Math (7-12)

theory, Algebra, Geometry, Calculus (Trigo.), Prob./statistics (Combinatorics)

* Notation: Set theory

$\{+, -, \div, \times\}$

$\in$ 'belongs to'

$\exists$

$\forall$

$\subset$

$\notin$ 'not belong to' $\equiv$ 'defined as'

Number th.

Number

- Real ($\mathbb{R}$)
  - Rational ($\mathbb{Q}$)
    - $\mathbb{W}$
    - $\mathbb{N}$
    - $\mathbb{Z}$
  - Irrational
- Complex ($\mathbb{C}$)

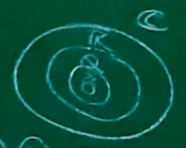
$i \equiv \sqrt{-1}$ ✓

$\boxed{x^2 + 4 = 0}$

$x^2 = -4$

$(2)^2 = 4$

$(-2)^2 = 4$



* $\mathbb{C} \supset \mathbb{R} \supset \mathbb{Q} \supset \mathbb{Z}$

$\Rightarrow$ 'Implies'

$\Leftrightarrow$

$\exists$: 'there exists'

Integer

* $\mathbb{Z} = \{-\infty, \ldots, -1, 0, 1, \ldots, \infty\}$, $\times$

* $a \in \mathbb{Z}, b \in \mathbb{Z} \Rightarrow (a \times b) \in \mathbb{Z}$ 'Closure'

$a \times b = b \times a$ 'Commutative'

$a \times (b \times c) = (a \times b) \times c$ 'Associative'

$\neq (a \times c) \times c + b \times (a \times c)$

$a \times (b \pm c) = (a \times b) \pm (a \times c)$ distributive over +

$a \times 1 = 1 \times a = a$ '$\exists$ multip. id.'

$a \times 0 = 0 \times a = 0$

* $(-1)^n \equiv \underbrace{(-1) \times (-1) \times \cdots \times (-1)}_{n \text{ times}} = \begin{cases} 1 & n \text{ even} \\ -1 & n \text{ odd} \end{cases}$

* $(+a_1) \times (+a_2) \times \cdots (+a_n) = \begin{cases} a_1 \times a_2 \times \cdots \times a_n & n \text{ even} \\ -(a_1 \times a_2 \times \cdots \times a_n) & n \text{ odd} \end{cases}$

$a_n \in \mathbb{Z}$

* $a, b \in \mathbb{Z} : a > b \Rightarrow \begin{cases} a \times c > b \times c, & c > 0 \\ a \times c < b \times c, & c < 0 \end{cases}$

* $1, 0$

* $\frac{4}{2} = 2 \Leftrightarrow 4 = 2 \times 2$ ✓

$\frac{0}{4} = 0 \Leftrightarrow 0 = 0 \times 4$

* $\boxed{\frac{n}{0} = x \Leftrightarrow n = 0 \times x}$ $x \in \mathbb{C}$

$x \longrightarrow \infty$

$\boxed{\infty \equiv \frac{n}{0}}$ 'Symbol' 'Indetermined form'

Theory

BODMAS

* $x = 24 - \underbrace{4 \div 2}\times 3 = 24 - \underbrace{2\times 3} = 18$

* $y = (-5) - \underbrace{(-48) \div (-16)} + (-2)\times 6 = -20$

* $z = 27 - \left[5 + \left\{28 - (29-7)\right\}\right] = 16$

* $x = \underbrace{(-1)\times(-1)\times\ (-1)}_{25\text{ times}}$, $y = (-3)\ (-3)(-3)$

$xy = 27$

fraction (Ratio / Rational #)

* $2\frac{1}{2} = \frac{5}{2}$

* $x = \frac{N}{D} = \frac{N\times a}{D\times a} = \frac{\frac{N}{a}}{\frac{D}{a}} \neq \frac{N\pm a}{D\pm a}$

* $\frac{a}{b} = \frac{c}{d} \iff ad = bc$

* $a \in \mathbb{Z} \longrightarrow \begin{cases} a \neq 0 \Rightarrow \frac{0}{a} = 0 \\ a = 0 \Rightarrow \frac{a}{0} : \text{Not may} \end{cases}$

$\underbrace{(a-b)-c} \neq \underbrace{a-(b\div c)}$

$\left(\frac{\frac{a}{b}}{c}\right) = \frac{a}{bc}$ $\quad \frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b}$

* $a, b \in \mathbb{Z} \longrightarrow \begin{cases} a > b \Rightarrow \frac{a}{c} > \frac{b}{c} & c > 0 \\ a < b \Rightarrow \frac{a}{c} < \frac{b}{c} & c < 0 \end{cases}$

* $\frac{5}{3} > \frac{3}{2} \xrightarrow{\text{TRICK}} 10 > 9$

Cross multipl

* $x = \frac{5}{3} + \frac{2}{5} = \frac{25+6}{15} = \frac{31}{15}$

* $y = \frac{4}{1} - \frac{7}{8} = \frac{32-7}{8} = \frac{25}{8}$

5	5 , 3
3	1 , 3
	1 , 1

* $\frac{a}{b}\times\frac{c}{d} = \frac{ac}{bd}$; $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b}\times\frac{d}{c} = \frac{ad}{bc}$

$\frac{a}{b} \longrightarrow \frac{1}{\frac{a}{b}} = \frac{b}{a}$ reciprocal

* $\frac{-5}{7} = \frac{x}{28}$

$(-1)^n = \begin{cases} 1 & n \text{ Even} \\ -1 & n \text{ odd} \end{cases}$

* $a^n = \underbrace{a\times a\times \cdots a}_{n\text{ times}}$, a base, n power

$(-4)^2 = 16$, $(-2)^3 = -8$

* $a\times a\times b\times b\times c\times d\times d = a^2b^2cd^2$

* $\left(\frac{-2}{3}\right)^3 = \frac{-8}{27}$

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

* 5^3 $\quad 3^5$ $\quad 10^{101}$ $\quad 9^{198}$ $\Big\}$ → logarithm

* $\left(2^{-2} - 4^{-3}\right)\times 2^3 = ?$

Rules (Empirical / Experimental)

* $2^3\times 3^5$

$5^2\times 5^3 = 5^{2+3} = 5^5$ $\Big\}$ $a^m\times a^n = a^{m+n}$

* $\frac{a^m}{a^n} = a^m\times a^{-n} = a^{m-n}$

$\frac{1}{a}\times a = 1$

* $(a^n)^m = a^{nm}$

* $a^n\times b^n = (a\times b)^n$

* $\boxed{(a+b)^n \neq a^n + b^n}$

* $\left(\frac{1}{4} - \frac{1}{64}\right)8$

$\left(2 - \frac{1}{8}\right) = \frac{15}{8}$

* $(2^{-2} - 4^{-3})\,2^3 = \left(2^{-2} - (2^2)^{-3}\right)2^3$

$\rightarrow \left(2^{-2} - 2^{-6}\right)2^3 = 2^{-2}\times 2^3 - 2^{-6}\times 2^3$

$\rightarrow 2 - 2^{-3} = 2\left(1 - \frac{2^{-3}}{2}\right) = 2\left(1 - 2^{-4}\right)$

$= 2\left(1 - \frac{1}{16}\right) = 2\times\frac{15}{16} = \frac{15}{8}$

Exponents / Consult on basic summation

* $\underbrace{a+a+a+\quad+a}_{n \text{ times}} = na$, $\underbrace{a\times \quad \times a}_{n \text{ times}} = a^n$

$-a-a \ldots -a = -na$

* $\frac{a}{b}+\frac{a}{b}+\cdots+\frac{a}{b} = \frac{na}{b}$, $\frac{a}{b}\times \quad \times\frac{a}{b} = \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

* $\left(\frac{-a}{b}\right)\times\left(\frac{-a}{b}\right)\times\cdots\left(\frac{-a}{b}\right) = \left(\frac{-a}{b}\right)^n = \left(-1\frac{a}{b}\right)^n = (-1)^n\left(\frac{a}{b}\right)^n = (-1)^n\frac{a^n}{b^n}$

$$= \begin{cases} \frac{a^n}{b^n} & n \text{ even} \\ -\frac{a^n}{b^n} & n \text{ odd} \end{cases}$$

Laws

* $a^n \times a^m = a^{n+m}$, fundamental

* $\frac{a^n}{a^m} = \begin{cases} a^n \times a^{-m} = a^{n-m} & n \neq m \\ 1 & n = m \end{cases}$

* $\underbrace{\frac{a^n}{a^n}}_{a^{n-n}=a^0} = 1 \Rightarrow a^0 = 1$

* $(a^n)^m = \underbrace{a^n \times a^n \times \times a^n}_{m \text{ times}} = a^{\overbrace{n+n+n+\cdots n}^{m \text{ times}}} = a^{nm}$

* $(a\times b)^n = \underbrace{(a\times b)(a\times b)\quad\times(a\times b)}_{n \text{ times}}$

$= \underbrace{(a\times a\times a\times\quad)}_{n}\times\underbrace{(b\times b\times\cdots\times b)}_{n} = a^n\times b^n$

Practice

* $\frac{\{(5^2)^3\times 5^4\}}{5^7} = 5^3$

* $\frac{3^n+3^{n+1}}{3^{n+1}-3^n} = \frac{\cancel{3^n}(1+3)}{\cancel{3^n}(3-1)} = \frac{4}{2} = 2$

* $\frac{16\cdot 2^{n+1}-4\cdot 2^n}{16\cdot 2^{n+2}-2\cdot 2^{n+2}} = \frac{\cancel{2^n}\,4(4\cdot 2-1)}{\cancel{2^{n+2}}\cdot 2(8-1)} = \frac{1}{2}$

* $\frac{10\cdot 5^{n+1}+25\cdot 5^n}{3\cdot 5^{n+2}+10\cdot 5^{n+1}} = \frac{\cancel{5^n}(2\cdot 5+5)}{\cancel{5^{n+1}}(3\cdot 5+10)} = \frac{15}{25} = \frac{3}{5}$

* $(2^2)^n = (2^3)^4$

* $2^{n-5}\times 5^{n-4} = 5 \Rightarrow 2^{n-5} = 5^{1-n+4} = 5^{5-n} = 5^{-(n-5)}$

$2^{n-5} = 5^{-(n-5)} \Rightarrow \frac{2^{n-5}}{5^{-(n-5)}} = 1$

$2^{n-5}\times 5^{n-5} = 1 \Rightarrow (2\cdot 5)^{n-5} = 1$

$10^{n-5} = 1 = (10)^0 \Rightarrow \boxed{n=5}$

$1 = a^0$

* $2^{n-7}\times 5^{n-4} = \underbrace{1250}_{2\times 5^4} \Rightarrow \frac{2^n}{2^7}\times\frac{5^n}{5^4} = 2\times 5^4$

$2^n\times 5^n = 2^8\times 5^8 \Rightarrow (2\cdot 5)^n = (2\cdot 5)^8 \Rightarrow \boxed{n=8}$

HW

* $25^{n-1}+100 = 5^{2n-1}$

* $\left(\frac{125}{5}\right)^5\cdot\left(\frac{125}{8}\right)^n = \left(\frac{5}{2}\right)^{18}$

* $\frac{9^n\cdot 3^2\cdot 3^n-(27)^n}{(3^3)^5\times 2^3}$

* $(6^{-1}-8^{-1})^{-1}+(2^{-1}-3^{-1})^{-1}$

* $\frac{(4^{-1}+8^{-1})}{\left(\frac{2}{3}\right)^{-1}}$

* $\left(\frac{1}{4}\right)^{-2}+\left(\frac{1}{2}\right)^{-2}+\left(\frac{1}{3}\right)^{-2}$

* $\left\{\left(\frac{3}{4}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right\}^{-1}$

* $\frac{\left[\left(\frac{1}{3}\right)^{-3}-\left(\frac{1}{2}\right)^{-3}\right]}{\left(\frac{1}{4}\right)^{-3}}$

* $\left[\left\{\left(\frac{-1}{5}\right)^{-2}\right\}^2\right]^{-1}$

Exponents

* $(25^2 - 15^2)^{3/2} = (5^4 - \underbrace{(5\times3)^2}_{5^2\cdot3^2})^{3/2}$

$= \left[5^2(5^2-3^2)\right]^{3/2} = \underbrace{(5^2)^{3/2}}_{5^{2\times\frac{3}{2}}}\cdot\underbrace{(5^2-3^2)^{3/2}}_{4^{2\times\frac{3}{2}}} = 5^3\cdot4^3 = 20^3$

* $\left(\frac{5}{3}\right)^6 = \left(\frac{5}{3}\right)^{8x} \Rightarrow 6 = 8x \Rightarrow x = \frac{3}{4}$

