

Battery operated, pipe bending device designed to bend 6mm pipes to an angle of 45 to 90 degrees with a bend radius of 15 mm.

Assignment 2

Detail Design

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1. Introduction

This report provides an overview of the detail design procedure followed for a battery-operated pipe-bending device. **Figure 1** shows the final concept developed in the basic design phase. The device developed in detail design is an embodiment of the final concept with some functional and layout changes:

- The device proposed in detail design was bi-directional, in that it could bend a pipe in two different configurations. This feature proved unfeasible as it would require duplicate parts increasing the cost of the device. Additionally, requiring two positions for the support die (as shown in the final concept sketch) complicates the angle control mechanism.
- The final concept made use of a key for positive engagement between the actuator arm and the custom shaft. Manufacturing keyways on shafts with a diameter less than 25 mm is generally not recommended [1]. The new approach skims the sides of the shaft. This is expanded in section 2.2.4.
- The battery was moved from under the motor to behind the motor. This has several advantages related to the ergonomics and cost of the device. The centre of gravity of the device was shifted closer to the middle of the device making it easier for the user to handle. The width of the handle halved making it easier to hold the device. These changes ensure that the device meets the User Requirement Statement (URS) constraint that the device should be operatable by a 95% percentile adult. The change also meant that the handle could be made from one piece of plastic instead of two halves. This has obvious cost benefits both in the prototype and mass production phase.

The sizing of the motor, gearbox, and number of cells, shown in **Table 1**, proposed in basic design is used in detail design.

Table 1: Sizing from Basic design

Gear ratio	No. of cells	Motor	T_m (Nm)	T_{arm} (Nm)
[12,36,12,84]	2	100 rpm	0.732	15.069

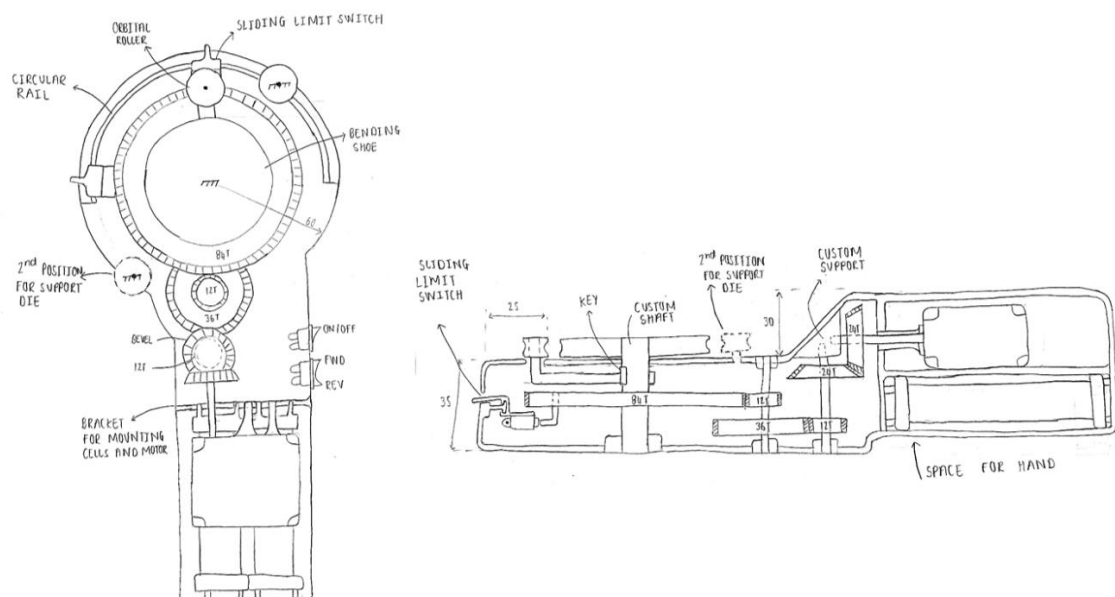


Figure 1: Final concept sketch

2. Detailed Design Description¹

2.1 Motor mounting, Shaft connection and Alignment

2.1.1 Motor mounting

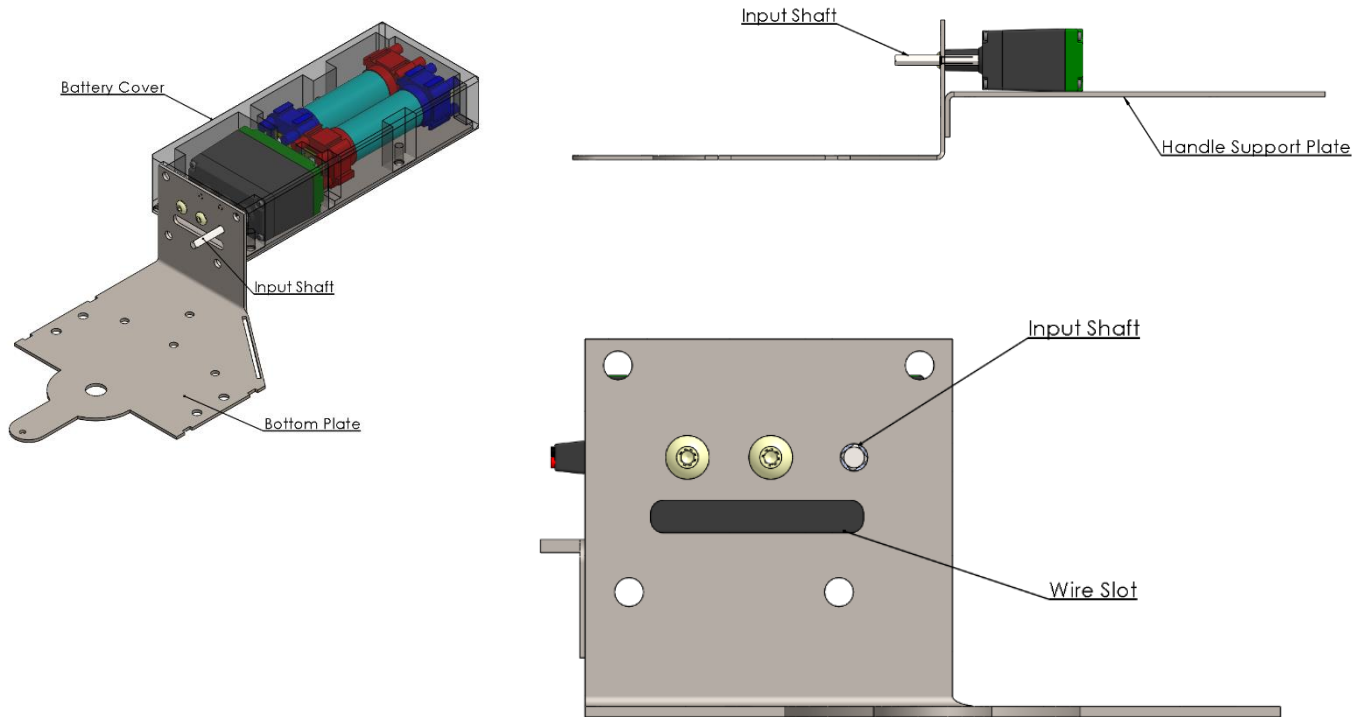


Figure 2: Motor mounting onto bottom plate from various views

The 100 rpm VEX “2-Wire Motor 393” is mounted using the two threaded insert stubs integrated with the motor. The stubs allow for accurate location with a pair of laser cut, 4.2 mm diameter holes cut into the bottom Plate. The accuracy of laser cutting technology (0.1 mm) is sufficient for providing accurate location for this application. The motor is mounted with the recommend MEC4124-006 (0.5 inch) screws. A ‘Wire slot’ allows motor and battery wires to run to the 3-way Rocker Switch this is expanded on in section 2.6. The handle support plate is positioned so that the lowest point of the motor is just resting on the plate.

¹ Sections 2.1 to 2.5 do not show any mounting screws in the interest of clarity

2.1.2 Shaft Connection

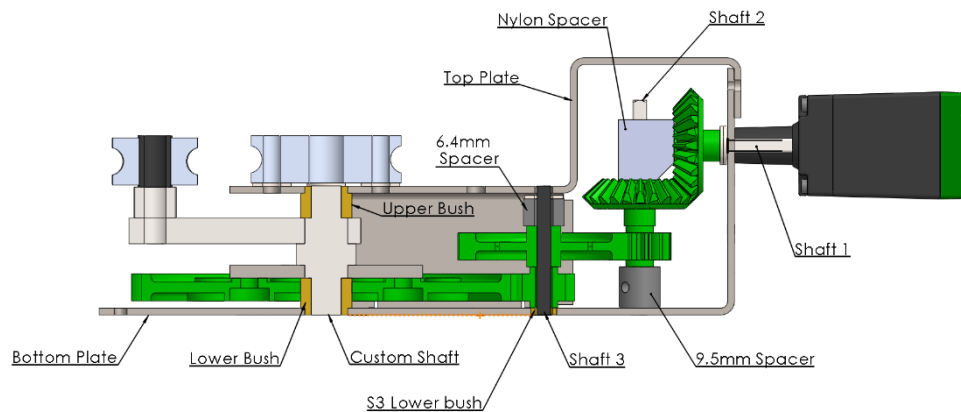


Figure 3: Gear train Front View

The pipe bending device makes use of **4** shafts. Three of the shafts are 0.125-inch square shaft of lengths 40, 50 and 30 mm respectively where the 30 mm shaft makes use of the heat-treated material. The **Output** shaft (S4) is a custom Plain Carbon steel shaft. The design of this shaft is shown in Error! Reference source not found.. The **Input** shaft (S1) is connected to the motor and turns a 24T Bevel gear (B1). Shafts **2** (S2) and **3** (S3) are **Intermediary** shafts where S2 is driven by the input shaft by means of an 24T Bevel gear (B2) meshing with the Input shaft. S3 is driven by S2 by a 36T (G2) gear that meshes with a 12T gear (G1) on S2. The Output shaft is driven by an 84T (G4) gear meshing with a 12T gear (G4) on S3. **Figure 3** shows that all the shafts are connected to the sheet metal casing of the device by means of the Top and Bottom plates.

2.1.3 Alignment

Proper alignment is necessary to ensure that gear are correctly meshing, maintain gear tooth strength and gear efficiency. In this regard, the **axial** and **centre-to-centre** distance alignment of the gears is important. The **Intermediary** (S2 and S3) and **output** (S4) shaft are mounted between the Top and Bottom plates. These support holes ensure the alignment on the **individual** plates to an accuracy of 0.1 mm but the alignment between plates requires extra consideration shown in 2.1.3.1.

2.1.3.1 Centre-to-Centre Distance

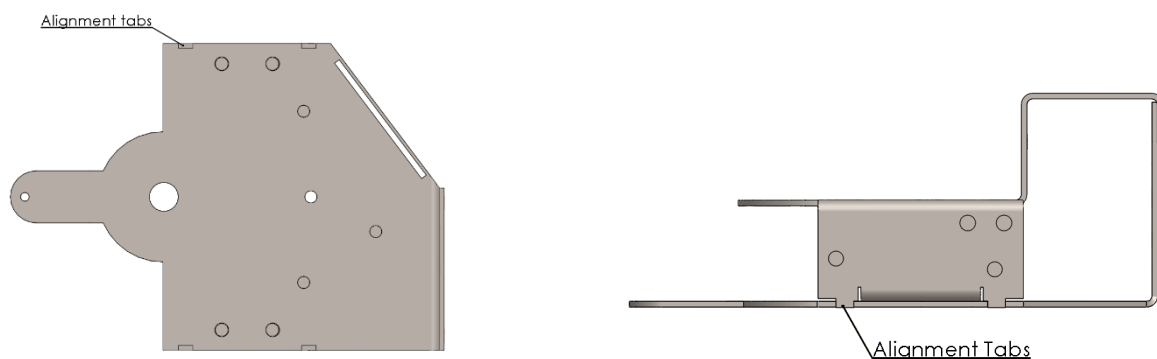


Figure 4: Alignment tabs used for accurate location between the top and bottom plates

Although the holes in the sheet metal plate can be assumed to be accurate, the two plates are secured together with self-tapping screws which cannot provide accurate location. To ensure alignment between the two plates, alignment tabs, shown in **Figure 4**, are used.

2.1.3.2 Axial Alignment

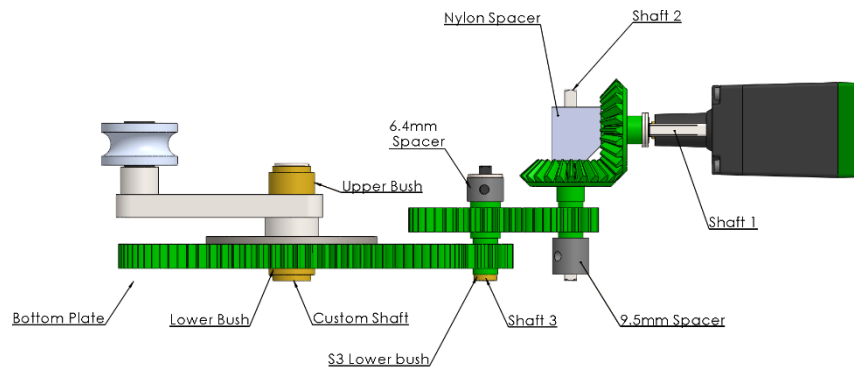


Figure 5: Gear train, shaft and shaft supports

For proper operation the midplanes of two meshing gears must be coincident. This is achieved with the use of spacers, bushes and washers. The 6.4mm and 9.5mm spacers provided axial location when used with a ST2.9 screw (not shown for clarity). This is acceptable since the axial forces of the bevel gears are not substantial. Gear B1 is constrained by means of a Nylon spacer and two washers. Gears B2 and G1 are constrained by the Nylon spacer and a 9.5 mm spacer. Gears G2 and G3 are constrained between the two sheet metal plates with a 6.4 mm spacer and washer. The alignment tabs shown in **Figure 5** are designed to be the same length as the thickness of the sheet metal (1.6mm) thus ensuring that the two sheet metal plates are separated by the correct distance.

2.2 Force/Torque Transfer and Shaft Support, Strength and Deflection

2.2.1 Force/Torque Transfer

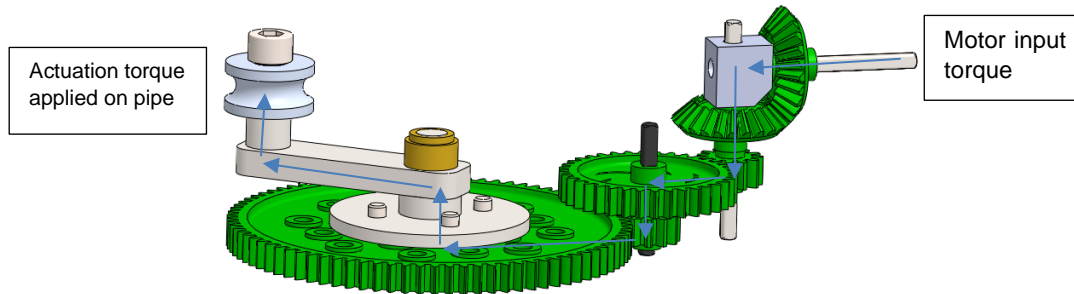


Figure 6: Torque propagation diagram

Torque is transferred from the motor through the 3-stage reduction gearbox to the output 84T gear. The torque is then transferred to the custom output shaft through a support plate that positively engages with the output shaft. The output torque is transferred to the actuator arm by means of the output shaft. Finally, this torque is transferred to the pipe through the small roller which is connected to the actuator arm. The torque propagation is shown in **Figure 6**.

