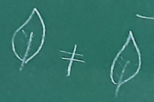
Lecture 1 (Algebra vol-2) (27/2/2022)

Inequalities

* "Every concept originates through our equating what is unequal"

— On Truth & Lies, Nietzsche



} = "LEAF"

forgetting differences/
Cancellation of difference/
Equalization of the inequality

⇒ ∃ Equality (=)

Concept / Category

Nature - unskilled hand

* "Every phenomenon refers to an inequality by which it is conditioned"

— Difference & Rep. (DR), Deleuze

Inequality ⇒ Phenomenon

* "The world 'happens' while God (~Nature) calculates; if the calculations were exact (=), there would be no world. (No phenomena)"

* "if $a \neq b$ is impossible, then $a = b$; if a is distant from every ≠ c which is distant from b then $a = b$"

— DR, Deleuze

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* a, b (2 obj) → a is not b ⇒ Difference

Immanent POV Sys. of thought	Dogmatic POV Poses a beyond
Vyas (~Gita)	Hegelian difference
Lucretius (~700bce)	Platonic
Heraclitus	Aristotle "
Nietzsche	"
Bergson (Mathem.)	99% Philosophers
Deleuze	~100% physicist
	~100% Mathematicians
Now 2022	(2022)
# ~10^2 people	# ~10^{15} people

* Difference = rel^n b/w identities A, B
(change) (Movement) (object) (Stasis)

1, 2 identities → 2 − 1 = 1
difference operator

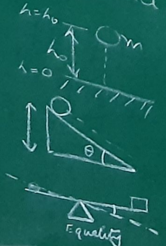
Problem ↓

Whence/How identities come from?

* "Everything is like the flight of an Eagle: overflights, suspension, & descent. Everything goes from high to low"

— DR

* Difference ⇒ Phenomena



∃ 'Height difference'

⇓

Phenomena (flow)

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* "Nietzsche attacks in science the scientific mania for seeking Balances (=), utilitarianism.

utility — Fruitful
└ Harmful

(Nietzsche's)

* His whole critique operates on 3 level: against

logical Identity ⇒ A = A (self-similar)

Mathematical Equality (=)

Physical Equilibrium (stasis)"

— Nietzsche & philosophy, Deleuze

* To deny differences ⇒ To deny life (Phenomena)

⇓

Death

* Duration → ∄ Origin
→ ∄ End } otherwise → Paradox/ Infinitely flawed

"there existed a time when there was no time"

Duration ≠ Time (= physics' Time)

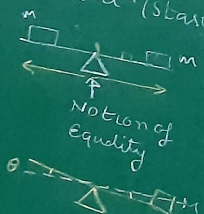
⇓

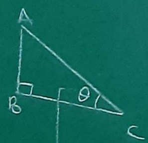
$\boxed{\text{Duration is Eternal}}$

Lecture 2 (28/2/2022)

Inequalities

* Equalization of the unequal ⇒ =
* Forgetting differences ⇒ =
* Inequality ⇒ Phenomenon / Processes
  Inexact "Calculation" ⇒ World / Reality
* $\boxed{\text{Difference} \Rightarrow \text{Phenomenon}}$
* Deny difference ⇒ deny 'life' ⇒ Death
* $\underbrace{\sum F}_{}= 0$ ⟶ Modified $\sum F > 0$, $\sum F < 0$
  'Vector' sum of all the forces
  ⇓
  Equi.^{m} (Stasis)
*





Cline = In$\widehat{\text{CLIN}}$ation
(Κλινω) = Bend / Angle ∢

* ∃ Minimal Angle ⇒ Movement
* "Thms. in statics generally reduce to 0 the angle of inclination & the inequalities that cause it: a negation of the same kind will reign over the science."
  — The Birth of Phys., Michel Serres

* Statics = a discourse on Inequality:
  deviations, measures, descriptions are brought back to 0

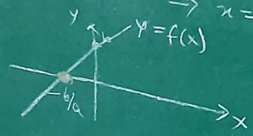
* $y = f(x) = \sum_{i=0}^{n} a_i x^i$
  - $\to n=0 \to f(x) = a$
  - $\to n=1 \to f(x) = ax+b$
  - $\to n=2 \to f(x) = ax^2+bx+c$

  Transformations / Machines

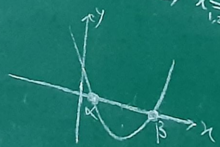
  ↓ Switch off

  $x = \alpha \Rightarrow \boxed{f(\alpha) = 0}$
  root / Sol^n / Zero.

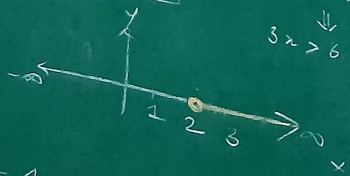
* $y = ax+b = 0 \Rightarrow x = -\frac{b}{a}$ 'point' $a, b > 0$



* $y = ax^2+bx+c = 0 \Rightarrow x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ Point



* $y = f(x) = 3x-6 = 0 \Rightarrow$ 'Root / Sol^n' Switch off Point $\boxed{x = 2}$
* Analyzing Inequality
  $\underbrace{3x-6}_{} > 0 \Rightarrow 3x-6+6 > 0+6$
  $\boxed{f(x) \geq 0}$
  ⇓
  $3x > 6 \Rightarrow \frac{3x}{3} > \frac{6}{3}$
  ⇓
  $\boxed{x > 2}$
  ⇓
  $x \in (2, \infty)$

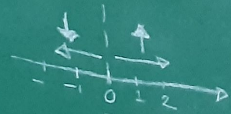


* Inequality ⟶ Analysis ⟶ Set / lines of flight
* ○ ≡ Excluded ≡ ( )
  ● ≡ Included ≡ [ ]

* Solve (Equality) ⟶ Analyze (Inequality)

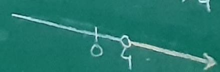
1. Rules

* $ax-b > 0 \Rightarrow \begin{cases} x > \frac{b}{a} & a > 0 \\ x < \frac{b}{a} & a < 0 \end{cases}$
* If $x > a \Rightarrow -x < -a$
  'change of Inequality when multiplied by −ve'
  $x = 2 \Rightarrow -x = -2$
  $5 > 2 \Rightarrow -5 < -2$



Ground / Eye of obs for judgement

* $-3x+12 < 0 \Rightarrow x > 4 \Rightarrow x \in (4, \infty)$



Lecture 3 (5/3/2022)

Inequalities ($\mathbb{R}$) "strict Inequality" $x > 2$

1. Rules

Multiplicative Inv.
$\exists\, a^{-1}: a^{-1} a = 1$

$\exists\, -1 : a - a = 0$
Additive Inv.

$f(x) > 0$
a, b, c distinct

* $a > b \Rightarrow a + c > b + c$
  $a - c > b - c$
  $ac > bc$
  $\frac{a}{c} > \frac{b}{c}$

  Transformations: Inequality is preserved

* $a > b \Rightarrow -a < -b$ 'Additive Inverse'
  $a > b \Rightarrow \frac{1}{a} < \frac{1}{b}$ 'Multiplicative Inv.'

  Transf: inequ. is not preserved

* $a > b \Rightarrow b < a$ 'invertible'
* $a_1 > b_1,\ a_2 > b_2 \quad a_n > b_n$
  $\Downarrow$
  $a_1 + a_2 + \dots + a_n > b_1 + b_2 + \dots b_n$
  $a_1 a_2 \dots a_n > b_1 b_2 \dots b_n$
* $a > b \Rightarrow \sqrt[p]{a} > \sqrt[p]{b} \Leftrightarrow a^{1/p} > b^{1/p} \Rightarrow a^{q/p} > b^{q/p}$
  $\Downarrow$
  $a^r > b^r$
* $a^r > b^r \Rightarrow \frac{1}{a^r} < \frac{1}{b^r} \Rightarrow a^{-r} < b^{-r}$
* $(a-b)^2 > 0 \Rightarrow a^2 + b^2 > 2ab \Rightarrow \frac{a^2+b^2}{2} > ab$

$$\frac{(a+b)^2 - 2ab}{2} > ab \Rightarrow \frac{(a+b)^2}{2} - ab > ab$$
$$\Downarrow$$
$$\frac{(a+b)^2}{2} > 2ab$$
$$\Downarrow$$
$$\boxed{\underbrace{\frac{a+b}{2}}_{A} > \underbrace{\sqrt{ab}}_{G}} \Leftarrow \left(\frac{a+b}{2}\right)^2 > ab$$

On AM / GM.

* $A = \frac{a+b}{2}$, $G = \sqrt{ab}$
  Arithmetic Mean, geometric Mean / Avg. / Typicality

* $A - G = \frac{a+b}{2} - \sqrt{ab} = \frac{a+b-2\sqrt{ab}}{2} = \frac{(\sqrt{a}-\sqrt{b})^2}{2} > 0$

$\boxed{A > G}$

* $ax^2 + bx + c = 0 \longrightarrow \alpha + \beta = -\frac{b}{a},\ \alpha\beta = \frac{c}{a}$
  $\downarrow$
* $\boxed{x^2 - 2Ax + G^2 = 0}$
  $\downarrow$
  $x_{1,2} = \frac{2A \pm \sqrt{4A^2 - 4G^2}}{2} = A \pm \sqrt{A^2 - G^2}$

$\longrightarrow$ AM, GM of 2 # given.

2 #s $= x_{1,2}$ (Can be calculated)

Lecture-4 (6/3/2022)

Inequalities ($\mathbb{R}$)

* $a^2+b^2 > 2ab$; $\overbrace{\frac{a+b}{2}}^{AM} > \overbrace{\sqrt{ab}}^{GM}$

* T.P : $a^2+b^2+c^2 > ab+bc+ca$, $a,b,c>0$

check

* b, c → $b^2+c^2 > 2bc$
  c, a → $c^2+a^2 > 2ca$
  a, b → $a^2+b^2 > 2ab$
  ⇓
  $2(a^2+b^2+c^2) > 2(bc+ca+ab)$ □

* T.P : $2(a^3+b^3+c^3) > bc(b+c)+ca(c+a)+ab(a+b)$

check

* $b^2+c^2 > \underbrace{2bc}_{bc+bc}$

  $\underbrace{b^2+c^2-bc} > bc$
  ⇓
  $\underbrace{(b^2+c^2-bc)(b+c)}_{b^3+c^3} > bc(b+c)$

$(a+b)^3 = a^3+b^3+3ab(a+b)$
$a^3+b^3 = (a+b)(\underline{a^2+b^2-ab})$

* $b^3+c^3 > bc(b+c)$
  $a^3+b^3 > ab(a+b)$
  $a^3+c^3 > ac(a+c)$
  $2(a^3+b^3+c^3) > ab(a+b)+bc(b+c)+ca(a+c)$ □

Taste of Calculus

* Real valued transf^n
  $f(x) = x^3+1$, $g(x) = x^2+x$ $x \in \mathbb{R}$
  Which out of these transf^n is 'greater'?
  $f(x) - g(x) = x^3+1-x^2-x$
  $= x^3-x^2+(1-x) = x^2(x-1)-(x-1)$

{transformation only possible within $\mathbb{R}$}

* $f(x)-g(x) = \underbrace{(x^2-1)}_{(x+1)(x-1)}(x-1) = \underbrace{(x-1)^2}_{+ve}(x+1)$

* $f(x) > g(x)$ if $x+1>0 \Rightarrow x>-1$
  $f(x) < g(x)$ if $x+1<0 \Rightarrow x<-1$
  $f(x) = g(x)$ if $x+1=0$ ⇓ $x=-1$ Critical pt.

Imp (hint of Calculus : Max/Min)

* $a, b > 0$; $S = a+b$, $P = ab$
  i) if Sum (Fixed / given) → Product is greatest (Max) when ?
  ii) if Product (given) → Sum is least (Min) when ?