* $\underbrace{(1+3+5+7+9+11)}_{\text{AP (arithmetic prog)}}^{3/2}$

$\frac{x^{abc}}{x^{abc}} = \frac{x^0}{x^0} = 1$ | $\frac{x^0}{x^0} = 1$

* $(\underbrace{33^2 - 3^2}_{})^{5/3} = 128^{5/3} = 2^{7\times\frac{5}{3}} = 2^5$

* $a = 3^{-3} - 3^3$, $b = 3^3 - 3^{-3} = -a \Rightarrow \frac{b}{a} = -1$

$\frac{a}{b} = -1 \Rightarrow \frac{a}{b} - \frac{b}{a} = 0$

Meth 2

$a^m \times a^n = a^{m+n}$

* $\frac{a}{b} = \frac{3^{-3} - 3^3}{3^3 - 3^3} = \frac{\cancel{3^3}(3^{-6} - 1)}{\cancel{3^3}(1 - 3^{-6})} = -\frac{\cancel{(1-3^{-6})}}{\cancel{(1-3^{-6})}} = -1$

* $3^x = 6561$, $3^{x-3} = 3^x\, 3^{-3} = \frac{3^{(8)}}{3^3} = 3^5$

3	6561
3	2187
3	729
3	243
3	81
3	27
3	9
3	3
	1

* $(8^4 + 8^2)^{1/2} = \left[8^2(8^2+1)\right]^{1/2} = 8^{2\times\frac{1}{2}}(8^2+1)^{1/2}$

$= 8\sqrt{65}$

Squares / roots / Cubes / roots

* $a \in \mathbb{R}$: $a^2 = a\times a \Rightarrow a$ is perfect sq

P1 $\nexists\ p, q \in \mathbb{N} : p^2 = 2q^2$

Check

Case I

p, q : odd (O) $\Rightarrow p^2, q^2$: odd

$2q^2$: Even $\Rightarrow p^2 = 2q^2$ Even

'By Contradiction' □ 'QED'

Case II

p : odd, q : Even $\Rightarrow p^2$: odd, q^2 : Even $\Rightarrow 2q^2$: Even

$p^2 = 2q^2$

'by Contradiction' □

Case III

p : E $\Rightarrow$ 2 is a factor of p

q : O $\Rightarrow q^2$: O

p^2 : E $\Rightarrow 2^2 (=4)$ is factor of p^2

$2q^2$: Even (smallest factor = 2)

$p^2 \neq 2q^2$

Case IV

p : E

q : E

$p = 22 = 11\times2$

$q = 14 = 7\times2$

$HCF(22, 14) = 2 = c$

$HCF(11, 7) = 1$

$n \quad m$

p : E $\Rightarrow$ 2 is factor of p

p^2 : E $\Rightarrow$ 4 is factor of p^2

q : E $\Rightarrow$ 2 is factor of q

q^2 : E $\Rightarrow$ 4 " " " q^2

$p = cn$, $q = cm$, $c = HCF(q, p)$

$HCF(n, m) = 1$

$p^2 = 2q^2 \Rightarrow \cancel{c^2}n^2 = 2\cancel{c^2}m^2 \Rightarrow n^2 = 2m^2$

'By Reduction'

atleast one of them is 0 or E

reduce to any of 3 cases

Lecture 3

* $\frac{1}{1+x^{a-b}} + \underbrace{\frac{1}{1+x^{b-a}}}_{\frac{1}{1+x^{-(a-b)}}} = \frac{1}{1+x^{a-b}} + \frac{1}{1+x^{-(a-b)}}$

$\rightarrow \frac{1}{1+z} + \underbrace{\frac{1}{1+\frac{1}{z}}}_{\frac{1}{\frac{1+z}{z}} = \frac{z}{1+z}} = \frac{1}{1+z} + \frac{z}{1+z} = \frac{1+z}{1+z} = 1 \quad \Big| \; z = x^{a-b}$

* P2. $(n+1)^2 - n^2 = (n+1)+n \quad \forall n \in \mathbb{N}$

check

* $(n+1)^2 = (n+1)(n+1) = n^2 + 2n + 1$

$(a+b)(a-b) = a^2 - \cancel{ab} + \cancel{ab} - b^2 = a^2 - b^2$

* $LHS = (n+1+n)\underbrace{(\cancel{n}+1-\cancel{n})}_{1} = n + (n+1)$

$n = 3, \quad 4^2 - 3^2 = 4+3 = 7$

$n = 98, \quad 99^2 - 98^2 = 99+98 = 197$

* P3 : $n^2 = 1+3+5+(2n-1) \quad \forall n \in \mathbb{N}$

$1^2 = 1$

$2^2 = 4 = 1+3$

$3^2 = 9 = 1+3+5$

notation (summation)

* $\sum_{i=1}^{n} i = 1+2+3+ \quad n = ?$ (AP) $n \in \mathbb{N}$

dummy variable

upper bound

$\sum_{i=1}^{m} i = 1+3+5+\underbrace{(2n-1)}_{m} \quad m = 2n-1$

lower bound/limit

$\sum_{i=0}^{2n+1} i = 1+3+ \cdots (2n+1)$

$\sum_{i \in \{\text{odd \#}\}}^{n} i = 1+3+5 \cdots +n$

* P4. $(a, b, c) : a^2+b^2=c^2$ Pythagorean triplet

* P5. $(2n, n^2-1, n^2+1)$ – pyth triplet $n \in \mathbb{N}, n \neq 1$

check

* $(2n)^2 + (n^2-1)^2 = 4n^2 + n^4 + 1 - 2n^2 = \underbrace{n^4}_{(n^2)^2} + 2n^2 + 1 = (n^2+1)^2$

$(a+b)^2 = a^2+b^2+2ab$

Square Roots

* $a \in \mathbb{R}$: $b = a^2 \Rightarrow a = \pm(b)^{1/2} = \pm\sqrt{b}$ sq root

$b = a^3 \Rightarrow a = \pm(b)^{1/3} = \pm\sqrt[3]{b}$ cube root

long div.

* $\sqrt{54756} = 234$

double it

$2 \rightarrow 4\,3 \rightarrow 46\,4$

$$\begin{array}{r|l} & 234 \\ \hline & 54756 \\ & 4 \\ \hline & 147 \\ & 129 \\ \hline & 1856 \\ & 656 \\ \hline & 0 \end{array}$$

Prime factorize (for integer)

* $\sqrt[3]{1728}$

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

$1728 = 2^6 \times 3^3$

$(1728)^{1/3} = (2^6 \times 3^3)^{\frac{1}{3}} = 2^{6 \times \frac{1}{3}} \times 3^{3 \times \frac{1}{3}} = 2^2 \cdot 3 = 12$

$(1728)^{1/2} = (2^6 \times 3^3)^{\frac{1}{2}} = 2^3 \cdot 3^{3/2} = 8\sqrt{27}$

* 2.000000000

Lecture-4

* $\dfrac{7^{n+2} - 3\cdot 7^{n+1}}{20\cdot 7^n - 2\cdot 7^n} = \dfrac{7^{n+1}(7-3)}{7^n\cdot 2(10-1)} = \dfrac{7}{2}\cdot\dfrac{\cancel{4}^{2}}{9} = \dfrac{14}{9}$

* $\dfrac{5^{n+3} - 6\cdot 5^{n+1}}{9\cdot 5^n - 2^2\cdot 5^n} = 19$

* if $\dfrac{9^n\cdot 3^2\, 3^n - 27^n}{3^{3m}\cdot 2^3} = \dfrac{1}{27} \xrightarrow{PT} m-n = 1$

$\dfrac{3^{2+3n} - 3^{3n}}{3^{3m}\cdot 2^3} = \dfrac{1}{3^3} \Rightarrow \dfrac{3^{3n}(\cancel{3^2-1})}{3^{3m}\cdot \cancel{2^3}} = 3^{-3} \Rightarrow 3^{3n-3m} = 3^{-3}$

$n-m = -1$

HW

* $\left(\dfrac{x^a}{x^b}\right)^c \left(\dfrac{x^b}{x^c}\right)^a \left(\dfrac{x^c}{x^a}\right)^b = \left(\dfrac{x^a}{x^b}\right)^{\frac{1}{ab}} \left(\dfrac{x^b}{x^c}\right)^{\frac{1}{bc}} \left(\dfrac{x^c}{x^a}\right)^{\frac{1}{ca}}$

$= \left(\dfrac{x^a}{x^b}\right)^{a+b} \left(\dfrac{x^b}{x^c}\right)^{b+c} \left(\dfrac{x^c}{x^a}\right)^{c+a}$

$= \left(\dfrac{x^a}{x^b}\right)^{a^2+ab+b^2} \left(\dfrac{x^b}{x^c}\right)^{b^2+bc+c^2} \left(\dfrac{x^c}{x^a}\right)^{c^2+ac+a^2} = \left(\dfrac{x^a}{x^b}\right)^{a+b-c} \left(\dfrac{x^b}{x^c}\right)^{b+c-a} \cdot \left(\dfrac{x^c}{x^a}\right)^{c+a-b}$

$= \dfrac{x^{a(b-c)}}{x^{b(a-c)}} = \dfrac{(x^{a+b})^2 (x^{b+c})^2 (x^{c+a})^2}{(x^a x^b x^c)^4}$

$= \dfrac{1}{\left(\dfrac{x^b}{x^a}\right)^c}$

$= \dfrac{1}{1+x^{b-a}+x^{c-a}} + \dfrac{1}{1+x^{a-b}+x^{c-b}} + \dfrac{1}{1+x^{b-c}+x^{a-c}} = 1$

* $\dfrac{1}{x^a(x^a+x^b+x^c)} + \dfrac{1}{x^b(x^b+x^a+x^c)} + \dfrac{1}{x^c(x^c+x^b+x^a)} = 1$

* $\dfrac{a^{-1}}{a^{-1}+b^{-1}} + \dfrac{a^{-1}}{a^{-1}-b^{-1}} = \dfrac{1}{a(a^{-1}+b^{-1})} + \dfrac{1}{a(a^{-1}-b^{-1})}$

$= \dfrac{1}{\frac{1}{a}+\frac{1}{b}} + \dfrac{1}{\frac{1}{a}-\frac{1}{b}} = \dfrac{b}{b+a} + \dfrac{b}{b-a} = \dfrac{b^2-\cancel{ba}+b^2+\cancel{ba}}{b^2-a^2}$

$= \dfrac{2b^2}{b^2-a^2}$

Lecture 6.

Practice - II

$(a+b)(a-b) = a^2 - b^2$

$(a-b)(a^2+ab+b^2) = a^3 - b^3$

$a^3 + \cancel{a^2b} + \cancel{ab^2} - \cancel{b^2a} - \cancel{ab^2} - b^3$

* $x = \sqrt[3]{28}$, $y = \sqrt[3]{27} = 3$

$$x+y - \frac{1}{(x^2+xy+y^2)} = (x+y) - \frac{(x-y)}{(x-y)(x^2+xy+y^2)} = x+y - \frac{(x-y)}{x^3-y^3}$$

$$= x+y-(x-y) = \cancel{x}+y-\cancel{x}+y = 2y = 6$$

$x = 3$

* $(x^{1/3}+x^{-1/3})\underbrace{(x^{2/3}+x^{-2/3}-1)}_{(x^{1/3})^2+(x^{-1/3})^2-(x^{1/3})(x^{-1/3})} = (x^{1/3})^3 + (x^{-1/3})^3$

$(a+b)(a^2-ab+b^2) = a^3+b^3$

$\rightarrow x + x^{-1} = 3 + \frac{1}{3} = \frac{10}{3}$

* $\underbrace{25^{x-1}}_{5^{2x-2}} = 5^{2x-1} - 100 \Rightarrow \underbrace{5^{2x-2} - 5^{2x-1}}_{5^{2x-2}(1-5^1)} = -100$

$5^{2x-2} = 5^2 \Rightarrow x = 2$

* $4^{2x} = \frac{1}{32} \Rightarrow x = -\frac{5}{4}$

* $\underbrace{\sqrt{\left(\frac{3}{5}\right)^{1-2x}}}_{\left(\frac{3}{5}\right)^{\frac{1-2x}{2}}} = 4\frac{17}{27} = \left(\frac{3}{5}\right)^{-3} \Rightarrow \frac{1-2x}{2} = -3 \Rightarrow x = \frac{7}{2}$

* $3(2^x+1) - 2^{x+2} + 5 = 0$

$\underbrace{3\cdot 2^x}+3 - \underbrace{4\cdot 2^x}+5 = 0 \Rightarrow \underbrace{3\cdot 2^x - 4\cdot 2^x}+8 = 0$

$(3a - 4a = -a)$

$2^x(3-4) = -2^x$

$8 = 2^x \Rightarrow x = 3$

* $3^x = 9\cdot 3^y$, $8\cdot 2^y = 4^x \longrightarrow x = 2+y$, $3+y-2x$ $\Big\}$ $x = 1$, $y = -1$

substitution method

* $a^x = b$, $b^y = c$, $c^z = a$, PT $xyz = 1$

$\downarrow$

$\underbrace{(c^z)^x}_{c^{zx}} = b \longrightarrow (b^y)^{zx} = b^1 \Rightarrow b^{yzx} = b^1 \Rightarrow xyz = 1$

* if $\underbrace{a^x = b^y = c^z}$, $b^2 = ac$ PT: $y = \frac{2xz}{x+z}$.

