Lecture-1 (19/8/2022): Trigonometry vol. 2 (11-12)

7/10

* $\dfrac{ax}{\cos\theta} + \dfrac{by}{\sin\theta} = a^2-b^2$

$\dfrac{ax\sin\theta}{\cos^2\theta} - \dfrac{by\cos\theta}{\sin^2\theta} = 0$

$\left.\right\}$ $(ax)^{2/3}+(by)^{2/3} = ?$

$\Downarrow$

$ax\sin^3\theta = by\cos^3\theta \Rightarrow \dfrac{ax}{\cos^3\theta} = \dfrac{by}{\sin^3\theta} \Rightarrow \dfrac{(ax)^{1/3}}{\cos\theta} = \dfrac{(by)^{1/3}}{\sin\theta}$

* $\dfrac{(ax)^{1/3}}{\cos\theta} = \dfrac{(by)^{1/3}}{\sin\theta}$

$\Downarrow$

$\dfrac{(ax)^{2/3}}{\cos^2\theta} = \dfrac{(by)^{2/3}}{\sin^2\theta}$

Classical Th. of Ratio

$\dfrac{a}{b} = \dfrac{c}{d}$

$\Downarrow$

$\dfrac{a}{b} = \dfrac{c}{d} = \dfrac{a+c}{b+d}$

Trick!

$\Downarrow$

$\dfrac{(ax)^{2/3}}{\cos^2\theta} = \dfrac{(by)^{2/3}}{\sin^2\theta} = \dfrac{(ax)^{2/3}+(by)^{2/3}}{\sin^2\theta+\cos^2\theta} = (ax)^{2/3}+(by)^{2/3}$ ✓

* $\cos^2\theta = \dfrac{(ax)^{2/3}}{(ax)^{2/3}+(by)^{2/3}} \Rightarrow \cos\theta = \dfrac{(ax)^{1/3}}{\sqrt{(ax)^{2/3}+(by)^{2/3}}}$ □

* $\sin\theta = \dfrac{(by)^{1/3}}{\sqrt{(ax)^{2/3}+(by)^{2/3}}}$ □

* $\underbrace{\dfrac{ax}{\cos\theta} + \dfrac{by}{\sin\theta}}_{(a^2-b^2)} = \left\{\dfrac{ax}{(ax)^{1/3}} + \dfrac{by}{(by)^{1/3}}\right\}\sqrt{(ax)^{2/3}+(by)^{2/3}}$

$= \left\{(ax)^{2/3}+(by)^{2/3}\right\}\sqrt{(ax)^{2/3}+(by)^{2/3}}$

$= \left[(ax)^{2/3}+(by)^{2/3}\right]^{3/2}$

* $\boxed{(ax)^{2/3}+(by)^{2/3} = (a^2-b^2)^{2/3}}$ □ Hence Proved!

4/10

* $\dfrac{\sin\alpha}{\sin\beta} = m$; $\dfrac{\cos\alpha}{\cos\beta} = n$; $\tan\alpha, \tan\beta = ?$

* $\dfrac{\sin\alpha}{\sin\beta}\cdot\dfrac{\cos\beta}{\cos\alpha} = \dfrac{m}{n} \Rightarrow \boxed{\dfrac{\tan\alpha}{\tan\beta} = \dfrac{m}{n}}$

given

* $\boxed{\sin\alpha = m\sin\beta} \Rightarrow \tan\alpha\cos\alpha = m\tan\beta\cos\beta$

$\dfrac{\tan\alpha}{\sqrt{1+\tan^2\alpha}} = m\dfrac{\tan\beta}{\sqrt{1+\tan^2\beta}}$

$\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}$

T-result

* $\sin\theta = \tan\theta\cos\theta = \dfrac{\tan\theta}{\sec\theta} = \dfrac{\tan\theta}{\sqrt{1+\tan^2\theta}}$

* $\boxed{\dfrac{\tan\alpha}{m} = \dfrac{\tan\beta}{n} \equiv \lambda}$ Ratios Trick!

