Crafting S.L. Market Expansion

Maximization of Profits

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Mathematics - Math 132A

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Contents

1 Introduction

This report is to investigate how Crafting S.L can maximize their weekly profit from sales to other states and not sell in California anymore. According to the marketing office of the company, the company could sell all that they produce at 30 dollars per meter of cloth, 9 dollars per skein of yarn, and 25 dollars per pound of crafting materials.

For Crafting S.L, Producing 1 meter of cloth needs 1 skein of yarn, 0.1 pounds of crafting materials, 200 yards of sewing thread, a cost about 4 dollars for other necessary materials and expenses, and 30 minutes of labor. The maximum capacity of cloth weekly production is 5,500 meters for the Crafting S.L.

For Crafting S.L, Producing 1 skein of yarn needs 0.05 pounds of crafting materials, 100 yards of sewing thread, a cost about 2.5 dollars for other necessary machinery, and 5 minutes of labor. The maximum capacity of yarn weekly production is 3,000 skeins for the Crafting S.L.

For Crafting S.L, Producing 1 pound of crafting material needs 0.75 skeins of yarn, 0.12 meters of cloth, a cost about 1.5 dollars for other necessary machinery, and 50 minutes of labor. The maximum capacity of crafting material weekly production is 500 pounds for the Crafting S.L.

The Crafting S.L has 2000 hours available each week. To start the production at the beginning of the week, the company can buy up to 1000 pounds of crafting materials and the company will lose 2 dollars per pound in this transaction due to the payment of interest to the bank. The company can also buy sewing thread at the price of 0.5 dollars per 100 yards.

2 Linear Model Justification

2.1 Linear Model In Lingo

x_1 is the meters of cloth produced by the company

x_2 is the skeins of yarn produced by the company

x_3 is the pounds of crafting materials produced by the company

y_1 is the meters of cloth sold by the company

y_2 is the skeins of cloth sold by the company

y_3 is the pounds of cloth sold by the company

c_3 is the pounds of crafting materials the company bought to start the production

Max

$$30y_1 + 9y_2 + 25y_3 - 5x_1 - 3x_2 - 1.5x_3 - 2c_3$$

Subject to

Labor Constraint:

$$30x_1 + 5x_2 + 50x_3 \leq 12000$$

Cloth Limit:

$$y_1 + 0.12x_3 - x_1 = 0$$

Yarn Limit:

$$y_2 + x_1 + 0.75x_3 - x_2 = 0$$

Crafting Materials:

$$y_3 + 0.1x_1 + 0.05x_2 - x_3 - c_3 = 0$$

Cloth Maximum Limit:

$$x_1 \leq 5500$$

Yarn Maximum Limit:

$$x_2 \leq 3000$$

Crafting Material Maximum Limit:

$$x_3 \leq 500$$

Buying Compounds Limit:

$$c_3 \leq 1000$$

Crafting Material Sale Limit:

$$y_3 - x_3 \leq 0$$

2.2 Justification of the Model

The objective function $30y_1 + 9y_2 + 25y_3 - 5x_1 - 3x_2 - 1.5x_3 - 2c_3$ is designed for calculating the weekly profit of the Crafting S.L. The coefficient of "y_1" is 30 because the Crafting S.L can sell a meter of cloth at 30 dollars. The coefficient of "y_2" is 9 because the Crafting S.L can sell a skein of yarn at 9 dollars. The coefficient of "y_3" is 25 because the Crafting S.L can sell a pound of crafting materials at 25 dollars. The coefficient of "x_1" is -5 because it costs the Crafting S.L 5 dollars to produce one meter of cloth other than the use of crafting material ($2 \cdot 0.5 + 4 = 5$, and "$2 \cdot 0.5$" is that producing a meter of cloth needs 200 yards of sewing thread which the company can buy at 0.5 dollars per 100 yards, 4 is the other necessary materials needed to produce a meter of cloth). The coefficient of "x_2" is -3 because it costs the Crafting S.L 3 dollars to produce one meter of cloth other than the use of crafting material.($1 \cdot 0.5 + 2.5 = 3$, and "$1 \cdot 0.5$" is that producing a meter of cloth needs 100 yards of sewing thread which the company can buy at 0.5 dollars per 100 yards, 2.5 is the cost of other necessary materials needed to produce a skein of yarn). The coefficient of "x_3" is -1.5 because it costs the Crafting S.L 1.5 dollars to produce one pound of cloth other than the use of yarn and cloth. The coefficient of "c_3" is -2 because the transaction of each pound of simple crafting material the company used to start the production at the start of the week will lead to a loss of 2 dollars.