$\left.\begin{array}{l} b = a^{x/y} \\ c = a^{x/z} \end{array}\right\} \xrightarrow{b^2 = ac} \underbrace{(a^{x/y})^2}_{a^{2x/y}} = \underbrace{a(a^{x/z})}_{a^{1+\frac{x}{z}}}$

$\frac{2x}{y} = 1 + \frac{x}{z} = \frac{x+z}{z} \Rightarrow 2xz = y(x+z) \Rightarrow \frac{2xz}{x+z} = y$

V. Std. Identities (Not var / just numbers) ; $a, b, c \in \mathbb{R}$

* $(a+b)^2 = (a+b)(a+b) = a^2 + ab + ba + b^2 = a^2 + 2ab + b^2$

* $(a-b)^2 = a^2 + b^2 - 2ab$.

* $(a+b)(a-b) = a^2 - b^2$

$a^n b^n = (ab)^n$

* $(\sqrt{a})^2 = a$; $\sqrt{a}\sqrt{b} = \sqrt{ab}$, $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

* $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) = a - b$

$(a+\sqrt{b})(a-\sqrt{b}) = a^2 - b$

$(\sqrt{a} \pm \sqrt{b})^2 = a + b \pm 2\sqrt{ab}$

* $(\sqrt{a}+\sqrt{b})(\sqrt{c}+\sqrt{d}) = \sqrt{ab} + \sqrt{ad} + \sqrt{bc} + \sqrt{bd}$

VI Rationalization (Math-9)

* $\sqrt[5]{16}\cdot\sqrt[5]{2} = \sqrt[5]{16\cdot 2} = \sqrt[5]{32} = 2^{5\times\frac{1}{5}} = 2$

* $(5+\sqrt{7})(2+\sqrt{5}) = 10 + 5\sqrt{5} + 2\sqrt{7} + \sqrt{35}$

Lecture - 7

* $\left(\sqrt{\frac{2}{3}}\right)^5 \left(\frac{6}{7}\right)^2 = \frac{4}{9} \cdot \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{6^2}{7 \cdot 7} = \left(\frac{4}{7}\right)^2 \cdot \sqrt{\frac{2}{3}} = \sqrt{\frac{2 \cdot 4^4}{3 \cdot 7^4}}$

$= \sqrt{\frac{512}{7203}}$

* $2^{5/2} = \sqrt{2^4 2} = 2^2 \sqrt{2} = 4\sqrt{2}$

* $\left(\frac{5^{-1} \cdot 7^2}{5^2 \cdot 7^{-4}}\right)^{7/2} \cdot \left(\frac{5^{-2} \cdot 7^3}{5^3 \cdot 7^{-5}}\right)^{-5/2} = \left(5^{-3} 7^6\right)^{7/2} \cdot \left(5^{-5} 7^8\right)^{-5/2}$

$\rightarrow \left(5^{-3}\right)^{7/2} \cdot \left(7^3\right)^7 \left(5^{-5}\right)^{-5/2} \cdot \left(7^4\right)^{-5}$

$= \underbrace{\left(5^{-\frac{21}{2}} \cdot 5^{\frac{25}{2}}\right)}_{5^2} \underbrace{\left(7^{21} \cdot 7^{-20}\right)}_{7} = 1 + 5$

* $2^x = 3^y = 12^z$ T.P $\frac{1}{z} = \frac{1}{y} + \frac{2}{x}$

Method 1

* $2^x = 3^y \Rightarrow \boxed{2 = 3^{y/x}}$

$3^y = 12^z = 3^z \cdot 2^{2z}$

$2^x = 12^z = 3^z \cdot 2^{2z}$

$2^x = 3^z \cdot \left(3^{y/x}\right)^{2z} = 3^z \cdot 3^{\frac{2yz}{x}} = 3^{z + \frac{2yz}{x}}$

$\left(3^{y/x}\right)^x = 3^y$

$y = z + \frac{2yz}{x}$ ✓

Rev. Eng. Check: (Be back n Trigo.)

$\frac{1}{z} = \frac{1}{y} + \frac{2}{x} \Rightarrow \frac{1}{z} = \frac{x + 2y}{xy} \Rightarrow z = \frac{xy}{x + 2y}$

$zx + 2yz = xy \Rightarrow z + \frac{2yz}{x} = y$

VI Rationalization (of denom) misleading

* $x = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3} = \frac{2}{3}\sqrt{3}$

* $x = \frac{1}{\sqrt{a}} = \frac{1}{\sqrt{a}} \cdot \frac{\sqrt{a}}{\sqrt{a}} = \frac{\sqrt{a}}{a}$

* $x = \frac{\sqrt{a}}{a} = \frac{\sqrt{a}}{\sqrt{a} \cdot \sqrt{a}} = \frac{1}{\sqrt{a}}$ ✓

* $x = \frac{3 + \sqrt{5}}{\sqrt{2}} = \frac{3\sqrt{2} + \sqrt{10}}{2}$

$(a+b)^2 = a^2 + b^2 + 2ab$

$(a+b)(a-b) = a^2 - b^2$

* $y = \frac{1}{2 + \sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{2\sqrt{3} + 3}$ (Not done)

$y = \frac{1}{2 + \sqrt{3}} \cdot \frac{2 + \sqrt{3}}{2 + \sqrt{3}} = \frac{2 + \sqrt{3}}{(2 + \sqrt{3})^2} = \frac{2 + \sqrt{3}}{4 + 3 + 4\sqrt{3}}$

$= \frac{2 + \sqrt{3}}{7 + 4\sqrt{3}}$ (Not done)

$y = \frac{1}{(2 + \sqrt{3})} \frac{2 - \sqrt{3}}{(2 - \sqrt{3})} = 2 - \sqrt{3}$ (Done)

* $y = \sqrt{4} - \sqrt{3} = 2 - \underbrace{\sqrt{3}}_{1.73} \neq 1$

* $x = \frac{1}{a \pm \sqrt{b}} \cdot \frac{a \mp \sqrt{b}}{a \mp \sqrt{b}} = \frac{a \mp \sqrt{b}}{a^2 - b}$ (Done)

$(a \pm b)(a \mp b) = a^2 - b^2$

Lecture-8

* $\alpha = \dfrac{1}{7+3\sqrt{2}} = \dfrac{7-3\sqrt{2}}{31}$

* $\beta = \dfrac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}} \cdot \dfrac{2\sqrt{2}-3\sqrt{3}}{2\sqrt{2}-3\sqrt{3}} = \dfrac{4\sqrt{6}-18-2\sqrt{10}+3\sqrt{15}}{8-27}$

$= \dfrac{-4\sqrt{6}+18+2\sqrt{10}-3\sqrt{15}}{19}$

* $\underbrace{\dfrac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}}_{} = a - b\sqrt{6} \Rightarrow a = 2,\ b = \dfrac{-5}{6}$

$\dfrac{12+5\sqrt{6}}{6} = \dfrac{12}{6} + \dfrac{5}{6}\sqrt{6} = 2 - \left(\dfrac{-5}{6}\right)\sqrt{6}$

* $x = \dfrac{4+\sqrt{5}}{4-\sqrt{5}} + \dfrac{4-\sqrt{5}}{4+\sqrt{5}} \equiv y + \dfrac{1}{y} = \dfrac{y^2+1}{y}$

$= \dfrac{(4+\sqrt{5})^2}{(4-\sqrt{5})} \dfrac{1}{(4+\sqrt{5})} + \dfrac{(4-\sqrt{5})^2}{(4+\sqrt{5})(4-\sqrt{5})} = \dfrac{(16+5)2}{16-5}$

$= \dfrac{42}{11}$

* $\dfrac{(a+b)^2}{2} + \dfrac{(a-b)^2}{2} = (a^2+b^2)$; $(a+b)(a-b) = a^2-b^2$

* $\gamma = \dfrac{3}{5-\sqrt{3}} + \dfrac{2}{5+\sqrt{3}} = \dfrac{25+\sqrt{3}}{22}$

* $\delta = \dfrac{\sqrt{5}-2}{\sqrt{5}+2} - \dfrac{\sqrt{5}+2}{\sqrt{5}-2} = (\sqrt{5}-2)^2 - (\sqrt{5}+2)^2 = -8\sqrt{5}$

* $(a+b)^2 - (a-b)^2 = 4ab$

* $\epsilon = \dfrac{1}{1+\sqrt{2}} + \dfrac{1}{\sqrt{2}+\sqrt{3}} + \dfrac{1}{\sqrt{3}+2} + \dfrac{1}{2+\sqrt{5}} + \dfrac{1}{\sqrt{5}+\sqrt{6}}$

$+ \dfrac{1}{\sqrt{6}+\sqrt{7}} + \dfrac{1}{\sqrt{7}+2\sqrt{2}} + \dfrac{1}{2\sqrt{2}+3}$

$= -(1-\cancel{\sqrt{2}} + \cancel{\sqrt{2}} - \cancel{\sqrt{3}} + \cancel{\sqrt{3}} - \cancel{\sqrt{4}} + \cancel{\sqrt{4}} - \cancel{\sqrt{5}} + \cancel{\sqrt{5}} - \cancel{\sqrt{6}} + \cancel{\sqrt{6}}$

$- \cancel{\sqrt{7}} + \cancel{\sqrt{7}} - \cancel{\sqrt{8}} + \cancel{\sqrt{8}} - \sqrt{9}) = 2$

* $\theta = \dfrac{1}{3-\sqrt{8}} - \dfrac{1}{\sqrt{8}-\sqrt{7}} + \dfrac{1}{\sqrt{7}-\sqrt{6}} - \dfrac{1}{\sqrt{6}-\sqrt{5}} + \dfrac{1}{\sqrt{5}-2}$

$= 3 + \cancel{\sqrt{8}} - \cancel{\sqrt{8}} - \cancel{\sqrt{7}} + \cancel{\sqrt{7}} + \cancel{\sqrt{6}} - \cancel{\sqrt{6}} - \cancel{\sqrt{5}} + \cancel{\sqrt{5}} + 2 = 5$

* $x = 2+\sqrt{3}$, $x^2 + \dfrac{1}{x^2} = ? = \left(x+\dfrac{1}{x}\right)^2 - 2 = 14$

$\dfrac{1}{x} = \dfrac{1}{2+\sqrt{3}} = 2-\sqrt{3}$

'1st App of Identity'

$\boxed{x + \dfrac{1}{x} = 4}$

$\underbrace{\left(x+\dfrac{1}{x}\right)^2}_{} = \underbrace{x^2 + \dfrac{1}{x^2}}_{} + 2$

$(a+b)^2 = a^2+b^2+2ab$

* $x = 1-\sqrt{2}$, $\left(x - \dfrac{1}{x}\right)^2 = (1-\sqrt{2}-(-1-\sqrt{2})) = 8$

$\dfrac{1}{x} = \dfrac{1+\sqrt{2}}{1-2} = -1-\sqrt{2}$

Lecture 9

* $x^y = 3^y = 6^z \longrightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

$x^y = 3^y \Rightarrow x = 3^{y/x}$

$3^y = 6^z = 2^z \cdot 3^z$

$2^x = 6^z = 2^z \cdot 3^z \Rightarrow 3^y = (3^{y/x})^z \cdot 3^z = 3^{-\frac{yz}{x} - z}$

$\Downarrow$

$y = -\frac{yz}{x} - z \Rightarrow y + z = -\frac{yz}{x}$

* $\frac{y+z}{yz} = -\frac{yz}{x} \cdot \frac{1}{yz} \Rightarrow \frac{1}{z} + \frac{1}{y} + \frac{1}{x} = 0$

* $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0 \rightarrow \frac{y+z}{yz} = -\frac{1}{x} \Rightarrow y + z = -\frac{yz}{x}$

$\Downarrow$

$y = -z - \frac{yz}{x}$

* $3^{(x-2)(2x-3)} = 3^{-1} \cdot 3^3 \Longleftrightarrow \begin{cases} x - 2 = 1 \\ 2x - 3 = 3 \end{cases} \Rightarrow x = 3$