$\tan\alpha = m\underset{\text{unknown}}{\lambda}$, $\tan\beta = n\overset{\text{unknown}}{\lambda}$

* $\dfrac{\tan\alpha}{\sqrt{1+\tan^2\alpha}} = m\dfrac{\tan\beta}{\sqrt{1+\tan^2\beta}} \Rightarrow \dfrac{\cancel{m}\cancel{\lambda}}{\sqrt{1+m^2\lambda^2}} = \dfrac{\cancel{m}n\cancel{\lambda}}{\sqrt{1+n^2\lambda^2}}$

$n\sqrt{1+m^2\lambda^2} = \sqrt{1+n^2\lambda^2} \Rightarrow n^2(1+m^2\lambda^2) = 1+n^2\lambda^2$

$n^2 + m^2n^2\lambda^2 = 1+n^2\lambda^2 \Rightarrow n^2-1 = (n^2-m^2n^2)\lambda^2$

$\lambda^2 = \dfrac{n^2-1}{n^2(1-m^2)} \Rightarrow \boxed{\lambda = \pm\dfrac{1}{n}\sqrt{\dfrac{n^2-1}{1-m^2}}}$

* $\tan\alpha = \pm\dfrac{m}{n}\sqrt{\dfrac{n^2-1}{1-m^2}}$, $\tan\beta = \pm\sqrt{\dfrac{n^2-1}{1-m^2}}$ □

5/10

given:

* $\dfrac{\sin^4\theta}{a} + \dfrac{\cos^4\theta}{b} = \dfrac{1}{a+b}$

Trick! Ratios

* $\dfrac{\sin^8\theta}{a^3} + \dfrac{\cos^8\theta}{b^3} = ? = \dfrac{(\sin^2\theta)^4}{a^3} + \dfrac{(\cos^2\theta)^4}{b^3}$

* $(a+b)\left(\dfrac{\sin^4\theta}{a} + \dfrac{\cos^4\theta}{b}\right) = 1$

$\underbrace{\cancel{\sin^4\theta} + \cancel{\cos^4\theta}} + \dfrac{a}{b}\cos^4\theta + \dfrac{b}{a}\sin^4\theta = \underset{\sim}{1}$

$(\sin^2\theta+\cos^2\theta)^2$

$\parallel$

$\cancel{\sin^4\theta} + \cancel{\cos^4\theta} + 2\sin^2\theta\cos^2\theta$

$\dfrac{a}{b}\cos^4\theta + \dfrac{b}{a}\sin^4\theta - \dfrac{2\sin^2\theta\cos^2\theta}{1} = 0$

$(a\cos^2\theta - b\sin^2\theta)^2 = 0 \Rightarrow \dfrac{\sin^2\theta}{\cos^2\theta} = \dfrac{a}{b}$

Ratios

$\dfrac{\sin^2\theta}{a} = \dfrac{\cos^2\theta}{b} \Rightarrow \dfrac{\sin^2\theta}{a} = \dfrac{\cos^2\theta}{b} = \dfrac{1}{a+b}$

$\sin^2\theta = \dfrac{a}{a+b}$, $\cos^2\theta = \dfrac{b}{a+b}$

* $\dfrac{\sin^8\theta}{a^3} + \dfrac{\cos^8\theta}{b^3} = \dfrac{1}{a^3}\left(\dfrac{a}{a+b}\right)^4 + \dfrac{1}{b^3}\left(\dfrac{b}{a+b}\right)^4$

$= \dfrac{a+b}{(a+b)^4} = \dfrac{1}{(a+b)^3}$

Lecture – 2 (21/8/2022): Trigonometry vol. 2 (11-12)

The dark side of Trig V-1

* $m^2+n^2+2mn\cos\theta=1$
  $p^2+q^2+2pq\cos\theta=1$
  $mp+nq+(mq+np)\cos\theta=0$

  Prove that $m^2+p^2=\operatorname{cosec}^2\theta$

  $\sin^2\theta(m^2+p^2)=1$

* $m^2+n^2+2mn\cos\theta=1$
  $\Downarrow$
  $(m+n\cos\theta)^2+\underbrace{n^2-n^2\cos^2\theta}_{n^2\sin^2\theta}=1$
  $(m+n\cos\theta)^2=1-n^2\sin^2\theta$

  $p^2+q^2+2pq\cos\theta=1$
  $\Downarrow$
  $(p+q\cos\theta)^2+\underbrace{q^2-q^2\cos^2\theta}_{q^2\sin^2\theta}=1$
  $(p+q\cos\theta)^2=1-q^2\sin^2\theta$

* $\underbrace{(m+n\cos\theta)^2(p+q\cos\theta)^2}=(1-n^2\sin^2\theta)(1-q^2\sin^2\theta)$
  $(mp+(mq+np)\cos\theta+nq\underbrace{\cos^2\theta}_{(1-\sin^2\theta)})^2$
  $(\underbrace{mp+nq+(mq+np)\cos\theta}_{0}-nq\sin^2\theta)^2$

* $\cancel{n^2q^2\sin^4\theta}=1-(n^2+q^2)\sin^2\theta+\cancel{n^2q^2\sin^4\theta}$
  $\boxed{n^2+q^2=\operatorname{cosec}^2\theta}$ □

Correct result (Mod.)