The "Labor Constraint" is $30x_1 + 5x_2 + 50x_3 <= 120000$. The Labor constraint is set that way because the company has 2000 available labor hours a week (2000 hours equals to 120000 minutes) and the corresponding time cost for making one unit of cloth, yarn, and crafting material are 30 minutes , 5 minutes , and 50 minutes correspondingly.

The "Cloth Limit" is "$y_1 + 0.12x_3 - x_1 = 0$". It is set like that because the company has to use up all the cloth it has produced, x_1 (the production of cloth is represented by x_1) to produce crafting material which cost $0.12x_3$ in total (0.12 meters of cloth has to be costed for producing

1 pound of crafting material) and directly sale y_1.

The "Yarn limit" is "$y_2 + x_1 + 0.75x_3 - x_2 = 0$" It is set like that because the company has to use up all the yarn it has produced, x_2 (the production of yarn is represented by x_2) to produce cloth and crafting material which cost $x_1 + 0.75x_3$ in total (1 skeins of yarn has to be consumed to produce 1 meter of cloth and 0.75 skeins of yarn has to be costed for producing 1 pound of crafting material) and directly sale y_2.

The "Crafting Material" is "$y_3 + 0.1x_1 + 0.05x_2 - x_3 - c_3 = 0$" . It is set like that because the company has to use up all the crafting materials it has produced or bought at the start of the week to start production, $x_3 + c_3$, (the production of crafting material is represented by x_3 and the pounds of simple crafting materials the company has bought at the start of the week is represented as c_3) to producing cloth and yarn which cost $0.1x_1 + 0.05x_2$ in total (0.1 pound of crafting material has to be consumed to produce 1 meter of cloth and 0.05 pound of crafting material has to be costed for producing 1 skein of yarn) and directly sale y_3.

The "Cloth Maximum Limit" is "$x_1 \leq 5500$". It is set like that because The maximum amount of cloth that the company can produce is 5500 meter per week.

The "Yarn Maximum Limit" is "$x_2 \leq 3000$". It is set like that because the maximum amount of yarn that the company can produce is 3000 skeins per week.

The "Crafting Material Maximum Limit" is "$x_3 \leq 500$". It is set like that because the maximum amount of crafting materials that the company can produce is 500 pounds per week.

The "Buying Compounds Limit" is "$c_3 \leq 1000$". It is set like that because the maximum amount of crafting materials that the company can buy to start the production is 1000 pounds per week.

The "Crafting Material Sale Limit" is "$y_3 - x_3 \leq 0$". It is set like that because since there is a transaction loss of 2 dollars in each pound of crafting material the company buy to start the production, the company should use all the bought crafting materials into producing other products and sell only crafting materials produced by itself(i.e y_3 should not surpass x_3)

Using Linear Programming for the model is appropriate for this problem since this problem meets all four assumptions of linear programming.

First, the cost of production or buying goods, revenue of selling products, and the assumption of each kind of resources to produce other products remain constant. Therefore, this problem meets the "Proportionality" assumption of linear programming.

Second, the combined effect of the decision variables in any constraint or in the objective function is the algebraic sum of their individual weighted effects. Therefore, this problem meets the "Additivity" assumption of linear programming.

Third, the number of money and the unit of products or resources can all take fractional value. Therefore, this problem meets the "Divisibility or Continuity" assumption.

Finally, all model parameters are known deterministically. Therefore, this problem meets the "Certainty" assumption.

Since this problem meets all four assumptions of linear programming, using Linear Programming for the model is appropriate for this problem.

3 Solutions and Discussion of Different Scenarios

3.1 Production and Sale for Max Profit

In order to maximize the profit, Crafting S.L. should produce 2625 meters of cloth, 3000 skeins of yarn, and 500 pounds crafting materials (The company should also buy 412.5 pounds of crafting materials at the start of the week). After consuming some of those products during the production (60 meters of cloth and all the yarns produced are consumed during the production), they should sell 2565 meters of cloth and 500 pounds of crafting materials. The optimal weekly profit will then be 65750 dollars.