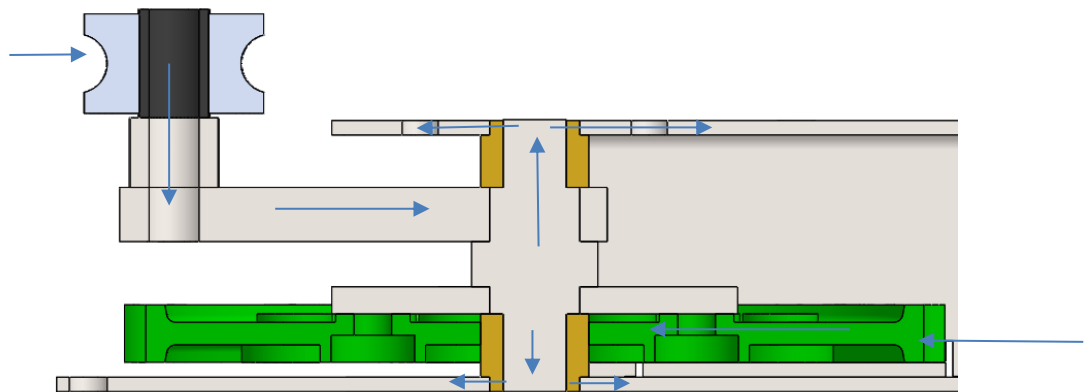


Figure 7: Force transfer for output shaft

A force transfer diagram is shown in **Figure 7** for the output shaft assembly. Force is transmitted onto the output gear (G4) due to meshing in the gear train. This force is transmitted through the shaft and is distributed into the sheet metal casing. The actuator arm also transmits force onto the output shaft which is distributed into the sheet metal casing. **Figure 7** is a good representation for how the force is transmitted throughout the geartrain and device.

2.2.2 Shaft strength and Deflection

Table 2: Shaft strength, safety factor and deflection

Shaft	Critical θ_{vm} (Mpa)	Safety factor	Deflection at critical section (mm)	Torque (Nm)
S1	151.73	2.996	0.003	0.675
S2	210.92	2.134	0.08	0.657
S3*	417.56	1.557	0.039	1.903
Custom	59.589	3.65	-	12.92

* Shaft 3 makes use of the heat treated 0.125-inch square shaft

The torques on S1 and S2 are below maximum allowable 5Nm for the 0.125-inch square shafts. Similarly, the torque on S3 is below the maximum allowable 13Nm for the heat-treated square shaft.

The table above shows the Von Mises equivalent stress (θ_{vm}) at the critical sections of the shaft and the safety factor for this section. Due to the largest stresses on S3 it was necessary to use the high strength 30 mm shaft to obtain reasonable safety factors.

Table 2 shows the deflections of the shafts at a critical section. The critical section is defined as the meshing location with the highest stress. Deflection at this point is important because it affects gear tooth strength and efficiency. The calculated deflections are very small thus deflection of the shafts should not be an issue in this application.

2.2.3 Shaft Supports

Table 3: Reaction forces on shafts

Shaft	Reaction forces (N)		Outcome
	A	B	
S1	24.28	30.59	Suitable
S2	69.518	64.381	Suitable
S3	77.507	271.265	Bushing
Custom	419.11	349.073	Bushing

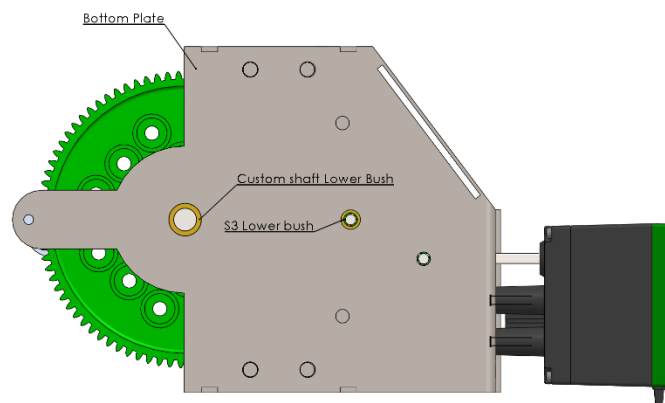


Figure 8: Brass Bushing for shaft support on the bottom plate

All shafts were modelled as simply supported beams. The **Table 3** shows the reaction forces for each shaft where A and B are the reaction forces on the top and bottom plate, respectively. The reaction forces for S1 and S2 were deemed **low** enough for the shafts to be supported by **4.2 mm holes** in the sheet metal. It should be noted S1 and S2 are also supported by a **Nylon spacer**. The reaction force for S3 of 271N was deemed sufficiently large that a small brass **bush** is used.

The large reaction forces acting on the custom shaft requires the shaft to be supported by two brass bushes that sit in between the top and bottom sheet metal plates. The big die roller sits on washers which provides a small gap allowing for the top of the shaft to freely rotate.

2.2.4 Custom Shaft

Table 4: Diameter and safety factor at different sections on custom output shaft

Section	Diameter (mm)	Safety factor
A	7	3.843
B	7	4.331
C	10	3.692
D	10	3.65
E	14	10.016

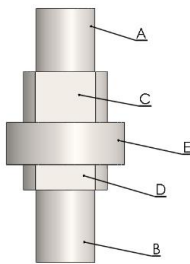


Figure 10: Custom output shaft labelling

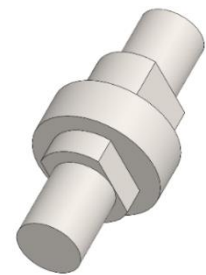


Figure 11: Isometric view of custom output shaft

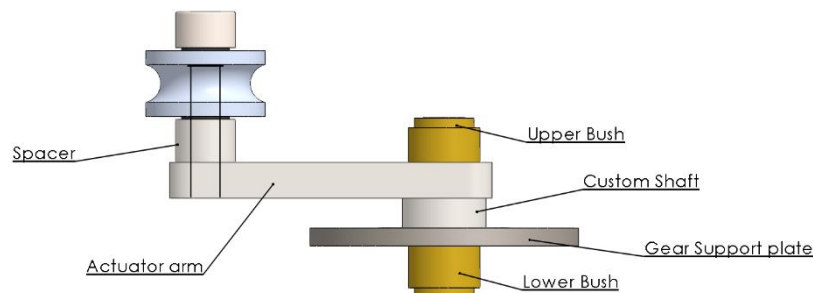


Figure 9: Assembled custom output shaft

Although the output torque is below the max allowable torque of the heat treated square shaft (13Nm), the forces due to the actuator arm and gear would result in stresses that exceed the yield stress of the shaft. As a result, a custom shaft was required.

The roller arm and gear support plate are act at sections C and D, respectively, in **Figure 10**. The diameter of the shaft at this section is 10mm. This would be too small to feasibly create a keyway. A better, and cheaper, solution is to skim the sides of the shafts at these sections. This profile can easily be laser cut into the actuator arm and the support plate. The safety factors, shown in the table above, for sections C and D are calculated for shafts with a circular cross section. Therefore, in reality the safety factor would be lower. Although the safety factors calculated are large enough to deem the discrepancy acceptable. The shaft is supported by two bushings which sit in between the top and bottom sheet metal plate.

2.3 Gearbox Design, Strength and Efficiency

2.3.1 Reduction ratio and Efficiencies

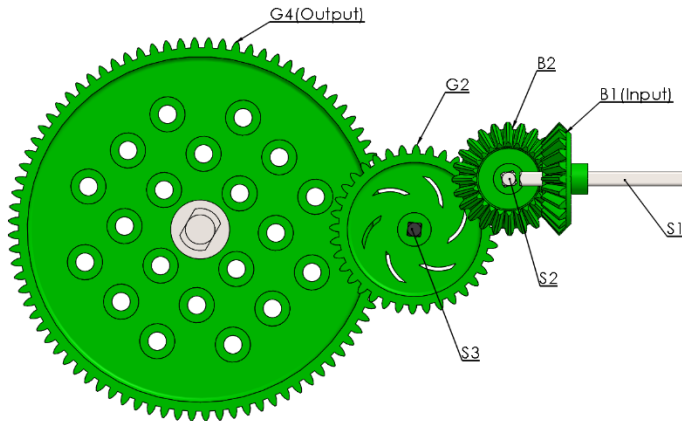


Table 5: Gear train Top View

Table 6: Individual gear sizes

Gears	
B1	24T
B2	24T
G1	12T
G2	36T
G3	12T
G4	84T

Table 7: Efficiency for each reduction step

Reduction step	Gear A	Gear B	Reduction Ratio		Efficiency
			Speed reduction (B/A)	Effective	
1	24	24	1	0.974	0.974
2	12	36	3	2.895	0.965
3	12	84	7	6.79	0.97
Total			21	19.146	0.912

A 3 three stage gearbox is used where B1 is the **input** gear and G4 is the **output** gear. The gearbox provides a speed reduction of **21**. The gear efficiency for each reduction step is shown in the **Table 7** the effective or torque reduction ratio of the gearbox is **19.146**.

2.3.2 Motor operation conditions

Table 8: Motor operating conditions

Motor Torque (Nm)	0.675
Trip time (s)	19.547
Estimated bend time (s)	6.147
Motor power(w)	3.621
Peak power (w)	3.64

Table 8 shows the operating conditions for the motor including the efficiencies related to the gearbox. There is 13.4 second buffer between the estimated bend time and motor trip time. In reality this buffer

might be less due to additional efficiencies and friction that is not considered in this design. The buffer is large enough to assume that the pipe bending operation will complete before the motor trips.

2.3.3 Gear Force/Torque analysis

Table 9: Force and torque on each gear

Gear	Teeth	F_T (N)	$F_{T(MAX)}$ (N)	T (Nm)	T_{MAX} (Nm)	Outcome
B1 (Input)	24	53.135	300	0.675	3	Suitable
B2	24	51.744	300	0.657	3	Suitable
G1	12	103.489	450	0.657	3	Suitable
G2	36	34.496	700	1.903	3.4	Suitable
G3	12	299.629	450	1.903	3	Suitable
G4 (Output)	84	290.664	700	12.92	4.1	Reinforcement

It shown in the appendix (7.2) that there is only one operating condition using the orbital roller mechanism. The torque required for bending was found to be 12.92 Nm which is also the torque that the final output shaft must produce to bend the pipe.

Using the required output gear torque of 12.92 Nm and effective gear ratio of 19.146. The torque and tangential force are calculated for all the gears with the results shown in **Table 9**. The only gear that requires reinforcement is the output gear (G4) due to the torque exceeding the maximum allowable.

2.3.4 Gear Reinforcement

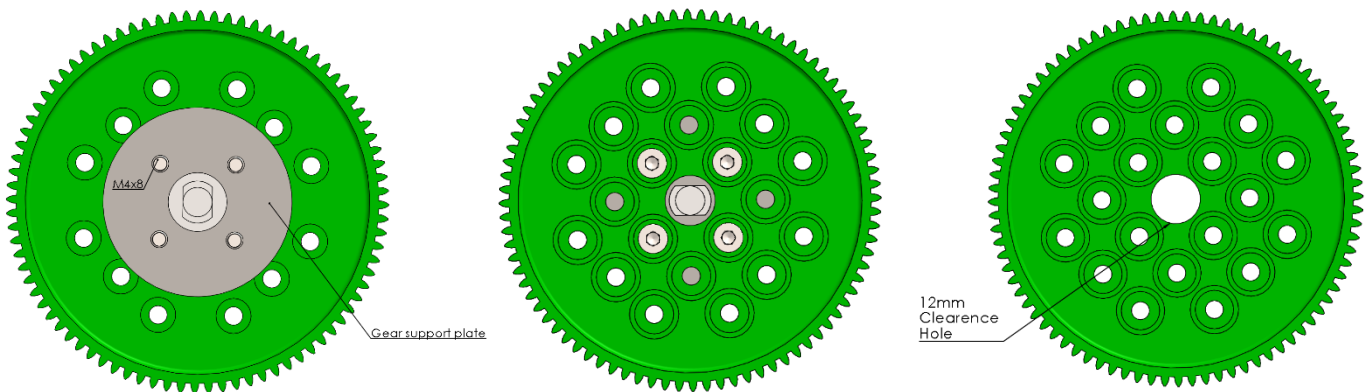


Figure 12: (Left) Reinforced output gear back. (Centre) Reinforced output gear front. (Right) Modified output gear

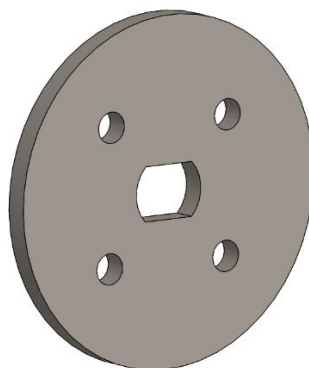


Figure 13: 3mm thick sheet metal output gear support plate

The 84T gear is reinforced using a 3mm plate made of sheet metal. The support plate is attached to the gear with the use of 4 M4x8 socket head bolts. The image above does not show the thread in the holes for the M4x8 bolts for the sake of clarity. The support plate positively engages with the custom shaft by means of the profile described in 2.2.4.

The motivation for using a 3mm thick plate makes the following assumptions:

- a) Allowable torque transmitted is proportional to the thickness of the shaft boss and yield material of the material
- b) Allowable torque is proportional to the cross-sectional area of the shaft

Using these assumptions, a proportional constant (shape factor) for the 0.125 inch square shaft/gear interface can be found. The shape factor can then be used to find an equivalent thickness of sheet metal plate that would be required to support the torque of the square shaft. The required thickness of the shaft was found to be 9 mm. Assumption (b) allows for an estimation to be made based on the relative cross-sectional areas of the square shaft and custom shaft. The cross-sectional area of the square shaft is 4.93 times larger than the 0.125 inch square shaft. Therefore, using this approach the required thickness is $9/4.93$ or 1.8 mm and justifies the use of 3mm thick sheet metal plate as the support for the output gear.

2.4 Actuator Design

2.4.1 Actuator Arm Design and Strength

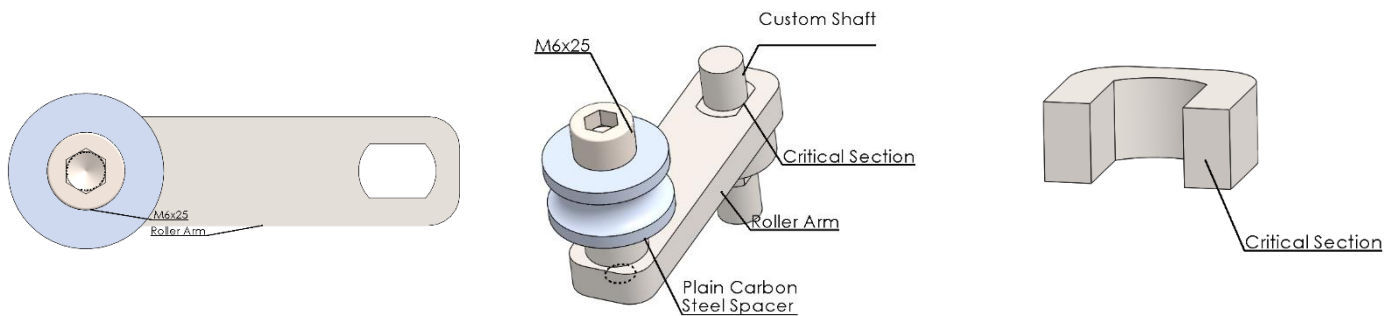


Figure 14: (Left) Top view of actuator arm (Centre) Isometric view of Actuator arm. (Right) Critical cross-section of the actuator arm

The final actuator is simply a “Roller Arm” carrying the small roller die. The roller arm is driven by the output (custom) shaft. The arm engages with the shaft due to the rectangular profile that is cut into the arm. The button screw that comes with small roller was found to be too small and replaced by M6x25mm socket head screw. This roller arm is made from 6mm Hot Rolled Mild Steel Plate and the feature are laser cut.

Table 10: Dimensions, strength, safety factor and deflection of the actuator arm.

Width (mm)	Thickness (mm)	θ_{vm} (Mpa)	θ_y (Mpa)	Safety factor	Torsional Deflection (Degrees)	Vertical Deflection (mm)
14	6	115.173	220	1.91	0.096	0.151

The θ_{vm} shown above relates to the critical section of the roller shown in **Figure 14**. A safety factor of 1.91 is reasonable given that the failure scenario is yield which does not present danger to the user. Additionally, the operating conditions of the device are well known therefore the calculated stress values should be accurate estimates of actual stresses experienced during operation. The torsional and vertical deflection calculated for the roller arm is small enough to assume that these deflections will not affect the operation of the device.

