• Chi-squared distribution

We compute quantity $Q = \sum_{i=1}^{N} z_i^2$
where $z_i = (x_i - \underline{\mu^0}) / \sigma_i$

Because μ is replaced by μ^0, the number of degrees of freedom becomes k=N-1

Q is still distributed as the chi-squared, now with k=N-1 deg. of freedom. For large k (say, >10 or so), the chi-squared distribution approximately morphs into good old Gaussian distribution $N(k, \sqrt{2k})$.

In practice, the "chi-squared per degree of freedom" is often used:

$$\chi^2_{dof} = \frac{1}{N-1} \sum_{i=1}^{N} \left(\frac{x_i - \overline{x}}{\sigma_i} \right)^2$$

Each point deviates "on average" by one "error bar"

We expect $\chi^2{}_{\mathrm{dof}}$ to be 1 to within a few $\sqrt{2/(N-1)}$

Note that $\chi^2{}_{\mathrm{dof}}$ is essentially an estimate of the standard deviation squared for the quantity z_i