3.2 Change of Profit by Increasing Price of Yarn

If the revenue for the yarn increases by 1 dollars, the weekly profit will not change. The increase in revenue for each skein of yarn by 1 dollars is within the allowable increase (15.8 dollars) and that means that the optimal mix does not change and the company should still sell no yarn.

3.3 The Allowable Increase of Yarn for Profits

The revenue for yarn sale price should at least increase to 24.8 dollars per skein ($15.8+9 = 24.8$) in order to be profitable (When the price of yarn rise to more than 24.8 dollars per skein, i.e increase over 15.8 dollars per skein, the optimal mix changes and the sale of yarn in the new optimal solution will transform from 0 to a positive number). Just selling good quality yarn at the price of 10 dollars per skein is not realistic.

3.4 Weekly Profit for Selling Yarn to a Good Client

If the company must sell 100 skeins of yarn with the price of 11.5 dollars per skein, the weekly profit will become 64420 dollars. Since the reduced cost coefficient for the sale of yarn (Y_2) is 15.8 dollars per skein, selling 100 skeins of yarn in price 9 will lead to a decrease in profit of 1580 dollars ($100 \cdot 15.8 = 1580$). Rising the sale price of yarn to 11.5 will make up the lose by 250 dollars ($11.5 - 9 = 2.5, 2.5 * 100 = 250$). Therefore, the new weekly profit for the Crafting S.L if 100 skeins of yarn must be sold with price 11.5 will be 64420 dollars. ($65750 - 1580 + 250 = 64420$).

3.5 Introduction of production-worthy products

The cloth should be used as the sparing resources if a new product is introduced since the resource cloth has the largest dual price which is 30 dollars. (The corresponding dual price for other three resources labor, yarn, and crafting materials are 0 dollars per hour, 24.8 dollars per skein, and 2 dollars per pound)That means that using more units of cloth to produce one unit of new product will make the product become less profitable compared to increasing the use of units of other three resources.

3.6 Satisfied Price of Cloth Bag

Since the dual prices for labor time, clothes, and crafting materials are 0 dollars per hour, 30 dollars per meter, and 2 dollars per pound correspondingly, the profitable price for this new bag which requires 15 minutes (0.25 hour), 0.3 meters cloth, and 0.01 pounds of crafting materials will be

$$0.25 * 0 + 0.3 * 30 + 0.01 * 2 = 9.02$$

So these bags must be sold for at least 9.25 dollars per bag to be profitable.

3.7 The Best Investment Choice

Investing this extra 5000 dollars in expanding weekly production of yarns by 30 skeins will be the best option compared to expanding the weekly production of cloth by 50 meters or expanding the weekly production of crafting materials by 40 pounds. The increase of 50 meters of cloth, 30 skeins of yarn, and 40 pounds of crafting materials are all within the allowable increase.(The corresponding allowable increase for cloth, yarn, and crafting materials are infinity, 35.71429, and 45.45455). In this case, the optimal mix does not change after the expansion and the resource production capacity for yarn has the highest dual price 21.7 dollars per unit change which will lead to an increase of the weekly profit by 651 dollars ($50 \cdot 0 = 0$). That is higher than the 0 or 52 dollars ($40 \cdot 1.3 = 52$) brought by expanding weekly production for resource cloth or resource crafting materials.

3.8 The production of Cloth when Cost Increase and Profit Change

If the thread used for producing cloth increased by 0.4 dollars per 100 units (That means that the cost of producing each meter of cloth increased by 0.8 dollars), the company should still keep to the current plan and produce the same amount of cloth. (The optimal mix does not change since the allowable decrease for the cost of producing one meter of cloth is 13.5 dollars) The profit will decrease by $2625 \cdot 0.8 = 2100$ dollars and the weekly profit in this scenario will be 63650 dollars (The original maximum weekly profit is 65750 dollars. It minus 2100 dollars leads to the optimal weekly profit in this scenario, 63650 dollars.)