The maximum strength on the M6x25 mm screw was calculated to be 278.65 MPa. The screw used is grade 8.8 with a yield strength of 660 MPa [2]. This gives a safety factor of 2.37 which is again acceptable for this application due to the reasons stated above.

2.4.2 Actuator Operation

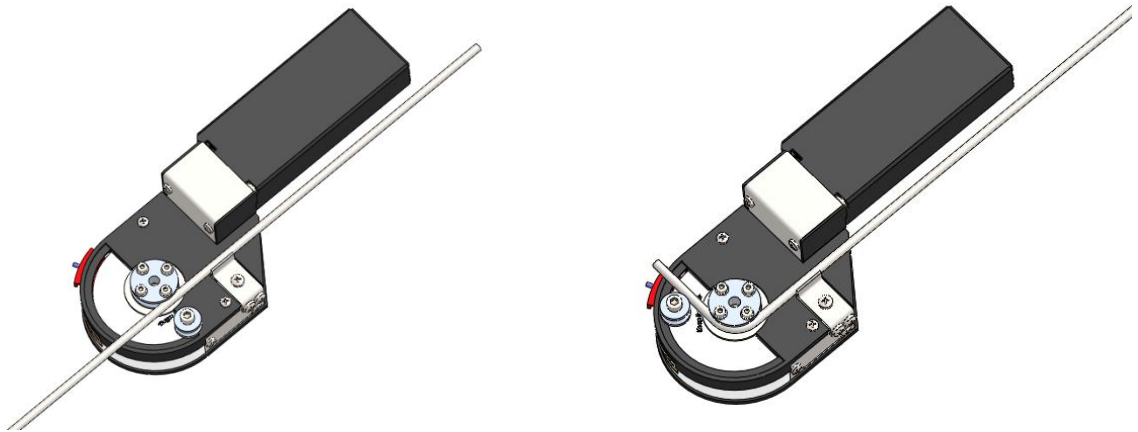


Figure 15: (Left) Start of bending operation (Right) End of bending operation

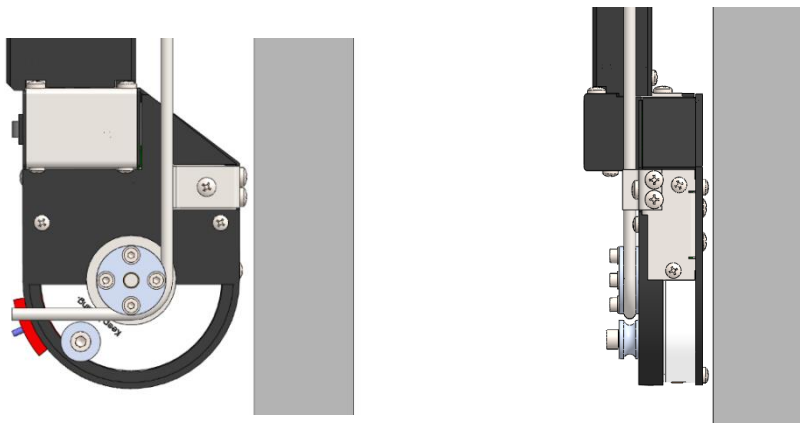


Figure 16: Clearance shown in two different configurations for a pipe 40 mm from a surface

The user manoeuvres the device to be parallel to the pipe that is to be bent as shown in **Figure 16**. The small roller must be on the opposite side of the pipe to the Big roller. The user can simply use the 3 way rocker switch to ensure that small roller is in the home position. Once the user has set the desired angle and is satisfied with the position of the pipe relatively to the device the user simply flips the rocker switch to begin the bending process. The actuator arm will stop once it has reached the limit switch trolley.

Figure 15 shows the loading (start) configuration the bending process. The actuator arm rotates with the final output gear, contacts the pipe and bends the pipe to a user defined angle (90 degrees in this case). The Big roller is fixed in place with 4 M4x16 hex bolts. The big roller is placed on 4 washers to provide clearance for the output shaft below.

Figure 14 shows a Plain Carbon Steel spacer that is used to ensure that the big and smaller roller are correctly aligned. The Small roller is fixed to the actuator arm by means of M6x25 hex bolted into the actuator arm.

The 'bent' end of the pipe shown in **Figure 16** is 60 mm long. This shows that the offset used between the Big and Small roller is sufficiently small to meet Feature F3 from the URS.

2.5 Angle Control Mechanism

2.5.1 Design

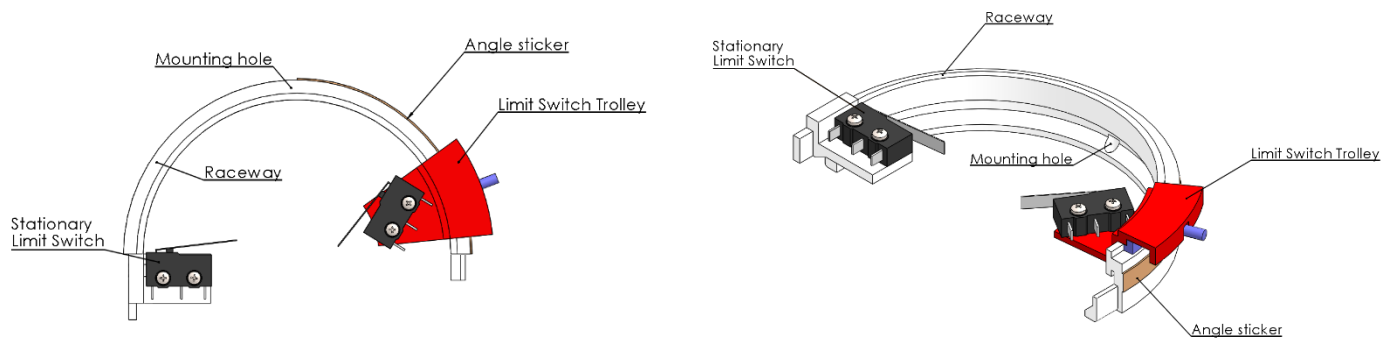


Figure 17: Angle control mechanism assembly

The angle control makes use of two limit switches that are attached to a 3D printed, PLA 'Raceway'. One of the switches is stationary and controls the 'home' or return position of the actuator arm. The second switch is mounted onto a 'Limit switch trolley'. The user can rotate the limit switch trolley to control the bend angle. In the testing phase of the prototype, a sticker containing angles ranging from 45 to 90 degrees can be added to the outer surface of the raceway allowing the user to set the desired angle. The limit switch assembly slides in from the front of the device during assembly and is secured using the mounting hole shown in **Figure 17**.

2.5.2 Functionality

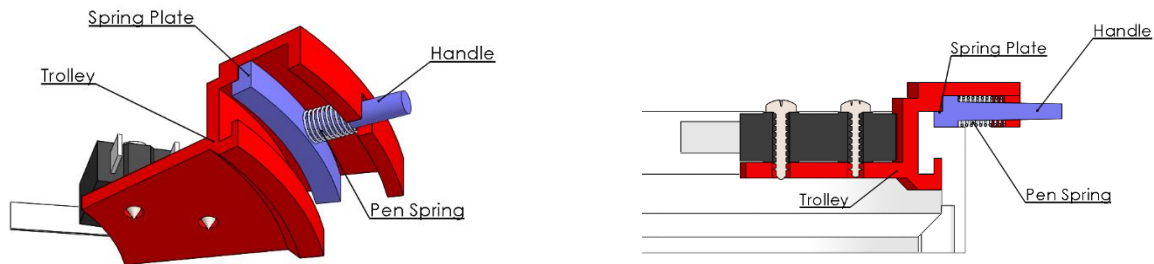


Figure 19: (Left) Limit switch trolley (Right) Cross-section of limit switch trolley and raceway.

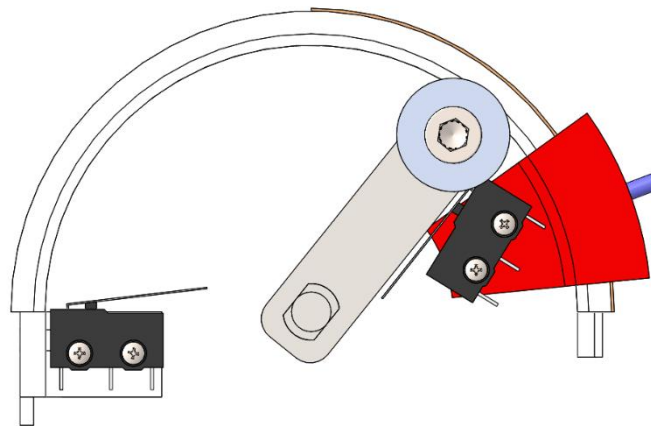


Figure 18: Angle control operation

The angle control is achieved using the 'Limit Switch Trolley'. This is a 3D printed part that is used to manipulate the bend angle. The Trolley is made up of 3 components, namely, the 'trolley', 'spring plate' and regular spring found in a pen.

To set the desired the angle the user simply pulls back on the handle and rotates the trolley along the raceway releasing the handle at the desired bend angle. The spring force ensures that the trolley does not slide as the actuator arm meets the limit switch.

The spring used for basic design was sourced from a common spring-loaded pen. The particular spring had solid height of 5mm and coil diameter of approximately of 3.5mm. These dimensions were used to design the Limit Switch Trolley. The user only needs to pull back the handle connected to the spring plate approximately 2mm before solid height is reached. The slot in which the spring plate handle moves through has a strategic open at the bottom to for the assembly of the limit switch trolley. Additionally, the raceway is open on the side opposite to the stationary limit switch. This allows for the trolley to be mounted on the raceway. In operation the trolley will only be able to move 90 degrees in which the device body limits this. This is acceptable since the pipe only needs to bend from 45 to 90 degrees.

2.5.3 Actuator Direction Control



Figure 20: (Left) 3 way Rocker switch CAD model (Right) Physical rocker switch from Rabtron [2]

The direction of the actuator arm is controlled in conjunction with the 3-way rocker switch and the limit switch. Once one of the limit switches are actuated the user can change the direction by flipping the rocker switch. This switch does not use any screws and simply clips into place. The 3 way switch is convenient as it allows the direction control and ON/OFF control with one switch.

2.6 Component Packaging

2.6.1 Sheet Metal Structure

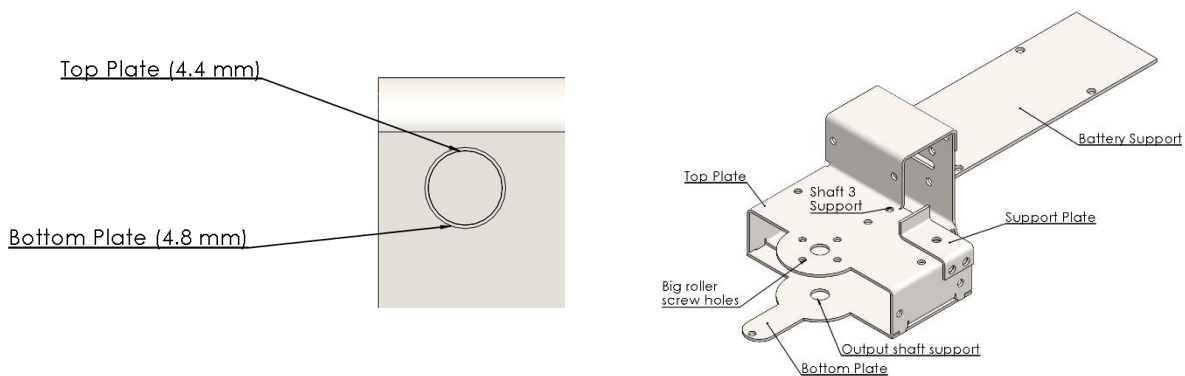


Figure 21: (Left) Inner and Outer holes in the sheet metal plates (Right) Isometric view of the sheet metal structure

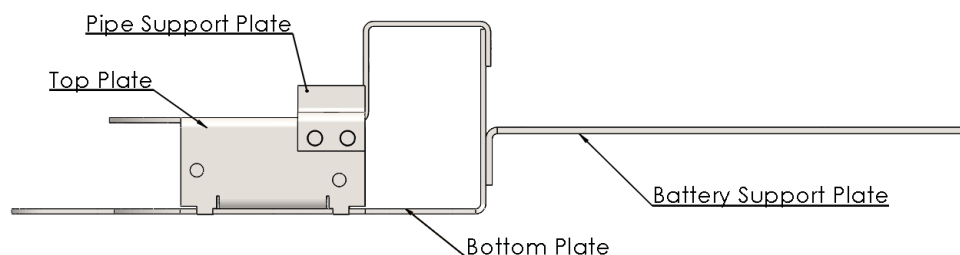


Figure 22: Side view of sheet metal casing

The sheet metal casing shown in **Figures 21** and **22** provides the structural support and rigidity. The sheet metal casing supports all the shafts, Big die roller, handle assembly and plastic covers. All sheet metal part are made of Hot Rolled Mild Steel 1.6 mm thick Plate. The main stresses acting on the sheet metal plate are contact pressure forces due to the various shafts and the big die roller. The hertzian contact stress was found using an online Amesweb calculator for the interface between the bottom plate and the lower bush of the output shaft as this is the largest reaction force acting on the sheet metal casing [3]. The results for this are shown in **Figure 23**. These results provide confidence in the use of 1.6mm sheet metal plate for this application.

INPUT PARAMETERS			
Parameter	Object-1	Object-2	Unit
Object shape	<div>Cylinder</div>	<div>Inner Cylinder</div>	
Poisson's ratio $[\nu_1, \nu_2]$	<div>0.28</div>	<div>0.28</div>	
Elastic modulus $[E_1, E_2]$	<div>200</div>	<div>200</div>	<div>GPa</div>
Diameter of object $[d_1, d_2]$	<div>10</div>	<div>10.1</div>	<div>mm</div>
Force $[F]$	<div>419.11</div>		<div>N</div>
Line contact length $[l]$	<div>1.6</div>		<div>mm</div>

RESULTS			
Parameter	Obj-1	Obj-2	Unit
Maximum Hertzian contact pressure $[p_{max}]$	133.8		<div>MPa</div>
Max shear stress $[\tau_{max}]$	<div>40.2</div>	<div>40.2</div>	
Depth of max shear stress $[z]$	0.979	0.979	<div>mm</div>
Rectangular contact area width $[2b]$	2.492		

Figure 23: (Left) Input parameters to online hertzian contact stress calculator (Right) Results of online calculator

Table 11: Max stress and deflection of handle support plate due to the weight of the device

Moment due to COM (Nm)	$\theta_{support}$ (Mpa)	$v_{support}$ (mm)
0.15	7.456	0.479

The limiting factor for the sheet metal in this case is rigidity. **Table 11** shows the stresses caused due to the weight of device analysed at the base of the battery support plate. The worst case is considered where the user is applying a moment at the tip of the battery support plate. A deflection of the 0.479 mm is sufficiently small to motivate using 1.6mm thick plate. Additionally, the plastic handle provides some rigidity as well that was not considered in this analysis. Sheet metal is ideal for this application because it can made into complex shapes and accurately cut at relatively low prices.