* $(n+m\cos\theta)^2=\ldots$
  $(q+p\cos\theta)^2=\ldots$

* $\frac{\sin^4\phi}{\lambda}+\frac{\cos^4\phi}{\mu}=\frac{1}{\lambda+\mu}$, $\frac{\sin^{4n}\phi}{\lambda^{2n-1}}+\frac{\cos^{4n}\phi}{\mu^{2n-1}}=?$

Test: /10

* $\frac{\sin^4\phi}{\lambda}+\frac{\cos^4\phi}{\mu}=\frac{1}{\lambda+\mu}$

* $\sin^2\phi=?$
  $\cos^2\phi=?$

  $\sin^2\theta+\cos^2\theta=1$

* $\frac{\sin^{4n}\phi}{\lambda^{2n-1}}+\frac{\cos^{4n}\phi}{\mu^{2n-1}}=?$

* $\frac{\mu\sin^4\phi+\lambda\cos^4\phi}{\lambda\mu}=\frac{1}{\lambda+\mu}$
  $\frac{\lambda\mu\sin^4\phi+\mu^2\sin^4\phi+\lambda^2\cos^4\phi+\lambda\mu\cos^4\phi}{\lambda\mu}=1$
  $\cancel{\sin^4\phi}+\cancel{\cos^4\phi}+\frac{\mu}{\lambda}\sin^4\phi+\frac{\lambda}{\mu}\cos^4\phi=\underbrace{(\sin^2\phi+\cos^2\phi)^2}_{\cancel{\sin^4\phi}+\cancel{\cos^4\phi}+2\sin^2\phi\cos^2\phi}$ ‖ Trick
  $\frac{\mu}{\lambda}\sin^4\phi+\frac{\lambda}{\mu}\cos^4\phi-2\sin^2\phi\cos^2\phi=0$
  $\mu^2\sin^4\phi+\lambda^2\cos^4\phi-2\lambda\mu\sin^2\phi\cos^2\phi=0$
  $(\mu\sin^2\phi-\lambda\cos^2\phi)^2=0\Rightarrow\frac{\sin^2\phi}{\lambda}=\frac{\cos^2\phi}{\mu}=\frac{1}{\lambda+\mu}$ $\boxed{\text{Ratio}}$
  $\sin^2\phi=\frac{\lambda}{\lambda+\mu}$ $\cos^2\phi=\frac{\mu}{\lambda+\mu}$
  $\frac{(\sin^2\phi)^{2n}}{\lambda^{2n-1}}+\frac{(\cos^2\phi)^{2n}}{\mu^{2n-1}}=\frac{1}{(\lambda+\mu)^{2n}}\cdot\frac{\lambda^{2n}}{\lambda^{2n-1}}+\frac{1}{(\lambda+\mu)^{2n}}\cdot\frac{\mu^{2n}}{\mu^{2n-1}}$
  $\frac{\lambda+\mu}{(\lambda+\mu)^{2n}}=\frac{1}{(\lambda+\mu)^{2n-1}}$

  $\frac{x^a}{x^b}=x^{a-b}$

HW

* $\frac{\cos^4\alpha}{\cos^2\beta}+\frac{\sin^4\alpha}{\sin^2\beta}=1$
  a. $\sin^4\alpha+\sin^4\beta=?$
  b. $\frac{\cos^4\beta}{\cos^2\alpha}+\frac{\sin^4\beta}{\sin^2\alpha}=?$

Lecture - 3 (22/8/2022): Trigonometry Vol. 2 (11-12)

The dark side of Trig V-1 (Joyous-side)

* $\dfrac{\cos^4\alpha}{\cos^2\beta} + \dfrac{\sin^4\alpha}{\sin^2\beta} = 1$, $\sin^2\theta + \cos^2\theta = 1$

a. $\sin^4\alpha + \sin^4\beta = ?$

* $\underbrace{\cos^4\alpha}_{(1-\sin^2\alpha)^2} \sin^2\beta + \sin^4\alpha \underbrace{\cos^2\beta}_{} = \underbrace{\cos^2\beta}_{}\sin^2\beta$

$(1-\sin^2\beta)\ (1-\sin^2\beta)$

$(1+\sin^4\alpha - 2\sin^2\alpha)\sin^2\beta + \sin^4\alpha - \cancel{\sin^4\alpha\sin^2\beta} = \sin^2\beta - \sin^4\beta$

$\cancel{\sin^2\beta} - 2\sin^2\alpha\sin^2\beta + \sin^4\alpha = \cancel{\sin^2\beta} - \sin^4\beta$

$\sin^4\beta + \sin^4\alpha - 2\sin^2\alpha\sin^2\beta = 0 \Rightarrow (\sin^2\alpha - \sin^2\beta)^2 = 0$