* $(a+b)^2 = a^2+b^2+2ab$ — ①
  $(a-b)^2 = a^2+b^2-2ab$ — ②
  $\underbrace{4ab}_{P} = \underbrace{(a+b)^2}_{S} - (a-b)^2$ ① − ②

* $4P = S - (a-b)^2 \xrightarrow{S:\ fix}$ P is max when $\boxed{a=b}$
  ⇓ fix
  $4P - S = -(a-b)^2$
  $S = 4P + (a-b)^2 \xrightarrow{P:\ fix}$ S is min when $\boxed{a=b}$
  ⇓ fixed
  $S - 4P = (a-b)^2$

* $4P - S = -(a-b)^2$

a	b	$-(a-b)^2$	P	S
1	2	-1	2	3
3	5	-4	15	8
2	3	-1	6	5
2	2	0	4	4
3	3	0	9	6
4	4	0	16	8

* (S fixed)

a	b	$-(a-b)^2$	P	S
1	3	-4	3	4
$\boxed{2}$	$\boxed{2}$	0	4	4
3	1	-4	3	4
$\frac{1}{2}$	$\frac{7}{2}$	-9	$\frac{7}{4}$	4

* $S - 4P = (a-b)^2$ $P_{max} = 4$ when $a=b=2$

(P fix)

a	b	$(a-b)^2$	P	S
1	6	25	6	7
2	3	1	6	5
6	1	25	6	7
12	$\frac{1}{2}$	$\left(\frac{23}{2}\right)^2$	6	$\frac{25}{2} = 12.5$
$\boxed{\sqrt{6}}$	$\boxed{\sqrt{6}}$	0	6	$\boxed{2\sqrt{6} = 4.89}$

$S_{min} = 4.89$ when $a=b$

Analyze / draw line of flight

* $5x - 3 < 3x + 1$
  $2x < 4 \Rightarrow x < 2 \Rightarrow x \in (2, \infty)$
  0 1 2

* $2(2x+3) - 10 \le 6(x-2)$
  $4x - 4 \le 6x - 12 \Rightarrow -2x \le -8 \Rightarrow x \ge 4$
  ⇓
  $x \in [4, \infty)$
  4

* $\frac{5x-2}{3} - \frac{7x-3}{5} > \frac{x}{4}$

* $\frac{1}{2}\left(\frac{3}{2}x+4\right) \ge \frac{1}{3}(x-6)$

* $\frac{3(x-2)}{5} \ge \frac{5(2-x)}{3}$

* $\frac{1}{x-2} < 0$

* $\frac{x+1}{x+2} \ge 1$

* $\frac{x-3}{x-5} > 0$

Lecture 5 (10/3/2022)

Inequalities ($\mathbb{R}$)

Analysis / lines of flight

* $\frac{5x-2}{3} - \frac{(7x-3)}{5} > \frac{x}{4}$

$\frac{5(5x-2)-3(7x-3)}{15} > \frac{x}{4} \Rightarrow x > 4 \Rightarrow x \in (4,\infty)$

* $\frac{1}{2}\underbrace{\left(\frac{3}{2}x+\frac{4}{1}\right)}_{\frac{3x+8}{2}} \geq \frac{1}{3}(x-6) \Rightarrow 3x+8 \geq \frac{4}{3}(x-6)$

$3x+8 \geq \frac{4}{3}x - 8$

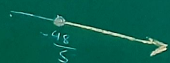
$\frac{3x}{1} - \frac{4}{3}x \geq -16$

$\Downarrow$

$x \geq -\frac{48}{5} \Leftarrow \frac{5}{3}x \geq -16$

$\Downarrow$

$x \in [-\frac{48}{5}, \infty)$



* $\frac{3(x-2)}{5} \geq \frac{5(2-x)}{3} \Rightarrow \frac{3x-6}{5} \geq \frac{10-5x}{3}$

$9x-18 \geq 50-25x \Rightarrow 34x \geq 68 \Rightarrow x \geq 2$

$\Downarrow$

$x \in [2,\infty)$

* $\frac{1}{x-2} < 0 \rightarrow \frac{a}{b} < 0,\ a>0 \Rightarrow x-2<0$

$\Downarrow$

$x<2$

* $\frac{x+1}{x+2} \geq 1 \Rightarrow \frac{x+1}{x+2} - \frac{1}{1} \geq 0$

$\Downarrow$

$\frac{\cancel{x}+1-\cancel{x}-2}{x+2} \geq 0 \Rightarrow \frac{-1}{x+2} > 0$

* $\frac{a}{b} > 0,\ a<0,\ b<0 \Rightarrow x+2<0 \Rightarrow x<-2 \Rightarrow x \in (-\infty,-2)$

2 More Rules | difference $\Rightarrow$ Phenomena

* $\frac{a}{b} < 0,\ a>0 \Rightarrow b>0$

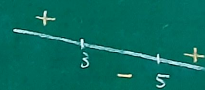
* $\frac{a}{b} > 0,\ a<0 \Rightarrow b<0$

Wavy Curve Method for Analysis

* $f(x) = ax+b,\ g(x) = cx+d$ Type 1 (linear)

$\underbrace{\frac{f(x)}{g(x)}}_{h(x)} > 0,\ \frac{-f(x)}{g(x)} < 0$

* $\frac{x-3}{x-5} > 0$



$f(x) = x-3 = 0 \Rightarrow x=3$

$g(x) = x-5 = 0 \Rightarrow \underbrace{x=5}$

Critical points

* $h(x) \equiv \frac{x-3}{x-5}$

$\boxed{h(x)>0}$ ✓

* $h(x<3) = \frac{1-3}{1-5} = \frac{-2}{-4} = +ve$

* $h(3<x<5) = \frac{4-3}{4-5} = -ve$

* $h(x>5) = \frac{6-3}{6-5} = +ve$

* $x \in (-\infty,3) \cup (5,\infty)$

"union"

HW

* $\frac{x-2}{x+5} > 2$

* $\frac{2x+4}{2x-1} \geq 5$

* $\frac{x+3}{x-2} \leq 2$

* $\frac{1-x^2}{5x-6-x^2} < 0$

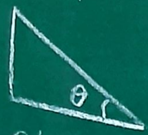
* Equality $\Rightarrow$ Inequality

Lecture 6 (13/3/2022)

Inequalities ($\mathbb{R}$)

* Equalization of the unequal $\Rightarrow$ =
* $\underbrace{|AB| = \text{const.}}_{\text{Equality}} \Rightarrow$ Rigidity $\Downarrow$ x-values

* $\nexists$ Rigidity $\Rightarrow$ $\nexists$ Equality
* Inequality $\Rightarrow$ phenomena
* Difference $\Rightarrow$ phenomena



Cline

2. More Rules (Contd)

* $\frac{a}{b} < 0,\ a>0 \Rightarrow b<0$
* $\frac{a}{b} > 0,\ a<0 \Rightarrow b<0$
* $ab>0,\ a>0 \Rightarrow b>0$
* $ab<0,\ a<0 \Rightarrow b>0$

$$ab>0 \Rightarrow \begin{cases} a>0,\ b>0 \\ a<0,\ b<0 \end{cases}$$

* $$ab<0 \Rightarrow \begin{cases} a>0,\ b<0 \\ a<0,\ b>0 \end{cases}$$

Wavy Curve Mtd. for Analysis

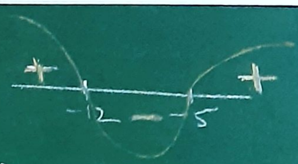
* $f(x) \gtrless 0$
* $\frac{x-2}{x+5} > 2 \Rightarrow \frac{x-2}{x+5} - \frac{2}{1} > 0$

$$\frac{x-2-2(x+5)}{x+5} > 0 \Rightarrow \frac{-x-12}{x+5} > 0 \Rightarrow \underbrace{\frac{x+12}{x+5}}_{f(x)} < 0$$

* $f(x) = \frac{x+12}{x+5} = \frac{g(x)}{h(x)}$

Critical point: $\left.\begin{array}{l} g(x) = x+12 = 0 \\ h(x) = x+5 = 0 \end{array}\right\} \Rightarrow x = -12, -5$

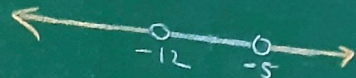
* $f(x>-5) = \frac{0+12}{0+5} = +$



$f(-12<x<5) = \frac{-10+12}{-10+5} = -$

$f(x<-12) = \frac{-15+12}{-15+5} = +$

$\boxed{f(x)>0}$



* $x \in (-\infty, -12) \cup (-5, \infty)$
* $4x^3 - 24x^2 + 44x - 24 > 0$

$4(x^3 - 6x^2 + 11x - 6) > 0$

* $(a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$

$a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$

by factorizn:

* $f(x) = 4(x^3 - 6x^2 + \underbrace{11x}_{11x+x-x} - 6 + 2 - 2)$

$= 4(\underbrace{x^3 - 6x^2 + 12x - 8} - x + 2)$

$= 4\{(x-2)^3 - (x-2)\}$

$= 4(x-1)(x-2)(x-3)$

* $\boxed{f(x)>0} \Rightarrow (x-1)(x-2)(x-3) > 0$

CP: $x = 1, 2, 3$

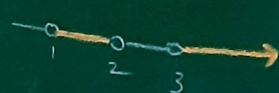
$f(x>3) = +$

$f(2<x<3) = (+)(+)(-) = -$

$f(1<x<2) = (+)(-)(-) = +$

$f(x<1) = (-)(-)(-) = -$

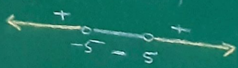
* $x \in (1,2) \cup (3, \infty)$



Lecture-7 (14/3/2022)

* $\frac{x}{x-5} > \frac{1}{2} \Rightarrow \frac{x}{x-5} - \frac{1}{2} > 0 \Rightarrow \underbrace{\frac{x+5}{2(x-5)}}_{f(x)} > 0$

$f(x) = \frac{x+5}{(x-5)}$



$f(x>5) = +$

$f(-5<x<5) = -$

$f(x<-5) = \frac{-}{-} = +$

$x \in (-\infty, -5) \cup (5, \infty)$

* $\frac{1}{x-1} \le 2 \Rightarrow \frac{1}{x-1} - 2 \le 0 \Rightarrow \frac{-2x+3}{x-1} \le 0$

$f(x) = \frac{-2x+3}{x-1} = \frac{g(x)}{h(x)}$

CP : $\left.\begin{array}{l} g(x)=0 \\ h(x)=0 \end{array}\right\} \Rightarrow x = \frac{3}{2}, 1$

$f(x>\frac{3}{2}) = \frac{-}{+} = -$

$f(1<x<\frac{3}{2}) = \frac{-\frac{5}{4}+3}{\frac{5}{4}-1} = \frac{+}{+} = +$

$f(x<1) = \frac{+}{-} = -$

$f(x=\frac{3}{2}) = 0 \le 0$ 'Included'

$f(x=-1) \to \infty$ 'excluded'

$x \in (-\infty, -1) \cup [\frac{3}{2}, \infty)$

Wavy curve steps

1. $f(x) > 0$, $f(x) < 0$
2. $f(x) = \frac{g(x)}{h(x)}$
   find Critical pts by $g(x)=0$, $h(x)=0$
3. Plot C.P.s on # line
4. check Inequality for $\underbrace{\text{pts} > \text{CP}, < \text{CP}}_{\text{Grey region}}$
5. select $f(x)>0$ or $f(x)<0$ as Ineq-flipped

* Type I : $f(x) = \frac{g(x) \to \text{linear}}{h(x) \to \text{linear}}$

* Type II : $f(x) = \frac{\text{Quad 1}}{\text{Quad 2}}$ # CP ↑

* $\frac{1-x^2}{5x-6-x^2} < 0$

$\Downarrow$

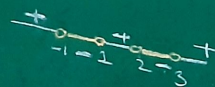
$\frac{\cancel{-}(x^2-1)}{\cancel{-}(x^2-5x+6)} < 0 \Rightarrow \frac{x^2-1}{x^2-5x+6} < 0$

* $f(x) = \frac{x^2-1}{\underset{-2x-3x}{x^2-5x+6}} = \frac{(x-1)(x+1)}{(x-2)(x-3)} = \frac{g(x)}{h(x)}$

* $g(x) = 0 \Rightarrow (x-1)(x+1) = 0 \Rightarrow x = 1, -1$

$h(x) = 0 \Rightarrow (x-2)(x-3) = 0 \Rightarrow x = 2, 3$

* $f(x>3) = \frac{++}{++} = +$



$f(2<x<3) = \frac{++}{+-} = -$

$f(1<x<2) = \frac{++}{--} = +$

$f(-1<x<1) = \frac{-+}{--} = -$

$f(x<-1) = \frac{--}{--} = +$

$x \in (-1, 1) \cup (2, 3)$

* $\frac{8x^2+16x-51}{2x^2+5x-12} > 3 \Rightarrow \frac{(x-\frac{5}{2})(x+3)}{(x+4)(x-\frac{3}{2})} > 0$

CP : $x = \frac{5}{2}, -3, -4, \frac{3}{2}$

$x \in (-\infty, -4) \cup (-3, \frac{3}{2}) \cup (\frac{5}{2}, \infty)$

HW

* $\frac{x^2-2x+5}{3x^2-2x-5} > \frac{1}{2}$

* $\frac{x^2-2x+24}{x^2-3x+4} \le 4$

* $\frac{x^2-4x+7}{x^2-7x+12} \ge \frac{2}{3}$

Lecture-8 (18/3/2022)

* $\overbrace{\frac{1}{x+1} - \frac{4}{(2+x)^2}}^{f(x)} > 0$ $\qquad x \neq -1, -2$, $\quad x \in \mathbb{R}$

$\frac{4+x^2+4x-4x-4}{(x+1)(2+x)^2} > 0 \Rightarrow \frac{x^2}{(2+x)^2(x+1)} > 0$

$\Downarrow$

$x > -1 \Leftarrow x+1 > 0 \Leftarrow \left(\frac{x}{x+2}\right)^2 \frac{1}{x+1} > 0$

$\Downarrow$

$x \in (-1, 0) \cup (0, \infty)$ $\quad$ | CP: $x = -1, -2, 0$

* $\frac{x^2-2x+24}{x^2-3x+4} \le 4 \Rightarrow \frac{x^2-2x+24}{x^2-3x+4} - \frac{4}{1} \le 0$

$\Downarrow$

$\frac{x^2-2x+24-4x^2+12x-16}{x^2-3x+4} \le 0$

$\Downarrow$

$\frac{3x^2-10x-8}{x^2-3x+4} \ge 0 \Leftarrow \frac{-3x^2+10x+8}{x^2-3x+4} \le 0$

$\Downarrow$

$\frac{(x-4)(3x+2)}{(x^2-3x+4)} \ge 0$

Comment on Deno.