2.6.2 Handle

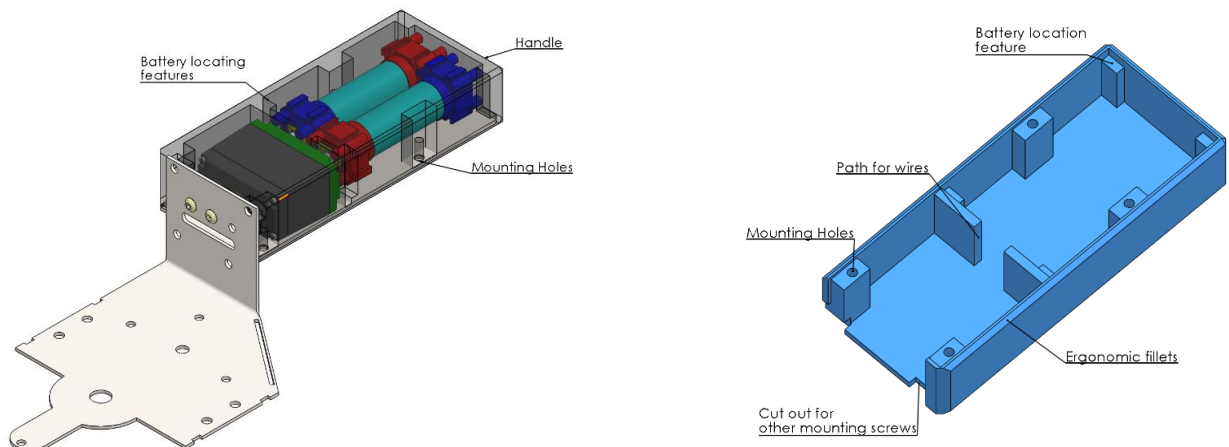


Figure 24: (Left) mounting and internal components of handle assembly (Right) Plastic handle

The handle assembly consists of a sheet metal support plate discussed in 2.6.1 and a plastic (3D printed) cover. The cover shown in **Figure 24** has internal features that provide location for the battery. The internal features are extrudes that use the M3 plastic stubs on the cell cap to locate the battery. The cover has a cut that allows battery wires to run to other compartments of the device. The plastic handle is mounted to the support plate using **ST4.2** screws and the support plate is mounted to the bottom plate using **ST4.8** screws.

2.6.3 Plastic Covers

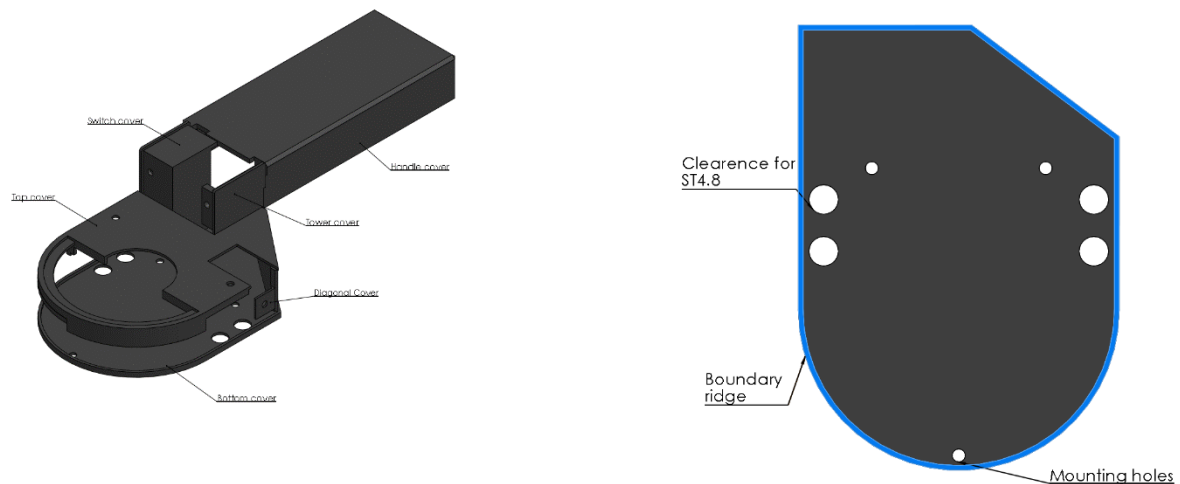


Figure 25: (Left) Assembly showing plastic covers (Right) Bottom plastic cover

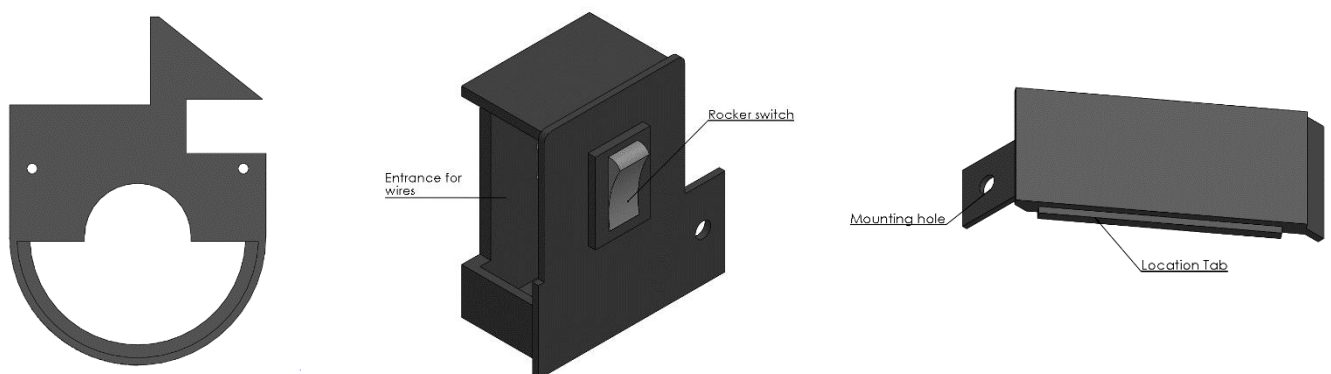


Figure 26: (Left) Top plastic cover (Centre) Switch cover (Right) Diagonal side cover

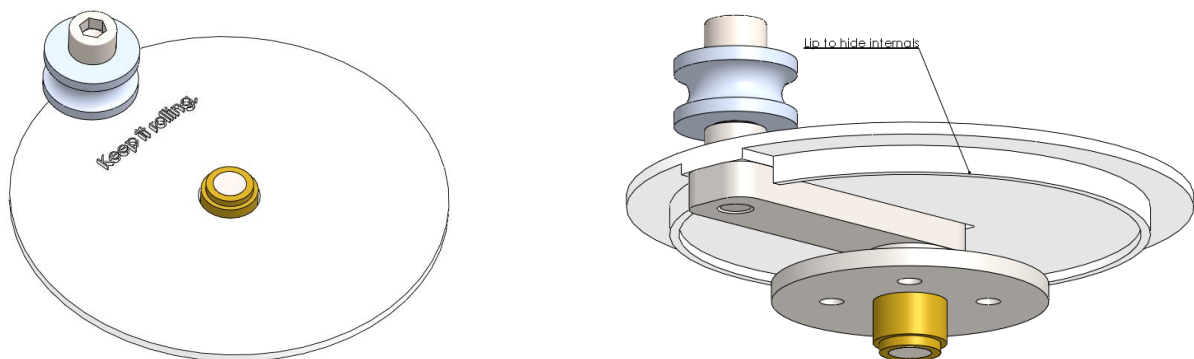


Figure 25: Actuator cover that covers the internals of the device from the user

The plastic covers serve the purpose of ensuring the device meets the safety and ergonomics requirements (E1 and E2) from the URS. The covers ensure that the user is not exposed to any moving part except for the actuator arm. The specifics of the ergonomic benefits of the plastic cover are discussed in 2.7.3.

All the plastic covers are to be PLA 3D printed devices. The covers are mounted to the sheet metal plates with the ST4.2 screws. **Figure 26** shows the 'switch cover'. This component holds the 3-way rocker switch and provides space for the wires and additional electronics that may be required.

2.7 Manufacturing, Assembly and Ergonomic considerations

2.7.1 Assembly

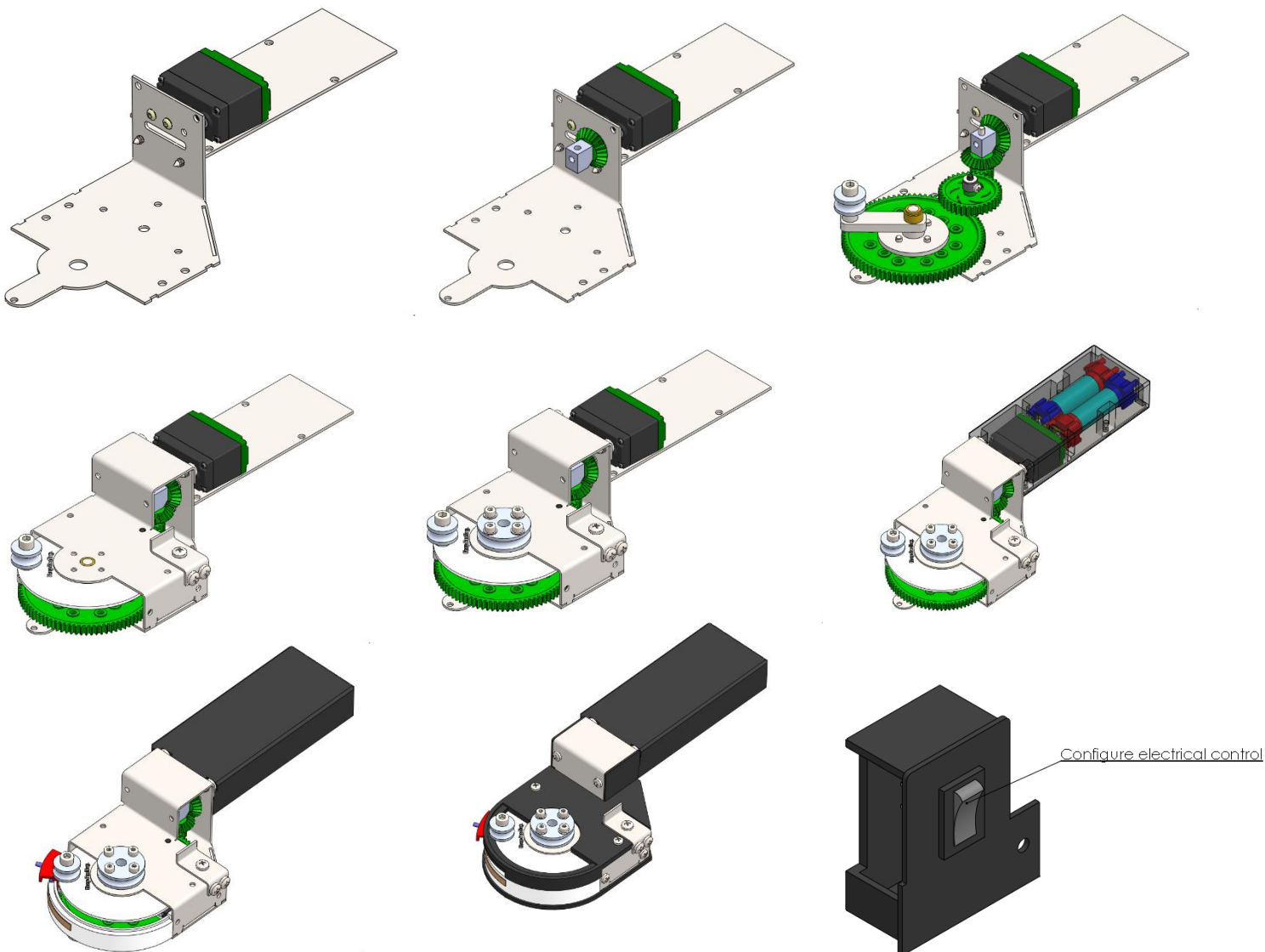


Figure 26: Overview of Assembly steps

Figure 26 shows a series of images that highlight the broad steps in the manufacturing steps. This shows that the individual components can be assembled in a reasonable manner. The alignment tabs used significantly aid the assembly process. Additionally, the plastic covers are all designed with lips that can be used to locate the covers quickly. The final step relates to the configure the electronics of

the device. The specifics are not considered in this design, but the plastic cover shown holds the space for electronics. The device is assembled with only requiring basic workshop tools that are readily available (screwdriver and allen key set).

2.7.2 Manufacturing

Table 12: Different manufacturing techniques used for the device

Manufacturing techniques	Parts
Machining (Lathe)	Custom shaft, Brass bushings, Roller arm spacer
Laser cutting (Sheet metal)	Sheet metal casing, Actuator Arm and Gear support plate
3D Printed	Plastic covers

The device makes use of 3 manufacturing techniques that are all readily used by the UCT workshop. The machining only requires the use of a lathe as all of the parts are cylindrical in nature. The machined parts will all require tolerance to ensure that there is proper clearance and allowance in the gearbox. None of the parts will require any additional surface treatments except for the cleaning of the 3D printed parts.

2.7.3 Ergonomic

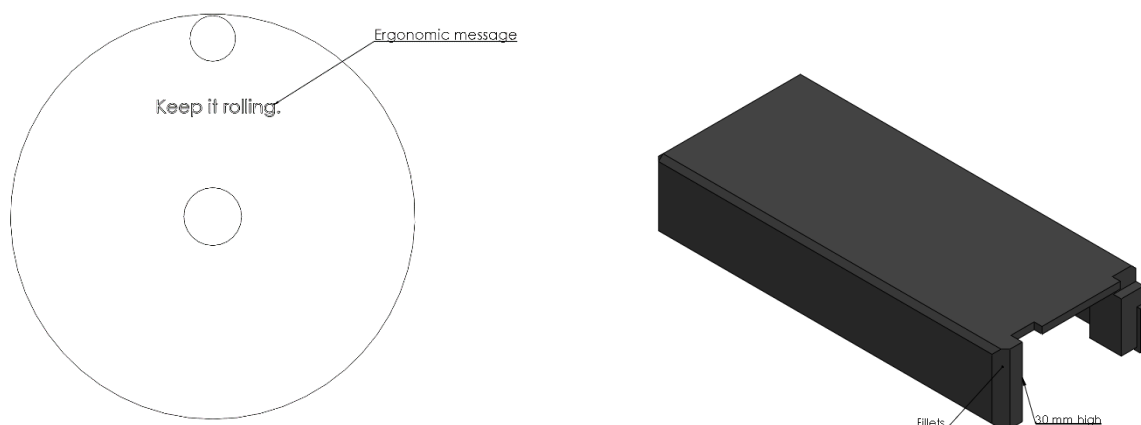


Figure 27: (Left) Ergonomic message on plastic cover (Right) Ergonomic considerations for the handle

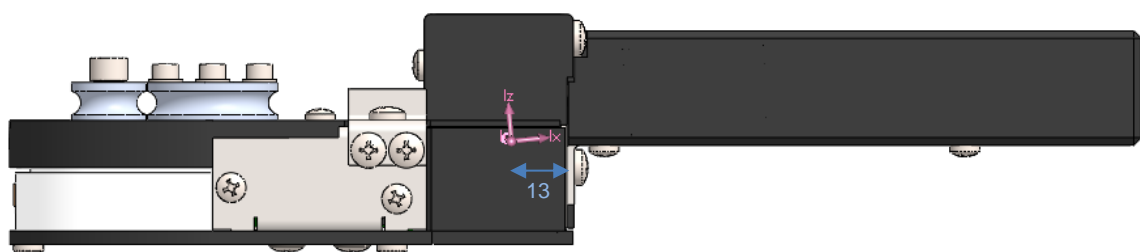


Figure 28: Centre of Mass of the device

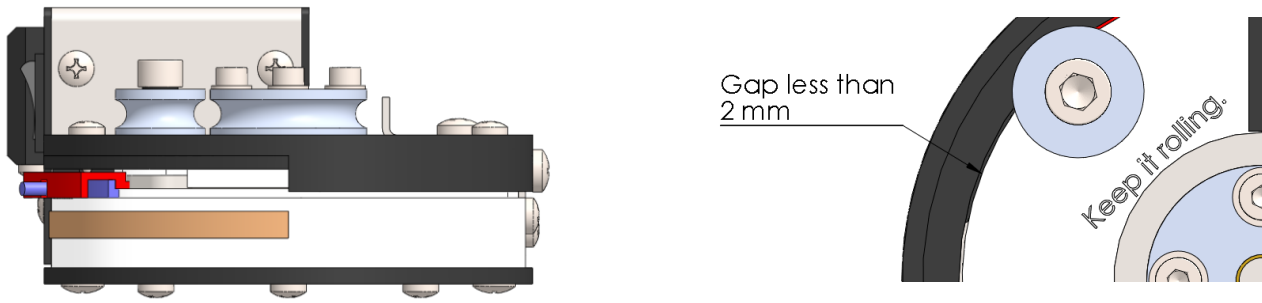


Figure 27: Plastic covers used to block the internals of the device

The plastic covers ensure that the device meets the ergonomic requirements of the URS. The functional purpose of the covers is to ensure that all gears and electronics are covered such that a road of 2mm diameter cannot contact these parts. **Figure 29** shows where covers are used to close block the internals of the device.

