfrac{a-b}{a+b}}\,\tan\dfrac{\varphi}{2}$

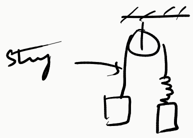
Conditional Identity

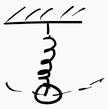
* $\because \cos\theta = \dfrac{1-\tan^2\theta/2}{1+\tan^2\theta/2} = \dfrac{1 - \frac{a-b}{a+b}\tan^2\varphi/2}{1 + \frac{a-b}{a+b}\tan^2\varphi/2} = \dfrac{a+b-(a-b)\tan^2\varphi/2}{a+b+(a-b)\tan^2\varphi/2}$

$= \dfrac{a\left(1-\tan^2\frac{\varphi}{2}\right) + b\left(1+\tan^2\frac{\varphi}{2}\right)}{a\left(1+\tan^2\frac{\varphi}{2}\right) + b\left(1-\tan^2\frac{\varphi}{2}\right)} = \dfrac{\cancel{\left(1+\tan^2\frac{\varphi}{2}\right)}\left\{a\underbrace{\dfrac{\left(1-\tan^2\frac{\varphi}{2}\right)}{1+\tan^2\varphi/2}}_{\cos\varphi} + b\right\}}{\cancel{\left(1+\tan^2\frac{\varphi}{2}\right)}\left\{a + b\underbrace{\dfrac{\left(1-\tan^2\varphi/2\right)}{1+\tan^2\varphi/2}}_{\cos\varphi}\right\}}$

$= \dfrac{a\cos\varphi + b}{a+b\cos\varphi}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= \cos^2\theta(1-\tan^2\theta)$
$\cos 2\theta = \dfrac{(1-\tan^2\theta)}{1+\tan^2\theta}$
$\cos\theta = \dfrac{1-\tan^2\theta/2}{1+\tan^2\theta/2}$ ✓

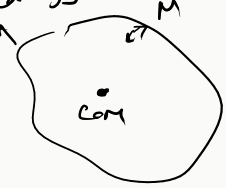




$F = -kx$

$1 e 2$ ✓
$1x + 2x$ ✓
$1x \times 2x$ ✓
$1x \div 2x$ ✓ 8

← Lagrange / Ham



HW

* if $\cos\theta = \cos\alpha\cos\beta \xrightarrow{\text{P.T}} \tan\dfrac{\theta+\alpha}{2}\tan\dfrac{\theta-\alpha}{2} = \tan^2\dfrac{\beta}{2}$ ✓

* if $\cos\theta = \dfrac{\cos\alpha - \cos\beta}{1-\cos\alpha\cos\beta} \xrightarrow{\text{P.T}} \tan\dfrac{\theta}{2} = \pm\tan\dfrac{\alpha}{2}\cot\dfrac{\beta}{2}$ ✓

* if $\cos\theta = \dfrac{\cos\alpha\cos\beta}{1-\sin\alpha\sin\beta} \xrightarrow{\text{P.T}} \tan\dfrac{\theta}{2} = \dfrac{\tan\frac{\alpha}{2} - \tan\frac{\beta}{2}}{1-\tan\frac{\alpha}{2}\tan\frac{\beta}{2}}$ ✓

Lecture-54 (22/Jan) 1:45

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ ✓

I am a crowd !

warm up :

✓ $\underbrace{\cos\dfrac{\pi}{12}}_{\frac{\pi}{4}-\frac{\pi}{6}} = \cos\dfrac{\pi}{4}\cos\dfrac{\pi}{6} + \sin\dfrac{\pi}{4}\sin\dfrac{\pi}{6} = \dfrac{1+\sqrt{3}}{2\sqrt{2}}$ $\left(\frac{1}{\sqrt{2}}\ \frac{\sqrt{3}}{2}\ \frac{1}{\sqrt{2}}\ \frac{1}{2}\right)$

* $\cos\dfrac{\pi}{8} = \sqrt{\dfrac{1+\sqrt{2}}{2\sqrt{2}}}$

* $\tan\dfrac{\pi}{8} = \sqrt{\dfrac{1-1/\sqrt{2}}{1+1/\sqrt{2}}} = \sqrt{\dfrac{\sqrt{2}-1}{\sqrt{2}+1}} = \sqrt{\dfrac{(\sqrt{2}-1)^2}{2-1}} = \sqrt{2}-1$ □