* $g(x) = x^2-3x+4$

$= x^2-3x+4+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2$

$= \left(x-\frac{3}{2}\right)^2 - \frac{7}{4}$

* $\underbrace{ax^2+bx+c}_{g(x)} = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ $\nearrow \alpha$ $\searrow \beta$

* $D = b^2-4ac = 9-16 = -ve \Rightarrow x \in \mathbb{C}$

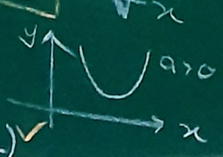
$\boxed{1x^2-3x+4 > 0}$ $\quad \forall x$

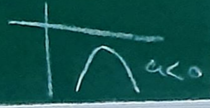
$D<0$, $a>0$

$\downarrow$

$y>0$ (Lec. 33 Vol 1) ✓

$\boxed{g(x)>0}$



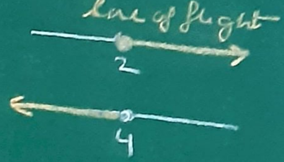


* $x \in \left(-\infty, -\frac{2}{3}\right] \cup [4, \infty)$

3. Machines driving other machines
a. # variables = 1

* $3x-6 \ge 0 \Rightarrow x \ge 2$

$4x-10 \le 6 \Rightarrow x \le 4$



Affected line of flight

$x \ge 2 \Rightarrow x \in [2, \infty)$

$x \le 4 \Rightarrow x \in (-\infty, 4]$

* Interaction $\Rightarrow \exists$ capacity to affect and (Simultaneous) be affected

$\downarrow$

$x \in (-\infty, 4] \underbrace{\cap}_{} [2, \infty)$

"intersection" $\equiv$ { Common things b/w 2 sets $\cap$ (habit) }

"Everywhere IT is machines – real ones, not figurative ones: machines driving other machines, machines being driven by other machines, with all the necessary couplings & connections"

– Anti-Oedipus, Deleuze

Lecture-9 (19/3/2022)

* $\dfrac{x^2-4x+7}{x^2-7x+12} \le \dfrac{2}{3} \Rightarrow \underbrace{\dfrac{(x+3)(x-1)}{(x-3)(x-4)}}_{f(x)} \le 0$

CP: $x = 1, 3, 4, -3$

* $f(x>4) = \dfrac{+}{+}$

$f(3<x<4) = \dfrac{++}{+-} = -$

$f(1<x<3) = \dfrac{++}{--} = +$

$f(-3<x<1) = \dfrac{+-}{--} = -$

$f(x<-3) = \dfrac{--}{--} = +$

$x \in [-3, 1] \cup (3, 4)$

a # var = 1, # Inequalities = 2

* $\dfrac{5x}{4} + \dfrac{3}{8}x > \dfrac{39}{8}$

$\dfrac{2x-1}{12} - \dfrac{(x-1)}{3} \le \dfrac{3x+1}{4}$

Affected line of flight?

* $x > 3$

$x \ge 0$

$x \in (3, \infty)$

common ($\cap$)

* $-11 \le 4x-3 \le 13$

"Condensed Notation of Interaction"

* $-11 \le 4x-3 \le 13$ → $-11 \le 4x-3$; → $4x-3 \le 13$

Method 1

Method 2

* $-11 \le 4x-3 \le 13$

$-11+3 \le 4x \le 13+3$

$\dfrac{-11+3}{4} \le x \le \dfrac{13+3}{4}$

$-2 \le x \le 4 \Rightarrow x \in [-2, 4]$

Get bare x
||
stripped/Naked
⇓
No coupling/connections

* $-5 \le \dfrac{2-3x}{4} \le 9$

$\Downarrow$

$\dfrac{-34}{3} \le x \le \dfrac{22}{3} \Rightarrow x \in \left[\dfrac{-34}{3}, \dfrac{22}{3}\right]$

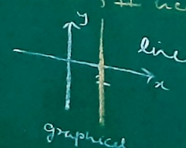
HW

* $\dfrac{x}{2x+1} \ge \dfrac{1}{4}$, $\dfrac{6x}{4x-1} < \dfrac{1}{2}$

b # var = 1 or 2, # inequality = 1

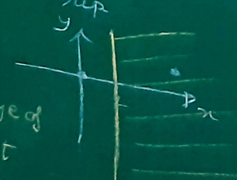
* $x = 2$

Algebraic rep



graphical rep

* $x \ge 2$



Surface Plowed/Carved region

O(0,0) eye of obs doesn't satisfy Inequality ($0 \not\ge 2$) ⇒ Origin Not included in the Carved Space

* $y \le -3$

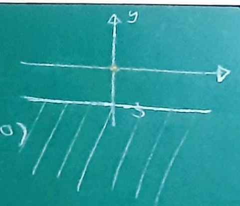
$y = -3$

Check O(0,0)

$0 \not\le -3$

$\Downarrow$

O doesn't satisfy



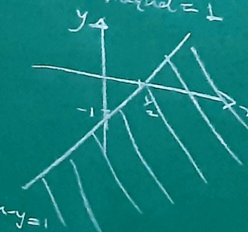
* $2x - y \ge 1$

var = 2
Inequal = 1

* $2x - y = 1$

$y = 2x - 1$

x	1/2	0
y	0	-1



Check O(0,0)

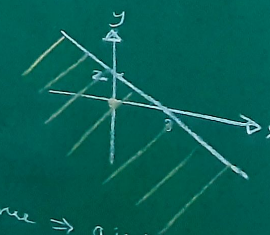
* $2(0) - 0 \ge 1 \Rightarrow 0 \ge 1$ False

O doesn't satisfy

* $2x + 3y \le 6$

$2x + 3y = 6$

x	3	0
y	0	2



$0 \le 6$ True ⇒ 0 is satisfy

Lecture 10 (20/3/2022)

$\mathbb{R}^2$

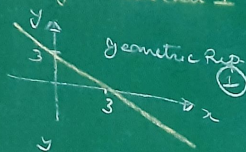
c. # var = 2, # inequality = More than 1

* $x+y=3$

Algebr rep

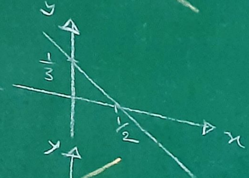
x	0	3
y	3	0

Geometric Rep (1)



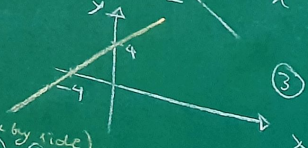
$2x+3y=1$

x	0	$1/2$
y	$1/3$	0



$-x+y=4$

x	0	-4
y	4	0



(3)

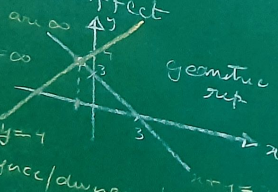
* Interaction of (1), (3) (set them side by side) $\Rightarrow$ Affect

$x+y=3 \rightarrow$ # pts are ∞

$-x+y=4 \rightarrow$ # O.P $=\infty$

$\downarrow$

$\exists$ a O.P pt of "convergence/divergence" (Common to both Eqⁿ)



Geometric rep

* Zoned-in by substitution

$x+y=3 \rightarrow x=3-y$ 'Intervene' $\equiv$ Interfere

$-x+y=4$

Put $x=3-y$ into $-x+y=4$

$-(3-y)+y=4 \Rightarrow \boxed{y=7/2}$

* ∞ # of O.P $\xrightarrow[\text{Interaction}]{\text{Intervene}}$ Zoned-in to 1 O.P (Common to both Eqⁿ)

$f(x,y)=x+y-3=0$

$g(x,y)=-x+y-4=0$ $\Big\} \rightarrow (x_0, y_0)$ zoned in pt

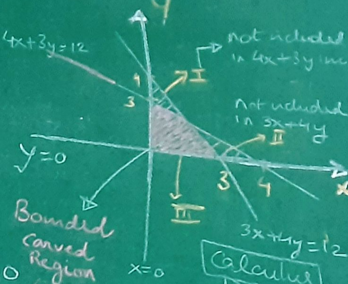
* $3x+4y\le 12$, $4x+3y\le 12$, $x\ge 0$, $y\ge 0$

* $3x+4y=12$

x	0	4
y	3	0

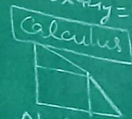
* $4x+3y=12$

x	0	3
y	4	0



Bounded Curved Region (Intersection/Common)

Area → Euclidean (finite) region
Area → Fractal ∞ (Mandelbrot) Hausdorff



No Direct formula

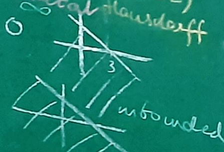
* $x=0$, $y=0$

$\downarrow$ ∥ to y, $\downarrow$ ∥ to x

Check O(0,0)

* $0+0\le 12 \Rightarrow$ region passes O

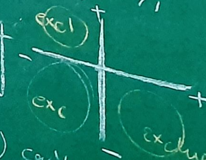
* $4(0)+3(0)\le 12 \Rightarrow$ ''



* $\boxed{\begin{array}{l} x\ge 0 \\ y\ge 0 \end{array}}$

$\Downarrow$

(x,y) can't be -ve



* $A=\{(x,y): 3x+4y\le 12, 4x+3y\le 12, x\ge 0, y\ge 0 \ \forall x,y\in\mathbb{R}\}$

$A\subset\mathbb{R}^2$

"tends to" Area $\rightarrow\infty$

* $3x+4y\ge 12$, $y\ge 1$, $x\ge 0$

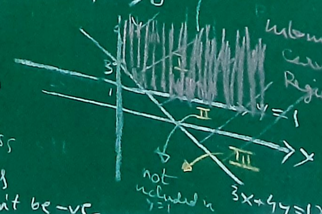
Check

* $0+0\ge 12$ $\Downarrow$ No pass thru O

* $0\ge 1 \Rightarrow$ No pass

* $0\ge 0 \Rightarrow$ passes

* $x\ge 0 \Rightarrow x$ can't be -ve

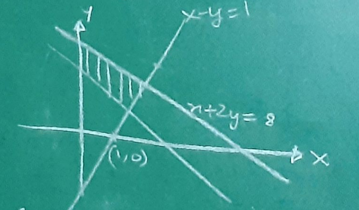


Unbounded Curved Region

HW

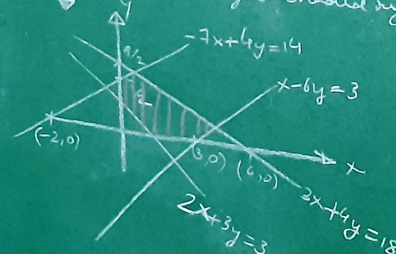
* $x+y\le 5$, $4x+y\ge 4$, $x+5y\ge 5$, $x\le 4$, $y\le 3$

*



Inequalities to get shaded region?