Additional ergonomic features such as fillets and messages on the Actuator cover fulfil the purpose of making the device aesthetically pleasing. **Figure 28** shows the COM of the device. The design tried to move the COM as close to the handle as possible. This aids in making the device easier to manoeuvre by the user. The COM is 13 mm away and the device has a mass of 1123 g meaning that there is bending moment of 0.15 Nm that the user has to resist. This is deemed acceptable as the handle has length of 147 mm. Assuming the user supports the device from the middle of the handle, only 2 N of force are required to balance this moment. This suggests that device will be able to be used with one hand by a 95% percentile adult.

2.8 URS Requirements Compliance table

Budget	
B1	Device costs R 2157
B2	Supplier price is less than target selling price
B3	BEP meet after 12 months of volume production
Constraints	
C1	Uses two cells
C2	All wires are contained in the device with feature to hold electrical components
C3	The bending is achieved by a mechanical arm powered by a motor
C4	Two limit switches are used angle control

Features	
F1	3 Way rocker switch is used for motor direction control
F2	There is clearance for a pipe that is 40 mm away from a wall (Figure 16)
F3	The offset between the big and small roller is sufficiently small to have a free end of minimum 60 mm (Figure 16)
F4	The device used a Die roller with inner radius of 15 mm to achieve the bend radius
Performance	
P1	Device is designed to bend 6mm pipes
P2	Angle control for a bend 45-to-90-degree bend is controlled with the limit switch
P3	The precision is purely dependent on the resolution of the sticker placed on the limit switch raceway and not a limit of the device.
P4	Device has a mass of 1.1 kg
P5	All critical parts have safety factors on yield
Safety and Ergonomics	
E1	Plastic covers hide the internals of the device to the user
E2	The thin handle and strategic COM position allows the device to be handled by a 95% percentile adult
E3	The COM position and handle shape allows the device to be easily manoeuvred by the user

3. Costing

3.1 Prototype costing

3.1.1 Prototype Costing Assumptions

- Costs for various materials are obtained from the provided 'Typical costing inputs' document provided.
- Masses of the individual parts are estimates generated by Solidworks the properties in the Solidworks material library.
 - 3D printed parts, made from PLA, use the ABS properties for mass estimates
 - Sheet Metal parts, made from Hot Rolled Mild Steel Plate, use Plain Carbon Steel for mass estimates
- Sheet metal cut lengths are obtained from the Solidworks sheet metal 'cutlist' feature.
- Machined parts:

Table 13: Machining time for various machined parts

Part	Equipment used	Time taken (min)
Spacers	Lathe	15
Shaft	Lathe	30

- All costs other than sheet metal parts are calculated using the per-unit normalized cost data of the material:

$$cost = (mass)(cost \text{ per } kg)$$

- Sundry costs:
 - Washers: 8 for 20 [4]
 - Hex screws: 146.59 for 50 [5]
 - Self-Tapping screws: 28.91 for 100 [6]

3.1.2 Costing Breakdown

A sample calculation is shown in the appendix (7.9) for the cost of a single sheet metal part shown in **Table 14**. The cost for the various parts in **Table 15** and **16** are calculated using the a simple cost formula:

Table 14: Sheet Metal parts cost breakdown

Part	Sheet Metal Cost				Cost (Rands)
	Mass (g)	Cut Length	Thickness (mm)	No. of Bends	
Bottom Plate	161.57	582.05	1.6	1	28.43
Top Plate	180.59	712.177	1.6	7	90.20
Support Plate	12.52	142.68	1.6	2	23.76
Battery Support Plate	148.08	450.97	1.6	1	24.24
Gear Support Plate	34.55	141.37	3	0	5.16
Actuator Arm	30.43	129.13	6	0	5.40
Total (Rands)					177.18

Table 15: 3D printed parts cost breakdown

Part	3D Printed Parts Cost	
	Mass (g)	Cost (incl. R 65 setup)
Plastic Handle	77.16	119.01
Bottom Cover	24.22	81.95
Top Cover	13.38	74.37
Switch Cover	14.33	75.03
Diagonal Cover	6.72	69.70
Tower Cover	3.34	67.34
Actuator cover	19.42	78.59
Limit switch trolley	2.47	66.73
Spring Plate	0.37	65.26
Raceway	15.02	75.51
Total (Rands)		773.50

Table 16: Machined parts cost breakdown

	Machined Parts					
Part	Mass (g)	Material	Material cost (/kg)	Machining time (mins)	Machining cost	Part cost (Rands)
Output shaft Lower Bush	4.98	Brass	155	15	75	75.77
Output shaft Upper Bush	4.22	Brass	155	15	75	75.65
Shaft 3 Lower bush	0.2	Brass	155	15	75	75.03
Custom Shaft	15.31	Plain Carbon Steel	25	30	150	150.38
Nylon Spacer	1.63	Nylon	92	15	75	75.15
Actuator Spacer	2.82	Plain Carbon Steel	25	15	75	75.07
				Total (Rands)	525	527.06

Table 17: Cost breakdown of sundry parts

Part	Sundry parts		
	Cost per unit (Rands)	Quantity	Part Cost (Rands)
ST2.2	0.29	4	1.16
ST4.2	0.29	14	4.05
ST4.8	0.29	11	3.18
M4 Washers	0.40	12	4.80
Socket Head M4x16	2.93	4	11.73
Socket Head M6x25	2.93	1	2.93
Total (Rands)			27.84

- Washers cost sourced from Gelmar [4]
- Socket head bolt cost sourced from Rs Components [5]
- Self-tapping screws cost sourced from AliExpress [6]
- Nylon cost sourced from Alibaba [7]

3.1.3 Bill of Materials

NO.	PART NUMBER	DESCRIPTION	SPECIFICATION	Mass	Cost	QTY
1.1	CLRDIN001-001	Bottom Plate	Hot Rolled Mild Steel Plate 1.6mm Sheet Metal	161.57	28.43	1
1.2	CLRDIN001-002	Top Plate	Hot Rolled Mild Steel Plate 1.6 mm thick	180.59	90.2	1
1.3	CLRDIN001-003	Support Plate	Hot Rolled Mild Steel Plate 1.6mm thick	12.52	23.76	1
1.4.1	MEC4124-030	Cell w Caps	Li-ion LFP 18650 cell with Vruzend connector caps	53.84	54	2
1.4.2	MEC4124-031	Cell link	0.5mm Plated Brass link (Vruzend)	1.02	1	1
1.4.3	MEC4124-032	Fused link	PCB with 5A resettable fuse	0.15	25	1
1.4.4	CLRDIN001-004	Plastic Handle	PLA 3D Printed	77.16	119.01	1
1.4.5	CLRDIN001-005	Battery Support Plate	Hot Rolled Mild Steel Plate 1.6mm Sheet Metal	148.08	24.24	1
1.5	CLRDIN001-006	Bottom Cover	PLA 3D Printed	24.22	81.95	1
1.6	CLRDIN001-007	Top Cover	PLA 3D Printed	13.38	74.37	1
1.7	CLRDIN001-008	Switch Cover	PLA 3D Printed	14.33	75.03	1
1.8	CLRDIN001-009	Diagonal Cover	PLA 3D Printed	6.72	69.7	1
1.9	CLRDIN001-010	Tower Cover	PLA 3D Printed	3.34	67.34	1
1.1	MEC4124-001	100rpm Motor	2-Wire Motor 393 (VEX 276-2177)	90	315	1
1.1.1	CLRDIN001-011	3 Way Rocker Switch	SPDT, 3 Pin Rocker Switch (Shalin Electronics)	3.59	15	1
1.1.2	MEC4124-006	6-32x1/2" screw	#6-32x1/2" button-head screw (VEX 275-1169)	1.279	3.5	2
2.1	MEC4124-010	12T Gear	12T spur gear, 24/"P (VEX 276-2169-001)	1.13	4.5	2
2.2	MEC4124-012	36T Gear	36T spur gear, 24/"P (VEX 276-2169-002)	5.54	18	1
2.3	MEC4124-015	84T Gear	84T spur gear, 24/"P (VEX 276-2169-004)	27.67	40.5	1
2.4	MEC4124-016	24T Bevel Gear	24T bevel gear, 24/"P (VEX 276-2184-001)	2.26	15	2
2.5	MEC4124-004-40	SQ shaft 40 long	Square Shaft 0.125" x 40mm (VEX 276-1149)	3.07	6.4	1
2.6	MEC4124-004-50	SQ shaft 50 long	Square Shaft 0.125" x 50mm (VEX 276-1149)	3.84	8	1
2.7	CLRDIN001-012	Nylon Spacer	Custom Nylon spacer	1.63	75.15	1
2.8	MEC4124-042	Spacer 9.5 long	3/8" OD x 0.375" Nylon Spacer (VEX 276-6340-003)	0.73	3	1
2.9	MEC4124-041	Spacer 6.4 long	3/8" OD x 0.25" Nylon Spacer (VEX 276-6340-002)	0.48	3	1
2.10.1	UCT-15005	Washer M4	ISO 7089 - 4.3 x 9 x 0.8THK, Carbon Steel Gr 5.8	0.306	0.8	2
2.11	CLRDIN001-013	Brass Bush	Lower Brass bush for Shaft 3. Outer diameter 6mm and Inner diameter 4.2mm	0.2	75	1
2.12	MEC4124-008-30	HS SQ shaft 30 long	Square Shaft 0.125" x 30mm, heat treated (VEX 276-1149)	2.3	4.8	1
3.1	CLRDIN001-014	Custom Shaft	Plain Carbon Steel. Outer diameter of 14mm	15.31	150.38	1
3.2	CLRDIN001-015	Actuator Arm	Hot Rolled Mild Steel Plate 6mm thick	30.43	5.4	1
3.3	CLRDIN001-016	Lower Bush	Brass Bush for custom shaft. 12 mm Outer diameter and 7mm Inner Diameter	4.98	75.77	1
3.4	CLRDIN001-017	Upper Bush	Brass bush for custom shaft. 12 mm Outer diameter and 7mm inner diameter	4.22	75.65	1
3.5	CLRDIN001-018	Output gear Support Plate	Hot Rolled Mild Steel Plate 3mm thick	34.55	5.16	1
3.6	MEC4124-051	Small roller set	15mm roller with bush and screw	6.05	32	1
3.7	CLRDIN001-019	Smaller roller spacer	Plain Carbon Steel spacer for small roller	2.82	75	1
3.8	CLRDIN001-020	Actuator Arm Cover	PLA 3D Printed	19.42	78.59	1
3.9	UCT-05073	Socket HD Bolt M6	ISO 4762 - M6 X 25 x 25, Carbon Steel Gr 8.8	1.098	2.93	1
4.1	CLRDIN001-021	Limit Switch Raceway	Circular PLA 3D printed raceway for limit switch trolley	15.02	75.51	1
4.2	MEC4124-036	Microswitch	Sub-Miniature Micro Switch 1C-SPDT(CO), 5A, 250VAC (Comm SS5GL111)	2.2	17	2
4.3.1	CLRDIN001-022	Limit Switch Trolley	PLA 3D Printed limit switch trolley	2.47	66.73	1

4.3.2	CLRDIN001-023	Plastic Spring Plate for limit switch trolley	PLA 3D Printed	0.37	65.26	1
4.3.3	CLRDIN001-024	Common pen spring used with the limit switch plate	Self-sourced pen spring. Solid height of 5mm and coiled diameter of 3.5mm	0.07	10	1
4.4	UCT-19502	Self tapping screw	DIN 7049 ST2.2 X 9.5, Carbon Steel Gr 8.8	0.2777	1.16	4
5	UCT-15005	Washer M4	ISO 7089 - 4.3 x 9 x 0.8THK, Carbon Steel Gr 5.8	0.306	3.2	8
6	MEC4124-052	Big roller	30mm roller, 6.5dia groove	22.31	65	1
7	UCT-05044	Socket HD Bolt M4	ISO 4762 - M4 X 16 x 16, Carbon Steel Gr 8.8	2.617	11.73	4
8	UCT-19516	Self tapping screw	DIN 7049 ST4.2 X 9.5, Carbon Steel Gr 8.8	1.404	4.05	14
9	UCT-19523	Self tapping screw	DIN 7049 ST4.8 X 9.5, Carbon Steel Gr 8.8	2.1783	3.18	11
10	UCT-19505	Self tapping crew	DIN 7049 ST4.8 X 9.5, Carbon Steel Gr 8.8	0.5259	0.58	2
11	UCT-05041	Socket HD Bolt M4	ISO 4762 - M4 X 8 x 8, Carbon Steel Gr 8.8	2.617	11.73	4
Total				1128.65	2157.69	

3.2 Production Costing

3.2.1 Production Costing assumptions

- Multiple covers can be machined with a single injection mould if the parts fit within the diagonal limit of the mould.
- Sheet Metal parts can be stamped where the cost of the die is 1.5 times the cost of the injection moulding tool.
- The custom shaft is the only part that will require machining where a fabrication of 1.5 is used since the shaft is simple.
- Copper bushings can be sourced from suppliers at 2 times material cost of copper.
- Sundry costs of washers and screws are assumed to remain constant.
- It is assumed that 5000 products will be produced. This is a reasonable assumption considering the large number of engineering applications that require pipe routing. The device is designed to be portable meaning that is likely for a company to buy more than one device.
- Indirect variable costs are neglected since they are difficult to accurately estimate.
- A 20 % increase is added to every part to cover for Indirect fixed costs such as rent.

3.2.2 Die Costs

Table 18: Costing of Sheet Metal moulds

Part	Sheet Metal Dies				
	Box length (mm)	Box width (mm)	Box diagonal (mm)	Base mould Cost (Rands)	Mould cost (Rands)
Top plate	178	106	208	52500	104416
Bottom plate	203	156	256	52500	116598
Gear Support	45	45	64	37500	53410
Pipe support					
battery Support	166	21	168	37500	79450
Total					353873

Table 19: Costing for Plastic injection moulding

Plastic Injection moulding					
Part	Box length (mm)	Box width (mm)	Box diagonal (mm)	Base mould Cost (Rands)	Mould cost (Rands)
Bottom Cover	148.5	109	184	35000	81052.45
Top cover	148.5	109	184	35000	81052.45
Side covers	132.3	52.18	142	25000	60554.573
Handle	151	69.13	166	35000	76518.036
				Total (Rands)	299177.51

3.2.3 Product cost

Table 20: Breakdown direct product costs for mass production

Part	Mass (g)	cost per part (Rands)*
Sheet Metal	567.74	14.19
Plastic for injection moulding	176.43	3.53
Brass machined	9.4	1.46
Plain Carbon Steel machined (incl. 1.5 factor)	18.13	0.68
Electronics		451.00
Gearbox		100.40
Sundry cost		27.84
Total (Rands)		599.10

Table 21: Cost of device for mass production

Product Cost		
Cost	Cost Category	Cost per device
Direct	Direct	598.87
Mould Costs	Indirect Fixed	130.61
Total		729.48
Fixed costs (20%)		145.90
Nett Cost		875.38

4. Development Plan

4.1 Project Costing Assumptions

Development Costs	R 160 000	2 Man-month engineering at R500/h over 3 months
Ramp-up costs	R 658 295	1 Prototype at R2145 and tooling at R656 050 over 3 months
Volume product cost per unit	R 875.38	See product costing
Sale price to distributor per product	R 1225.53	40 % markup. The target selling price is R1400. This allows a markup of 22% for suppliers
Steady state production/ Sales per month	277	5000 products will be produced and sold over a selling period of 1.5 years

4.2 Cash-Flow Schedule

Table 22: Cashflow analysis showing Break-even-point

Month	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Expenses [kR]	53	53	273	219	219	44	131	242	242	242	242	242	242	242	242	242	242	242	242	242	242	242	242	242
Development [kR]	53	53	53																					
Ramp-up [kR]			219	219	219																			
Production						44	131	242	242	242	242	242	242	242	242	242	242	242	242	242	242	242	242	242
Quantity						50	150	277	277	277	277	277	277	277	277	277	277	277	277	277	277	277	277	277
Income [kR]								245	339	339	339	339	339	339	339	339	339	339	339	339	339	339	339	339
Sales [kR]								245	339	339	339	339	339	339	339	339	339	339	339	339	339	339	339	339
Quantity								200	277	277	277	277	277	277	277	277	277	277	277	277	277	277	277	277
Nett Cost [kR]	-53	-107	-379	-599	-818	-862	-993	-990	-893	-796	-699	-602	-505	-408	-311	-214	-117	-19	78	175	272	369	466	563
Stock Inventory						50	200	277	277	277	277	277	277	277	277	277	277	277	277	277	277	277	277	277

The BEP occurs in the 19 months of the product life or **12** months after the start of volume production.