$\Downarrow$

$\boxed{\sin^2\alpha = \sin^2\beta}$ ✓

a. $\sin^4\alpha + \sin^4\beta = (\sin^2\alpha - \sin^2\beta)^2 + 2\sin^2\alpha\sin^2\beta$

$= 0 + 2\sin^2\alpha\sin^2\beta$

$= 2\sin^4\alpha = 2\sin^4\beta$

$\sin^4\alpha = \sin^4\beta$

$(\sin^2\alpha - \sin^2\beta)^2 = \sin^4\alpha + \sin^4\beta - 2\sin^2\alpha\sin^2\beta$

b. $\dfrac{\cos^4\beta}{\cos^2\alpha} + \dfrac{\sin^4\beta}{\sin^2\alpha} = 1$

1.5/10

Doesn't belong here, sorry!

$\sin^2\alpha = \sin^2\beta$

$\Downarrow$

$\cancel{1}\cos^2\alpha = \cancel{1}\cos^2\beta$

$\cos^2\alpha = \cos^2\beta$

* $\sin\alpha + \cos\alpha = \lambda$

$\sin^6\alpha + \cos^6\alpha = ?$

Constraint Eq^n: $\lambda^2 \le 2$

$(\sin^2\alpha)^3 + (\cos^2\alpha)^3$

$(\sin^2\alpha + \cos^2\alpha)^3 - 3\sin^2\alpha\cos^2\alpha(\sin^2\alpha + \cos^2\alpha)$

$1 - 3\sin^2\alpha\cos^2\alpha = 1 - \frac{3}{4}(\lambda^2-1)^2$

$\dfrac{4 - 3(\lambda^2-1)^2}{4}$

$a^3 + b^3 = (a+b)^3 - 3ab(a+b)$

$\underbrace{\sin^2\alpha + \cos^2\alpha}_{1} + 2\sin\alpha\cos\alpha = \lambda^2$

$\Downarrow$

$\sin\alpha\cos\alpha = \dfrac{\lambda^2-1}{2}$

Comment:

* $\lambda^2 \le 2 \rightarrow \underbrace{\sin^6\alpha + \cos^6\alpha}_{X} = \dfrac{4 - 3(\lambda^2-1)^2}{4}$

$\lambda = 1 \rightarrow X = 1$

$\lambda = 2 \rightarrow X = \dfrac{4-27}{4} = \dfrac{-23}{4}$

$\underbrace{\lambda}_{\sin\alpha + \cos\alpha = 3} = 3 \rightarrow X = \dfrac{4-192}{4} = \dfrac{-188}{4} = -17$

* $f(\alpha) = \sin\alpha + \cos\alpha$

Max $(\sin\alpha) = 1$ at $\alpha = \pi/2$

Max $(\cos\alpha) = 1$ at $\alpha = 0$

Max $f(\alpha) = $ Max$(\sin\alpha) + $ Max$(\cos\alpha) = 2 < 3$ ← Not possible

Warmdown.

* $\dfrac{\tan\alpha + \sec\alpha - 1}{\tan\alpha - \sec\alpha + 1} = \dfrac{1+\sin\alpha}{\cos\alpha}$, Prove that!

$\dfrac{\tan\alpha + \sec\alpha - (\sec^2\alpha - \tan^2\alpha)}{\tan\alpha - \sec\alpha + 1}$

$1 + \tan^2\alpha = \sec^2\alpha$

$\sec^2\alpha - \tan^2\alpha = 1$

$\dfrac{(\sec\alpha + \tan\alpha)[1 - \cancel{(\sec\alpha - \tan\alpha)}]}{\cancel{\tan\alpha - \sec\alpha + 1}} = \dfrac{1+\sin\alpha}{\cos\alpha}$

Lecture-4 (28/8/2022): Trigonometry vol. 2 (11-12)

1. Transcendence & Reduction

A. Metric Space → Euclidean space

* $x^a = \{x^1, x^2, \ldots x^n\}$ Basis, $x^a \in \mathbb{R}$, $a \in \mathbb{Z}^+$
* $s^2 = g_{ab} x^a x^b$ ↑ Metric ; $g_{ab} = \delta_{ab} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ Kronecker delta | $\Delta s^2 = g_{ab}\Delta x^a \Delta x^b$

Euclidean Space

* $\mathbb{R}^2 \equiv \mathbb{R}\times\mathbb{R} = \{(x,y) : \overbrace{\exists\, s^2 = x^2+y^2}^{\text{Rule}}, x\in\mathbb{R}, y\in\mathbb{R}\}$
* $s^2 = \delta_{ab}x^a x^b = (x)^2 + (y)^2$
* $s = \sqrt{x^2+y^2}$ Pythagoras thm.
* $s = s(x,y)$ "distance"/ "st line" is a machine with 2 i/p & 1 o/p