*

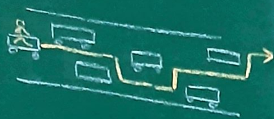


Lecture-11 (21/3/2022)

→ Optimization Problem / Math "programming"

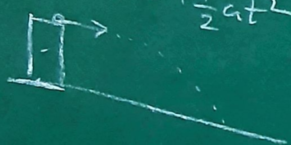
a feeling

Drive fast : No bumping
Follow rules
(Constraints)



$$S = ut + \frac{1}{2}at^2$$

Morality / Ethics



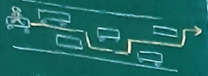
Archery

Perception (POV) - Metaphysics, Bergson-

Duration	Space
'driver'	Observer
feeling	Representation outside the obj
Emotions (heartbeat / BP / Pulse)	Relative
Be inside it	Description
Quality	Frame of Reference
Simple act	Symbol / frame dependent
flawless	$\exists$ ground for judgement (a priori / pre-given)
No textbook / rules	⫴
$\nexists$ x-val / t-val	Eye of observers
$\nexists$ symbol	= origin of CS
Grade with the Person himself	Quantity
Solid town. = original (perfect)	$\sum$ (photographs of town) = Representation Imperfect
Gold Coin	$\sum$ small changes $\neq$ Gold coin
$\boxed{\text{Intuition}}$ ✓	$\boxed{\text{Analysis}}$ = logical process
Uncountable ✓	Obj $\xrightarrow{\text{reducible}}$ known element
Absolute	Investigation
DURATION	Countable / Relative
Instinctive	SPACE-TIME

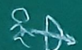
Lecture-12 (24/3/2022)

* Optimization Problem / Math "programming" (utilitarian)
a. feeling
* Drive fast : No bumping
Follow rules
(Constraints)



$S = ut + \frac{1}{2}at^2$

Morality / Ethics / physics
not required



Archery

* Math/physics "madness" . Quality ⟶ Quantity
(uncountable) (countable)
Duration SpaceTime
Intuition Analysis

b. Representation ↱ Algorithmic - George Dantzig (LP)
→ Iterative / Matrix
→ Calculus Based

(Graphical Method)
Linear Optimization / Programming

* Furniture dealer
2 product ↱ Chairs
↳ Tables
2 machines ↱ A 2hr 4hrs
↳ B 6hr 2hrs
chair table

* # avail hrs/day on A = 16
" " B = 20
Profit n chair = \$30
" " Table = \$50

↱ Least cost
→ Max. Profit
→ Highest Margin
→ Least time

$\boxed{\text{Max Profit}}$
at how many chair/table

step 1
* Representation (tabular)

Items	Chair	Table	Allotted t
Mach A	2	4	16
Mach B	6	2	20
Profit	30	50	

* # chairs/day = x
# tables/day = y

* Mach A : $2x + 4y \leq 16$
Mach B : $6x + 2y \leq 20$
$x \geq 0,\ y \geq 0$

* Profit = $30x + 50y = Z$

step 2
* Analysis of Inequalities

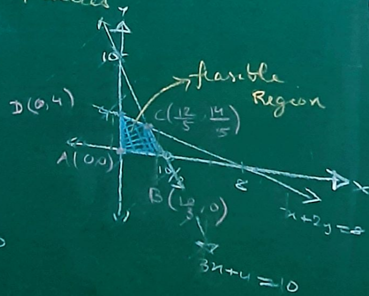
* $x + 2y = 8$

x	0	8
y	4	0

* $3x + y = 10$

x	0	10/3
y	10	0

* $3(8 - 2y) + y = 10$
$24 - 6y + y = 10$
$y = \frac{14}{5}$, $x = 8 - \frac{28}{5} = \frac{12}{5}$



step 3 (Optimization by Corner pt Method)
a) get vertices of Polygon
Vertices ≡ extreme pts of feasible sol^n
b) Put vertices one by one in Z to see Max Z

	Z
A(0,0)	0
B(0,4)	200
C($\frac{10}{3}$,0)	100
D($\frac{12}{5}$, $\frac{14}{5}$)	212

* Z(Profit) : Max at $\left(\underbrace{\frac{12}{5}}_{\sim 2}, \underbrace{\frac{14}{5}}_{\sim 3}\right)$

optimal Sol^n to LPP
optimal value $= Z_{max}$

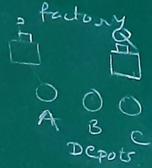
Terminology
* $Z = c_1x_1 + c_2x_2 + \dots c_nx_n$
Max/Min
objective function
* $a_{11}x_1 + a_{12}x_2 + \dots a_{1n}x_n \gtrless b_1$
$a_{21}x_1 + a_{22}x_2 + \dots a_{2n}x_n \gtrless b_2$
⋮
$a_{n1}x_1 + a_{n2}x_2 + \dots a_{nn}x_n \gtrless b_n$
Constraints
$x_1, x_2, x_3 \dots x_n \geq 0$
Non negativity Restriction

* Sol^n set $= \{x_1, x_2 \dots x_n\}$
↓
Convex Polygon
↓
optimal Z value

Lecture-13 (28/3/2022)

c. Practice (Math. formulation)

* weekly requirᵐ of A, B, C
  5, 5, 4 units resp
* Production capacity at P, Q
  8 & 6 units resp



From \ To	A	B	C
		Cost ($)	
P	16	10	15
Q	10	12	10

→ of transportation / unit

units to transp.
: Cost = Minimized

step 1

discussion at the end of this lecture

optimal pdt line prob.

Factory → 2 pdts → P_1 → 2 hr Mould, → 3 hr Grind, → 4 hr Polish
→ P_2 → 4 M, → 2 G, → 2 P

M availability = 20 hrs
G " = 24 hrs
P " = 13 hrs
Profit / unit of P_1 = \$5
" " " P_2 = \$3

Max Profit

step 1

Mach \ Pdt	P_1	P_2	Availab
M	2	4	20
G	3	2	24
P	4	2	13
Profit	5	3	

* # of unit of $P_1 = x$
  # of " " $P_2 = y$
  $Z = 5x + 3y$ Profit
* $2x + 4y \le 20$
  $3x + 2y \le 24$
  $4x + 2y \le 13$
  $x \ge 0,\ y \ge 0$

Diet prob.

* Diet: atleast
  400 unit of carbs
  500 " " fats
  300 " " protein } Min req.

2 Foods avail → F_1 cost/unit 2\$
→ F_2 4\$

units

F_1 → 10 carbs, → 20 fats, → 15 pro
F_2 → 25 carbs, → 10 fats, → 20 prot.

Min Cost.

step 1

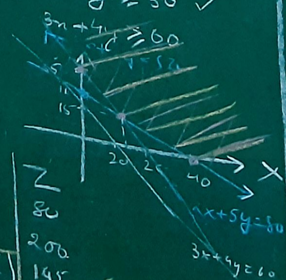
* # of $F_1 = x$
  # of $F_2 = y$
  $Z = 2x + 4y$ cost
* $10x + 25y \ge 400$
  $20x + 10y \ge 500$
  $15x + 20y \ge 300$
  $x \ge 0,\ y \ge 0$
  → $2x + 5y \ge 80$ ✓
  $2x + y \ge 50$ ✓
  $3x + 4y \ge 60$

*
x	0	40
y	16	0

*
x	0	25
y	50	0

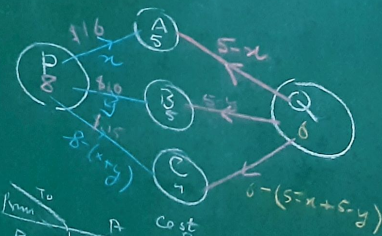
x	0	20
y	15	0

Vertices	Z
(40, 0)	80
(0, 50)	200
$\boxed{\left(\frac{85}{4}, \frac{15}{2}\right)}$	$\frac{145}{2}$ ✓ Min cost



* $2x + 5y = 80$
  $-\ 2x + y = 50$
  $4y = 30 \Rightarrow y = \frac{15}{2}$
  $2x + \frac{15}{2} = 50 \Rightarrow 2x = 50 - \frac{15}{2} \Rightarrow x = \frac{85}{4}$

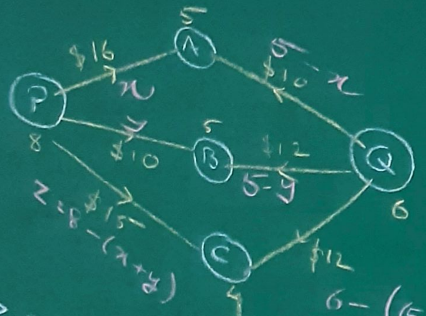
Transportation prob.



From \ To	A	Cost B	C
P	16	10	15
Q	10	12	12

Lecture 14 (31/3/2022)

Practice (Math formulation)
Transportation prob.



$6-\left((5-x)+(5-y)\right) = x+y-4$

* $x \geq 0,\ y \geq 0,\ z \geq 0$

$8-x-y \geq 0 \Rightarrow -(x+y) \geq -8 \Downarrow x+y \leq 8$

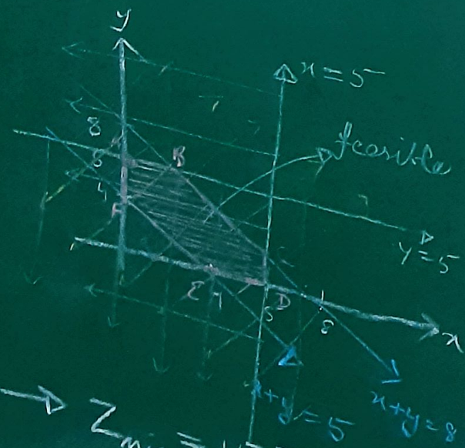
* $5-x \geq 0 \Rightarrow x \leq 5$, $5-y \geq 0 \Rightarrow y \leq 5$, $x+y-4 \geq 0 \Rightarrow x+y \geq 4$

* $Z = 16x + 10y + 15\{8-(x+y)\} + 10(5-x) + 12(5-y) + 12\{6-\underbrace{[5-x+5-y]}_{x+y-4}\}$
cost

$Z = 3x - 5y + 182$

Step 2

	Z
* A (0,5)	157
B (3,5)	166
C (5,3)	182
D (5,0)	197
E (4,0)	194
F (0,4)	162



* $x = 0,\ y = 5 \longrightarrow Z_{min} = 157$ ✓
Minimized Value of Z

Method 2 (Iso-profit line)

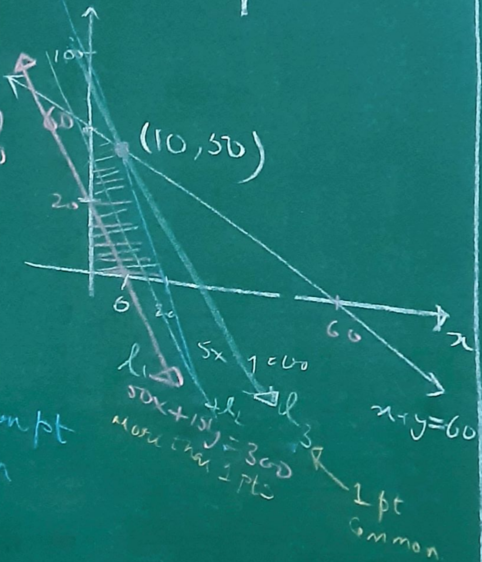
* Maximize $Z = 50x + 15y$
subject to
$5x + y \leq 100$
$x + y \leq 60$
$x, y \geq 0$

Anticipated Ans.
(10, 50)
$Z_{max} = 1250$

* $50x + 15y = 300$ (say)

x	0	6
y	20	0

Move farther from 0 and draw ∥ lines till 1 common pt in feasible region



Step

1. get feasible region & vertices
2. choose convenient val of Z (Z_0)
draw the line $ax + by = Z_1$ Iso-profit line
3. $ax + by = Z_2$

Z → Max: draws lines ∥ to Iso profit, obtain line farther from origin
Z → Min: draw lines ∥ to Iso profit, obtain lines nearest to origin

till 1 pt common to feasible Region

* Common pt = Optimal Sol^n

Lecture 15 (2/4/2022)

c. Practice

* Minimize $Z = 3x + 5y$
subject to

$-2x + y \leq 4$

$x + y \geq 3$

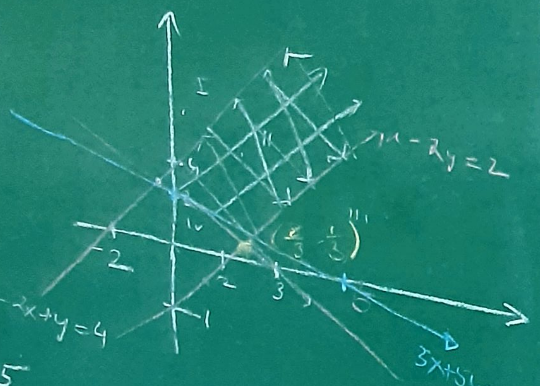
$x - 2y \leq 2$, $x \geq 0$, $y \geq 0$

* $-2x + y = 4$

x	0	-2
y	4	0

$x - 2y = 2$

x	0	2
y	-1	0



* $3x + 5y = 15$

x	0	5
y	3	0

$ax + by = Z_0$ || LCM(a, b)

$x = \frac{8}{3}$, $y = \frac{1}{3}$ $\Rightarrow$ $\boxed{Z_{min} = \frac{29}{3}}$

5. Theory of Ratios (classical)

* $Q = \left\{ \frac{p}{q} : p, q \in \mathbb{Z} \right\}$ Parts understood in terms of whole Rational Set

a. Rules

* $x = \frac{a}{b} = \frac{ma}{mb} = \frac{na}{nb}$ "$a : b$"

$x = \frac{\frac{a}{m}}{\frac{b}{m}} = \frac{a}{m} \times \frac{m}{b} = \frac{ma}{mb}$

* Law of Compounding $\Rightarrow$ Multiplying the ratio

$\frac{2a}{3b}, \frac{6ab}{5c^2}, \frac{c}{a} \longrightarrow \frac{4a}{5c}$

* $x = \frac{a}{b} \xrightarrow[\text{with itself}]{\text{Compounded}} y = a^2/b^2$ 'duplicate ratio'