* $\sin\dfrac{\pi}{24} = \sqrt{\dfrac{1-\cos\pi/12}{2}} = \sqrt{\dfrac{2\sqrt{2}-\sqrt{3}-1}{4\sqrt{2}}}$ □

$\cos 2\theta = \cos^2\theta - \sin^2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$
$= 1-2\sin^2\theta$
$= 2\cos^2\theta - 1$

$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$\dfrac{\cos 2\theta}{1} = \dfrac{1-\tan^2\theta}{1+\tan^2\theta} \Rightarrow \dfrac{\cos 2\theta + 1}{\cos 2\theta - 1} = \dfrac{2}{-2\tan^2\theta}$

$\tan\theta = \sqrt{\dfrac{1-\cos 2\theta}{1+\cos 2\theta}}$ ☆☆☆

HW Method 1

* $\underbrace{\cot\frac{\pi}{24}}_{\sim 7.6} = \frac{\cos \pi/24}{\sin \pi/24} = \sqrt{\frac{1+\cos\pi/12}{1-\cos\pi/12}} = \sqrt{\frac{1+\frac{1+\sqrt3}{2\sqrt2}}{1-\frac{(1+\sqrt3)}{2\sqrt2}}} = \sqrt{\frac{2\sqrt2+1+\sqrt3}{2\sqrt2-1-\sqrt3}}$

Trouble

$$\sqrt{\frac{2\sqrt2+\sqrt3+1}{2\sqrt2-(\sqrt3+1)}} = \sqrt{\frac{2\sqrt2+(\sqrt3+1)}{2\sqrt2-(\sqrt3+1)}\times\frac{2\sqrt2+(\sqrt3+1)}{2\sqrt2+(\sqrt3+1)}} = \frac{2\sqrt2+\sqrt3+1}{\sqrt{8-(\sqrt3+1)^2}} = \frac{2\sqrt2+\sqrt3+1}{\sqrt{8-3-1-2\sqrt3}} = \frac{2\sqrt2+\sqrt3+1}{\sqrt{2(2-\sqrt3)}}$$

$$= \frac{1}{\sqrt2}\left\{\frac{2\sqrt2+\sqrt3+1}{\sqrt{2-\sqrt3}}\,\frac{\sqrt{2+\sqrt3}}{\sqrt{2+\sqrt3}}\right\} = \frac{1}{\sqrt2}\cdot\frac{(2\sqrt2+\sqrt3+1)\sqrt{2+\sqrt3}}{\sqrt{4-3}} = \frac{(2\sqrt2+\sqrt3+1)\sqrt{2+\sqrt3}}{\sqrt2}$$ Stuck!

$$= \left(2+\sqrt{\frac32}+\frac{1}{\sqrt2}\right)\sqrt{2+\sqrt3}$$ Correct but not simplified!

* $\cot\frac{\pi}{24} = \frac{\cos\pi/24}{\sin\pi/24} = \frac{1+\cos\pi/12}{\sin\pi/12} = \sqrt2+\sqrt3+\sqrt4+\sqrt6$

$\sin2\theta = 2\sin\theta\cos\theta$

$\cos2\theta = 2\cos^2\theta-1$

$\cos^2\theta = \frac{1+\cos2\theta}{2}$

$$\boxed{\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{2\cos^2\theta}{2\sin\theta\cos\theta} = \frac{2\cos^2\theta}{\sin2\theta} = \frac{1+\cos2\theta}{\sin2\theta} = \frac{\sin2\theta}{1-\cos2\theta}}$$

HW

* if $\sin\alpha+\sin\beta = a$, $\cos\alpha+\cos\beta = b$ $\to$ $\cos(\alpha-\beta) = ?$, $\to$ $\tan\frac{\alpha-\beta}{2} = ?$

* $a\cos\theta+b\sin\theta = c$ has 2 distinct roots α, β $\longrightarrow$ P.T. $\sin(\alpha+\beta) = \frac{2ab}{a^2+b^2}$