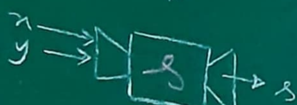
$x = (x_2, x_1)$

$y = (y_2, y_1)$

Properties/Axioms

* $s : \mathbb{R}\times\mathbb{R} \longrightarrow \mathbb{R}$

↑ function/Map (Rule) that takes an OP & spits out a #



* $s = \sqrt{\underbrace{(x_2-x_1)^2}_{\Delta x} + \underbrace{(y_2-y_1)^2}_{\Delta y}} \geq 0$ · Positivity condition

* $s(x,y) = s(y,x)$ Symmetry Condition
* $s(x,y) + s(y,z) \geq s(x,z)$ "Triangle Inequality"

* for a Metric Space: (g_{ab}, s)

Formal def

* A, $s : A\times A \longrightarrow \mathbb{R}$ is distance fun^/Metric if
* $s(x,y) \geq 0$

$s(x,x) = 0$

$s(x,y) = s(y,x)$

$\boxed{s(x,z) \leq s(x,y) + s(y,z)}$

* (A, s) Metric Space
* $A = \mathbb{R}$; $s(x,y) = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

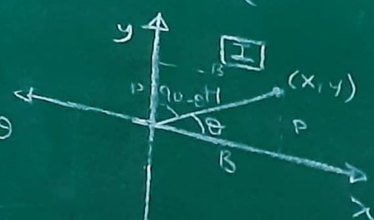
$(\mathbb{R}^2, s)$ Euclidean 2-space

B. Going Beyond

* I : (+x, +y)
* $\sin(90-\theta) = \frac{B}{H} = \cos\theta$

$\tan(90-\theta) = \cot\theta$

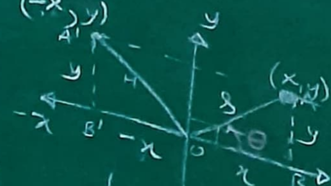
$\mathrm{Cosec}(90-\theta) = \sec\theta$



B.1 2nd quadrant T-Machines

* $\sin(90+\theta) = ?$

HW



$\sin\theta = \frac{y}{s}$

$\cos\theta = \frac{x}{s}$

Known

$\alpha = 90-\theta \Leftarrow \alpha + 90 + \theta = 180°$

Lecture - 5 (3/9/2022)

1. Transcendence & Reduction
2. Going Beyond / 2^{nd} Most fundamental Identity

Comment on Need to study Trigo. (Feeling)

* $\sin\theta \Rightarrow$ trigonometry

only function to understand fully

* $\exists$ difference of degree b/w $\sin\theta$ & $\cos\theta$

$\sin(90-\theta) = \cos\theta$

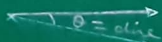
$\tan\theta = \frac{\sin\theta}{\cos\theta}$

* Any 2D shape $= \sum$ Most atomic 2D obj = Triangle = "Atom" of 2D geome (Qualitatively)

$\in$ Flat geometry

Matter $= \sum$ Atoms

*



Slightest deviation $\Rightarrow$ Emergence of Triangle

$\Delta\theta = \theta_2 - \theta_1$

* Plane 2D shape $= \sum$ Triangle

$f(x) = \sum \sin + \sum \cos$

decomposition into sin & Cosines (1760s)

Fourier Transformation / decomposition

$C = A + B$

dome

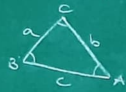
$\exists$ level of Precision

Representations

Geometrical

Algebraic

* $\Delta = \Delta$(sides, Angles)



visual manifestation of a Δ

Trigo map : $\theta \to$ side

Inter-relationship b/w sides & angle [Beneath surface]

* Surface information

* $\exists$ threshold / Resolving Power (Optical)

$\Downarrow$

No "accurate" measurement

* Macro

* Can't go "infinitesimal"

Trig = that which holds it together / like a glue

Micro.

∞ precision to Analysis

Comment on use of Fourier decomp

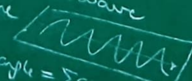
* Types of Motion

Rectilinear

Circular

Oscillatory

EM wave

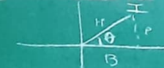


Plane = $\sum$ triangle $= \sum \sin + \sum \cos$

B.2 Problematic $\to$ 2^{nd} fundamental Id

* $\sin(90+\theta) = ?$

Problematic



* $\theta > 90° \Rightarrow \nexists$ Right Angle Δ

Transcendence

$\Downarrow$

No Trigo.