* $x = \frac{a}{b} \xrightarrow[\text{ratio}]{\text{triplicate}} y = \frac{a^3}{b^3}$

$\xrightarrow[\text{duplicate}]{\text{sub}} y = \frac{a^{1/2}}{b^{1/2}}$

* $x = \frac{a}{b}$ Antecedent, Consequent ; $y = \frac{a+x}{b+x}$, $a, b \in \mathbb{Z}^+$

the gelicon difference

$x - y = \frac{a}{b} - \left(\frac{a+x}{b+x}\right) = \frac{\cancel{ab} + ax - \cancel{ab} - bx}{b(b+x)}$

$= \frac{x(a-b)}{b(b+x)}$

$a > b$ greater inequality

$x - y > 0$

$\Downarrow$

$\boxed{\frac{a}{b} > \frac{a+x}{b+x}}$

$a < b$

$x - y < 0$

$\Downarrow$

$\boxed{\frac{a}{b} < \frac{a+x}{b+x}}$

* $x = \frac{a}{b}$, $y = \frac{a-x}{b-x}$

$x - y = \frac{a}{b} - \left(\frac{a-x}{b-x}\right) = \frac{\cancel{ab} - ax - \cancel{ab} + bx}{b(b-x)}$

$= \frac{x(b-a)}{b(b-x)}$

$b > a \Rightarrow N : +ve$

$b > x \Rightarrow D : +ve$

$\boxed{\frac{a}{b} > \frac{a-x}{b-x}}$

$b < a \Rightarrow N : -ve$

$b < x \Rightarrow D : -ve$

Similarly

$b > a \Rightarrow N : +ve$

$b < x \Rightarrow D : -ve$

$b < a \Rightarrow N : -ve$

$b > x \Rightarrow D : +ve$

$\frac{a}{b} < \frac{a-x}{b-x}$

Lecture 16 (3/4/2022)

5. Classical theory of Ratios

a. Rules

* $x = \frac{a}{b}$ $\to a>b \Rightarrow \frac{a}{b} > \frac{a+x}{b+x}$

$\to a<b \Rightarrow \frac{a}{b} < \frac{a+x}{b+x}$

* $x = \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots$

$\Downarrow$

$\frac{a}{b} = \frac{c}{d} = \dots = \left(\frac{pa^n + qc^n + re^n \dots}{pb^n + qd^n + rf^n \dots}\right)^{\frac{1}{n}}$ p, q, r any #

side remark

* if $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} \Rightarrow \frac{a}{b} = \frac{c}{d} = \frac{e}{f} \overset{?}{=} \frac{a+c+e}{b+d+f}$

check

* $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots = K$

$\frac{a}{b} = K \Rightarrow a = Kb \Rightarrow a^n = K^n b^n \Rightarrow pa^n = pK^n b^n$

$\frac{c}{d} = K \Rightarrow c = Kd \Rightarrow c^n = K^n d^n \Rightarrow qc^n = qK^n d^n$

$\frac{e}{f} = K \Rightarrow e = Kf \Rightarrow e^n = K^n f^n \Rightarrow re^n = rK^n f^n$

$pa^n + qc^n + re^n \dots = K^n(pb^n + qd^n + rf^n \dots)$

* $\frac{pa^n + qc^n + re^n \dots}{pb^n + qd^n + rf^n + \dots} = K^n$

* $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots = K = \left(\frac{pa^n + qc^n + re^n \dots}{pb^n + qd^n + rf^n \dots}\right)^{\frac{1}{n}}$

Method 1

* $p = q = r = n = 1$

$\Downarrow$

$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} \Rightarrow \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a+c+e}{b+d+f}$

Ex.

* if $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, PT $\frac{a^3b + 2c^2e - 3ae^2f}{b^4 + 2d^2f - 3bf^3} = \frac{ace}{bdf}$

* $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = K \Rightarrow \begin{cases} a = bK \\ c = dK \\ e = fK \end{cases}$

* $\frac{a^3b + 2c^2e - 3ae^2f}{b^4 + 2d^2f - 3bf^3} = \frac{b^3K^3b + 2d^2K^2fK - 3bKf^2K^2f}{b^4 + 2d^2f - 3bf^3}$

$= K^3\left(\frac{\cancel{b^4 + 2d^2f - 3bf^3}}{\cancel{b^4 + 2d^2f - 3bf^3}}\right) = K^3 = K \cdot K \cdot K$

$= \frac{ace}{bdf}$

Ex:

* if $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$

PT. $\frac{x^2+a^2}{x+a} + \frac{y^2+b^2}{y+b} + \frac{z^2+c^2}{z+c} = \frac{(x+y+z)^2 + (a+b+c)^2}{x+y+z+a+b+c}$

check

* $\frac{x}{a} = \frac{y}{b} = \frac{z}{c} = K \Rightarrow \begin{cases} x = aK \\ y = bK \\ z = cK \end{cases}$

$\frac{x^2+a^2}{x+a} = \frac{a^2K^2 + a^2}{aK + a} = \frac{a(K^2+1)}{K+1}$

$\frac{y^2+b^2}{y+b} = \frac{b(K^2+1)}{K+1}$, $\frac{z^2+c^2}{z+c} = \frac{c(K^2+1)}{K+1}$

* $\frac{x^2+a^2}{x+a} + \frac{y^2+b^2}{y+b} + \frac{z^2+c^2}{z+c} = \frac{(K^2+1)(a+b+c)}{(K+1)}$

* $\frac{x}{a} = \frac{y}{b} = \frac{z}{c} = \frac{x+y+z}{a+b+c} = K$

* $\frac{x^2+a^2}{x+a} + \frac{y^2+b^2}{y+b} + \frac{z^2+c^2}{z+c} = \frac{(K^2+1)(a+b+c)}{K+1}$

$= \frac{\left[\left(\frac{x+y+z}{a+b+c}\right)^2 + \frac{1}{1}\right]\left[a+b+c\right]}{\frac{(x+y+z)}{a+b+c} + \frac{1}{1}}$

$= \frac{\frac{(x+y+z)^2 + (a+b+c)^2}{\cancel{(a+b+c)^2}} \cdot \cancel{(a+b+c)}}{\frac{x+y+z+a+b+c}{\cancel{(a+b+c)}}}$

HW

* if $\frac{x}{b+c-a} = \frac{y}{c+a-b} = \frac{z}{a+b-c}$

PT. $\frac{x+y+z}{a+b+c} = \frac{x(y+z) + y(z+x) + z(x+y)}{2(ax+by+cz)}$

Lecture 17 (4/4/2022)

5 Classical theory of Ratios

a. Rules

* $\dfrac{x}{b+c-a} = \dfrac{y}{c+a-b} = \dfrac{z}{a+b-c}$

PT: $\dfrac{x+y+z}{a+b+c} = \dfrac{x(y+z)+y(z+x)+z(x+y)}{2(ax+by+cz)}$

* $\dfrac{x}{b+c-a} = \dfrac{y}{c+a-b} = \dfrac{z}{a+b-c} = \dfrac{x+y+z}{a+b+c}$ (Identity)

* $\Downarrow$

$\underbrace{\dfrac{x(y+z)}{(y+z)(b+c-a)}}_{A} = \underbrace{\dfrac{y(z+x)}{(z+x)(c+a-b)}}_{B} = \underbrace{\dfrac{z(x+y)}{(x+y)(a+b-c)}}_{C}$

$\Downarrow$

$A = B = C = \dfrac{x(y+z)+y(z+x)+z(x+y)}{(y+z)(b+c-a)+(z+x)(c+a-b)+(x+y)(a+b-c)}$

* Deno $= yb + \cancel{yc} - \cancel{ya} + \cancel{zb} + zc - \cancel{za} + zc + \cancel{za} - \cancel{zb} + \cancel{xc} + xa - \cancel{xb} + xa + \cancel{xb} - \cancel{xc} + \cancel{ya} + yb - \cancel{yc}$

$= 2(xa+yb+zc)$

Imp if

Example

* $\dfrac{x}{l(mb+nc-la)} = \dfrac{y}{m(nc+la-mb)} = \dfrac{z}{n(la+mb-nc)}$ Δ

P.T. $\dfrac{l}{x(by+cz-ax)} = \dfrac{m}{y(cz+ax-by)} = \dfrac{n}{z(ax+by-cz)}$

* if $\Delta \Rightarrow \dfrac{\frac{x}{l}}{mb+nc-la} = \dfrac{\frac{y}{m}}{nc+la-mb} = \dfrac{\frac{z}{n}}{la+mb-nc}$

$= \dfrac{\frac{y}{m}+\frac{z}{n}}{2la} = \dfrac{\frac{x}{l}+\frac{y}{m}}{2nc} = \dfrac{\frac{x}{l}+\frac{z}{n}}{2mb}$ Sum taken 2 at a time

$\dfrac{\frac{1}{l}\left(\frac{y}{m}+\frac{z}{n}\right)}{2a} = \dfrac{\frac{1}{n}\left(\frac{x}{l}+\frac{y}{m}\right)}{2c} = \dfrac{\frac{1}{m}\left(\frac{x}{l}+\frac{z}{n}\right)}{2b}$

$\dfrac{\frac{yn+mz}{\cancel{lmn}}}{\cancel{2}a} = \dfrac{\frac{xm+ly}{\cancel{lmn}}}{\cancel{2}c} = \dfrac{\frac{lz+xn}{\cancel{lmn}}}{\cancel{2}b}$

* $\dfrac{yn+mz}{a} = \dfrac{xm+ly}{c} = \dfrac{lz+xn}{b}$

$\dfrac{\overset{\mathrm{I}}{x(yn+mz)}}{ax} = \dfrac{\overset{\mathrm{II}}{z(xm+ly)}}{cz} = \dfrac{\overset{\mathrm{III}}{y(lz+xn)}}{by}$

$= \dfrac{\overset{\mathrm{III}+\mathrm{II}-\mathrm{I}}{\cancel{2}lyz}}{by+cz-ax} = \dfrac{\overset{\mathrm{II}+\mathrm{I}-\mathrm{III}}{\cancel{2}mxz}}{cz+ax-by}$

$= \dfrac{\overset{\mathrm{I}+\mathrm{III}-\mathrm{II}}{\cancel{2}nxy}}{ax+by-cz}$

divide by xyz

* $\square$ Hence proved

Reminder

* $\dfrac{a}{b} = \dfrac{c}{d} = \dfrac{e}{f} = k \Rightarrow \left(\dfrac{pa^n+qc^n+\dots}{pb^n+qa^n\dots}\right)^{\frac{1}{n}} = k$

* $\dfrac{a}{b} = \dfrac{c}{d} = \dfrac{e}{f} = k \Rightarrow k = \dfrac{a+c-e}{b+d-f} = \dfrac{a-c+e}{b-d+f} = \dfrac{a-c-e}{b-d-f}$

$\dfrac{a+c-e}{b+d-f} = \dfrac{k\cancel{(b+d-f)}}{\cancel{(b+d-f)}} = k$

Lecture 8 (9/4/2022)

5 Classical theory of Ratios

a. Rules

* $\frac{a_1}{b_1}, \frac{a_2}{b_2}, \ldots \frac{a_n}{b_n}$ unequal fractions

$\Downarrow$

$$\underbrace{\frac{a_r}{b_r}}_{\text{least fraction}} < \frac{a_1+a_2+\cdots+a_n}{b_1+b_2+\cdots b_n} < \underbrace{\frac{a_s}{b_s}}_{\text{greatest fraction}}$$

check

* $\frac{a_r}{b_r} = k \Rightarrow a_r = k\,b_r$

$\frac{a_1}{b_1} > k \Rightarrow a_1 > k\,b_1$

$\frac{a_2}{b_2} > k \Rightarrow a_2 > k\,b_2$

$\vdots$

$\frac{a_n}{b_n} > k \Rightarrow a_n > k\,b_n$

$a_1+a_2+\cdots a_n > k(b_1+b_2+\cdots b_n)$

$\Downarrow$

$$\underbrace{k}_{\frac{a_r}{a_s}} < \frac{a_1+a_2+\cdots a_n}{b_1+b_2+\cdots b_n}$$

Similarly, $\frac{a_1+a_2+\cdots a_n}{b_1+b_2+\cdots b_n} < \frac{a_s}{b_s}$

b. Variation

* $X = \{x_1, x_2, x, x_n\}$ Quantity

$Y = \{y_1, y_2, y, y_n\}$ Countable/Relational $\exists$ Intensities ><

* $\frac{x}{x_1} = \frac{y}{y_1}, \frac{x}{x_2} = \frac{y}{y_2}, \frac{x}{x_3} = \frac{y}{y_3} \ldots$

$\downarrow$

$$\frac{x_1}{y_1} = \frac{x}{y} = \frac{x_2}{y_2} = \frac{x_3}{y_3} = \ldots$$

$$\frac{x_1}{y_1} = \frac{x_2}{y_2} = \cdots = \frac{x}{y} = k$$

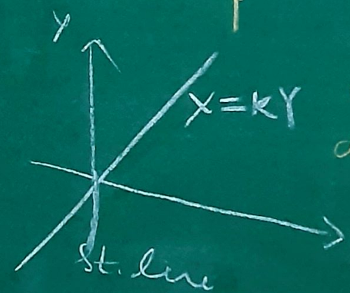
Convenient Notation

$$\frac{X}{Y} = k \Rightarrow \boxed{X = k.Y}$$

\#

$X \sim Y \Updownarrow X \propto Y$

Direct Variation/ 'directly proportional'



X	2	4	-2	0
Y	2a	4a	-2a	0

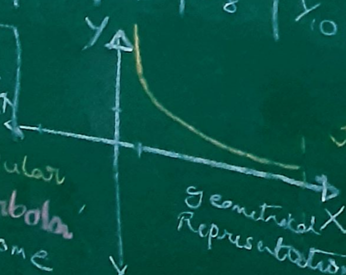
* $X = \frac{k}{Y} \Longleftrightarrow X \sim \frac{1}{Y}$ 'inverse proportional'

k=1

					tendencies	
X	2	4	8	10	0	∞
Y	$\frac{1}{2}$	$\frac{1}{4}$	$1/8$	$1/10$	∞	0

(representation) Inherently incomplete $\nexists$ stopping/starting pt.

$$\boxed{Y = \frac{k}{X}}$$



'Rectangular Hyperbola' Geometrical Representation

* Absolute/Rhizome $\infty \equiv \frac{1}{0}$ 'Quantity'

pure Quality

"Crisis of classical Ratio"

* Quality $\longrightarrow$ Quantity

Lecture 9 (10/4/2022)

< Proportions

* $\frac{a}{b} = \frac{c}{d} \Rightarrow$ Proportional.