It was stated in the product assumptions that the selling period would be over 1.5 years. The green highlighted block shows that after 18 months of selling the product the nett income is estimated to be 563 kR.

5. Discussion

5.1 Risks

The main safety risks related to the device are to do with the moving actuator. The nature of the application requires a moving actuator to bend the pipe. This device has the small roller moving on the outside of the casing. This poses a risk of a user getting a finger caught in the device. A possible scenario where this could happen is when the user is adjusting the limit switch since it is in the vicinity of the actuator arm. The relatively high output torque means that this could cause serious injury.

The structural casing of the device is made of sheet metal with a mass of 1.12 kg. If the device is dropped from a user's hand it could cause injury if the person does not have covered shoes. This is a portable device therefore it is highly likely that the device can be carried to elevated positions above the ground for a particular application. This becomes a larger concern if the device is dropped from this elevated position.

The manufacturing techniques used are simple engineering activities and the parts are simple in design. This minimises the risk of the prototype not meeting the URS which was the most important constraint in Basic Design. Although, there are still concerns with this device that could lead to failure to meet URS requirements. Namely, if there is not enough clearance for parts to freely rotate and if the designed parts are not strong enough to withstand the forces.

5.2 Drawbacks and further modifications

The device has a length, width and height 304, 109 and 61mm respectively. This is relatively large considering the device is bending a 6mm pipe. Additionally, it may prove difficult to fit the device in a toolbox. The configuration of the device means that the user will hold the

The plastic covers are all fastened to the device using screws this adds time to the assembly process. A better solution might be to design the plastic covers with clips.

Possible modifications to the device:

- A safety device can be developed to ensure that the user cannot flip the rocker switch by accident which could result in injury.
- A material loop could be integrated with the handle to ensure the user cannot accidentally drop the device.
- A removable handle can be designed which allows the battery to detach in order to save space and fit in a toolbox.
- There are various areas on the sheet metal plates that are not being substantially loaded. This is especially apparent with the battery support plate. Cut outs could be made in this plate to reduce the device mass.

6. Conclusion

The basic design process shows that the designed device is both feasible and successful in meeting all the requirements in the URS. The approach used: of detailed engineering calculations and motivations based on engineering training and industry standards ensures that the information presented in the detail design report is an accurate reflection of the product in reality. The device makes use of a sleek design that centred around meeting the performance, budget and ergonomics requirements of the URS. Simple engineering manufacturing techniques were used with relatively simple individual components as this ensures less risk of the prototype failing.

7. Appendix

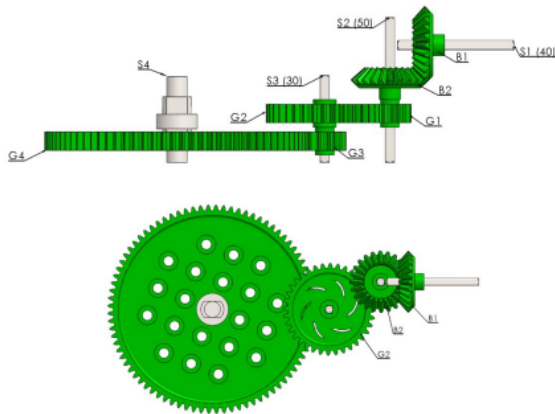
Assignment 2: Detailed Design

Dino Claro

CLRDIN001

-7.1 Sizing from basic design:

- 2 cells
- 100 rpm motor
- Gearbox transmission ratio: A three stage gearbox was proposed in basic design and was used in detail design. The image below shows the layout of the gearbox.



Gears		Shafts	
B1	24T	S1	40 mm
B2	24T	S2	50 mm
G1	12T	S3	30 mm
G2	36T	S4	Custom
G3	12T		
G4	84T		

$$Z_{B1} := 24 \quad Z_{B2} := 24$$

$$Z_{G1} := 12 \quad Z_{G2} := 36 \quad Z_{G3} := 12 \quad Z_{G4} := 84$$

$$r_{tot} := \left(\frac{Z_{B1}}{Z_{B2}} \right) \cdot \left(\frac{Z_{G2}}{Z_{G1}} \right) \cdot \left(\frac{Z_{G4}}{Z_{G3}} \right) = 21$$

7.2 Operating Conditions:

Using the orbital roller mechanism, the torque required by shaft 4 (S4) to bend the pipe is the torque that will initiate bending in the outer fibers if the pipe.

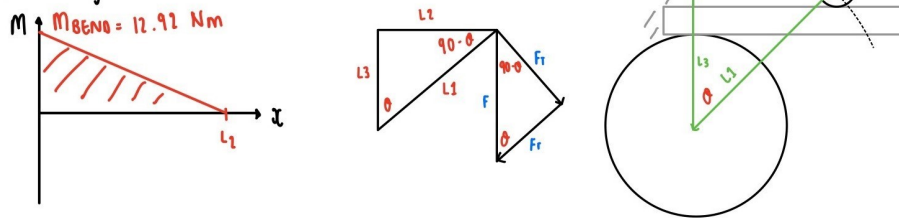
The torque required for bending, using the orbital roller mechanism, is constant throughout the bending process. This is shown below

$$\theta_y := 650 \text{ MPa} \quad d_o := 6 \text{ mm} \quad d_i := 3 \text{ mm} \quad y := 3 \text{ mm}$$

$$I := \left(\frac{\pi}{64} \right) (d_o^4 - d_i^4) = (5.964 \cdot 10^{-11}) \text{ m}^4$$

$$M_{yield} := \frac{\theta_y \cdot I}{y} = 12.922 \text{ N} \cdot \text{m}$$

Modelling as a clamped beam:



From triangles in the figure above:

$$M_{bend} = F \cdot L_2$$

$$F_t = F \cdot \cos(90 - \theta) = F \cdot \sin(\theta)$$

$$\sin(\theta) = \frac{L_2}{L_1}$$

$$T_{bend} = F_T \cdot L_1$$

$$= F \cdot \sin(\theta) \cdot L_1$$

$$= F \cdot \left(\frac{L_2}{L_1} \right) \cdot L_1$$

$$= \frac{M_{bend}}{L_2} \cdot \frac{L_2}{L_1} L_1$$

$$T_{bend} = M_{yield} = 12.922 \text{ N} \cdot \text{m}$$

This shows that there is effective **one operating condition** for the device since the torque remains constant.

The return trip of the roller arm is not analysed since there are no meaningful forces acting on the device.

7.3 Gearbox efficiency:

- Assuming the coefficient of kinetic friction of dry Acetal on Acetal is 0.2
- The gearbox shown above consists of three reduction stages. As a result there are three interfaces where efficiency must be applied.

$$\mu := 0.1$$

$$\eta_1 := 1 - \mu \cdot \pi \cdot \left(\frac{1}{24} + \frac{1}{24} \right) = 0.974$$

$$\eta_2 := 1 - \mu \cdot \pi \cdot \left(\frac{1}{12} + \frac{1}{36} \right) = 0.965$$

$$\eta_3 := 1 - \mu \cdot \pi \cdot \left(\frac{1}{12} + \frac{1}{84} \right) = 0.97$$

$$\eta_{tot} := \eta_1 \cdot \eta_2 \cdot \eta_3 = 0.912$$

7.4 Motor Operating Conditions:

As shown above there is only one operating condition. As a result there is **only one motor** operating condition

$$K_T := 0.0041 \, \text{N} \cdot \frac{\text{m}}{\text{A}} \quad I_{sat} := 1.2 \, \text{A} \quad r_g := 156.8 \quad \eta_g := 0.7 \quad T_o := 0.110 \, \text{N} \cdot \text{m}$$

$$V_b := 2 \cdot (3.2) \, \text{V} = 6.4 \, \text{V}$$

$$T_{bend} := 12.92 \, \text{N} \cdot \text{m}$$

$$T_m := \frac{T_{bend}}{\eta_{tot} \cdot r_{tot}} = 0.675 \, \text{N} \cdot \text{m}$$

$$I_m := \frac{T_m + T_o}{K_T \cdot r_g \cdot \eta_g} = 1.744 \, \text{A}$$

$$R_a := \left(1.61 + 0.15 \cdot \left(\left(\frac{I_m}{\text{A}} \right) - 1.2 \right) \right) \cdot \Omega = 1.692 \, \Omega$$

Since $I_m > I_{sat}$:

$$\omega_m := \frac{V_b - R_a \cdot I_m}{K_T \cdot r_g} = 5.366 \, \frac{\text{rad}}{\text{s}}$$

-It is important to check if a full 90 degree bend can be achieved before the motor tripping. The return trip of the actuator roller draws very little current which is deemed insignificant

$$t_{trip} := \left(140 \cdot \left(\frac{I_m}{\text{A}} \right)^{-3.54} \right) \cdot \text{s} = 19.547 \, \text{s}$$

$$\omega_{act} := \frac{\omega_m}{21} = 0.256 \, \frac{\text{rad}}{\text{s}}$$

$$t_{bend} := \frac{\pi}{2 \cdot \omega_{act}} = 6.147 \, \text{s}$$

$$t_{buffer} := t_{trip} - t_{bend} = 13.4 \, \text{s}$$

Since $t_{bend} < t_{trip}$ the motor should not trip.

$$P_m := T_m \cdot \omega_m = 3.621 \, \text{W}$$

7.5 Gearbox calculations:

- Pressure angle is 20 degrees for all gears
- Cone angle is 45 degrees for Bevel gears

$$\phi := 20 \cdot \frac{\pi}{180} \text{ rad} \quad \beta := 45 \cdot \frac{\pi}{180} \text{ rad}$$

Shaft 1:

$$T_{S1} := T_m = 0.675 \text{ N} \cdot \text{m} \quad (\text{Below } T_{max} = 3\text{Nm})$$

24T Bevel Gear 1 (B1):

$$r_{B1} := \frac{25.4 \text{ mm}}{2}$$

$$F_{t_{B1}} := \frac{T_{S1}}{r_{B1}} = 53.135 \text{ N} \quad (\text{Below } F_{T_{max}} = 300\text{N})$$

$$F_{r_{B1}} := F_{t_{B1}} \cdot \tan(\phi) \cdot \cos(\beta) = 13.675 \text{ N}$$

$$F_{a_{B1}} := F_{t_{B1}} \cdot \tan(\phi) \cdot \sin(\beta) = 13.675 \text{ N}$$

Shaft 2:

$$T_{S2} := T_{S1} \cdot \left(\frac{Z_{B2}}{Z_{B1}} \right) \cdot \eta_1 = 0.657 \text{ N} \cdot \text{m} \quad (\text{Below } T_{max} = 3\text{Nm})$$

24T Bevel Gear 2 (B2):

$$r_{B2} := \frac{25.4 \text{ mm}}{2}$$

$$F_{t_{B2}} := \frac{T_{S2}}{r_{B2}} = 51.744 \text{ N} \quad (\text{Below } F_{T_{max}} = 300\text{N})$$

$$F_{r_{B2}} := F_{t_{B2}} \cdot \tan(\phi) \cdot \cos(\beta) = 13.317 \text{ N}$$

$$F_{a_{B2}} := F_{t_{B2}} \cdot \tan(\phi) \cdot \sin(\beta) = 13.317 \text{ N}$$

12T spur gear (G1):

$$r_{G1} := \frac{12.7 \text{ mm}}{2}$$

$$F_{t_{G1}} := \frac{T_{S2}}{r_{G1}} = 103.489 \text{ N} \quad (\text{Below } F_{T_{max}} = 450\text{N})$$

$$F_{r_{G1}} := F_{t_{G1}} \cdot \tan(\phi) = 37.667 \text{ N}$$

Shaft 3:

$$T_{S3} := T_{S2} \cdot \left(\frac{Z_{G2}}{Z_{G1}} \right) \cdot \eta_2 = 1.903 \text{ N}\cdot\text{m} \quad (\text{Below } T_{max} = 3\text{Nm})$$

36T spur gear (G2):

$$r_{G2} := \frac{38.1 \text{ mm}}{2}$$

$$F_{t_{G2}} := \frac{T_{S2}}{r_{G2}} = 34.496 \text{ N} \quad (\text{Below } F_{T_{max}} = 700\text{N})$$

$$F_{r_{G2}} := F_{t_{G2}} \cdot \tan(\phi) = 12.556 \text{ N}$$

12T spur gear (G3):

$$r_{G3} := \frac{12.7 \text{ mm}}{2}$$

$$F_{t_{G3}} := \frac{T_{S3}}{r_{G3}} = 299.629 \text{ N} \quad (\text{Below } F_{T_{max}} = 450\text{N})$$

$$F_{r_{G3}} := F_{t_{G3}} \cdot \tan(\phi) = 109.056 \text{ N}$$

Shaft 4:

$$T_{S4} := T_{S3} \cdot \left(\frac{Z_{G4}}{Z_{G3}} \right) \cdot \eta_3 = 12.92 \text{ N}\cdot\text{m} \quad (\text{Above } T_{max} = 4.1 \text{ Nm})$$

84T spur gear (G4):

$$r_{G4} := \frac{88.9 \text{ mm}}{2}$$

$$F_{t_{G4}} := \frac{T_{S4}}{r_{G4}} = 290.664 \text{ N} \quad (\text{Below } F_{T_{max}} = 700\text{N})$$

$$F_{r_{G4}} := F_{t_{G4}} \cdot \tan(\phi) = 105.793 \text{ N}$$

Gear Reinforcement:

The thickness of the gear support is motivated by comparing the thickness of an equivalent support required for a steel plate for a 0.125 inch square

$$S_{y_{acetal}} := 66 \text{ MPa} \quad T_{max} := 4.1 \text{ N} \cdot \text{m} \quad t := 9.525 \text{ mm}$$

$$k := \frac{T_{max}}{S_{y_{acetal}} \cdot t} = (6.522 \cdot 10^{-6}) \text{ m}^2$$

$$t_{req_steel} := \frac{T_{S4}}{k \cdot 220 \cdot \text{MPa}} = 9.005 \text{ mm}$$

A steel plate would have to be 9mm thick to transmit 12.93 Nm of torque with the 0.125 inch shaft. The custom shaft support has much larger cross sectional area. The area ratio is calculated below

$$A := \frac{(7.1 \text{ mm})(7 \text{ mm})}{(0.125 \text{ in})^2} = 4.93$$

Assuming that area in contact is proportional to thickness required. The gear support plate thickness can be calculated:

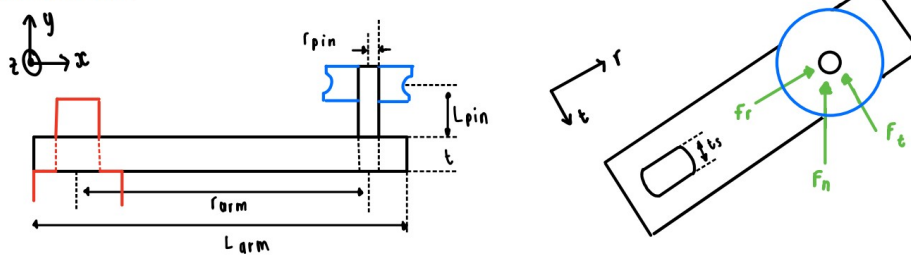
$$t_{plate} := \frac{t_{req_steel}}{A} = 1.826 \text{ mm}$$

7.6 Actuator Calculations:

The actuator arm experiences a reaction support from the pipe that resists bending. Since a roller is used the reaction force is simple a point normal force (F_n). This force can be described by a tangential (F_t) and radial force (F_r). The orbital roller mechanism is convenient in the sense that F_n stays constant throughout the bending process since the bending arm does not change. Additionally, the resistance from the pipe is assumed not to increase after yielding.