$\Downarrow$

Need to "Reduce" to 1^{st} Quadrant. (Symmetry Exploitation) $\to$ Periodicity

$\sin\theta = \frac{P}{H}$

$0 \le \theta \le 90$

More Algebraic, less geometry

* $\sin(A+B) = \sin(AOC)$

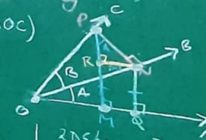
$= \frac{PM}{OP} = \frac{PR + RM}{OP}$

$= \frac{PR}{OP} + \frac{RM}{OP}$, $RM = NQ$

$= \frac{PN\cos A}{OP} + \frac{ON\sin A}{OP}$

$= \sin B\cos A + \cos B\sin A$

2^{nd} fund. Ident.



2D shape $= \sum(\Delta)$

1) Create a RAΔ ΔPMO
2) Create Δ PNO
3) " Δ NQO
4) " Δ NRP

Remark.

$\exists$ fixed relationships b/w Triangles

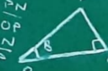
Connected up at the nodes, even if A or B ≥ 90

$\Downarrow$

Trigo. still holds

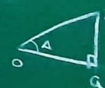
$\sin B = \frac{PN}{OP}$

$\cos B = \frac{ON}{OP}$



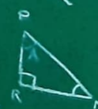
$\sin A = \frac{NQ}{ON}$

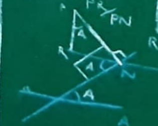
$\cos A = \frac{OQ}{ON}$



$\sin A = \frac{RN}{PN}$

$\cos A = \frac{PR}{PN}$



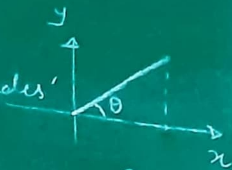


Lecture - 6 (4/9/2022)

1. Transcendence & Reduction $(\theta < 90^\circ)$ $\theta > 90$

* 2D shape = $\sum$ triangles 'Archimedes'

* $f(x) = \sum \sin + \sum \cos$



Comment on Silly circularity   fourier decomp.   Ended

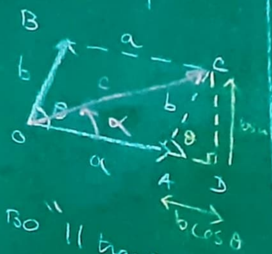
* ΔADC, use "Pythagoras"

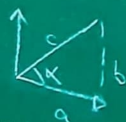
$c^2 = (a + b\cos\theta)^2 + b^2\sin^2\theta$

$c = \sqrt{a^2 + b^2 + 2ab\cos\theta}$

$\downarrow \theta = 90^\circ$

$\boxed{c^2 = a^2 + b^2}$ "derived" Pythagoras





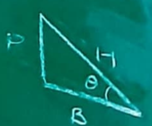
BO "|| transported" to AC

OA || transported to BC

* Postulate

$\sin\theta \equiv \frac{P}{H}$, $\cos\theta \equiv \frac{B}{H}$

$\sin^2\theta + \cos^2\theta = \frac{P^2 + B^2}{H^2} = \frac{H^2}{H^2} = 1$



θ Most funda || cline $\longrightarrow$ Pythagoras Space

(Running research)

Postulate

$g_{ab} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\downarrow$

$H^2 = P^2 + B^2$

$r^2 = x^2 + y^2$

* Most fundamental

$\boxed{\sin(A+B) = \sin A\cos B + \cos A\sin B}$

$\cos(A-B) = \sin(90 - (A-B))$

$= \sin((90 - A) + B)$

$= \sin(90-A)\cos B + \cos(90-A)\sin B$

$= \cos A\cos B + \sin A\sin B$

* $\boxed{\cos(A-B) = \cos A\cos B + \sin A\sin B}$

* $\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(90+A) = \sin A\cancel{\cos 90} + \cos A\sin 90 = \cos A$

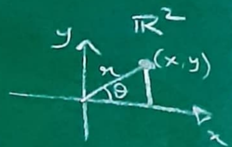
$\boxed{\sin(90+\theta) = \cos\theta} \xrightarrow{\theta \to -\theta} \underbrace{\sin(90-\theta) = \cos(-\theta)}_{\cos\theta}$

* $\boxed{\cos(-\theta) = \cos\theta}$

* $\sin(-\theta) = ?$

Lecture - 7 (10/9/2022)

* $\mathbb{R}^2 = \mathbb{R}\times\mathbb{R} = \{(x,y) : x\in\mathbb{R}, y\in\mathbb{R}\}$



$f(x)$

2D shape $\longrightarrow \sum \triangle \longrightarrow \sum \sin\theta + \sum \cos\theta$

Atom

fourier series / decom.