3.9 The Impact on Weekly Profit by Renting Warehouse

Having the warehouse capable of storing 500 skeins of yarns will decrease the right hand-side of the yarn-limit by 500. That is because in this case the company not only needs to use yarns to produce other two products or directly sell those yarns, but also the company should put 500 skeins of yarn they produced into the warehouse. A decrease of 500 skeins of yarn is within the allowable decrease for yarn's sale and production constraint named the "Yarn Limit" (The allowable decrease is 2565 skeins of yarn in total for "Yarn Limit " constraint). In this scenario, that week's profit will be 53050 dollars ($65750 - 24.8 \cdot 500 - 300 = 53350 - 300 = 53050$, 300 dollars for renting the warehouse, 24.8 is the dual price for yarn's sale and production constraint named "Yarn Limit".)

3.10 Consideration of Existing Inventories and Desired Inventories

It is possible to consider the existing inventories at the start of the week and the desired inventories at the end of the week by modifying the LP model.

Anytime an existing inventory appears at the start of the week, the right-hand-side of the corresponding resource constraint should increase by the same amount of value/units as the inventory's. For example, if there is an existing inventory of 500 skeins of yarn, the constraint named as "yarn limit" will transform from "$y_2 + x_1 + 0.75x_3 - x_2 = 0$" to "$y_2 + x_1 + 0.75x_3 - x_2 = 500$"

Anytime a desired inventory appears at the end of the week, the right-hand-side of the corresponding resource constraint should decrease by the same amount of value/units as the inventory's. For example, if there is an existing inventory of 500 skeins of yarn, the constraint named as "yarn limit" will transform from "$y_2 + x_1 + 0.75x_3 - x_2 = 0$" to "$y_2 + x_1 + 0.75x_3 - x_2 = -500$".

3.11 The Influence of Strike to Weekly Profit

In the best scenario where the available labor hours will reduce by 500 minutes, the weekly profit is assumed by this team to change nothing. In the worst scenario where the available labor hours will reduce by 1300 minutes, the weekly profit is assumed by this team to decrease by 11.0196075 dollars.

It is an estimate instead of an exact answer since as a resource weekly available labor's constraint known as "Labor Constraint" has 0 as its dual price and 500 minutes decrease is inside of its allowable decrease (The allowable decrease is 1250 minutes). Therefore, the weekly optimal profit will not change after the available weekly labor hour decreases by 500 minutes.

To find an estimated dual price for this constraint, the teams considered components that got affected by labor reduction. Labor time needed for cloth, yarn, and crafting materials correspondingly are 30 minutes, 5 minutes, and 50 minutes (The time cost proportion for cloth, yarn, and crafting material will therefore be 6:1:10). Therefore, the dual price estimate for labor constraint out of the allowable decrease will be the combination of dual price of cloth production limit known as "Cloth Limit", yarn production limit known as "Yarn Limit", and crafting material production limit known as "Crafting Materials" with their corresponding proportionality. Under this logic, the dual price estimate will be 13.223529 dollars per hour change ($30 \cdot 6/17 + 24.8 \cdot 1/17 + 2 \cdot 10/17 = 13.223529$, this number has to be divided by 60 if the time changing is in minutes).

Using this estimate of labor constraint out of its allowable decrease, the decrease of weekly profit under 1300 minutes reduction of labor hours will be 11.0196075 dollars ($1300 - 1250 = 50, 50 * 13.223529/60 = 11.0196075$).

3.12 Unconsidered Situation to Profit

3.12.1 Appropriate Cost to Increase the Weekly Maximum Yarn Production by 30 Skeins

Crafting S.L should be willing to pay up to 651 dollars to make this increase. Since increase of 30 skeins of yarn is within the allowable increase of the maximum weekly yarn production (35.71429) known as "Yarn Maximum Limit" and the dual price for this constraint is 21.7 dollars per skein, the cost that the company is willing to pay for will be up to 651 dollars for this increase. $(21.7 \cdot 30 = 651)$

3.12.2 Appropriate Cost to Increase the Weekly Maximum Crafting Material Production by 30 Pounds

Crafting S.L should be willing to pay up to 651 dollars to make this increase. Since increase of 30 pounds of crafting material is within the allowable increase of the maximum weekly crafting materials production known as "Crafting Material Maximum Limit" and the dual price for this constraint is 1.3 dollars per skein, the cost that the company is willing to pay for will be up to 39 dollars for this increase. $(1.3 \cdot 30 = 39)$