The simplified roller arm design is shown below. The force calculations will assume static equilibrium. This is a reasonable approximation since the angular velocity of the actuator arm is relatively small. The button screw is approximated as a pin.

Actuator arm



$$d_{pin} := 6 \text{ mm} \quad r_{arm} := 40 \text{ mm} \quad L_{arm} := 54 \text{ mm}$$

$$l_1 := r_{arm}$$

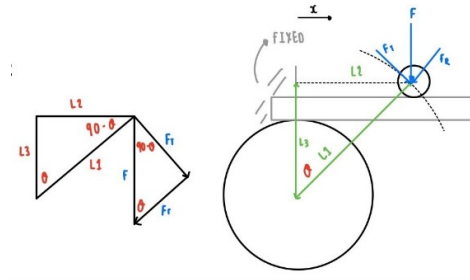
$$l_3 := 29 \text{ mm} \quad l_2 := \sqrt{l_1^2 - l_3^2} = 27.55 \text{ mm}$$

$$\theta := \arccos\left(\frac{l_3}{l_1}\right) = 0.76$$

$$F_n := \frac{T_{S4}}{l_2} = 468.966 \text{ N}$$

$$F_t := F_n \cdot \sin(\theta) = 323 \text{ N}$$

$$F_r := F_n \cdot \cos(\theta) = 340.001 \text{ N}$$



Considering the PIN:

Consider the maximum bending stress on the pin by finding the moment caused due to the normal force.

$$L_{pin} := 12.6 \text{ mm} \quad r_{pin} := \frac{d_{pin}}{2} = 3 \text{ mm}$$

$$F_{pin} := F_n = 468.966 \text{ N}$$

$$M_{pin} := F_n \cdot L_{pin} = 5.909 \text{ N} \cdot \text{m}$$

$$I_{pin} := \frac{\pi \cdot r_{pin}^4}{4} = (6.362 \cdot 10^{-11}) \text{ m}^4$$

$$\theta_{pin} := \frac{M_{pin} \cdot r_{pin}}{I_{pin}} = 278.65 \text{ MPa}$$

Consider the ARM:

$$t := 6 \text{ mm} \quad w := 14 \text{ mm} \quad S_y := 220 \text{ MPa}$$

- radial direction

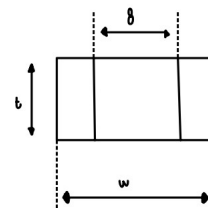
$$M_r := F_r \cdot L_{pin} = 4.284 \text{ N} \cdot \text{m}$$

Radial force causes a point moment

The cross-section of the arm at the shaft is considered.

$$I_r := \frac{w \cdot t^3}{12} - \frac{(8 \cdot \text{mm}) \cdot t^3}{12} = (1.08 \cdot 10^{-10}) \text{ m}^4$$

$$\theta_r := \frac{M_r \cdot \frac{t}{2}}{I_r} + \left(\frac{F_r}{t \cdot w - 8 \text{ mm} \cdot t} \right) = 128.445 \text{ MPa}$$



- Tangential direction

$$M_t := F_t \cdot l_1 = 12.92 \text{ N} \cdot \text{m}$$

$$I_t := \frac{L_{arm} \cdot t^3}{12}$$

$$\theta_t := \frac{M_t \cdot \frac{t}{2}}{I_t} = 39.877 \text{ MPa}$$

(J for a rectangular section)

$$J_{arm} := \frac{w \cdot t \cdot (t^2 + w^2)}{12} - \frac{(8 \cdot \text{mm}) \cdot t \cdot (t^2 + (8 \text{ mm})^2)}{12} = (1.224 \cdot 10^{-9}) \text{ m}^4$$

$$T_{arm} := F_t \cdot L_{pin} = 4.07 \text{ N} \cdot \text{m}$$

$$\tau_{arm} := \frac{T_{arm} \cdot \left(\frac{t}{2}\right)}{J_{arm}} = 9.975 \text{ MPa}$$

The approximation of the rectangular cross section is acceptable since the torsional shear stress are more than a factor of 10 smaller than the normal stresses.

Using maximum principal stress theory:

$$\theta_1 := \left(\frac{1}{2}\right) \left(\theta_r + \theta_t + \sqrt{(\theta_r - \theta_t)^2 + 4 \tau_{arm}^2}\right) = 129.554 \text{ MPa}$$

$$\theta_2 := \left(\frac{1}{2}\right) \left(\theta_r + \theta_t - \sqrt{(\theta_r - \theta_t)^2 + 4 \tau_{arm}^2}\right) = 38.767 \text{ MPa}$$

$$\theta_{vm} := \sqrt{\theta_1^2 - \theta_1 \cdot \theta_2 + \theta_2^2} = 115.173 \text{ MPa} \quad (\text{Von Mises equivalent stress})$$

$$N_{arm} := \frac{S_y}{\theta_{vm}} = 1.91$$

Torsional Deflection:

$$G := 79 \text{ GPa}$$

$$\phi_{arm} := \frac{r_{arm} \cdot T_{arm}}{G \cdot J_{arm}} = 0.096 \text{ deg}$$

Deflection due to Mr:

$$E := 210 \text{ GPa}$$

$$v_r := \frac{M_r \cdot r_{arm}^2}{2 \cdot E \cdot I_r} = 0.151 \text{ mm} \quad (\text{Deflection downwards})$$

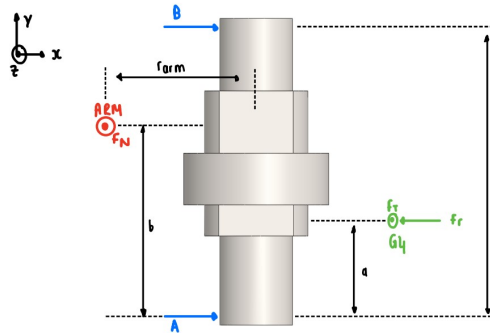
Deflection due to Mr:

$$v_t := \frac{M_r \cdot r_{arm}^2}{2 \cdot E \cdot I_t} = 0.017 \text{ mm} \quad (\text{Deflection in opposite direction to bending})$$

7.6 Custom Shaft Calculations

$$a := 9.8 \text{ mm} \quad b := 19.3 \text{ mm} \quad c := 28.5 \text{ mm}$$

$$S_{y_shaft} := 220 \text{ MPa}$$



The custom shaft was first designed with dimensions that suited the configuration of the gearbox. This analysis shows that the final dimensions used are adequate.

Shaft Reaction Forces due to Actuator arm:

F_{t_G4} always acts in the negative z and F_{r_G4} acts in the negative x direction. The worst loading case is approximated to be the position of the roller as bending starts. This is a fair assumption because the normal force acts purely in the negative z direction coinciding with F_{t_G4} which is the larger gear force component.

$$F_{Cz} := F_n = 468.966 \text{ N}$$

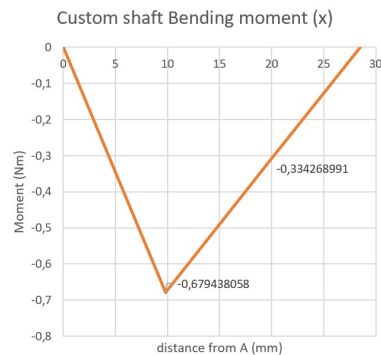
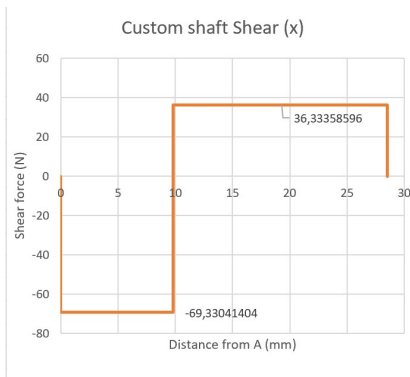
$$F_{Cx} := 0 \text{ N}$$

Shaft Support Reaction Forces

- x direction

$$F_{Ax} := \frac{a \cdot F_{r_G4}}{c} = 36.378 \text{ N} \quad (\text{Moments about b})$$

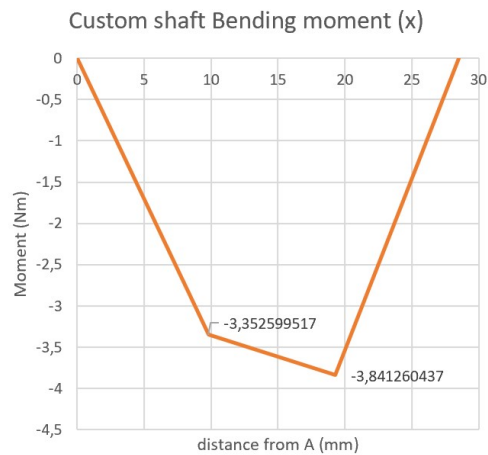
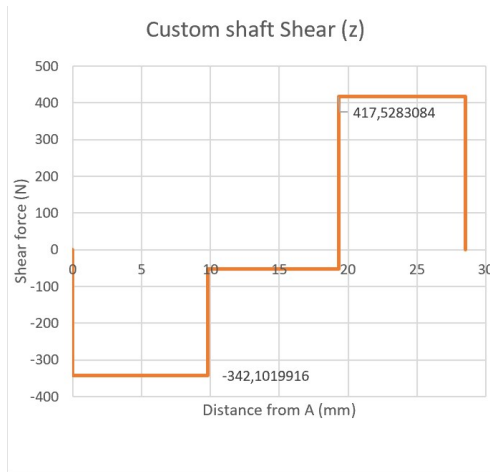
$$F_{Bx} := F_{r_G4} - F_{Ax} = 69.415 \text{ N} \quad (\text{Sum of forces})$$



- z direction

$$F_{Az} := \frac{a \cdot F_{t_{G4}} + b \cdot F_{Cz}}{c} = 417.528 \text{ N} \quad (\text{Moments about b})$$

$$F_{Bz} := -F_{Az} + F_{t_{G4}} + F_{Cz} = 342.102 \text{ N} \quad (\text{Sum of forces})$$



$$M_{Cx} := -0.679438058 \text{ N} \cdot \text{m}$$

$$M_{Dx} := -0.33426899 \text{ N} \cdot \text{m}$$

$$M_{Cz} := 3.352599517 \text{ N} \cdot \text{m}$$

$$M_{Dz} := 3.841260437 \text{ N} \cdot \text{m}$$

$$M_C := \sqrt{M_{Cx}^2 + M_{Cz}^2} = 3.421 \text{ N} \cdot \text{m}$$

$$M_D := \sqrt{M_{Dx}^2 + M_{Dz}^2} = 3.856 \text{ N} \cdot \text{m}$$

$$F_A := \sqrt{F_{Az}^2 + F_{Ax}^2} = 419.11 \text{ N}$$

$$F_B := \sqrt{F_{Bz}^2 + F_{Bx}^2} = 349.073 \text{ N}$$

The largest moment acts at point D. The torque is constant over section CD. Therefore, **Point D** is the most stressed point on the custom shaft.

Choosing the diameter of at A to be 7 mm, the safety factor on this section is found below

Let: $d_A := 7 \text{ mm}$

$$I_A := \frac{\pi \cdot (d_A)^4}{32} = (2.357 \cdot 10^{-10}) \text{ m}^4$$

$$\theta_{A_norm} := \frac{M_D \cdot \left(\frac{d_A}{2}\right)}{(I_A)} = 57.252 \text{ MPa}$$

$$J_A := \frac{\pi \cdot (d_A)^4}{16} = (4.714 \cdot 10^{-10}) \text{ m}^4$$

$$T_A := 0 \text{ N}\cdot\text{m} \quad (\text{Torque is zero at the supports})$$

$$\tau_A := \frac{T_A \cdot \left(\frac{d_A}{2}\right)}{J_A}$$

$$\theta_{A_1} := \left(\frac{1}{2}\right) \left(\theta_{A_{norm}} + \sqrt{\theta_{A_{norm}}^2 + 4 \tau_A^2}\right) = 57.252 \text{ MPa}$$

$$\theta_{A_2} := \left(\frac{1}{2}\right) \left(\theta_{A_{norm}} - \sqrt{\theta_{A_{norm}}^2 + 4 \tau_A^2}\right) = 0 \text{ MPa} \quad (\text{Von Mises equivalent stress})$$

$$\theta_{vm} := \sqrt{\theta_{A_1}^2 - \theta_{A_1} \cdot \theta_{A_2} + \theta_{A_2}^2} = 57.252 \text{ MPa}$$

$$N_A := \frac{S_{y_shaft}}{\theta_{vm}} = 3.843$$

This analysis can be repeated for the other sections of the shaft:

$$N_B := 4.331$$

$$N_C := 3.692$$

$$N_D := 3.65$$

The shaft is skimmed on the edges at C and D. The second moment of area would be smaller than the circular approximation made above. In reality, the maximum stresses will be higher at C and D. This should not be an issue since the safety factors are relatively large at the sections.

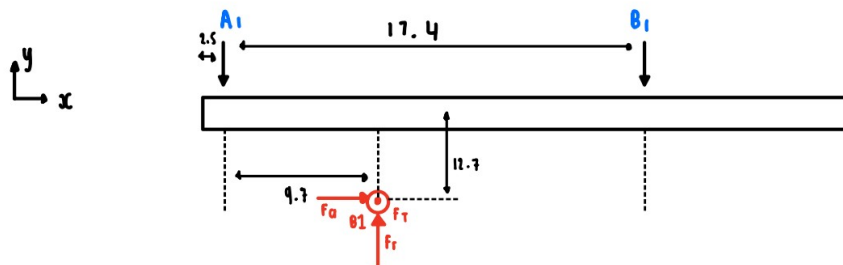
7.7 Intermediate Shaft Calculations:

Shaft 1, 2 and 3 can be made from the provided shaft the torque transmitted by the gears does not exceed neither maximum allowable torque of the shaft (5Nm). The main concern for the intermediate shafts is deflection due to the radial and tangential force.

$$t_{shaft} := 0.125 \text{ in} \quad S_{y_shaft} := 450 \text{ MPa} \quad S_{y_hard} := 650 \text{ MPa}$$

$$J_{shaft} := \frac{2 t_{shaft}^4}{12} = (1.694 \cdot 10^{-11}) \text{ m}^4 \quad I_{shaft} := \frac{t_{shaft}^4}{12} = (8.468 \cdot 10^{-12}) \text{ m}^4$$

Shaft 1:



Combining the radial and tangential of B1 into an effective force

$$F_{eff_S1} := \sqrt{F_{r_B1}^2 + F_{t_B1}^2} = 54.867 \text{ N}$$

$$a_{S1} := 9.7 \text{ mm} \quad L_{S1} := 17.4 \text{ mm} \quad b_{S1} := L_{S1} - a_{S1}$$

$$F_{a_S1} := \frac{F_{eff_S1} \cdot (L_{S1} - a_{S1})}{L_{S1}} = 24.28 \text{ N}$$

$$F_{b_S1} := F_{eff_S1} - F_{a_S1} = 30.587 \text{ N}$$

Strength:

$$M_{eff_S1} := F_{eff_S1} \cdot a_{S1} = 0.532 \text{ N} \cdot \text{m}$$

$$M_a := F_{a_B1} \cdot \left(\frac{25.4 \text{ mm}}{2} \right) = 0.174 \text{ N} \cdot \text{m} \quad \text{Point moment due to bevel axial force}$$

$$M_{norm_S1} := \sqrt{M_a^2 + M_{eff_S1}^2} = 0.56 \text{ N} \cdot \text{m}$$

$$\theta_{norm_S1} := \frac{M_{norm_S1} \cdot \frac{t_{shaft}}{2}}{I_{shaft}} = 104.949 \text{ MPa}$$

$$\tau_{S1} := \frac{T_{S1} \cdot \left(\frac{t_{shaft}}{2} \right)}{J_{shaft}} = 63.252 \text{ MPa}$$

$$\theta_{1_S1} := \left(\frac{1}{2} \right) \left(\theta_{norm_S1} + \sqrt{\theta_{norm_S1}^2 + 4 \tau_{S1}^2} \right) = 134.66 \text{ MPa}$$

$$\theta_{2_S1} := \left(\frac{1}{2} \right) \left(\theta_{norm_S1} - \sqrt{\theta_{norm_S1}^2 + 4 \tau_{S1}^2} \right) = -29.711 \text{ MPa (Von Mises equivalent stress)}$$