* $\sin\theta = \frac{P}{H}$, $0 \le \theta \le 90$

* $\sin(A+B) = \sin A\cos B + \cos A \sin B$

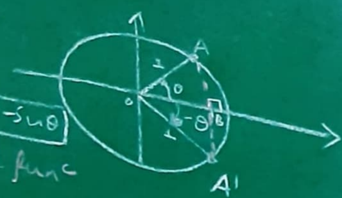
$\cos(A-B) = \cos A\cos B + \sin A\sin B$

* $\cos(-\theta) = \cos\theta$ Even fun^n (later)

* $\sin\theta = \frac{|AB|}{1}$

$\sin(-\theta) = \frac{|A'B|}{1}$

$\Rightarrow \boxed{\sin(-\theta) = -\sin\theta}$ odd func



$|AB| = -|A'B|$

* $\sin(A-B) = \sin A \underbrace{\cos(-B)}_{\cos B} + \cos A \underbrace{\sin(-B)}_{-\sin B}$

$= \sin A\cos B - \cos A\sin B$

* $\cos(A+B) = \cos A\cos(-B) + \sin A \underbrace{\sin(-B)}_{-\sin B}$

$= \cos A\cos B - \sin A\sin B$

$$\boxed{\begin{aligned}\sin(A\pm B) &= \sin A\cos B \pm \cos A\sin B\\ \cos(A\pm B) &= \cos A\cos B \mp \sin A\sin B\end{aligned}}$$

B.2 Symmetry exploitation / Periodicity

* $\sin(90-\theta) = \cos\theta \rightarrow \operatorname{cosec}(90-\theta) = \sec\theta$ $\quad 0 \le \theta \le 90°$

$\cos(90-\theta) = \sin\theta$

$\tan(90-\theta) = \cot\theta$

II

* $\sin(90+\theta) = \cos\theta \rightarrow \operatorname{cosec}(90+\theta) = \sec\theta$

$\cos(90+\theta) = -\sin\theta$

$\tan(90+\theta) = -\cot\theta$

* $\sin(180-\theta) = \sin(\widetilde{90}+\widetilde{90-\theta}) = \sin\theta$

$\cos(180-\theta) = -\cos\theta$

$\tan(180-\theta) = -\tan\theta$

III

* $\sin(180+\theta) = -\sin\theta$

$\cos(180+\theta) = -\cos\theta$

$\tan(180+\theta) = \tan\theta$

* $\sin(270-\theta) = -\cos\theta$

$\cos(270-\theta) = -\sin\theta$

$\tan(270-\theta) = \cot\theta$

IV

* $\sin(270+\theta) = -\cos\theta$

$\cos(270+\theta) = \sin\theta$

$\tan(270+\theta) = -\cot\theta$

* $\sin(360-\theta) = -\sin\theta$

$\cos(360-\theta) = \cos\theta$

$\tan(360-\theta) = -\tan\theta$

I

* $\sin(360+\theta) = \sin\theta$

$\cos(360+\theta) = \cos\theta$

$\tan(360+\theta) = \tan\theta$

HW

* $\cos(-480)$
* $\sin(-1125)$
* $\tan\left(\frac{19\pi}{3}\right)$
* $\sin\left(-\frac{11\pi}{3}\right)$
* $\cot\left(-\frac{15\pi}{4}\right)$
* $\cot(570)$
* $\cos(510)\cos(330) + \sin(390)\cos(120)$
* $\cos\left(\frac{3\pi}{2}+\alpha\right)\cos(2\pi+\alpha)\left\{\cot\left(\frac{3\pi}{2}-x\right) + \cot(2\pi+x)\right\}$

$\sin 0 = 0$

$\cos 0 = 1$

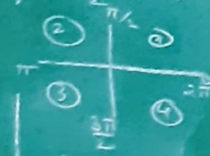
$\sin\frac{\pi}{2} = 1$

$\cos\frac{\pi}{2} = 0$

$\sin\pi = 0$

$\cos\pi = -1$

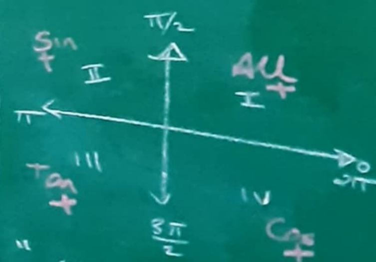
$\sin\frac{3\pi}{2} = -1$



$\cos\frac{3\pi}{2} = 0$

$\sin 2\pi = 0$

$\cos 2\pi = 1$



" All S T C "

School to college

Lecture - 8 (11/9/2022)