4 Recommendation for Development of Company

To yield the maximum weekly profit, Crafting S.L. should produce 2625 meters of cloth, 3000 skeins of yarn, and 500 pounds crafting materials and sell 2565 meters of cloth and 500 pounds of crafting materials. In order to produce and sell this set of production, the company should buy 412.5 pounds of crafting materials at the start of the week and the weekly profit will be 65750 dollars. Considering the situation of Crafting S.L, it will be really profitable for the company to spend money on expanding the maximum production capacity of yarn, much less profitable for the company to invest in expanding the maximum production capacity of crafting materials, and no benefit at all to increase the maximum production capacity of cloth. To increase the profit, the company could consider accepting inventories. Inventories in cloth will be the most profitable inventory to the company and inventories in yarn will be the second most profitable inventory to the company but inventories in crafting materials will not be so profitable. All the yarn should be used to produce crafting material and cloth unless its price rises over 24.8 dollars.

5 Computer Reports

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LINGO/OSX64 19.0.46 (1 Sep 2021 ), LINDO API 13.0.4099.299

Licensee info: Eval Use Only
License expires: 14 JUL 2022

Global optimal solution found.
Objective value:                              65750.00
Infeasibilities:                              0.000000
Total solver iterations:                             3
Elapsed runtime seconds:                          0.08

Model Class:                                        LP

Total variables:                     7
Nonlinear variables:                 0
Integer variables:                   0

Total constraints:                  10
Nonlinear constraints:               0

Total nonzeros:                     28
Nonlinear nonzeros:                  0


                     Variable           Value        Reduced Cost
                          Y_1        2565.000            0.000000
                          Y_2        0.000000            15.80000
                          Y_3        500.0000            0.000000
                          X_1        2625.000            0.000000
                          X_2        3000.000            0.000000
                          X_3        500.0000            0.000000
                          C_3        412.5000            0.000000

                          Row    Slack or Surplus      Dual Price
                       PROFIT        65750.00            1.000000
            LABOR_CONSTRAINT        1250.000            0.000000
                CLOTH_LIMIT        0.000000            30.00000
                 YARN_LIMIT        0.000000            24.80000
           CRAFTING_MATERIALS        0.000000            2.000000
          CLOTH_MAXIMUM_LIMIT        2875.000            0.000000
           YARN_MAXIMUM_LIMIT        0.000000            21.70000
CRAFTING_MATERIAL_MAXIMUM_LIMIT        0.000000            1.300000
          BUYING_COMPOUNDS_LIMIT        587.5000            0.000000
     CRAFTING_MATERIAL_SALE_LIMIT        0.000000            23.00000
```

Figure 1: LP Result

Ranges in which the basis is unchanged:

Objective Coefficient Ranges:

Variable	Current Coefficient	Allowable Increase	Allowable Decrease
Y_1	30.00000	1.494253	15.80000
Y_2	9.000000	15.80000	INFINITY
Y_3	25.00000	INFINITY	1.300000
X_1	-5.000000	1.733333	15.80000
X_2	-3.000000	INFINITY	21.70000
X_3	-1.500000	INFINITY	1.300000
C_3	-2.000000	17.33333	23.00000

Righthand Side Ranges:

Row	Current RHS	Allowable Increase	Allowable Decrease
LABOR_CONSTRAINT	120000.0	INFINITY	1250.000
CLOTH_LIMIT	0.000000	INFINITY	2565.000
YARN_LIMIT	0.000000	41.66667	2565.000
CRAFTING_MATERIALS	0.000000	412.5000	587.5000
CLOTH_MAXIMUM_LIMIT	5500.000	INFINITY	2875.000
YARN_MAXIMUM_LIMIT	3000.000	35.71429	2565.000
CRAFTING_MATERIAL_MAXIMUM_LI	500.0000	45.45455	500.0000
BUYING_COMPOUNDS_LIMIT	1000.000	INFINITY	587.5000
CRAFTING_MATERIAL_SALE_LIMIT	0.000000	587.5000	412.5000

Figure 2: Range Report