$\Updownarrow$

$a:b :: c:d \Leftrightarrow a:b = c:d$

means

extremes

* $\frac{a}{b} = \frac{c}{d} \Rightarrow ad = bc$ 'Cross multiply'

* $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} =$ 'continued prop'

$\Downarrow$

$ac = b^2$

* if $\frac{a}{b} = \frac{b}{c}$ (a,b,c) cont. prop.

$\downarrow$

$\frac{a}{c} = \frac{a}{b} \times \underbrace{\frac{b}{c}}_{\frac{a}{b}} = \frac{a^2}{b^2}$

* $\frac{a}{b} = \frac{c}{d}, \ \frac{e}{f} = \frac{g}{h}$

$\Downarrow$

$\frac{ae}{bf} = \frac{cg}{dh}$

$\frac{a}{b} = \frac{c}{d}, \ \frac{b}{x} = \frac{d}{y} \Rightarrow \frac{a}{x} = \frac{c}{y}$

Powerful Tool

* $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{a} = \frac{d}{c}$ 'Division by 1' 'Invertendo'

* $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{c} = \frac{b}{d}$ 'Alternando'

* $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{b} + 1 = \frac{c}{d} + 1$

$\frac{a+b}{b} = \frac{c+d}{d}$ 'Componendo'

* $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{b} - 1 = \frac{c}{d} - 1$

$\Downarrow$

$\frac{a-b}{b} = \frac{c-d}{d}$ 'Dividendo'

* componendo & dividendo

$\boxed{\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}}$

check (Rev. Eng.)

RHS = $\cancel{ac} - ad + bc - \cancel{bd} = \cancel{ac} + ad - bc - \cancel{bd}$

$= 2ad = 2bc \Rightarrow \frac{a}{b} = \frac{c}{d}$

* $\frac{\sqrt{x+1} + \sqrt{x-1}}{\sqrt{x+1} - \sqrt{x-1}} = \frac{4x-1}{2}$ $\quad x = ?$

Method 1

* $\frac{x+\cancel{1}+x-\cancel{1}+2\sqrt{x^2-1}}{x+\cancel{1}+x-\cancel{1}-2\sqrt{x^2-1}} = \frac{(4x-1)^2}{4}$

$\frac{\cancel{2}\left(x+\sqrt{x^2-1}\right)}{\cancel{2}\left(x-\sqrt{x^2-1}\right)} = \frac{16x^2+1-8x}{4}$

Now reach the Answer.

Method 2 $\frac{a}{b} = \frac{c}{d} \rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$

* $\frac{\sqrt{x+1}+\cancel{\sqrt{x-1}}+(\sqrt{x+1}-\cancel{\sqrt{x-1}})}{\cancel{\sqrt{x+1}}+\sqrt{x-1}-(\cancel{\sqrt{x+1}}-\sqrt{x-1})} = \frac{4x-1+2}{4x-1-2}$

* $\frac{\sqrt{x+1}}{\sqrt{x-1}} = \frac{4x+1}{4x-3}$

$\downarrow$

$\frac{x+1}{x-1} = \left(\frac{4x+1}{4x-3}\right)^2 = \frac{16x^2+1+8x}{16x^2+9-24x}$

C & D $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

* $\frac{x+\cancel{1}+x-\cancel{1}}{\cancel{x}+1-(\cancel{x}-1)} = \frac{16x^2+1+8x+(16x^2+9-24x)}{\cancel{16x^2}+1+8x-(\cancel{16x^2}+9-24x)}$

$x = \frac{32x^2-16x+10}{32x-8}$

$\Downarrow$

$\cancel{32x^2}-8x = \cancel{32x^2}-16x+10$

$\downarrow$

$x = 5/4$

HW

* If $(2ma+6mb+3nc+9nd)\cdot(2ma-6mb-3nc+9nd) = (2ma-6mb+3nc-9nd)\cdot(2ma+6mb-3nc-9nd)$

PT: $\frac{a}{b} = \frac{c}{d}$.

Lecture 20 (11/4/2022)

Linear Equality/Equation

* "Nietzsche believes that science, in the way it handles quantities (x, v, E) always tends to equalise them, to make up for inequalities." ....

"Nietzsche attacks on science, the scientific mania for seeking balances. His whole critique operates on 3 levels

i) against logic Identity $A=A$ self similar

ii) against Mathematical Equality $(=)$

iii) Against Phy. Equil^m (stasis)"

— Nietzsche & phil. Deleuze

* Difference $\Rightarrow$ Phenomena

forgetting difference $\Rightarrow$ $=$

$\Downarrow$

deny life $\Rightarrow$ Death

1. # Var = 1, # Eq^n = 1 (LE in 1 var)

Formul. or Ineq.

* $3x-6 \geq 0 \Rightarrow x \geq 2$

$\Downarrow$

'Analysis' $x \in [2, \infty)$

line of flight

2

∃ direction (Quality)

* $4x-10 \leq 6 \Rightarrow x \leq 4$

4

Interaction (Simultaneous) $\Rightarrow$ ∃ Capacity to affect & be affect

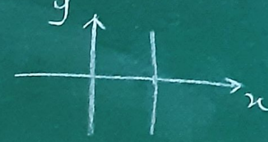
$\Downarrow$

Affected line of flight

"Affect"

Solve

* $3x-6=0 \Rightarrow x=2$



line
∄ direction
∄ Quality

* $3(x+3)-2(x-1)=5(x-5)$

$\Downarrow$

$x=9$

* $\frac{x}{2}-1=\frac{x}{3}+4 \Rightarrow x=30$

* $\frac{2x-1}{3}+1=\frac{x-2}{3}+2$

$\Downarrow$

$x=2$

* $\frac{12}{7}(x-5)=24+8x \Rightarrow x=-\frac{57}{11}$

* $\frac{x-6}{4}-\frac{x-4}{6}=1-\frac{x}{10} \Rightarrow x=10$

* $\frac{3}{4}(7x-1)-\left(2x-\frac{1-x}{2}\right)=x+\frac{3}{2}$

$\Downarrow$

$x=1$

* $\frac{x}{2}+\frac{x}{3}-\frac{x}{4}=7 \Rightarrow x=12$

* $\frac{3t-2}{4}-\frac{2t+3}{3}=\frac{2}{3}-t \Rightarrow t=2$

* $\frac{x+2}{6}-\left(\frac{11-x}{3}-\frac{1}{4}\right)=\frac{3x-4}{12} \Rightarrow x=11$

HW

$\alpha-\frac{2\alpha+8}{3}=\frac{1}{4}\left(\alpha-\frac{2-\alpha}{6}\right)-3$

$\Downarrow$

$\alpha=-10$

* $\frac{2}{5\gamma}-\frac{5}{3\gamma}=\frac{1}{15}$

* $(2\beta+5)^2+(2\beta-5)^2=(8\beta+6)(\beta-1)+22$

* $\frac{\mu+b}{a-b}=\frac{\mu-b}{a+b}$

* $\frac{1}{\alpha+1}+\frac{1}{\alpha+2}=\frac{2}{\alpha+10}$

Lecture 21 (14/4/2022)

Linear equation $\qquad \psi$ - psi

* $\dfrac{6x^2+13x-4}{2x+5} = \dfrac{12x^2+5x-2}{4x+3} \Rightarrow x = 1$

* $\dfrac{4\psi+17}{18} - \dfrac{13\psi-2}{17\psi-32} + \dfrac{\psi}{3} = \dfrac{7\psi}{12} - \dfrac{\psi+16}{36}$

$\Downarrow$

$\psi = 4$

2. $\mathbb{R}^2$ # var = 2 , # eqn = 2 (L.E in 2 var)

* $x+y=3$

Algebraic rep.

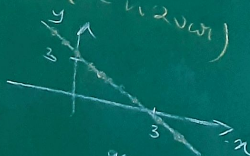
$\downarrow$

'∞ set of ordered pairs'

$R_1 = \{(x,y) : x+y=3$ (constraint), $x,y \in \mathbb{R}\}$

$R_1 = \{(0,3), (3,0), (1,2), \ldots\}$

'Broadening out'



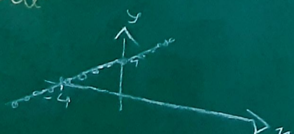
geometric rep

* $-x+y=4$

$\downarrow$

∞ set of O.P

$R_2 = \{(-4,0), (0,4) \ldots\}$



$\infty = \dfrac{1}{0}$

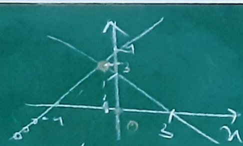
* (Simultaneous) Interaction $\Rightarrow$ Affect

$\downarrow$

Zoning-in/Narrowing-in/Cut-out (Common to both/Intersection)

* $x+y=3$

$-x+y=4$



∃ pt of convergence (special place) } → (Curve sketching) graphical way of Zoning-in

+ ↓ calculus 'Advanced'

$\downarrow$

Algebraic way of Zoning-in (very specific/Pin-point)

∃ 'Boundary' of Solvability

$x^3+\sqrt{y}+z=0$

$x+y=7$

Algebraic Methods for Zoning-in

a. Zoning in by Plug-in (Substitution)

* $f(x,y) = a_1x+b_1y+c_1 = 0$

$g(x,y) = a_2x+b_2y+c_2 = 0$ } Zoned-in pt → (x_0, y_0)

function = transform = Movement = Becoming = map

* $a_1x+b_1y+c_1=0$, $a_2x+b_2y+c_2=0$

$x = -\dfrac{(c_1+b_1y)}{a_1}$

$-\dfrac{a_2}{a_1}(c_1+b_1y)+b_2y+c_2=0$ $\quad$ LE in 1 var

* $-a_2(c_1+b_1y)+b_2a_1y+a_1c_2=0$

$y(a_1b_2-a_2b_1) = a_2c_1-a_1c_2$ } $\Rightarrow y = \dfrac{a_2c_1-a_1c_2}{a_1b_2-a_2b_1}$

* $x = -\dfrac{(c_1+b_1y)}{a_1}$

$= -\dfrac{\left(\dfrac{c_1}{1}+b_1\left(\dfrac{a_2c_1-a_1c_2}{a_1b_2-a_2b_1}\right)\right)}{a_1}$

$= -\left(\dfrac{c_1a_1b_2-\cancel{c_1a_2b_1}+\cancel{b_1a_2c_1}-b_1a_1c_2}{a_1(a_1b_2-a_2b_1)}\right)$

$= -\dfrac{\cancel{a_1}(c_1b_2-b_1c_2)}{\cancel{a_1}(a_1b_2-a_2b_1)}$

* $x = \dfrac{b_1c_2-c_1b_2}{a_1b_2-a_2b_1}$

$y = \dfrac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1}$

Lecture-22 (16/4/2022)

b. Zoning in by destruction (Elimination)

* $a_1x+b_1y+c_1=0$ } $\times a_2$

  $a_2x+b_2y+c_2=0$ } $\times a_1$

  $a_1a_2x+b_1a_2y+c_1a_2=0$

  $(-)\; a_1a_2x+b_2a_1y+c_2a_1=0$

  $(b_1a_2-b_2a_1)y+c_1a_2-c_2a_1=0$

* $y=\dfrac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1}$, $x=\dfrac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1}$