* $x = 2^{1/3} + 2^{-1/3} \longrightarrow x^3 - 6x = ? = 6$

$= 2^{1/3}(1 + 2^{-1})$

$x^3 = 2(1 + 2^{-1})^3 = 2(1 + 2 + 3 \cdot 2^{1/3} + 3 \cdot 2^{-2/3})$

* $(a+b)^3 = (a+b)(a+b)^2 = (a+b)(a^2 + b^2 + 2ab)$

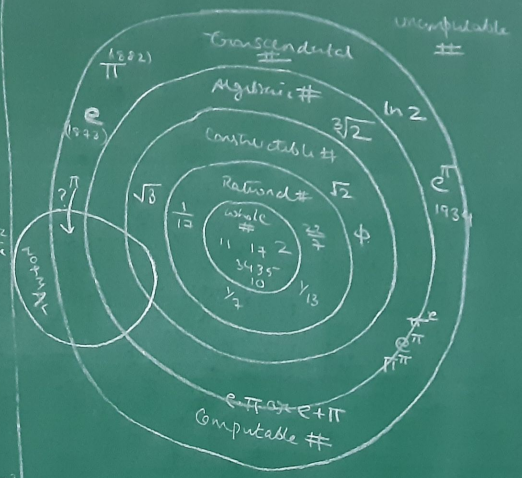
$= a^3 + ab^2 + 2a^2b + ba^2 + b^3 + 2ab^2$

$= a^3 + b^3 + 3a^2b + 3ab^2$

$= a^3 + b^3 + 3ab(a+b)$, $a, b \in \mathbb{R}$

$x^3 - 6x = (2 + 4 + 3 \cdot 2^{1/3} + 3 \cdot 2^{-1/3}) - \underbrace{6 \cdot 2^{1/3}}_{3 \cdot 2^{1/3}} - \underbrace{6 \cdot 2^{-1/3}}_{3 \cdot 2^{-1/3}} = 6$

Cutting Edge ideas of Number Theory:



* Constructible # : construct using ruler / pencil / compass

* $x^3 - 2 = 0 \Rightarrow x = \sqrt[3]{2}$

Algebraic eqn — Algebraic #

* Computable # : Rules to get the # finite Algorithm

Algo. time could be ∞

* Champernowne's const

$0.\underbrace{123456789101112131415 16 \ldots}_{\text{all val. \#}}$

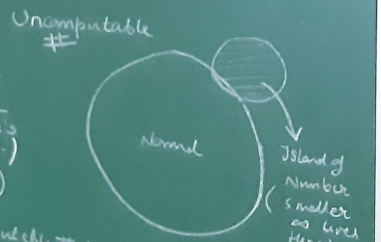
valid #, $\underbrace{\text{Computable}}_{\exists \text{ rule}}$, Transcendental, Artifical (Normal)

* Copeland Erdős

$0.\underbrace{23571113171923 29 31 \ldots}_{\text{all primes}}$

∃ rule (const)

* Normal # : No test for them



* Uncomputable # : No Eqn / Rule
No algo.

* Turing's Paper (1936) : ∃ # outside computable #

Lecture-10

* $(13)^{5x} = 4^4 - 3^4 - 6$

* $\underbrace{9^{x+2}}_{9^x 9^2} = 240 + 9^x \Rightarrow 9^x \underbrace{(81-1)}_{84} = 3^3 \cdot 10 \Rightarrow 3^{2x} = 3^1$

$\Downarrow$

$x = \frac{1}{2}$

Cantor-Dedekind Axiom (cornerstone of Coord geo)

* "$\mathbb{R}$ is order-isomorphic to linear continuum "essentially the same" of geometry"

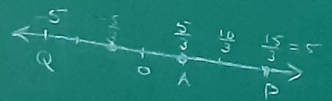
$= \{\pi, 0, \frac{-1}{12}\}$

(there exists) $\exists$ 1 on 1 correspondence b/w $\mathbb{R}$ & pts. on a line

Lt. line (1D)

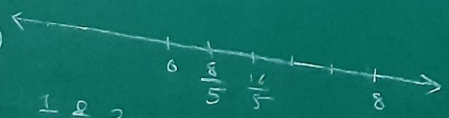
* Descartes' Blend - Real Number line

* $x = \frac{5}{3}, -\frac{5}{3}$



$x = \frac{1}{2}$

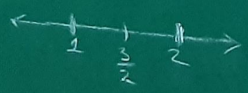
$x = \frac{8}{5}$ (choose as footing)



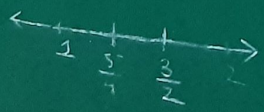
* x : lies b/w 1 & 2 , $x \in \mathbb{R}$

Algorithm (Newton's Bisection Method) simplest ver.

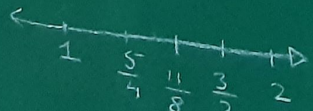
$x_1 = \frac{1+2}{2} = \frac{3}{2}$



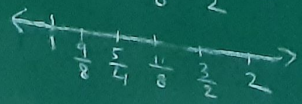
$x_2 = \frac{1 + \frac{3}{2}}{2} = \frac{5}{4}$



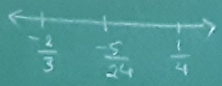
$x_3 = \frac{\frac{5}{4} + \frac{3}{2}}{2} = \frac{11}{8}$



$x_4 = \frac{1 + \frac{5}{4}}{2} = \frac{9}{8}$



* $x \in \mathbb{R}$: lies b/w $-\frac{2}{3}, \frac{1}{4}$



* $x, y \in \mathbb{Q}$: $x < y \Rightarrow \exists \left(\frac{x+y}{2}\right) : x < \frac{x+y}{2} < y$

'Algorithm'

* $\mathbb{Q} = \left\{ \underbrace{\frac{p}{q}}_{\text{Broken}} : q \neq 0,\ p, q \in \mathbb{Z} \right\}$ Essence (~Atoms)

Broken | fraction | fractured | Fractal

$\boxed{\text{Whole} \Rightarrow \text{Broken}}$

* $x = \frac{7}{8} = \frac{7 \times 10}{8 \times 10} = \frac{70}{8} \times \frac{1}{10}$

$= \frac{700}{8} \times \frac{1}{10^2} = \frac{7000}{8} \times \frac{1}{10^3}$

$= \frac{875}{1000} = 0.875$

Decimal Re-Presentation

$\boxed{\text{Iterative process}}$

line is modelled

$$\begin{array}{r} 875 \\ 8\overline{)7000} \\ 64 \\ \hline 60 \\ 56 \\ \hline 40 \\ 40 \\ \hline \times \end{array}$$

'T.M Halts' 100% finite step finite time

* $x = -\frac{17}{8} = -\frac{17000}{8} \times \frac{1}{10^3}$

$= -2.125$

$$\begin{array}{r} 2125 \\ 8\overline{)17000} \\ 160 \\ \hline 10 \\ 8 \\ \hline 20 \\ 16 \\ \hline 40 \\ 40 \\ \hline \times \end{array}$$

* $\underbrace{\text{time}}_{\text{recurrence/cycle/periodicity}} \Rightarrow$ counting

Lecture-11

Time + Counting + Algorithm
(Physics) (Maths) (CS)

* Time ⇒ Counting

* $x = \frac{7}{8}$ 'division process' (Rule)

↓

Halts

Rules : Known

steps - finite
time - finite

* $x = \frac{8}{3} = 2.666\ldots$

$$\begin{array}{r} 26 \\ 3\,\overline{)\,80} \\ 6 \\ \hline 20 \\ 18 \\ \hline 2 \end{array}$$

↓

No Halt

Rules : Known.

steps : infinite
time : infinite

'Repeat after 1 step'

* $x = \frac{2}{11} = 0.181818\ldots$ Repeat after 2 steps.

* $x = \frac{1}{7} = 0.\underbrace{142857}142857.$ Repeat after 6 steps

* $x = \frac{16}{45} = 0.3555\ldots$ Repeat after 2 steps

* $x = \frac{1}{17} = 0.\underbrace{0588235294117647}$ Repeats after 16 step.

* Hilbert's problem (1900s)
23
↓
3rd ques.

Entscheidungsproblem (Decision problem)

Is math. decidable?

↓

Turing's Halting Problem.

* input → $\boxed{\text{program}}$ —True→ Returns "Halts"

$X \equiv (a, b)$

will P halt with i/p X ?

—False→ Return "Never"

{Halting's Problem.

unsolvable decision problem
⇓
P can't handle it

* $x = \frac{8}{3} = 2.\overline{6}$, $y = \frac{16}{45} = 0.3\overline{5}$

* $x = 0.\overset{\downarrow}{1}11 = 0.\overline{1}$

$10x = 1.111\ldots$

$9x = 1 \Rightarrow x = \frac{1}{9}$

* $x = 0.2222\ldots$

$10x = 2.2\ldots$

$9x = 2 \Rightarrow x = \frac{2}{9}$

* $x = 0.\overline{8} = \frac{8}{9}$

* $x = 0.3535\ldots$

$100x = 35.35\ldots$

$99x = 35 \Rightarrow x = \frac{35}{99}$

* $x = 0.\overline{585}$

$1000x = 585.\overline{585}$

$x = \frac{585}{99}$

* $x = 0.\overset{\swarrow}{3}222\ldots$

$100x = 32.2222\ldots$ } won't work

Pattern

$0.\overline{1} = \frac{1}{9}$

$0.\overline{2} = \frac{2}{9} = 2 \times 0.\overline{1}$

$0.\overline{3} = 3 \times 0.\overline{1}$

$0.\overline{4} = 4 \times 0.\overline{1}$

$0.\overline{9} = 9 \times 0.\overline{1}$

Lecture 12

Time + Counting + Algorithm
(physics) (Maths) (C.S)

* Time ⇒ Counting

* $4^{2x} = \underbrace{\left(\sqrt[3]{16}\right)^{-6/y}}_{} = 8$

$(16)^{-\frac{2}{y}\cdot\frac{1}{3}} = 8 \Rightarrow (2^4)^{-\frac{2}{y}} = 2^3 \Rightarrow y = -\frac{8}{3}$

$\sqrt[n]{a} = a^{1/n}$

$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c}$

* $\dfrac{\left(a+\frac{1}{b}\right)^m \cdot \left(a-\frac{1}{b}\right)^n}{\left(b+\frac{1}{a}\right)^m \left(b-\frac{1}{a}\right)^n} = \dfrac{\left(\frac{ab+1}{b}\right)^m \cdot \left(\frac{ab-1}{b}\right)^n}{\left(\frac{ab+1}{a}\right)^m \left(\frac{ba-1}{a}\right)^n}$

$\rightarrow \left(\dfrac{\frac{ab+1}{b}}{\frac{ab+1}{a}}\right)^m \cdot \left(\dfrac{\frac{ab-1}{b}}{\frac{ab-1}{a}}\right)^n = \left(\frac{a}{b}\right)^m \left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)^{m+n}$

* $x = \dfrac{1}{\sqrt{12}} \cdot \dfrac{\sqrt{12}}{\sqrt{12}} = \dfrac{\sqrt{12}}{12} = \dfrac{2\sqrt{3}}{12} = \dfrac{\sqrt{3}}{6}$

* $\dfrac{b^2}{\sqrt{a^2+b^2}+a} \cdot \dfrac{\sqrt{a^2+b^2}-a}{\sqrt{a^2+b^2}-a} = \dfrac{b^2\sqrt{a^2+b^2} - ab^2}{a^2+b^2-a^2}$

$\rightarrow \sqrt{a^2+b^2} - a$

* $\dfrac{5+2\sqrt{3}}{7+4\sqrt{3}} \cdot \dfrac{7-4\sqrt{3}}{7-4\sqrt{3}} = 35 - 20\sqrt{3} + 14\sqrt{3} - 24$

$= 11 - 6\sqrt{3}$

* $\dfrac{1+\sqrt{2}}{3-2\sqrt{2}} \cdot \dfrac{3+2\sqrt{2}}{3+2\sqrt{2}} = 3 + 2\sqrt{2} + 3\sqrt{2} + 4 = 7 + 5\sqrt{2}$

* $\dfrac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}} \cdot \dfrac{3\sqrt{5}+2\sqrt{6}}{3\sqrt{5}+2\sqrt{6}} = \dfrac{6\sqrt{30}+24-15-2\sqrt{30}}{45-24}$

$\rightarrow \dfrac{9+4\sqrt{30}}{21}$

* $\dfrac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}} = \dfrac{4\sqrt{3}+5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}} \cdot \dfrac{4\sqrt{3}-3\sqrt{2}}{4\sqrt{3}-3\sqrt{2}}$

$\rightarrow \dfrac{48-12\sqrt{6}+20\sqrt{6}-30}{48-18} = \dfrac{18+8\sqrt{6}}{30} = \dfrac{9+4\sqrt{6}}{15}$

Lecture-13

* $x = 0.\overline{1} = 0.1111\ldots$
  $10x = 1.11\ldots$
  $9x = 1 \Rightarrow x = \frac{1}{9}$

$\underbrace{0.nnn\ldots}_{0.\overline{n}} = \frac{n}{9}$

$0.\overline{nm} = \frac{nm}{99}$

* $x = 0.3222\ldots$
  $100x = 32.222\ldots$ 'won't work'

  $x = 0.3222\ldots$
  $10x = 3.222\ldots = \underset{\substack{\uparrow \\ \text{int}}}{3} + \underbrace{0.22\ldots}_{\frac{2}{9}} = \frac{29}{9} \Rightarrow x = \frac{29}{90}$
  (split decimal)