Eq^n

Lecture-55 (27/Jan) 2

HW

* if $\cos\theta = \cos\alpha\cos\beta \xrightarrow{P.T} \underbrace{\tan\underbrace{\frac{\theta+\alpha}{2}}_{\frac{\theta}{2}+\frac{\alpha}{2}}\tan\underbrace{\frac{\theta-\alpha}{2}}_{\frac{\theta}{2}-\frac{\alpha}{2}}} = \tan^2\frac{\beta}{2}$ ✓

done by Penum Engineering

$$\frac{2\sin\frac{\theta+\alpha}{2}\sin\frac{\theta-\alpha}{2}}{2\cos\frac{\theta+\alpha}{2}\cos\frac{\theta-\alpha}{2}} = \frac{\cos\alpha-\cos\theta}{\cos\theta+\cos\alpha}$$

$\Downarrow$

$$\frac{\cos\theta}{\cos\alpha} = \frac{1-\tan^2\beta/2}{1+\tan^2\beta/2} = \cos\beta$$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$\tan(A+B) = \frac{\tan A+\tan B}{1-\tan A\tan B}$

$S+S = 2SC$
$S-S = 2CS$
$C+C = 2CC$
$C-C = 2SS^*$

A+B A-B ^ B

$\underbrace{\frac{a}{b} = \frac{c}{d}} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$

* if $\cos\theta = \frac{\cos\alpha - \cos\beta}{1-\cos\alpha\cos\beta} \xrightarrow{P.T} \tan\frac{\theta}{2} = \pm\tan\frac{\alpha}{2}\cot\frac{\beta}{2}$

$\downarrow$

$$\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} = \frac{\cos\alpha-\cos\beta}{1-\cos\alpha\cos\beta} \xrightarrow{C+D} \frac{1}{-\tan^2\theta/2} = \frac{(1+\overbrace{\cos\alpha-\cos\beta-\cos\alpha\cos\beta}^{\cos\beta(1+\cos\alpha)})}{(\cos\alpha-\cos\beta-1+\cos\alpha\cos\beta)} = \frac{(1+\cos\alpha)(1-\cos\beta)}{(\cos\alpha-1)(1+\cos\beta)}$$

$\cos\beta(\cos\alpha-1)$

$\Downarrow$

$$\tan^2\theta/2 = \frac{(1-\cos\alpha)(1+\cos\beta)}{(1+\cos\alpha)(1-\cos\beta)} = \tan^2\frac{\alpha}{2}\cdot\frac{1}{\tan^2\beta/2}$$

$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$\tan^2\frac{\theta}{2} = \frac{\cos\theta-1}{\cos\theta+1}$ CD

$A^2 = B^2 \Rightarrow A = \pm B$

* if $\cos\theta = \frac{\cos\alpha\cos\beta}{1-\sin\alpha\sin\beta} \xrightarrow{P.T} \tan\frac{\theta}{2} = \frac{\tan\frac{\alpha}{2}-\tan\frac{\beta}{2}}{1-\tan\frac{\alpha}{2}\tan\frac{\beta}{2}}$

detour to ITF (Comment)

* $\sin(A+B) = \sin A\cos B + \cos A\sin B$; $\sin A = x \Rightarrow A = \sin^{-1}(x)$ (deep meaning)

$\sin(A+B) = x\sqrt{1-y^2} + y\sqrt{1-x^2}$; $\sin B = y \Rightarrow B = \sin^{-1}(y)$

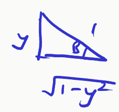
$\Downarrow$

$A+B = \sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right)$

$\Downarrow$

$$\boxed{\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right)}$$ ✓

$\boxed{\text{ITF}} \mapsto$



$\cos B = \sqrt{1-y^2}$

$\cos A = \sqrt{1-x^2}$

C D $\frac{C+D}{2}$ $\frac{C-D}{2}$

$S+S = 2SC$

$\sin C + \sin D$ = Known

* if $\underbrace{\cos\theta} = \underbrace{\frac{\cos\alpha\cos\beta}{1-\sin\alpha\sin\beta}} \xrightarrow{P.T} \tan\frac{\theta}{2} = \underbrace{\frac{\tan\frac{\alpha}{2}-\tan\frac{\beta}{2}}{1-\tan\frac{\alpha}{2}\tan\frac{\beta}{2}}}$