* $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ 'funda. Id.'
* $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

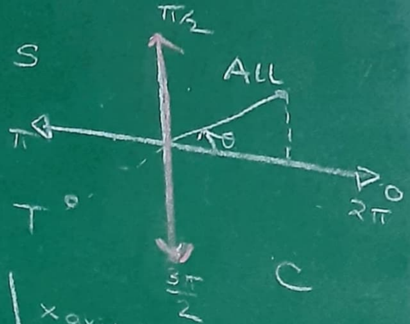
Trick to Remember:

* $\sin\left(\frac{\pi}{2} + \theta\right) = \cos\theta$

$\cos\left(\frac{3\pi}{2} + \theta\right) = \sin\theta$

$\sin(\pi - \theta) = \sin\theta$

$\cos(2\pi - \theta) = \cos\theta$



x axis : No change
y axis : change func

* $$\sin\left(n\frac{\pi}{2} + \theta\right) = \begin{cases} \cos\theta & n=1 \\ -\sin\theta & n=2 \\ -\cos\theta & n=3 \\ \sin\theta & n=4 \end{cases}$$

Ex. (HW)

* $$\cos\left(\frac{3\pi}{2} + \alpha\right)\cos(2\pi + \alpha)\left\{\cot\left(\frac{3\pi}{2} - \alpha\right) + \cot(2\pi + \alpha)\right\} = 1$$

8.3 Ratio table for $0 \le \theta \le 2\pi$ (Extended)

	0	30 $\frac{\pi}{6}$	45 $\frac{\pi}{4}$	60 $\frac{\pi}{3}$	90 $\frac{\pi}{2}$	120 $\frac{2\pi}{3}$	135 $\frac{3\pi}{4}$	150 $\frac{5\pi}{6}$	180 π
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0

	180 π	210 $\frac{7\pi}{6}$	225 $\frac{5\pi}{4}$	240 $\frac{4\pi}{3}$	270 $\frac{3\pi}{2}$	300 $\frac{5\pi}{3}$	315 $\frac{7\pi}{4}$	330 $\frac{11\pi}{6}$	360 2π
sin	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0
cos	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0

Lecture - 9 (17/9/2022)

B.4 Derivatives from fundamental (Primary)

* $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
  $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ I.

* $\tan(A+B) = \dfrac{\sin(A+B)}{\cos(A+B)} = \dfrac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}$ II.
  $= f(\tan A, \tan B)$

  $= \dfrac{1}{\cos A \cos B}\left(\dfrac{\sin A \cos B + \cos A \sin B}{1 - \dfrac{\sin A \sin B}{\cos A \cos B}}\right)$

* $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$ III.

* $\cot(A \pm B) = f(\cot A, \cot B) = \dfrac{\cot A \cot B \mp 1}{\cot A \pm \cot B}$

$\sin(-\theta) = -\sin\theta$
$\cos(-\theta) = \cos\theta$

$\dfrac{a+b}{c} = \dfrac{a}{c} + \dfrac{b}{c}$

$\sin^2\theta + \cos^2\theta = 1$

Multiples of θ:

* $\sin 2\theta = \sin(\theta+\theta) = 2\sin\theta\cos\theta$
  $= f(\tan\theta)$
  $= 2\dfrac{\sin\theta}{\cos\theta}\cos^2\theta$
  $= 2\tan\theta\,(\underbrace{\cos^2\theta}_{\frac{1}{\sec^2\theta} = \frac{1}{1+\tan^2\theta}})$

* $\sin 2\theta = 2\sin\theta\cos\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$ IV.

* $\cos 2\theta = \cos^2\theta - \sin^2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$ V.
  $= f(\tan\theta)$
  $= 1 - 2\sin^2\theta$
  $= 2\cos^2\theta - 1$

  $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

  $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$ $\tan^2\theta = \dfrac{1-\cos 2\theta}{1+\cos 2\theta}$

* $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$, $\cot 2\theta = \dfrac{\cot^2\theta - 1}{2\cot\theta}$

Example

* $\sin(A+B+C) = \sin A \cos B \cos C - \sin A \sin B \sin C + \cos A \sin B \cos C + \cos A \cos B \sin C$

* $\cos(A+B+C) = \cos A \cos B \cos C - \cos A \sin B \sin C - \sin A \cos B \sin C - \sin A \sin B \cos C$

Practice 1

* $\sin 75 = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$
* $\sin 15^\circ = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$
* $\tan 15 = \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

* $\sin(n+1)A \cdot \sin(n+2)A + \cos(n+1)A\cos(n+2)A$
  $\cos[(\cancel{n}+1)A - (\cancel{n}+2)A] = \cos A$