* $x = 0.12\overline{3}$
  $100x = 12.\overline{3} = 12 + 0.\overline{3} = 12 + \frac{1}{3} = \frac{37}{3}$

  $x = \frac{37}{300}$

* $x = 4.3\overline{2}$
  $10x = 43.\overline{2} = 43 + \frac{2}{9} = \frac{387+2}{9} = \frac{389}{9}$

  $x = \frac{389}{90}$

* $x = 15.7\overline{12}$
  $10x = 157.\overline{12} = 157 + \frac{\cancel{12}^{4}}{\cancel{99}_{33}} = \frac{5185}{33} \Rightarrow x = \frac{5185}{330}$

* $x = 0.2\overline{35} = \frac{233}{990}$

* $x = \frac{p}{q} = \begin{cases} \text{terminate} \\ \text{Non terminate} \rightarrow \text{Repeat} \end{cases}$

* $x = \frac{7}{8} = \frac{7}{2^3} \times \frac{5^3}{5^3} = \frac{7 \times 5^3}{(10)^3} = \frac{125 \times 7}{(10)^3}$
  $= \frac{875}{(10)^3} = 0.875$

* $x = \frac{49}{500} = \frac{49 \times 2}{1000} = 0.098$

Lecture-14

* $x = \frac{3-\sqrt{5}}{3+2\sqrt{5}} \times \frac{3-2\sqrt{5}}{3-2\sqrt{5}} = \frac{9-6\sqrt{5}-3\sqrt{5}+10}{9-20}$

$= \frac{19-9\sqrt{5}}{-11} = \frac{9\sqrt{5}-19}{11}$

* $x = \frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}}$

$= \frac{(7+3\sqrt{5})(3-\sqrt{5}) - (7-3\sqrt{5})(3+\sqrt{5})}{4}$

$= \frac{-7\sqrt{5}+9\sqrt{5}-(7\sqrt{5}-9\sqrt{5})}{4} = \frac{(2-7+9)\sqrt{5}}{4} = \sqrt{5}$

* $x = 3+\sqrt{8}$, $x^2 + \frac{1}{x^2} = ?$

$\frac{1}{x} = \frac{1}{3+\sqrt{8}} \frac{3-\sqrt{8}}{3-\sqrt{8}} = 3-\sqrt{8}$

Method 2

$\left(x+\frac{1}{x}\right)^2 = \underbrace{x^2 + \frac{1}{x^2}}_{\equiv A} + 2$ (TRICK)

$A = \left(x+\frac{1}{x}\right)^2 - 2 = \left(3+\cancel{\sqrt{8}} + 3-\cancel{\sqrt{8}}\right)^2 - 2$

$= 34$

* $x = 2+\sqrt{3}$, $x^3 + \frac{1}{x^3}$, $\frac{1}{x} = 2-\sqrt{3}$

$(a+b)^3 = a^3+b^3+3ab(a+b)$

$\left(x+\frac{1}{x}\right)^3 = \underbrace{x^3+\frac{1}{x^3}}_{A} + 3\left(x+\frac{1}{x}\right)$

$\left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right) = A$

* $A = 4^3 - 3(4) = 64-12 = 52$

(Math-10)

* $x = \frac{p}{q} = \begin{cases} \text{Prime fact of } q : 2^m 5^n \Leftrightarrow \text{terminates after } k \text{ places} \\ k = \text{Large}(m,n) \\ \text{q. fact not of form } 2^m \cdot 5^n \Rightarrow \text{Non terminate/Repeat} \end{cases}$ Theorem I

Checks

Case I : $m=n$

* $x = \frac{p}{q} = \frac{p}{2^n \cdot 5^m} = \frac{p}{(10)^m} \Rightarrow$ terminates after m places

Case II : $m>n$ $(m-n=a)$

* $x = \frac{p}{q} = \frac{p}{2^m \cdot 5^n} = \frac{p}{2^{n+a} 5^n} = \frac{p \cdot 5^a}{(10)^n \cdot 2^a \cdot 5^a} = \frac{p\,5^a}{(10)^{n+a}}$

$x = \frac{c}{(10)^m}$; $c = p \cdot 5^a \Rightarrow$ terminates after m places

Case III : $n>m$ $(n-m=a)$

* $x = \frac{p}{q} = \frac{p}{2^m 5^n} = \frac{p}{2^m \cdot 5^{m+a}} = \frac{p \cdot 2^a}{(10)^m \cdot 5^a 2^a} = \frac{2^a p}{(10)^{m+a}} = \frac{2^a p}{(10)^n}$

$x = \frac{c}{(10)^n} \Rightarrow$ terminates after n places

without division, Evaluate

* $\alpha = \frac{17}{8} = \frac{17 \cdot 5^3}{2^3 \cdot 5^3} = \frac{17 \times 125}{(10)^3}$

* $\beta = \frac{64}{455} = \frac{64}{5 \times 7 \times 13} \Rightarrow$ Non term./Repeat $(\sim 0.\overline{n})$

* $\gamma = \frac{29}{343} = \frac{29}{7^3} \Rightarrow$ Non term / Repeat

Lecture-15 (Math-10)

iff = if & only if

$\notin$ = doesn't belong

Divisibility (Remainder = 0)

* $a|b$ iff $\exists c : b = ca$

"a divides b"
"b divisible by a"
"a factor b"
"b is multiple of a"

divisor, dividend, quotient

$5\overline{)20}(4$

Remainder

'divisibility concept'

$a, b, c \in \mathbb{Z}$

$\frac{b}{a} = c \longrightarrow a|b$: translation

* $5|20$ true iff $\exists c : 20 = c \cdot 5 \Rightarrow \boxed{c = 4}$

$c \in \mathbb{Z}$

* $13|-25$ true iff $\exists c : -25 = c \cdot 13 \Rightarrow c = \frac{-25}{13}$

It's false $\Leftarrow c \notin \mathbb{Z}$

c: DNE (doesn't exist)

Properties

* $\pm 1 | a \quad \forall \; a \in \mathbb{Z} - \{0\}$
* $a | 0 \quad \forall \; a \in \mathbb{Z} - \{0\}$
* $0 | a$ DNE
* $a|b, b|a \Rightarrow a = \pm b$

check

* $a|b \Rightarrow \frac{b}{a} = k$, $b|a \Rightarrow \frac{a}{b} = l$ $\Big\} \Rightarrow \frac{b}{a} \cdot \frac{a}{b} = kl \Rightarrow l = \frac{1}{k}$ $\quad k, l \in \mathbb{Z}$

$l, k \in \mathbb{Z}$

$\boxed{l, k = \pm 1} \Rightarrow \frac{a}{b} = \pm 1 \Rightarrow \boxed{a = \pm b}$

* $a|b, a|c \Rightarrow \begin{cases} a | b \pm c \\ a | bc \\ a | bx \end{cases}$

check

* $\frac{b}{a} = k, \frac{c}{a} = l, k, l \in \mathbb{Z}$

$\frac{b \pm c}{a} = k \pm l \in \mathbb{Z} \Rightarrow a | b \pm c$

* $\frac{bc}{a^2} = kl \Rightarrow \frac{bc}{a} = kla \in \mathbb{Z} \Rightarrow a | bc$
* $\frac{b}{a} = k \Rightarrow \frac{bx}{a} = kx \in \mathbb{Z} \Rightarrow a | bx$
* $a|b, c|d \Rightarrow ac | bd$
* $ac | bc \Rightarrow a | b$

Euclid division lemma

* $2 \overset{?}{|} 20 \longrightarrow 2\overline{)20}(10 \longrightarrow 20 = 2 \times 10 + 0$

$0 \le 0 < 2$

$\frac{20}{2} = 10$

* $3 \overset{?}{|} 20 \longrightarrow 3\overline{)20}(9 \longrightarrow 20 = 3 \times 9 + 2$

$0 \le 2 < 3$

* $5 \overset{?}{|} 37 \longrightarrow 5\overline{)37}(7$

$37 = 5 \times 7 + 2$

$0 \le 2 < 5$

$4 \overset{?}{|} 53 \longrightarrow 4\overline{)53}(13$

$53 = 4 \times 13 + 1$

$0 \le 1 < 4$

(Extension) Conviction to "above" rotation

$b = c \cdot a$

* $\boxed{(a, b) \in \mathbb{Z}, \exists (q, r) : a = bq + r \quad 0 \le r < b}$

'Euclid div lemma'

Lecture-16 (Math-10)

* $x = 0.4777\ldots$

$10x = 4.777\ldots = 4 + 0.\overline{7} = 4 + \frac{7}{9} = \frac{43}{9}$

$x = \frac{43}{90}$

* $a \mid b$ iff $\exists c : b = c \cdot a \longleftrightarrow \frac{b}{a} = c$

"a divides b" "divisibility" $a, b, c \in \mathbb{Z}$

* $(a, b) \in \mathbb{Z}$, $\exists (q, r) : a = bq + r$, $0 \le r < b$

"Euclid's division lemma"

Comment

* $v = u + at = at + u$; $a = \frac{v}{t} \Rightarrow v = at$

(1st Eqn of Motion)

* $(a - r) = bq \Rightarrow \frac{(a-r)}{b} = q \Rightarrow b \mid a - r$

* $r = 0 \Rightarrow a = bq \Rightarrow \frac{a}{b} = q \Rightarrow b \mid a$

Proof of lemma.

*

$a - 2b, a - b, a, a + b, a + 2b \ldots$ "Common diff $= b$" | A.P.

$\exists$ a smallest non-negative int. $q : a - qb = r \Rightarrow a = bq + r$

$0 \le r < b$

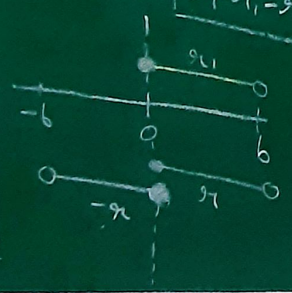
Uniqueness of $(q, r) \Rightarrow q_1 = q$, $r_1 = r$

check

* $a = bq + r$, $a = bq_1 + r_1$, $0 \le r < b$, $0 \le r_1 < b$

$bq + r = bq_1 + r_1 \Rightarrow (r_1 - r) = b(q - q_1)$

$\frac{r_1 - r}{b} = q - q_1 \Rightarrow \boxed{b \mid r_1 - r}$, $0 \le r < b$



$0 \le r_1 < b$

(Inequality)

$0 \le r_1 - r \le 0$

$\boxed{r_1 = r}$

$q_1 = q$

* +ve int divisible by 2(b)

$\underset{\text{number}}{\#} = 2q, 2q + 1$ $\quad 0 \le r < 2$

(a)

* +ve int. div. by 3(b)

$\underset{(a)}{\#} = 3q, 3q + 1, 3q + 2$ $\quad 0 \le r < 3$

* +ve int. div by 4(b)

$\underset{a}{\#} = 4q, 4q + 1, 4q + 2, 4q + 3$ $\quad 0 \le r < 4$

* T.P. Every Even int is of form $2q$

" Odd " " " $2q + 1$

Claus

* let a be any int

$b = 2$

ED lemma $\Rightarrow a = 2q + r$ | $a \mid b \to \frac{b}{a} = c$

$a = 2q + 0 \to$ even

$a = \underbrace{2q}_{E} + \underbrace{1}_{O} \to$ odd

$0 \le r < 2$

$\Downarrow$

$0 \le r \le 1$

$\Downarrow$

$r = 0, 1$

Lecture 17

T.P = To prove / show that

* $x \in \mathbb{Z}^+$, T.P. $x \sim 3q$ or $3q+1, 3q+2$, $q \in \mathbb{Z}$ "is of form"

* $(a,b) \in \mathbb{Z}$; $\exists (q,r)$: $a = bq + r$, $0 \le r < b$ 'E-D-lemma'

* a, $b = 3$

$(a,3) \in \mathbb{Z}$; $\exists (q,r)$: $a = 3q + r$, $0 \le r < 3$ $\Downarrow$ $0 \le r \le 2$

$a = 3q$
$a = 3q+1$
$a = 3q+2$

* $x \in \mathbb{Z}^+_{odd}$, TP $x \sim 4q+1$, $4q+3$, $q \in \mathbb{Z}$

$a^2 - b^2 = (a+b)(a-b)$
$(a+b)^2 = a^2 + b^2 + 2ab$

a, $b = 4$ + 'E D lemma' (proof)

* $n \in \mathbb{Z}^+_{odd}$ T.P $8 \mid n^2 - 1$

"$n^2 - 1$ is divisible by 8"

Proof

$n = 4q+1$ (case I) or $4q+3$ (case II) $n \in \mathbb{Z}^+_{odd}$

Case I

$n^2 - 1 = (4q+1)^2 - 1^2 = (4q+2)4q = 8\underbrace{q(2q+1)}_{\in \mathbb{Z}}$ $\Downarrow$ $8 \mid n^2 - 1$