$S+S = 2SC$
$S-S = 2CS$
$C+C = 2CC$
$C-C = 2SS^*$

$$\frac{\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} - \frac{\sin\frac{\beta}{2}}{\cos\frac{\beta}{2}}}{1-\frac{\sin\alpha/2\,\sin\beta/2}{\cos\alpha/2\,\cos\beta/2}} = \frac{\sin\frac{\alpha}{2}\cos\frac{\beta}{2}-\cos\frac{\alpha}{2}\sin\frac{\beta}{2}}{\cos\frac{\alpha}{2}\cos\frac{\beta}{2}-\sin\frac{\alpha}{2}\sin\frac{\beta}{2}} = \frac{\sin\left(\frac{\alpha}{2}-\frac{\beta}{2}\right)}{\cos\left(\frac{\alpha}{2}+\frac{\beta}{2}\right)}$$

$$\tan^2\frac{\theta}{2} = \frac{1-\cos(\alpha+\beta)}{1+\cos(\alpha+\beta)} = \frac{1-\cos\alpha\cos\beta+\sin\alpha\sin\beta}{1+\cos\alpha\cos\beta-\sin\alpha\sin\beta}$$

$$\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\cos^2\frac{\theta}{2} = \frac{1+\cos\theta}{2}$$

identity

$$* \because \tan^2\frac{\theta}{2} = \frac{1-\cos\theta}{1+\cos\theta} = \frac{1-\dfrac{\cos\alpha\cos\beta}{1-\sin\alpha\sin\beta}}{1+\dfrac{\cos\alpha\cos\beta}{1-\sin\alpha\sin\beta}}$$

$$\frac{1-\dfrac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta}}{\left(\dfrac{\cos\alpha\cos\beta}{-\sin\alpha\sin\beta}-1\right)}$$

Lecture-56 (28/Jan) 2

All syllabus Olympiad

Feb – Mar

9–10th Math/Chem. ph-1

Stats
Δ, O
quadrilateral
Coord geo.
Sur. Area/vol

Apr–Oct: just ques.

nothing dun at all!

$$2\sin^2\theta = 3\cos\theta \longleftrightarrow 0 \le \theta \le 2\pi$$

Trig Eqn

$f(\sin\theta) = 0 / \frac{\sqrt{3}}{2}, \frac{1}{2} \ldots = \sin\theta_0$

known

S | A
T | C

$$2(1-\cos^2\theta) - 3\cos\theta = 0$$

Quad Eqn

$*$ if $\sin\alpha+\sin\beta = a$, $\cos\alpha+\cos\beta = b$

$\to \cos(\alpha-\beta) = ? = \frac{a^2+b^2-1}{2}$ ✓

$\to \tan\frac{\alpha-\beta}{2} = ?$

$\theta = \alpha-\beta$

$$\tan^2\frac{\theta}{2} = \frac{1-\cos\theta}{1+\cos\theta} = \frac{1-\left(\dfrac{a^2+b^2-2}{2}\right)}{1+\left(\dfrac{a^2+b^2-2}{2}\right)} = \frac{2-a^2-b^2+2}{\cancel{2}+a^2+b^2-\cancel{2}} \Rightarrow \tan\frac{\alpha-\beta}{2} = \pm\sqrt{\frac{4-a^2-b^2}{a^2+b^2}}$$

$$\frac{\cos 2\theta}{1} = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$\frac{\cos 2\theta+1}{\cos 2\theta - 1} = \frac{2}{-2\tan^2\theta}$$

$*$ $a\cos\theta + b\sin\theta = c$ has 2 distinct roots α, β $\longrightarrow$ $\sin(\alpha+\beta) = \frac{2ab}{a^2+b^2}$

Eqn

1st attempt

$$a\cos\theta = c - b\sin\theta \Rightarrow a^2\cos^2\theta = c^2 + b^2\sin^2\theta - 2bc\sin\theta$$