  ↓

* $\dfrac{y}{c_1a_2-c_2a_1}=\dfrac{1}{a_1b_2-a_2b_1}=\dfrac{x}{b_1c_2-b_2c_1}$

* $\dfrac{x}{(b_1c_2-b_2c_1)}=\dfrac{-y}{a_1c_2-a_2c_1}=\dfrac{1}{a_1b_2-a_2b_1}$

  $\dfrac{x}{\begin{vmatrix} b_1 & b_2\\ c_1 & c_2\end{vmatrix}}=\dfrac{-y}{\begin{vmatrix} a_1 & a_2\\ c_1 & c_2\end{vmatrix}}=\dfrac{1}{\begin{vmatrix} a_1 & a_2\\ b_1 & b_2\end{vmatrix}}$ "Cramer's rule"

* $|-a|$ = absolute value of #
* $A=\{a_1,a_2,b_1,b_2\}$ 'representation'

  Set ~ juxtaposing thing with their 'Rules'

* $A=\begin{pmatrix} a_1 & a_2\\ b_1 & b_2\end{pmatrix}$ Matrix ~ juxtaposition with their 'Rules'

  $|A|$ = "Value" of a matrix = Determinant

* $|A|=\begin{vmatrix} a_1 & a_2\\ b_1 & b_2\end{vmatrix}=a_1b_2-a_2b_1$ = Number

Cramer's rule (1760)

(J. Sylvester, 1850)

* Matrix = womb = Seed

  mater = Mother

$B=\begin{pmatrix} a & b\\ c & d\end{pmatrix} \xrightarrow{\text{create}} \begin{vmatrix} a & b\\ c & d\end{vmatrix}=ad-bc$

Matrix 2×2 Determinant

$C=\begin{pmatrix} a & b & c\\ e & f & g\\ h & i & j\end{pmatrix} \longrightarrow \begin{vmatrix} a & b & c\\ e & f & g\\ h & i & j\end{vmatrix}=?$ later!

Matrix 3×3

* $A=\begin{pmatrix} a_{11} & a_{12}\\ a_{21} & a_{22}\end{pmatrix}$

  Matrix

  element (#)

  Element a_{ij}: i^{th} row, j^{th} column

c. Solvability Conditions

* $x=\dfrac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1}$, $y=\dfrac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1}$

  $x=\dfrac{\begin{vmatrix} b_1 & b_2\\ c_1 & c_2\end{vmatrix}}{\begin{vmatrix} a_1 & a_2\\ b_1 & b_2\end{vmatrix}}$, $y=-\dfrac{\begin{vmatrix} a_1 & a_2\\ c_1 & c_2\end{vmatrix}}{\begin{vmatrix} a_1 & a_2\\ b_1 & b_2\end{vmatrix}}$

$\boxed{\text{Case 1}}$

* $\begin{vmatrix} a_1 & a_2\\ b_1 & b_2\end{vmatrix}=a_1b_2-a_2b_1\neq 0 \Rightarrow \boxed{\dfrac{a_1}{a_2}\neq\dfrac{b_1}{b_2}}$

  (x,y): unique solⁿ / intersecting

* $a_1b_2-a_2b_1=0 \Rightarrow \dfrac{a_1}{a_2}=\dfrac{b_1}{b_2}=K$

  $a_1=a_2K$, $b_1=b_2K$

  $a_1x+b_1y+c_1=0$, $\underbrace{a_2x+b_2y}+c_2=0$

  $K(\underbrace{a_2x+b_2y})+c_1=0$

  $K(-c_2)+c_1=0 \Rightarrow \boxed{K=\dfrac{c_1}{c_2}}$

* $a_1=Ka_2$, $b_1=Kb_2$, $c_1=Kc_2$

  ↓

  $a_1x+b_1y+c_1=0$

  $K(a_2x+b_2y+c_2)=0$

  ⇓

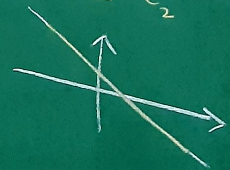
  $a_2x+b_2y+c_2=0$

* $\dfrac{a_1}{a_2}=\dfrac{b_1}{b_2}=\dfrac{c_1}{c_2} \Rightarrow$ ∞ many solⁿ

  ∞ # of o.p.

  ⇓

  ∄ zoning-in



Coincident lines

Lecture-23 (17/4/2022)

* $a_1x+b_1y+c_1=0$, $a_2x+b_2y+c_2=0$ (system of simultan. L E)

$x=\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1}$, $y=-\frac{(a_1c_2-a_2c_1)}{a_1b_2-a_2b_1}$

$$\boxed{x=\frac{D_1}{D},\quad y=-\frac{D_2}{D}}$$

$D_1=\begin{vmatrix} b_1 & b_2\\ c_1 & c_2\end{vmatrix}$, $D_2=\begin{vmatrix} a_1 & a_2\\ c_1 & c_2\end{vmatrix}$, $D=\begin{vmatrix} a_1 & a_2\\ b_1 & b_2\end{vmatrix}$

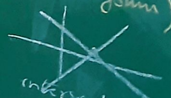
Case I

* $D\neq 0 \Rightarrow (x,y)$ unique/one-n ordered pair

$\Downarrow$

$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

(Consistent system)



intersecting line

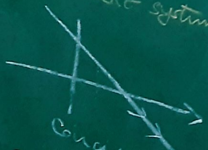
Case II

* $D=0$, $D_1=D_2=0 \Rightarrow \infty$ many soln/ordered pair

$\Downarrow$

$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

(Consistent system)



Coincident lines

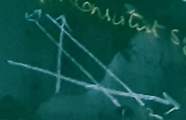
Case III

* $D=0$, D_1 or $D_2\neq 0 \Rightarrow$ No soln/ordered pair/commonness

$\Downarrow$

$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

(Inconsistent sys)



Practice!

* $3x-5y=-1$, $x-y=-1$ $\Rightarrow (x,y)=(-2,-1)$

* $\frac{2x}{a}+\frac{y}{b}=2$, $\frac{x}{a}-\frac{y}{b}=4$ $\Rightarrow (x,y)=(2a,-2b)$

* $\frac{1}{2x}-\frac{1}{y}=-1$, $\frac{1}{x}+\frac{1}{2y}=8$, $x,y\neq 0$ $\Rightarrow (x,y)=\left(\frac{1}{6},\frac{1}{4}\right)$

* $\frac{1}{2(2x+3y)}+\frac{12}{7(3x-2y)}=\frac{1}{2}$, $\frac{7}{2x+3y}+\frac{4}{3x-2y}=2$ $\Rightarrow (x,y)=(2,1)$

Trick

$\frac{1}{2x+3y}=\alpha$, $\frac{1}{3x-2y}=\beta$

* $\frac{\alpha}{2}+\frac{12\cdot 2\beta}{7\cdot 2}=\frac{1}{2}$, $7\alpha+4\beta=2$

$\alpha+\frac{24}{7}\beta=1 \Rightarrow \alpha=1-\frac{24}{7}\beta$

$7\left(\frac{1}{1}-\frac{24}{7}\beta\right)+4\beta=2 \Rightarrow 7-24\beta+4\beta=2$

$\Downarrow$

$-20\beta=-5 \Rightarrow \beta=\frac{1}{4}$

$\alpha=1-\frac{6}{7}=\frac{1}{7}$

* $2x+3y=7$, $3x-2y=4 \Rightarrow x=\frac{2y+4}{3}$

$\frac{4y+8}{3}+\frac{3y}{1}=7 \Rightarrow \frac{3y+8}{3}=7 \Rightarrow y=1$

* $\frac{5}{x+y}-\frac{2}{x-y}=1$

$\frac{15}{x+y}+\frac{7}{x-y}=10$

* $3(2u+v)=7uv$

$3(u+3v)=11uv$

* $4x+\frac{6}{y}=15$

$6x-\frac{8}{y}=14$

$(x,y)=?$

$p=?$: $y=px-2$

* $\frac{2}{x}+\frac{3}{2y}=\frac{1}{6}$

$\frac{3}{x}+\frac{2}{y}=0$

$(x,y)=?$

$a=?$: $y=ax-4$

Lecture-24 (18/4/2022)

Practice-2

* $6u+3v=7uv \rightarrow \frac{6}{v}+\frac{3}{u}=7$

$3u+9v=11uv \Rightarrow \frac{3}{v}+\frac{9}{u}=11$

$\frac{1}{v}=x,\ \frac{1}{u}=y \Rightarrow 6x+3y=7$

$3x+9y=11$

Types (symmetry exploitation) Powerful with large #

* $ax+by=c$

$bx+ay=d$

$(+)\quad (a+b)x+(a+b)y=c+d \Rightarrow (a+b)(x+y)=c+d$

$\Downarrow$

$x+y=\frac{c+d}{a+b}$

$(-)\quad (a-b)x+(b-a)y=c-d$

$\Downarrow$

$(a-b)(x-y)=c-d \Rightarrow x-y=\frac{c-d}{a-b}$

* $217x+131y=913$

$131x+217y=827$

$\cancel{348}(x+y)=\cancel{1740}\ 5 \Rightarrow x+y=5$

$\cancel{86}(x-y)=\cancel{86} \Rightarrow x-y=1$

$x=3,\ y=2$

* $37x+41y=70$

$41x+37y=86$ $\Big\} \Rightarrow (x,y)=(3,-1)$

* $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$

$\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$ $\Big\} \Rightarrow (x,y)=(4,9)$

* $2x+3y-17=0\ )\times 3 \Rightarrow 6x+9y=51$

$3x-2y-6=0\ )\times 2 \Rightarrow 6x-4y=12$ $(-)$

$13y=39$

$\Downarrow$

$y=3$

$3x=12 \Rightarrow x=4$

* $2x-y-3=0$

$4x+y-3=0$

$\begin{pmatrix} a_1x+b_1y+c_1=0 \\ a_2x+b_2y+c_2=0 \end{pmatrix}$

$$x=\frac{D_1}{D}=\frac{\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}}=\frac{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$$y=-\frac{D_2}{D}=-\frac{\begin{vmatrix} a_1 & a_2 \\ c_1 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}}=-\frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

Trick to Remember

$$\frac{x}{D_1}=\frac{-y}{D_2}=\frac{1}{D}$$

* $\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}=\begin{vmatrix} 2 & 4 \\ -1 & 1 \end{vmatrix}=6$

* $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}=\begin{vmatrix} 2 & -1 \\ 4 & 1 \end{vmatrix}=6$

* $$x=\frac{\begin{vmatrix} -1 & -3 \\ 1 & -3 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ 4 & 1 \end{vmatrix}}=\frac{3+3}{2+4}=1$$

$$y=-\frac{\begin{vmatrix} 2 & -3 \\ 4 & -3 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ 4 & 1 \end{vmatrix}}=\frac{-(-6+12)}{6}=-1$$

* $\frac{x}{a}+\frac{y}{b}=a+b$

$\frac{x}{a^2}+\frac{y}{b^2}=2$

* $x+y=a+b$

$ax-by=a^2-b^2$

* $\frac{x}{a}+\frac{y}{b}=2$

$ax-by=a^2-b^2$

Lecture-25 (21/4/2022)

Practice 3

* $\frac{2}{x} + \frac{3}{2y} = \frac{1}{6}$, $\quad \frac{3}{x} + \frac{2}{y} = 0$ $\Rightarrow$ $2\alpha + \frac{3}{2}\beta = \frac{1}{6}$, $\quad 3\alpha + 2\beta = 0$, $\quad 3\alpha = -2\beta$

$\frac{2}{3}\underbrace{(3\alpha)}_{-2\beta} + \frac{3}{2}\beta = \frac{1}{6} \Rightarrow \left(-\frac{4}{3} + \frac{3}{2}\right)\beta = \frac{1}{6}$

$\boxed{\beta = 1}$ $\quad \boxed{\alpha = -\frac{2}{3}}$

* LHS $2\left(-\frac{2}{3}\right) + \frac{3}{2}(1) = \frac{-8+9}{6} = \frac{1}{6}$ = RHS

* $\frac{x}{a} + \frac{y}{b} = a+b$, $\quad \frac{x}{a^2} + \frac{y}{b^2} = 2$

Subs

* $x = \left(a+b-\frac{y}{b}\right)a$

* $\frac{1}{a^2}\cancel{a}\left(a+b-\frac{y}{b}\right) + \frac{y}{b^2} = 2$

$1 + \frac{b}{a} - \frac{y}{ab} + \frac{y}{b^2} = 2$

$y\underbrace{\left(\frac{1}{b^2} - \frac{1}{ab}\right)}_{\frac{1}{b}\left(\frac{1}{b} - \frac{1}{a}\right)} = \frac{1}{1} - \frac{b}{a} = \frac{a-b}{a}$

$y\,\frac{1}{b}\left(\frac{a-b}{ab}\right) = \frac{\cancel{(a-b)}}{\cancel{a}} \Rightarrow \boxed{y = b^2}$

* $x = a^2$

* $ax + by = a - b$, $\quad bx - ay = a + b$ $\rightarrow$ $ax + by - (a-b) = 0$, $\quad bx - ay - (a+b) = 0$

* $x = \frac{D_1}{D} = \frac{\begin{vmatrix} b & -(a-b) \\ -a & -(a+b) \end{vmatrix}}{\begin{vmatrix} a & b \\ b & -a \end{vmatrix}}$

$= \frac{\cancel{-ab} - b^2 - a^2 + \cancel{ab}}{-a^2 - b^2} = 1$

* $y = -\frac{D_2}{D} = \frac{\begin{vmatrix} a & -(a-b) \\ b & -(a+b) \end{vmatrix}}{\begin{vmatrix} a & b \\ b & -a \end{vmatrix}} = -1$

Adv. qns.