$n^2 - 1 = (4q+3)^2 - 1^2 = (4q+4)(4q+2) = 8\underbrace{(q+1)(2q+1)}_{\in \mathbb{Z}}$ $\Downarrow$ $8 \mid n^2 - 1$

* $x, y \in \mathbb{Z}^+_{odd}$; T.P. $x^2 + y^2$ is Even / $4 \nmid x^2 + y^2$

$x = 2q_1 + 1$, $y = 2q_2 + 1$, $x, y \in \mathbb{Z}^+_{odd}$

$x^2 + y^2 = 4q_1^2 + 1 + 4q_1 + 4q_2^2 + 1 + 4q_2$
$= 4(q_1^2 + q_2^2) + 4(q_1 + q_2) + 2$

Formula Sheet (1st steps to prove)

* $x \sim 2q$ $x \in \mathbb{Z}^+_{even}$
$x \sim 2q+1$ $x \in \mathbb{Z}^+_{odd}$

* $x \sim 3q$, $3q+1$, $3q+2$ $x \in \mathbb{Z}^+$

* $x \sim 4q+1$, $4q+3$ $x \in \mathbb{Z}^+_{odd}$

* $x \sim 4q$, $4q+2$ $x \in \mathbb{Z}^+_{even}$

* $x^2 + y^2 = 4\underbrace{\{(q_1^2 + q_2^2) + (q_1 + q_2)\}}_{\equiv q} + 2 = 4q + 2$

$\frac{x^2+y^2}{2} = 2q+1 \in \mathbb{Z} \Rightarrow 2 \mid x^2 + y^2 \Rightarrow x^2+y^2$ Even

$\frac{x^2+y^2}{4} = q + \frac{1}{2} \notin \mathbb{Z}^+ \Rightarrow 4 \nmid x^2 + y^2$

* $n \in \mathbb{Z}^+$ P.T $2 \mid n^2 - n$

Proof

Case I

* $n = 2q$ $n \in \mathbb{Z}^+_{even}$, $q \in \mathbb{Z}^+$

$n^2 - n = 4q^2 - 2q = 2(2q^2 - q) \Rightarrow \frac{n^2-n}{2} = 2q^2 - q \in \mathbb{Z}^+$ $\Downarrow$ $2 \mid n^2 - n$

* Case II

$n = 2q+1$

$n^2 - n = (2q+1)^2 - (2q+1) = (2q+1)(2q+\cancel{1} - \cancel{1})$
$= (2q+1)2q = 2\underbrace{q(2q+1)}_{\equiv t} = 2t$

$\frac{n^2-n}{2} = t \in \mathbb{Z} \Rightarrow 2 \mid n^2 - n$

Lecture - 18

* $(a,b) \in \mathbb{Z}$, $\exists\ (q,r) : a = bq + r$, $0 \le r < b$
'E.D lemma'

* $x \sim 2q$ $\quad x \in \mathbb{Z}^+_{even}$
$2q+1$ $\quad x \in \mathbb{Z}^+_{odd}$

$x \sim 3q$
$3q+1$ $\quad x \in \mathbb{Z}$
$3q+2$

$x \sim 4q+1$
$4q+3$ $\quad x \in \mathbb{Z}^+_{odd}$

$x \sim 4q$
$4q+2$ $\quad x \in \mathbb{Z}^+_{even}$

* $x \in \mathbb{Z}^+$, T.P $x^2 \sim 3q$ or $3q+1$

proof

Case I.

* $x = 3m \Rightarrow x^2 = 3(\underbrace{3m^2}_{=q}) = 3q$

Case II

* $x = 3m+1 \Rightarrow x^2 = 9m^2 + 1 + 6m = 3(\underbrace{3m^2 + 2m}_{\equiv a}) + 1 = 3a+1$

Case III.

* $x = 3m+2 \Rightarrow x^2 = 9m^2 + \underbrace{4}_{3+1} + 12m = 3(\underbrace{3m^2 + 4m + 1}_{\equiv b}) + 1$
$= 3b+1$

* $x \in \mathbb{Z}^+$, T.P $x^3 \sim 9q, 9q+1, 9q+8$

* $x \in \mathbb{Z}^+$, T.P $x^3 \sim 4q, 4q+1, 4q+3$

* $x \in \mathbb{Z}^+$, T.P $x^2 \not\sim 5q+2, 5q+3$ (HW)

ED lemma
$b = 5 \Rightarrow 0 \le r < 5 \Rightarrow r = 0, 1, 2, 3, 4$
$x = 5q, 5q+1, 5q+2, 5q+3, 5q+4$

* $n \in \mathbb{Z}^+$, $3 \mid n$, $3 \mid n+2$, $3 \mid n+4$
which one is true in Cases?

Case 2
$n = 3q+1 \Rightarrow n+2 = 3q+3 = 3(q+1) = 3m$
$\frac{n+2}{3} = m \in \mathbb{Z} \Rightarrow 3 \mid n+2$

* $(a,b) \in \mathbb{Z}^+$: $a = bq + r \Rightarrow \overbrace{C.D}^{\text{common divisor}}(a,b) = CD(b,r)$
'ED lemma' 'HCF theorem'

proof.

* $CD(a,b) = c \Rightarrow c \mid a \Rightarrow \frac{a}{c} = q_1 \Rightarrow a = cq_1$
$c \mid b \Rightarrow \frac{b}{c} = q_2 \Rightarrow b = cq_2$

* $a = bq + r \Rightarrow r = a - bq = cq_1 - cbq_2$
$= c(q_1 - bq_2)$

$\frac{r}{c} = q_1 - bq_2 \Rightarrow \boxed{c \mid r,\ c \mid b}$

$a \,/\, \overset{\text{dividend}}{b} \,\big|\, c$ (divisor)
$\quad\ \ b$
$\quad\ \ \overline{x}$

$\frac{42}{7} = 6$

$\frac{63}{7} = 9$

$CD(42,63) = 7$

Lecture-9

* $(a,b) \in \mathbb{Z}^+ \quad \exists\ (q,r) : a = bq + r, \quad 0 \le r < b$

ED lemma

* $(a,b) \in \mathbb{Z}^+ : a = bq + r \Rightarrow CD(a,b) = CD(b,r)$

'HCF Thm.'

* $a = 117,\ b = 45 \qquad a > b$

$117 = 45 \times 2 + 27 \rightarrow a = bq_1 + r_1,\ 0 \le r_1 < b \quad CD(a,b) = CD(b, r_1)$

$45 = 27 \times 1 + 18 \rightarrow b = r_1 q_2 + r_2,\ 0 \le r_2 < r_1 \quad CD(b, r_1) = CD(r_1, r_2)$

$27 = 18 \times 1 + 9 \rightarrow r_1 = r_2 q_3 + r_3,\ 0 \le r_3 < r_2 \quad CD(r_1, r_2) = CD(r_2, r_3)$

$18 = 9 \times 2 + 0 \rightarrow r_2 = r_3 q_4 + \underset{0}{r_4},\ 0 \le r_4 < r_3 \quad CD(r_2, r_3) = CD(r_3, r_4)$

$$\boxed{CD(a,b) = CD(r_3, r_4)}$$

$CD(a,b) = r_3 = GCD = HCF$

$A = B,\ B = C \Rightarrow A = C$ transitivity

'old-school'

$$\begin{array}{r|l} 45 & 117 \ (2 \\ & 90 \\ \hline 27 & 45 \ (1 \\ & 27 \\ \hline 18 & 27 \ (1 \\ & 18 \\ \hline 9 & 18 \ (2 \\ & 18 \\ \hline & 0 \end{array}$$

HCF = 9

Euclid div algorithm

* $HCF(a,b) = ? \qquad a > b$

1. find (q,r) : $a = bq + r \quad 0 \le r < b$

2. $r = \begin{cases} = 0 \Rightarrow HCF(a,b) = b \\ \neq 0 \Rightarrow \text{apply lemma to } (b,r) \end{cases}$

* $HCF(6, 20) \qquad a > b$

$20 = 6 \times q + r,\ 0 \le r < 6 \quad CD(20,6) = CD(6,2)$

$= 6 \times 3 + 2$

$6 = 2 \times 3 + 0 \qquad 0 \le 0 < 2 \quad CD(6,2) = CD(2,0) = 2 = HCF$

* $HCF(17, 7) = ?$

$17 = 7 \times 2 + 3 \qquad 0 \le 3 < 7 \quad CD(17,7) = CD(7,3)$

$7 = 3 \times 2 + 1 \qquad 0 \le 1 < 3 \quad CD(7,3) = CD(3,1)$

$3 = \underbrace{1}_{HCF} \times 3 + 0 \qquad 0 \le 0 < 1 \quad CD(3,1) = 1 = HCF$

Fundamental thm. of Arithmetic

* $\boxed{p \in \mathbb{Z}^+,\ p \neq 1;\ p \text{ is prime} \Rightarrow 1 \mid p,\ p \mid p}$

* $32760 = 2^3 \cdot 3^2 \cdot 5 \cdot 7 \cdot 13$

unique except the order

$ab = ba,\ a, b \in \mathbb{C}$

(Commutative / Abelian)

$$\begin{array}{r|l} 2 & 32760 \\ \hline 2 & 16380 \\ \hline 2 & 8190 \\ \hline 3 & 4095 \\ \hline 3 & 1365 \\ \hline 5 & 455 \\ \hline 7 & 91 \\ \hline 13 & 13 \\ \hline & 1 \end{array}$$

* $x \in \mathbb{Z}^+$ then $\begin{cases} x \text{ is prime} \Rightarrow 1 \mid x,\ x \mid x \\ x = p_1 \cdot p_2 \cdots p_n,\ p_i \text{ prime} \quad p_1 < p_2 < \cdots < p_n \end{cases}$

$\equiv \underbrace{\prod_{i=1}^{n} p_i}$

representation is unique but order isn't

{Gauss' fundamental th of Arth. Thm. 1

$$\boxed{\frac{100}{0} = x \Rightarrow 100 = x \times 0}$$

∞

$\frac{x}{100} = \frac{1}{0}$

$x \times 0 = 100 \times 1 = 100$

Lecture-20 (5/12/2021)

* $(a,b) \in \mathbb{Z}^+$ $\exists\ (q,r) : a = bq + r,\ 0 \le r < b$

ED lemma

* $(a,b) \in \mathbb{Z}^+ : a = bq + r \Rightarrow CD(a,b) = CD(b,r)$

'HCF Thm.'

* $x \in \mathbb{Z}^+$, $b = 6$ | $a^2 + b^2 + 2ab = (a+b)^2$

$x \sim 6q,\ 6q+1,\ 6q+2,\ 6q+3,\ 6q+4,\ 6q+5$

Case I : $x = 6q$

$x(x+1)(x+2) = 6q(6q+1)(6q+2)$

$\frac{x(x+1)(x+2)}{6} = q(6q+1)(6q+2) \in \mathbb{Z}^+ \Rightarrow 6 \mid x(x+1)(x+2)$

Case II : $x = 6q+1$

$x(x+1)(x+2) = (6q+1)\underbrace{(6q+2)}_{2(3q+1)}\underbrace{(6q+3)}_{3(2q+1)} = 6(6q+1)(3q+1)(2q+1)$

$\frac{x(x+1)(x+2)}{6} = (6q+1)(3q+1)(2q+1) \in \mathbb{Z}^+ \Rightarrow 6 \mid x(x+1)(x+2)$

* $x \in \mathbb{Z}^+$, $b = 3$

$x \sim 3q,\ 3q+1,\ 3q+2$

$x(x+1)(x+2) = 3q(3q+1)(3q+2)$

$\frac{x(x+1)(x+2)}{3} = q(3q+1)(3q+2) \in \mathbb{Z}^+ \Rightarrow 3 \mid x(x+1)(x+2)$

$\Downarrow$

$6 \mid x(x+1)(x+2)$

$x(x+1)(x+2) = 3(3q+1)(3q+2)(q+1) \Rightarrow 3 \mid x(x+1)(x+2)$

* $A, B \in \mathbb{Z}^+_{odd}$

$X = \frac{A+B}{2}$, $Y = \frac{A-B}{2}$

Check (O/E) (O/E)

* $b = 4 \rightarrow A = 4q_1+1,\ 4q_1+3$ odd

$B = 4q_2+1,\ 4q_2+3$ odd

$X = \frac{A+B}{2} = \frac{4(q_1+q_2)+2}{2} = 2(q_1+q_2)+1$

* $x = \frac{A+B}{2} = \frac{4(q_1+q_2)+6}{2} = 2(q_1+q_2)+3$

$Y = \frac{A-B}{2} = \frac{4(q_1-q_2)}{2} = 2(q_1-q_2)$

Fundamental thm. of Arithmetic:

* $x \in \mathbb{Z}^+$ then $\begin{cases} x \text{ is prime} \Leftrightarrow 1 \mid x,\ x \mid x \\ x = p_1 p_2 \cdots p_n \quad p_1 < p_2 \cdots < p_n \end{cases}$

$= \underbrace{\prod_{i=1}^{n} p_i}_{\text{unique but order not imp.}}$

'Gauss thm 1'

* $\boxed{p \text{ prime},\ a \in \mathbb{Z}^+,\ \text{if } p \mid a^2 \Rightarrow p \mid a}$

Check

Thm. 2

* $a = p_1 p_2 \cdots p_n \Rightarrow a^2 = (p_1 \cdot p_2 \cdots p_n)^2 = p_1^2 \cdots p_n^2$

* $p \mid a^2$ $\xrightarrow[\text{Thm 1}]{\text{PF are unique}}$ PF of a^2: p_1, p_2, p_n

(p divides a^2)

p is P.F of a^2

$\downarrow$

p is one of $p_1, p_2 \cdots p_n$

$\Downarrow$

$p \mid p_1 \cdot p_2 \cdots p_n \Rightarrow p \mid a$

* $a = 2 \cdot 3 \cdot 5$

$a^2 = 2^2 \cdot 3^2 \cdot 5^2$ } oldschool example

$\boxed{p = 7} \Rightarrow 7 \nmid a^2 \longrightarrow 7 \nmid a$

$p = 2 \Rightarrow 2 \mid a^2 \longrightarrow 2 \mid a$

$p = 3 \Rightarrow 3 \mid a^2 \longrightarrow 3 \mid a$

Interesting Questions

* T.P. $\exists$ ∞ many +ve primes

* $n \in \mathbb{Z}^+$, $n \neq 1$ TP $n = (\text{non-vetoumed})^2$ (odd #)

* $n \in \mathbb{Z}^+$, T.P n is prime if no prime p less than or equal to $\sqrt{n}$ divides n.

wrongly defined

$2 + 3 = 23$

"Sum"

Lecture 21 (12/12/2021)

$a|b \Rightarrow \exists c : b = ac$

$a|b \Leftrightarrow \frac{b}{a} = c$

* T.P : $\exists$ ∞ many +ve primes

Check

* let $\exists$ finite # of primes $p_1, p_2 \cdots p_n : p_1 < p_2 \cdots < p_n$

* $x \equiv 1 + p_1 p_2 \cdots p_n$

$$\left.\begin{array}{l} p_1 | p_1 p_2 \cdots p_n \\ p_2 | p_1 p_2 \cdots p_n \\ \vdots \\ p_n | p_1 p_2 \cdots p_n \end{array}\right\} \Rightarrow \underbrace{\begin{array}{l} p_1 \nmid x \\ p_2 \nmid x \\ \vdots \\ p_n \nmid x \end{array}}$$

$\Downarrow$

$\left\{\begin{array}{l} x \text{ is a prime / has primes} \\ \text{other than } p_1, \cdots p_n \end{array}\right. \Rightarrow \exists$ ∞ many primes.

$a|b \Rightarrow \frac{b}{a} = c \in \mathbb{Z}$

$a \overset{?}{|} b+1 \Rightarrow \frac{b+1}{a} = \underbrace{\frac{b}{a}}_{c} + \frac{1}{a} \notin \mathbb{Z}$

$a|b \Rightarrow a \nmid b+1$

* $n \in \mathbb{Z}^+$, $n \neq 1$ T.P $n = (\text{non-ve power of } 2)(\text{odd } \#)$

Check

* $n = p_1^{a_1} p_2^{a_2} \cdots p_k^{a_k}$

Fundamental Thm.

$p_1 < p_2 < p_k$

$a_1, a_2 \quad a_k \in \mathbb{Z}^+$

Case I

* $p_1 = 2$, $p_2 \; p_k \in \mathbb{Z}^+_{odd}$

$\downarrow$

$n = \underbrace{2^{a_1}}_{\substack{\text{+ve power of 2} \\ (a_1 > 0)}} \underbrace{(p_2^{a_2} \cdots p_k^{a_k})}_{\text{odd \#}}$

Case II

$p_1, \; p_k \in \mathbb{Z}_{odd}$

$\downarrow$

$n = \underbrace{2^0}_{\substack{\text{non-ve power of 2} \\ (a=0)}} \underbrace{(p_1^{a_1} \cdots p_k^{a_k})}_{\text{odd \#}}$

* $n \in \mathbb{Z}^+$. T.P n is prime, if no prime p less than or equal to $\sqrt{n}$, divide n

Check

* let n not prime : no prime less than or equal to $\sqrt{n}$ divide n 'Assumption'

$n \in \mathbb{Z}^+$

(not a prime)

* $n = ab$, $1 < a \leq b$

$\Downarrow$

$\sqrt{n} = \sqrt{ab} = \sqrt{a}\sqrt{b}$

$n = 375$
$= \underset{a}{15} \times \underset{b}{25}$

$a = 15 = \underset{p}{3} \times 5$

$1 < a < b \Rightarrow a < a^2 \leq \underbrace{ab}_{n} \Rightarrow a < a^2 \leq n \Rightarrow \boxed{a \leq \sqrt{n}}$

$\Downarrow$

$b < \underbrace{ab}_{n} \leq b^2 \Rightarrow n \leq b^2 \Rightarrow \boxed{b \geq \sqrt{n}}$

* p : PF of $a \Rightarrow \boxed{p \leq a \leq \sqrt{n}}$

Prime factor

prime less than or equal $\sqrt{n}$

$3 < 15 < \sqrt{375}$

$5 < 15 < \sqrt{375}$

$3|15$

$5|15$

$p|a \Leftrightarrow \frac{a}{p} = k \in \mathbb{Z}$

$\underbrace{p|ab}_{} \Leftrightarrow \frac{ab}{p} = kb = l \in \mathbb{Z}$

$\Downarrow$

$\boxed{p|n}$ 'p divide n'

"Proof by Contradiction"

Assumption Contradicted $\Rightarrow n$ is a prime (Assump. wrong)

Lecture 22 (17/12/2021)

HCF as linear combination (type)

* HCF(81, 237) → d = 3

$237 = 81 \times 2 + 75$
$81 = 75 \times 1 + 6$
$75 = 6 \times 12 + 3$
$6 = 3 \times 2 + 0$

reverse Engineering.

* $d = xa + yb = ax + by + ab - ab$
$= a(x+b) + (y-a)b$

{ d written as Linear Combination of a & b

* $3 = x \times 237 + y \times 81$ linear comb^n of 81, 237

check

* $237 = 81 \times 2 + 75 \Rightarrow 75 = 237 - 81 \times 2$
$81 = 75 \times 1 + 6 \Rightarrow 6 = 81 - 75 \times 1$
$75 = 6 \times 12 + 3 \Rightarrow 3 = 75 - 6 \times 12$
$6 = 3 \times 2 + 0$

$(81 - 75 \times 1)$

$3 = 75 - (81 - 75 \times 1) \times 12$
$= 75 - 81 \times 12 + 75 \times 12$
$= 13 \times 75 - 81 \times 12$
$= 13 \times (237 - 81 \times 2) - 81 \times 12$
$= 13 \times 237 - 81 \times \underbrace{13 \times 2}_{26} - 81 \times 12$
$3 = 13 \times 237 - 38 \times 81 = a \cdot 237 + b \cdot 81$

$a(b+c) = ab + ac$

* " Coeff of 237" is 13
" 81 is -38

Trick: Successive Elimination of previous Remainder

* T.P. $\nexists\, x \in \mathbb{N} : 4^n$ ends with 0

check

* $x \in \mathbb{Z}^+$, $x = 4^n = (2^2)^n = 2^{2n}$
$2 \mid x$ (2 is only prime)
$5 \nmid x \Rightarrow$ # doesn't end in 0

APP. on HCF/LCM (Fundamental thm)

* HCF(6, 20) = ?, LCM(6, 20) = ?

Euclid

* $20 = 6 \times 3 + 2$
$6 = \underbrace{2}_{\text{HCF}} \times 3 + 0$

Orthodox

$6 \overline{)20}\,(3$
18
$2 \overline{)6}\,(3$
HCF $\quad 6$
$\quad\quad X$

* $20 = 2 \times 2 \times 5 = 2^2 \cdot 5$
$6 = 2 \times 3 = 2^1 \cdot 3$

2	20
2	10
5	5
	1

2	6
3	3
	1

* HCF: Common P.F $\quad$ least Power
2 $\quad$ 1

$\boxed{HCF(2, 20) = 2^1 = 2}$

2	20, 6
2	10, 3
5	5, 3
3	1, 3
	1, 1

* LCM: Prime factor (PF) $\quad$ Highest Power
2 $\quad$ 2
3 $\quad$ 1
5 $\quad$ 1

$\boxed{LCM(2, 20) = 2^2 \cdot 3^1 \cdot 5^1 = 60}$

* $\boxed{HCF(a, b) \times LCM(a, b) = a \times b}$ valid only for #s

Trick

* HCF = Pdt. of smallest Power of each common P.F
LCM = Pdt. of highest power of Prime factor all

* (a, b) = (90, 144) ; HCF(a, b) = ?
LCM(a, b) = ?

* $a = x^3 y^2$, $b = xy^3$
$HCF(a, b) = x^1 \cdot y^2$
$LCM(a, b) = x^3 \cdot y^3 = (xy)^3$ } $= LCM \cdot HCF = x^4 y^5$
$= ab$

* $\boxed{HCF \cdot LCM = a \cdot b}$

* $\frac{x}{0} = y$ ✗
$x^2 + 1 = 0$ ✓

Lecture 23 (18/12/2021)

theory, Algebra
- Identities / Factorisation
- Polynomial
- Linear Eq^n
- Quad. Eq^n (10)

Lag, 2D Geometry (Pre-trig)
- Lines / Angles (7, 9)
- Δ (7, 9, 10) $\cong$, $\sim$
- Poly / Quad / Lim (8, 9)
- Circles (9, 10)

3D geometry surAr / vol. (7, 8, 9, 10)

Stat / Prob (7, 8, 9, 10)

$\boxed{\text{Trigonometry}}$

Pre-Set (10) → Post-Set / Function (11, 12)

Set / Relations / Function (11, 12)

2D Geometry (Post-Trig) Conic Section (11) (11th, 12th Now onwards)

Calculus (11, 12)

Stat / Probability

Irrational #s ($\mathbb{Q}'$)

* $x = \sqrt{2}$

```
                L.4 1
                2.00000000
                1
                100
                 96
                 400
                 281
                 11900
```

double it 1 → 2 4 → double it 2 8 → double it 2 8 2

* $\sqrt{2} = 1.4142135\ldots$
* T.P: $\sqrt{2}$ is I.R (Irrational)

Check

* let $\sqrt{2} \in \mathbb{Q}$ (Assumption)
* $\sqrt{2} = \frac{a}{b}$: HCF$(a, b) = 1$

$2 = \frac{a^2}{b^2} \Rightarrow a^2 = 2b^2 \Rightarrow \frac{a^2}{2} = b^2 \Rightarrow 2 \mid a^2 \xrightarrow{\text{Euclid (Gauss's 2nd Thm)}} 2 \mid a$

$2 \mid a \Rightarrow \frac{a}{2} = k \Rightarrow \underbrace{a^2}_{2b^2} = 4k^2 \Rightarrow b^2 = 2k^2 \Rightarrow \frac{b^2}{2} = k^2$

$2 \mid b \xleftarrow[\text{2nd}]{\text{Gauss'}} 2 \mid b^2$

* $2 \mid a, 2 \mid b \Rightarrow$ HCF$(a, b) \neq 1$

Contradiction to Assumption $\Rightarrow$ Assump wrong

Remark on negation

* Assumption $\xrightarrow[\text{wrong}]{\text{Assumption}}$ $x \notin \mathbb{Q}$

$x \in \mathbb{Q}$

x is not rational

$\downarrow$???

x is irrational

* $\boxed{\text{Negation} \not\Rightarrow \text{Novelty / Creation / Novel}}$

Useful Results

* $x \in \mathbb{Q}' \Rightarrow -x \in \mathbb{Q}'$

Check

* let $-x \in \mathbb{Q}$ (Assumption) $\Rightarrow -(-x) = x \in \mathbb{Q}$ $\xrightarrow{\text{Contradiction}}$ $x \in \mathbb{Q}'$

* $x \in \mathbb{Q}, y \in \mathbb{Q}' \Rightarrow x + y \in \mathbb{Q}'$

Check

* let $x + y \in \mathbb{Q} \Rightarrow \cancel{x} + y - \cancel{x} \in \mathbb{Q} \Rightarrow \boxed{y \in \mathbb{Q}}$ $\xrightarrow{\text{Contradict}}$ $y \in \mathbb{Q}'$