$(1-\sin^2\theta)$

$$a^2 - a^2\sin^2\theta = c^2+b^2\sin^2\theta - 2bc\sin\theta \Rightarrow (a^2+b^2)\sin^2\theta + 2bc\sin\theta + (a^2-c^2) = 0$$

$$ax^2+bx+c=0$$
$$\alpha+\beta = -\frac{b}{a}$$
$$\alpha\beta = \frac{c}{a}$$

$$\sin\alpha + \sin\beta = \frac{-2bc}{a^2+b^2}$$

2nd attempt

$$\boxed{a\cos\alpha + b\sin\alpha = c, \quad a\cos\beta + b\sin\beta = c}$$

$$* \; a(\cos\alpha - \cos\beta) + b(\sin\alpha - \sin\beta) = 0 \Rightarrow a\sin\frac{\alpha+\beta}{2} = b\cos\frac{\alpha+\beta}{2} \Rightarrow \tan\frac{\alpha+\beta}{2} = \frac{b}{a}$$

$$-\cancel{2}\sin\frac{\alpha+\beta}{2}\,\cancel{\sin\frac{\alpha-\beta}{2}} \qquad \cancel{2}\cos\frac{\alpha+\beta}{2}\,\cancel{\sin\left(\frac{\alpha-\beta}{2}\right)}$$

$S+S = 2SC$
$S-S = 2CS$
$C+C = 2CC$
$C-C = 2SS$

$$* \; \sin(\alpha+\beta) = \frac{2\tan\frac{\alpha+\beta}{2}}{1+\tan^2\frac{\alpha+\beta}{2}} = \frac{\frac{2b}{a}}{1+\frac{b^2}{a^2}} = \frac{2ab}{a^2+b^2}$$

$$\cos 2\theta = C^2 - S^2 = \frac{1-\tan^2}{1+\tan^2}$$

$$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2}$$

$$\frac{S}{C} = \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

* if $\tan\frac{\theta}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{\phi}{2} \xrightarrow{\text{P.T.}} \boxed{\cos\phi = \frac{\cos\theta - e}{1 - e\cos\theta}}$

$\cos 2\theta = f(\tan\theta)$

$\cos\phi - e\cos\phi\cos\theta = \cos\theta - e \Rightarrow (e\cos\phi + 1)\cos\theta = \cos\phi + e$

$\Downarrow$

$\boxed{\cos\theta = \frac{\cos\phi + e}{1 + e\cos\phi}}$

Last ques. of Dark Side :

* $\underbrace{\tan 142\frac{1}{2}^\circ}_{\left(\frac{285}{2}\right)^\circ} = 2 + \sqrt{2} - \sqrt{3} - \sqrt{6}$

$\tan\left(\frac{180 + 105}{2}\right) = \tan\left(90 + \frac{105}{2}\right) = -\cot\frac{105}{2} = -\cot\left(\frac{90 + 15}{2}\right) = -\cot\left(45 + \frac{15}{2}\right) = \frac{-1}{\tan\left(45 + \frac{15}{2}\right)}$

S | A

T | C

~142.

* $\tan\frac{285}{2} = \pm\sqrt{\frac{1 - \cos 285}{1 + \cos 285}} = \sqrt{\frac{1 - \cos 75}{1 + \cos 75}} = \sqrt{\frac{1 - \sin 15}{1 + \sin 15}\frac{(1 - \sin 15)}{(1 - \sin 15)}} = \frac{(1 - \sin 15)}{\sqrt{1 - (\sin 15)^2}} = \frac{1 - \overset{\frac{\sqrt{3}-1}{2\sqrt{2}}}{\sin 15}}{\cos 15}$ ✓

$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}}$

$\cos 285 = \cos(360 - 75)$

$= \cos 75$

$= \frac{1 - \frac{(\sqrt{3} - 1)}{2\sqrt{2}}}{\frac{(\sqrt{3} + 1)}{2\sqrt{2}}} = \frac{2\sqrt{2} - \sqrt{3} + 1}{\sqrt{3} + 1}\frac{(\sqrt{3} - 1)}{(\sqrt{3} - 1)} = \frac{2\sqrt{6} - 2\sqrt{2} - 3 + \sqrt{3} + \sqrt{3} - 1}{3 - 1}$

$= \sqrt{6} - \underbrace{\sqrt{2} - 2} + \sqrt{3}$