* $(a-b)x + (a+b)y = a^2 - 2ab - b^2$

$(a+b)(x+y) = a^2 + b^2$

* $y = -\frac{D_2}{D} = -\frac{\begin{vmatrix} a-b & -(a^2-2ab-b^2) \\ a+b & -(a^2+b^2) \end{vmatrix}}{\begin{vmatrix} a-b & a+b \\ a+b & a+b \end{vmatrix}}$

$= -\left\{\frac{-(\cancel{a^3} + \cancel{ab^2} - a^2b - \cancel{b^3}) + (\cancel{a^3} - \cancel{2a^2b} - ab^2 + \cancel{ba^2} - 2ab^2 - \cancel{b^3})}{(\cancel{a^2} - b^2 - \cancel{a^2} - b^2 - 2ab)}\right\}$

$= -\left\{\frac{-4ab^2}{-2b^2 - 2ab}\right\} = \left\{\frac{-\cancel{4}ab^2}{\cancel{2b}(a+b)}\right\} = \frac{-2ab}{(a+b)}$

Solve by Matrix Method (warm up for matrices)

* $\frac{a}{x} - \frac{b}{y} = 0$

$\frac{ab^2}{x} + \frac{a^2b}{y} = a^2 + b^2$

* $ax + by = 1$

$bx + ay = \frac{(a+b)^2}{a^2+b^2} - 1$

* $a(x+y) + b(x-y) = a^2 - ab + b^2$

$a(x+y) - b(x-y) = a^2 + ab + b^2$

* $ax + by = c$

$bx + ay = 1 + c$

Lecture-26 (23/4/2022)

Practice 4

*

$$\left.\begin{aligned}\frac{a}{x}-\frac{b}{y}&=0\\ \frac{ab^2}{x}+\frac{a^2b}{y}&=a^2+b^2\end{aligned}\right\} \rightarrow \begin{aligned} au-bv&=0\\ ab^2u+a^2bv&=a^2+b^2\end{aligned}$$

$$u=\frac{D_1}{D}=\frac{\begin{vmatrix}-b & 0\\ a^2b & -(a^2+b^2)\end{vmatrix}}{\begin{vmatrix}a & -b\\ ab^2 & a^2b\end{vmatrix}}$$

$\Downarrow$

$x=a,\ y=b.$

*

$$ax+by=1$$

$$bx+ay=\frac{(a+b)^2}{a^2+b^2}-1=\frac{2ab}{a^2+b^2}$$

$$x=\frac{D_1}{D}=\frac{\begin{vmatrix}b & -1\\ a & -\frac{2ab}{a^2+b^2}\end{vmatrix}}{\begin{vmatrix}a & b\\ b & a\end{vmatrix}}$$

$$=\frac{-\frac{2ab^2}{a^2+b^2}+\frac{a}{1}}{a^2-b^2}=\frac{-2ab^2+a^3+ab^2}{(a^2-b^2)(a^2+b^2)}$$

$$=\frac{a^3-ab^2}{a^2-b^2}=\frac{a\cancel{(a^2-b^2)}}{\cancel{(a^2-b^2)}(a^2+b^2)}=\frac{a}{a^2+b^2}$$

$$y=\frac{b}{a^2+b^2}$$

* $a(x+y)+b(x-y)=a^2-ab+b^2$

$a(x+y)-b(x-y)=a^2+ab+b^2$

Recall

* $x=\dfrac{D_1}{D},\ y=-\dfrac{D_2}{D}$

$D\neq 0$

$\Downarrow$

Consistent sys (unique sol^n)

$\dfrac{a_1}{a_2}\neq\dfrac{b_1}{b_2}$

$D=D_1=D_2=0$

$\Downarrow$

∞ many sol^n

$\dfrac{a_1}{a_2}=\dfrac{b_1}{b_2}=\dfrac{c_1}{c_2}$

$D=0$

D_1 or $D_2\neq 0$

No sol^n

$\dfrac{a_1}{a_2}=\dfrac{b_1}{b_2}\neq\dfrac{c_1}{c_2}$

Lecture-27 (24/4/2022)

Practice 5

* $x = \dfrac{-b - \widehat{bc + ac}}{(a-b)(a+b)} = \dfrac{c(a-b) - b}{(a-b)(a+b)}$

$= \dfrac{c}{(a+b)} - \dfrac{b}{a^2 - b^2}$

*
* $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ $x, y \in \mathbb{R}$ Algebraic rep.

$x = \dfrac{D_1}{D}$, $y = -\dfrac{D_2}{D}$

$D \neq 0$

$D = D_1 = D_2 = 0$

$D = 0$, D_1 or $D_2 \neq 0$

Matrix rep.

geometrical rep

* Physics $\neq$ Math

* What is Mathematics?
(Mathematician POV)

Gödel

Schools of Thought

Nonspatio/Non temporal
Eternal

$\exists$ Platonic Heaven

world of perfection
World of IDEAS/FORM
$\exists$, x, y, $\sin\theta$
perfect ○, □, △
π The circle

Fictionalist

$\exists$ Axioms
symbols/formal system
No meaning
characters in a Novel

POV

Posed a Beyond
Platonic Heaven
(Identity) (Form
Perfect
eternal
original)

Copies/Corrupted
Copies/always imperfect
||
Representation in physical World.

* copies without original
||
False participants
|||
Simulacra

* $\nexists$ original $\Rightarrow \exists$ Fake
$\Downarrow$
$\exists$ copies of a Fake
$\Downarrow$
$\boxed{\exists \text{ Only Simulacra}}$

* "You dont invent math, you discover it"

P.H. $\rightarrow$ discover
'spatial'
||
(Picture)
$\exists$ pre-given/all at once Reality

Math = symbol Manipulation

"Don't embrace Math, use it like toilet paper (Tool)"

?

* (P.H)
World of forms
location (Where?)
$\downarrow$
Non spatio
Non temp.
$\Downarrow$
Eternal

Lecture-28 (25/4/2022)

Reading 2 (How to Slow-Read "Is God a Mathematic.?")

* "Every word has to be commented on" - Deleuze on Nietzsche

* "The univ Superficial appears to have been designed (static) Blue print given all at once by a pure Mathematician" - James Jeans (phys.)

* Appearance ⇒ Judgement value

* Pure Mathematics = Abstract Mathematics = Group theory = $\boxed{\text{set theory}}$

Universe = { Rohan, Dukar, Anubhav, Car, helicopter, Galaxy, Louse, cells, ... }

Closed/given

Non change/Non temporal/

Eternal set

∃ a boundary

Anthropocentric stance ("humans are center")

* "God bestowed Geometry on Mankind. Seeking Truth / "Right" ordering / Pre-defined Concept given all at once" - Hobbes, Phil

* "mechanics necessarily deals with Eq^n s and that an Algebraic Eq^n always expresses something already done. Thus, algebra can represent the results in Space but Not duration & Motion themselves."

* Duration & Motion are necessarily left out of the Equation

- Time & free will, Bergson

Lecture-29 (30/4/2022) 'Critiques'

Reading 2 ("How to Slow-Read
"Is God a Mathematician?")

* Reality = Given all at once / Closed
= Abstract Mathematics
(~ Set theory)
'James Jeans'

* Reality = Geometry = Pre-defined
Concept
'Hobbes'

* ∃ 3 real diff. worlds"

Conscious perception
faces
Emotions
feelings
Quality
uncountable
Mind
Consciousness

Physical World
Atoms'
Planets
Galaxies
People
matter
Countable

Platonic Heaven.
ℂ
Geometry
NLM
String theory.
QM
laws

Quality $\xrightarrow{\text{Nuts}}$ Quantity
Nietzsche

Roger Penrose

* Matter $\xrightarrow{?}$ Mind $\xrightarrow{?}$ Math forms

- Wigner

2 Sides of Math

Active
Nature suggests
↓
Frameworks are Created.
Newtonian Mech.
Special Rel.
General Rel.
QM (Math.)

Passive
Pure Math
No reason
No Application

Lecture-30 (1/5/2022)

Reading-2 (Is God a Mathematician) How to Slow Read.

2 sides of Math

Active

Nature suggests

↓

"unbelievably accurate" Math "laws of Nature"

Passive

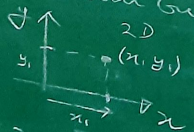
Pure Mathematics (Symbols)

Not connected to Reality

But ↓ later

Reincarnated in the real world

* (Descartes) Cartesian Geometry → $\boxed{\text{Newton}}$



$\{x, y\}$ → Basis for judgment

"Axis" ↓

x-val.

* Assumption : Euclid's Postulate

$\boxed{\parallel \text{ lines Postulate}}$ (Demand)

* $\boxed{\text{Metric}}$ → (Local behaviour) "Nature of distance b/w 2 pts" → Nature of Space

* $\parallel$ lines postulate (Demand) → Relaxed

↓

Non-Euclidean Geometry

Riemannian / Gaussian

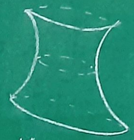
Differential Geometry

=

Calculus + Geometry

"Space"

Spherical geom.



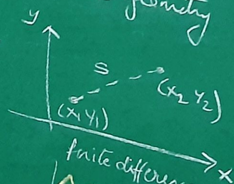
Hyperbolic geometry

Euclidean geo.

* $s^2 = \underbrace{(x_2 - x_1)^2}_{} + (y_2 - y_1)^2$

$$\boxed{s^2 = (\Delta x)^2 + (\Delta y)^2}$$

metric (Algebraic representation)



finite difference

Δ = delta (D)

δ = d

* $$\boxed{s^2 = \sum_{a,b=1,2 \text{ (labels)}} g_{ab}\, x^a x^b}$$ → Indices

distance b/w 2 pts

$a, b = 1, 2$

square

$$s^2 = g_{11}(x^1)(x^1) + g_{12}(x^1)(x^2) + g_{21}(x^2)(x^1) + g_{22}(x^2)(x^2)$$

* $\{x^1, x^2\}$ label → (x, y) Cartesian label

* $s^2 = g_{11} x^2 + g_{12}\, xy + g_{21}\, yx + g_{22}\, y^2$

Power   Power

* $s^2 = g_{11} x^2 + g_{12}\, xy + g_{21}\, yx + g_{22}\, y^2$

↓

$s^2 = x^2 + y^2$ (Pythagoras) Demand

guess :

g_{ab} – ∃ 4 number to completely define it / Juxtaposition

– Condensed Notation

– ∃ a specific way of transformation

* $$g_{ab} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$ Matrix Representation of Metric tensor

$g_{11} = 1,\ g_{12} = 0,\ g_{21} = 0,\ g_{22} = 1$

* Metric = Metric $\underbrace{\text{tensor}}_{}$ g_{ab}

tensor constant

* $$s^2 = \sum_{ab} g_{ab}\, x^a x^b$$

$$\boxed{s^2 = g_{ab}\, x^a x^b}$$ Einstein's summation Notation

Assume Sum over a & b

Lecture - 31 (3/5/2022)

Reading - 2 (Is God a Mathematician)
How to Slow Read.

Puzzle

∃ Platonic Heaven
Posing a Beyond.
$\boxed{\text{Discovery}}$
forms.
Seductive

fictionalists.
$\boxed{\text{Invention}}$
Formal
||
form Analysis

* Idealization / Abstraction of
Physical World
||
Human Invention

* "our Math" = 1 flavor of Math
= Soup of Math.

* Platonic World >>> Physical World
- Platonism

* Math = real / immutable / omnipresent
$t = -\infty$ to ∞

$\boxed{\text{Universe} \subset \text{Math}}$

* God ≠ Mathematician
Mathematics = God

* Reality $\xrightarrow[???]{\text{reducible}}$ Mathematical operators
"Math. Ques"

World of Forms.

location
Non Spatio
Non tempo.
Eternal
(Constant)
||

How forms emerged
(Non sp.
Non temp)
in my mind
(Spatio temp)
Entity

" What is before $\boxed{\text{existence}}$?
- It doesn't exist in a place & a time which are not even supposed to be there "

* if ∃ x which precedes existence, in a place & time which are supposedly not even there
- Hume on Platonism