(given) $x \in \mathbb{Q}$

* $x \in \mathbb{Q}, y \in \mathbb{Q}' \Rightarrow xy \in \mathbb{Q}'$
* $x \in \mathbb{Q}', y \in \mathbb{Q}' \Rightarrow$ $\left.\begin{matrix} x \pm y \\ xy \\ \frac{x}{y} \end{matrix}\right\}$ rational / Irrational

$\sqrt{2}, -\sqrt{2} \rightarrow \sqrt{2} - \sqrt{2} = 0 \in \mathbb{Q}$

$\sqrt{18}$ $\sqrt{2} \rightarrow \sqrt{18} \times \sqrt{2} = \sqrt{36} = 6 \in \mathbb{Q}$

$\sqrt{12}$ $\sqrt{3} \rightarrow \frac{\sqrt{12}}{\sqrt{3}} = \sqrt{\frac{12}{3}} = \sqrt{4} = 2 \in \mathbb{Q}$

Lecture 24 (19/12/2021)

* $n \in \mathbb{Q} \Rightarrow \sqrt{n} \in \mathbb{Q}'$ iff n is not a perfect square
* $\sqrt{2}, \sqrt{3}, \sqrt{5} \in \mathbb{Q}'$
* TP : $5-\sqrt{3} \in \mathbb{Q}'$

Check

* let $5-\sqrt{3} = \frac{a}{b} \in \mathbb{Q} \Rightarrow \frac{5}{1} - \frac{a}{b} = \sqrt{3}$, $a, b \in \mathbb{Z}^+$

$\frac{5b-a}{b} = \sqrt{3} \Rightarrow \sqrt{3} \in \mathbb{Q} \xrightarrow{\text{contradiction}} 5-\sqrt{3} \in \mathbb{Q}'$

* TP : $3\sqrt{2} \in \mathbb{Q}'$ HW
* TP : $3+2\sqrt{5} \in \mathbb{Q}'$ HW
* TP : $\sqrt{2}+\sqrt{5} \in \mathbb{Q}'$

Check

* let $\sqrt{2}+\sqrt{5} = \frac{a}{b} \in \mathbb{Q} \Rightarrow \frac{a}{b} - \sqrt{2} = \sqrt{5}$

$\left(\frac{a}{b} - \sqrt{2}\right)^2 = (\sqrt{5})^2 \Rightarrow \frac{a^2}{b^2} + 2 - 2\frac{a}{b}\sqrt{2} = 5$

$\underbrace{\frac{a^2}{b^2} - \frac{3}{1}}_{\frac{a^2-3b^2}{b^2}} = \frac{2a}{b}\sqrt{2} \Rightarrow \frac{a^2-3b^2}{2ab} = \sqrt{2} \in \mathbb{Q}$

contradict

$\sqrt{2}+\sqrt{5} \notin \mathbb{Q}$

Lecture Questions

* TP : $\nexists\, n : \sqrt{n-1} + \sqrt{n+1} \in \mathbb{Q}$, $n \in \mathbb{Z}^+$

Check

* let $n : \sqrt{n-1} + \sqrt{n+1} = \frac{a}{b} \in \mathbb{Q}$ Assumption $a, b \in \mathbb{Z}^+$

Ques :

* $\frac{b}{a} = \frac{1}{(\sqrt{n-1}+\sqrt{n+1})} \times \frac{\sqrt{n-1}-\sqrt{n+1}}{(\sqrt{n-1}-\sqrt{n+1})}$ | $x-y = -(y-x)$

$\frac{b}{a} = \frac{\sqrt{n-1}-\sqrt{n+1}}{(\cancel{n}-1)-(\cancel{n}+1)} = \frac{\sqrt{n-1}-\sqrt{n+1}}{-2}$

* $\frac{2b}{a} = \sqrt{n+1} - \sqrt{n-1}$
* $\frac{a}{b} = \sqrt{n+1} + \sqrt{n-1}$

+ (Add)

* $\frac{2b}{a} + \frac{a}{b} = \sqrt{n+1} + \sqrt{n+1} = 2\sqrt{n+1}$

(−) subtract

* $\frac{2b}{a} - \frac{a}{b} = \sqrt{n+1} - \sqrt{n-1} - (\sqrt{n+1} + \sqrt{n-1}) = -2\sqrt{n-1}$

Linear Eqn

$x+y = a$

$x-y = b$

$x+x+\cancel{y}-\cancel{y} = a+b$

$2x = a+b$

* $2\sqrt{n+1} = \frac{2b}{a} + \frac{a}{b} \Rightarrow \sqrt{n+1} = \frac{2b^2+a^2}{2ab} \in \mathbb{Q}$

$2\sqrt{n-1} = \frac{a}{b} - \frac{2b}{a} \Rightarrow \sqrt{n-1} = \frac{a^2-2b^2}{2ab} \in \mathbb{Q}$

* $\sqrt{n+1} = \left(\frac{l}{m}\right)$, $\sqrt{n-1} = \left(\frac{k}{p}\right)$

$\Downarrow$

$(n+1) = \left(\frac{l}{m}\right)^2$, $(n-1) = \left(\frac{k}{p}\right)^2$

$1^2 = 1$
$2^2 = 4$
$3^2 = 9$
$4^2 = 16$
$5^2 = 25$

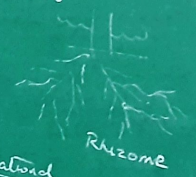
contradiction $\Longrightarrow$ Assumption wrong

* Perfect squares must diff. by odd int 2

* $a, b, c, d \in \mathbb{Q}^+$: $a+\sqrt{b} = c+\sqrt{d}$

TP $\nearrow a = c$, $b = d$

$\searrow$ b & d are squares of rational



* Case I : $a = c \Rightarrow \cancel{a} + \sqrt{b} = \cancel{a} + \sqrt{d} = \sqrt{b} = \sqrt{d} \Rightarrow b = d$
* Case II : $a \neq c$, $a - c = k$
* $\underbrace{a}_{c+k} + \sqrt{b} = c + \sqrt{d} \Rightarrow \cancel{c} + k + \sqrt{b} = \cancel{c} + \sqrt{d} \Rightarrow k + \sqrt{b} = \sqrt{d}$

$k + \sqrt{b} = \sqrt{d} \Rightarrow k^2 + b + 2k\sqrt{b} = d$

$2k\sqrt{b} = d - k^2 - b \Rightarrow \sqrt{b} = \underbrace{\frac{d-k^2-b}{2k}}_{} \in \mathbb{Q}$

$b = \left(\frac{d-k^2-b}{2k}\right)^2 = (\text{rational})^2$

* $\sqrt{d} = \underbrace{k}_{} + \underbrace{\sqrt{b}}_{\text{ration}} \in \mathbb{Q} \Rightarrow d = (\text{rational})^2$

* $a, b \in \mathbb{Z}^+$ TP $\frac{a}{b} < \sqrt{2} < \frac{a+2b}{a+b}$ HW
* $a, b, c, p \in \mathbb{Q}$: p is not perfect cube

$p \neq (k)^3 \Rightarrow (p)^{1/3} \in \mathbb{Q}'$

if $a + bp^{1/3} + cp^{2/3} = 0 \xrightarrow{\text{TP}} a = b = c = 0$

HW

Lecture 25 (25/12/2021)

* $a, b \in \mathbb{Z}^+$ T.P: $\frac{a}{b} < \sqrt{2} < \frac{a+2b}{a+b}$, $\frac{a}{b} > \sqrt{2} > \frac{a+2b}{a+b}$

Check

* $\frac{a}{b} - \frac{a+2b}{a+b} = \frac{a^2+\cancel{ab}-\cancel{ab}+2b^2}{b(a+b)} = \frac{a^2-2b^2}{b(a+b)}$

$3 < 10 < 15$

$15 - 3 = 12 > 10$

Cases

* $\frac{a}{b} - \frac{a+2b}{a+b} = \frac{a^2-2b^2}{b(a+b)} > 0$

$a^2 - 2b^2 > 0$

$\Downarrow$

$a^2 > 2b^2$

$\Downarrow$

$a > b\sqrt{2}$

$\Downarrow$

$\boxed{\frac{a}{b} > \sqrt{2}}$

$\boxed{\frac{a}{b} > \frac{a+2b}{a+b}}$

$\frac{a}{b} - \frac{a+2b}{a+b} = \frac{a^2-2b^2}{b(a+b)} < 0$

$\boxed{\frac{a}{b} < \sqrt{2}}$

Algebra "speeds up" Calculation

* $a^2 > 2b^2$; $\left(\frac{a+2b}{a+b}\right)^2 = \frac{a^2+4b^2+4ab}{a^2+b^2+2ab}$

Trick (completing the square)

$a^2 + \underbrace{a^2+2b^2+4ab}_{} > 2b^2 + \underbrace{a^2+2b^2+4ab}_{}$

$2(a^2+b^2+2ab) > a^2+4b^2+4ab$

$2(a+b)^2 > (a+2b)^2$

$\Downarrow$

$\boxed{\sqrt{2} > \frac{a+2b}{a+b}}$

* $\boxed{\frac{a}{b} > \sqrt{2} > \frac{a+2b}{a+b}}$

$\frac{a}{b} < \sqrt{2} < \frac{a+2b}{a+b}$

* $a, b, c, p \in \mathbb{Q}$; $p \neq (m)^3 \Rightarrow m \neq \sqrt[3]{p}$

Not perfect cube

if $a + bp^{1/3} + cp^{2/3} = 0 \xrightarrow{T.P} a = b = c = 0$

hint / speculation : $x = p^{1/3}$

* $cx^2 + bx + a = 0 \longrightarrow x = \frac{-b \pm \sqrt{b^2-4ac}}{2c}$

* $cx^2 + bx + a = 0 \;\} \times x \Rightarrow cx^3 + bx^2 + ax = 0 \;\} \times c$

$\Downarrow$

$c^2x^3 + bcx^2 + acx = 0$

* $cx^2 + bx + a = 0 \;\} \times b \Rightarrow bcx^2 + b^2x + ab = 0$

* $c^2x^3 + \cancel{bcx^2} + acx = 0$

$-\;\; \cancel{b^2x + bcx^2} + ab = 0$

$c^2x^3 - b^2x + acx - ab = 0$

* $c^2x^3 - ab - (b^2-ac)x = 0 \Rightarrow \underbrace{c^2x^3 - ab}_{=0} = \underbrace{(b^2-ac)}_{\neq 0}x$

$x \in \mathbb{Q}'$

$\Downarrow$

$x = \frac{c^2x^3 - ab}{b^2 - ac} \left(\frac{m}{0}\right)$

VVImp

{ Can't do this if $x \in \mathbb{Q}'$ divide by (b^2-ac) Fallacy }

* $x = p^{1/3} \Rightarrow p = x^3 \longrightarrow \boxed{\underbrace{p^{1/3}}_{x} = \frac{c^2p - ab}{b^2 - ac} \in \mathbb{Q}}$

(Not possible)

* $\because p^{1/3} \in \mathbb{Q}' \Rightarrow \boxed{\begin{matrix} b^2 - ac = 0 \\ c^2x^3 - ab = 0 \end{matrix}}$ $\Rightarrow b^2 = ac$, $\Rightarrow c^2\underbrace{x^3}_{p} = ab$

$c^2p = ab \rightarrow c^4p^2 = \underbrace{a^2b^2}_{ac} = a^3c \Rightarrow c^4p^2 - a^3c = 0$

$\Downarrow$

$(p^2c^3 - a^3)c = 0$

$p^2c^3 - a^3 = 0$

$\Downarrow$

$p^2 = \left(\frac{a}{c}\right)^3$

$\boxed{c = 0}$

$\underbrace{(\underbrace{p^{1/3}}_{x})^2}_{\in \mathbb{Q}} = \frac{a}{c} \Leftarrow (p^2)^{1/3} = \left(\frac{a}{c}\right)$

Not possible

$\therefore x \in \mathbb{Q}'$

* $b^2 = ac = 0 \Rightarrow b = 0$

$a + b\underbrace{p^{1/3}}_{0} + c\underbrace{p^{2/3}}_{0} = 0 \Rightarrow a = 0$

$x \times 0 = 0$

$x = \frac{0}{0}$

$A \cdot B = 0$

$A = 0$

$B = 0$

Either Not both Simultaneously

theory, Algebra
- identities / Factorisation
- Polynomial
- Linear Eqn
- Quad. Eqn (10)

Lag, 2D Geometry (Pre-trig)
- Lines / Angles (7, 9)
- Δ (7, 9, 10) ≅, ~
- Poly / Quad / Hom (8, 9)
- circles (9, 10)

3D geometry surAr / Vol.

Stat / Prob (7, 8, 9, 10)

(7, 8, 9, 10)

Trigonometry

Pre-Set (10) → Post-Set / Function (11, 12)

Set / Relations / Function (11, 12)

2D Geometry (Post-Trig)

Cone section (11)

(11th, 12th Now onwards)

Calculus (11, 12)

Stat / Probability