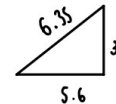
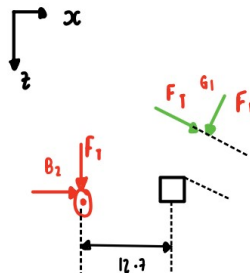
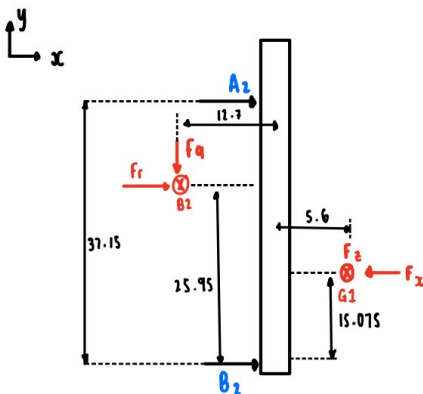
$$\theta_{vm_S1} := \sqrt{\theta_{1_S1}^2 - \theta_{2_S1} \cdot \theta_{1_S1} + \theta_{2_S1}^2} = 151.713 \text{ MPa}$$

$$N_{S1} := \frac{S_{y_shaft}}{\theta_{vm_S1}} = 2.966$$

Deflection:

$$v_{S1} := \frac{F_{eff_S1} \cdot a_{S1} \cdot b_{S1}}{6 \cdot E \cdot I_{shaft} \cdot L_{S1}} \left((L_{S1})^2 - (b_{S1})^2 - (a_{S1})^2 \right) = 0.003 \text{ mm}$$

Shaft 2:



$$L_{S2} := 37.15 \text{ mm}$$

The tangential and radial forces of the 12T gear do not coincide with the x,y and z plane described due to the offset of the shaft

$$\gamma := \arcsin\left(\frac{12}{25.4}\right) = 0.492$$

$$F_{x_{G1}} := F_{t_{G1}} \cdot \sin(\gamma) - F_{r_{G1}} \cdot \cos(\gamma) = 15.694 \text{ N}$$

$$F_{z_{G1}} := F_{t_{G1}} \cdot \sin(\gamma) + F_{r_{G1}} \cdot \cos(\gamma) = 82.09 \text{ N}$$

Finding the reaction forces:

X-direction:

$$F_{ax_{S2}} := \frac{F_{x_{G1}} \cdot (15.075 \text{ mm}) - F_{r_{B2}} \cdot (25.95 \text{ mm})}{L_{S2}} = -2.934 \text{ N}$$

$$F_{bx_{S2}} := -F_{ax_{S2}} - F_{x_{G1}} + F_{r_{B2}} = 0.557 \text{ N}$$

Z-direction:

$$F_{az_{S2}} := \frac{F_{z_{G1}} \cdot (15.075 \text{ mm}) + F_{t_{B2}} \cdot (25.95 \text{ mm})}{L_{S2}} = 69.456 \text{ N}$$

$$F_{bz_{S2}} := -F_{az_{S2}} + F_{z_{G1}} + F_{t_{B2}} = 64.379 \text{ N}$$

$$F_{a_{S2}} := \sqrt{F_{ax_{S2}}^2 + F_{az_{S2}}^2} = 69.518 \text{ N}$$

$$F_{b_{S2}} := \sqrt{F_{bx_{S2}}^2 + F_{bz_{S2}}^2} = 64.381 \text{ N}$$

Find a combined effective force:

$$F_{x_{eff}} := F_{x_{G1}} + F_{r_{B2}} = 29.011 \text{ N}$$

$$x_{eff_{S2}} := \frac{F_{bx_{S2}} \cdot L_{S2}}{F_{x_{eff}}} = 0.713 \text{ mm} \quad (\text{Distance from A})$$

$$F_{z_{eff}} := F_{z_{G1}} + F_{t_{B2}} = 133.835 \text{ N}$$

$$z_{eff_{S2}} := \frac{F_{bz_{S2}} \cdot L_{S2}}{F_{z_{eff}}} = 17.87 \text{ mm}$$

Strength:

- Finding the maximum moments in respective directions

$$M_{z_{S2}} := F_{bz_{S2}} \cdot 15.075 \text{ mm} = 0.971 \text{ N} \cdot \text{m}$$

$$M_{x_{S2}} := F_{bx_{S2}} \cdot 15.075 \text{ mm} = 0.008 \text{ N} \cdot \text{m}$$

$$M_{norm_{S2}} := \sqrt{M_{z_{S2}}^2 + M_{x_{S2}}^2} = 0.971 \text{ N} \cdot \text{m}$$

$$\theta_{norm_{S2}} := \frac{M_{norm_{S2}} \cdot \frac{t_{shaft}}{2}}{I_{shaft}} = 181.944 \text{ MPa}$$

$$\tau_{S2} := \frac{T_{S2} \cdot \left(\frac{t_{shaft}}{2} \right)}{J_{shaft}} = 61.597 \text{ MPa}$$

$$\theta_{1_{S2}} := \left(\frac{1}{2} \right) \left(\theta_{norm_{S2}} + \sqrt{\theta_{norm_{S2}}^2 + 4 \tau_{S2}^2} \right) = 200.836 \text{ MPa}$$

$$\theta_{2_{S2}} := \left(\frac{1}{2} \right) \left(\theta_{norm_{S2}} - \sqrt{\theta_{norm_{S2}}^2 + 4 \tau_{S2}^2} \right) = -18.892 \text{ MPa} \text{ (Von Mises equivalent stress)}$$

$$\theta_{vm_{S2}} := \sqrt{\theta_{1_{S2}}^2 - \theta_{2_{S2}} \cdot \theta_{1_{S2}} + \theta_{2_{S2}}^2} = 210.917 \text{ MPa}$$

$$N_{S2} := \frac{S_{y_{shaft}}}{\theta_{vm_{S2}}} = 2.134$$

Deflection in the respective directions:

-x direction

$$a_{S2_x} := x_{eff_{S2}} \quad b_{S2_x} := L_{S2} - a_{S2_x}$$

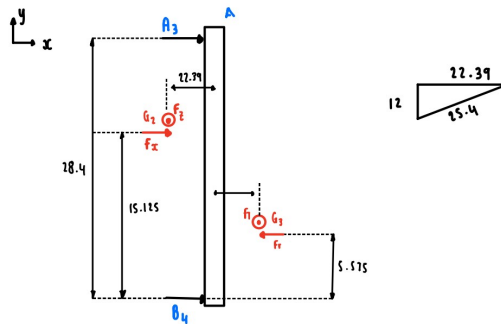
$$v_{s2_x} := \frac{F_{x_{eff}} \cdot a_{S2_x} \cdot b_{S2_x}}{6 \cdot E \cdot I_{shaft} \cdot L_{S2}} \left((L_{S2})^2 - (b_{S2_x})^2 - (a_{S2_x})^2 \right) = (9.882 \cdot 10^{-5}) \text{ mm}$$

- z direction

$$a_{S2_z} := z_{eff_{S2}} \quad b_{S2_z} := L_{S2} - a_{S2_z}$$

$$v_{s2_z} := \frac{F_{z_{eff}} \cdot a_{S2_z} \cdot b_{S2_z}}{6 \cdot E \cdot I_{shaft} \cdot L_{S2}} \left((L_{S2})^2 - (b_{S2_z})^2 - (a_{S2_z})^2 \right) = 0.08 \text{ mm}$$

Shaft 3:



$$L_{S3} := 28.4 \text{ mm}$$

The tangential and radial forces of the 12T gear do not coincide with the x,y and z plane described due to the offset of the shaft

$$\gamma := \arcsin\left(\frac{12}{25.4}\right) = 0.492$$

$$F_{x_{G2}} := F_{t_{G2}} \cdot \sin(\gamma) - F_{r_{G2}} \cdot \cos(\gamma) = 5.231 \text{ N}$$

$$F_{z_{G2}} := F_{t_{G2}} \cdot \sin(\gamma) + F_{r_{G2}} \cdot \cos(\gamma) = 27.363 \text{ N}$$

Finding the reaction forces:

X-direction:

$$F_{ax_{S3}} := \frac{-F_{x_{G2}} \cdot (15.125 \text{ mm}) + F_{r_{G3}} \cdot (5.575 \text{ mm})}{L_{S3}} = 18.622 \text{ N}$$

$$F_{bx_{S3}} := -F_{ax_{S3}} - F_{x_{G2}} + F_{r_{G3}} = 85.203 \text{ N}$$

Z-direction:

$$F_{az_{S3}} := \frac{F_{z_{G2}} \cdot (15.125 \text{ mm}) + F_{t_{G3}} \cdot (5.75 \text{ mm})}{L_{S3}} = 75.237 \text{ N}$$

$$F_{bz_{S3}} := -F_{az_{S3}} + F_{z_{G2}} + F_{t_{G3}} = 257.536 \text{ N}$$

$$F_{a_{S3}} := \sqrt{F_{ax_{S3}}^2 + F_{az_{S3}}^2} = 77.507 \text{ N}$$

$$F_{b_{S3}} := \sqrt{F_{bx_{S3}}^2 + F_{bz_{S3}}^2} = 271.265 \text{ N}$$

Find a combined effective force:

$$F_{x_{eff}} := F_{x_{G2}} - F_{r_{G3}} = -103.824 \text{ N}$$

(Distance from A)

$$x_{eff_S3} := \frac{-F_{bx_S3} \cdot L_{S3}}{F_{x_eff}} = 23.306 \text{ mm}$$

$$F_{z_eff} := F_{z_G2} + F_{t_G3} = 326.992 \text{ N}$$

$$z_{eff_S3} := \frac{F_{bz_S3} \cdot L_{S3}}{F_{z_eff}} = 22.368 \text{ mm}$$

Strength:

- Finding the maximum moments in respective directions

$$M_{z_S3} := F_{bz_S3} \cdot 5.525 \text{ mm} = 1.423 \text{ N} \cdot \text{m}$$

$$M_{x_S3} := F_{bx_S3} \cdot 5.525 \text{ mm} = 0.471 \text{ N} \cdot \text{m}$$

$$M_{norm_S3} := \sqrt{M_{z_S3}^2 + M_{x_S3}^2} = 1.499 \text{ N} \cdot \text{m}$$

$$\theta_{norm_S3} := \frac{M_{norm_S3} \cdot \frac{t_{shaft}}{2}}{I_{shaft}} = 280.961 \text{ MPa}$$

$$\tau_{S3} := \frac{T_{S3} \cdot \left(\frac{t_{shaft}}{2} \right)}{J_{shaft}} = 178.339 \text{ MPa}$$

$$\theta_{1_S3} := \left(\frac{1}{2} \right) \left(\theta_{norm_S3} + \sqrt{\theta_{norm_S3}^2 + 4 \tau_{S3}^2} \right) = 367.504 \text{ MPa}$$

$$\theta_{2_S3} := \left(\frac{1}{2} \right) \left(\theta_{norm_S3} - \sqrt{\theta_{norm_S3}^2 + 4 \tau_{S3}^2} \right) = -86.543 \text{ MPa}$$

$$\theta_{vm_S3} := \sqrt{\theta_{1_S3}^2 - \theta_{2_S3} \cdot \theta_{1_S3} + \theta_{2_S3}^2} = 417.557 \text{ MPa}$$

$$N_{S3} := \frac{S_{y_hard}}{\theta_{vm_S3}} = 1.557 \quad (\text{Note using 30mm hardened still})$$

Deflection in the respective directions:

-x direction

$$a_{S3_x} := x_{eff_S3} \quad b_{S3_x} := L_{S3} - a_{S3_x}$$

$$v_{S3_x} := \frac{F_{x_eff} \cdot a_{S3_x} \cdot b_{S3_x}}{6 \cdot E \cdot I_{shaft} \cdot L_{S3}} \left((L_{S3})^2 - (b_{S3_x})^2 - (a_{S3_x})^2 \right) = -0.01 \text{ mm}$$

- z direction

$$a_{S3_z} := z_{eff_S3}$$

$$b_{S3_z} := L_{S3} - a_{S3_z}$$

$$v_{S3_z} := \frac{F_{z_eff} \cdot a_{S3_z} \cdot b_{S3_z}}{6 \cdot E \cdot I_{shaft} \cdot L_{S3}} \left((L_{S3})^2 - (b_{S3_z})^2 - (a_{S3_z})^2 \right) = 0.039 \text{ mm}$$

7.8 Casing calculations:

- Stresses and deflection due to COM

$$L_{mg} := 13.62 \text{ mm}$$

$$m_{tot} := 1.119 \text{ kg}$$

$$g := 9.81 \text{ (m} \cdot \text{s}^{-2}\text{)}$$

$$t_{sheet} := 1.6 \text{ mm}$$

$$w_{sheet} := 47 \text{ mm}$$

$$L_{handel} := 147 \text{ mm}$$

$$I_{sheet} := \frac{w_{sheet} \cdot t_{sheet}^3}{12}$$

$$M_{rxn} := m_{tot} \cdot g \cdot L_{mg} = 0.15 \text{ N} \cdot \text{m}$$

$$\theta_{rxn} := \frac{M_{rxn} \cdot \left(1.6 \frac{\text{mm}}{2} \right)}{I_{sheet}} = 7.456 \text{ MPa}$$

Deflection:

$$v_{max} := \frac{M_{rxn} \cdot L_{handel}^2}{2 \cdot E \cdot I_{sheet}} = 0.479 \text{ mm}$$

7.9 Costing Calculations:

Sheet metal Parts:

Bottom plate

$$M := 0.1615 \text{ kg} \quad L := 0.58205 \text{ m} \quad t := 0.0016 \text{ m} \quad B := 1$$

$$Cost := 2.2 \cdot \left(\frac{M}{\text{kg}} \right) + (114) \left(\frac{L}{\text{m}} \right) \cdot \left(\frac{t}{\text{m}} \right)^{0.2} + 9.6 \cdot B = 28.265$$

8. References

- [1 Engineering Product Design, "Engineering Product Design," [Online]. Available:
] <https://engineeringproductdesign.com/knowledge-base/shaft-key-selection-keyway-design-guide/>.
 [Accessed 10 04 2023].
- [2 Fabory, Fabory, [Online]. Available: <https://www.fabory.com/en/hexagon-socket-head-cap-screw-iso-4762-steel-zinc-plated-8-8-m6x25/p/CM74234043>. [Accessed 10 04 2023].
- [3 Amesweb, Amesweb, [Online]. Available:
] <https://amesweb.info/HertzianContact/HertzianContact.aspx>. [Accessed 11 04 2023].
- [4 Gelmar, "Gelmar," [Online]. Available: <https://www.gelmar.co.za/flat-washers-m4-x-12mm-20-pieces.html>. [Accessed 11 04 2023].
- [5 RS za, "RS za," [Online]. Available: <https://za.rs-online.com/web/p/socket-screws/0281035>.
] [Accessed 10 04 2023].
- [6 Aliexpress, "Aliexpress," [Online]. Available:
] https://www.aliexpress.com/item/1005004030762920.html?spm=a2g0o.productlist.main.1.30ca1f64fEQc7W&algo_pvid=e6bdc44f-1bc0-4e8e-9b81-972037d4e465&algo_exp_id=e6bdc44f-1bc0-4e8e-9b81-972037d4e465-0&pdp_npi=3%40dis%21ZAR%2150.34%2127.68%21%21%21%21%21%40211b. [Accessed 11 04 2023].
- [7 Alibaba, "Alibaba," [Online]. Available: https://www.alibaba.com/product-detail/Cast-Nylon-sheet-Nylon-board-MC_1600494204764.html. [Accessed 11 04 2023].
- [8 Rabtron, "Rabtron," [Online]. Available: <https://www.rabtron.co.za/63-rocker-switches>. [Accessed
] 11 04 2023].
- [9 Matweb, [Online]. Available:
] <https://www.matweb.com/search/datasheet.aspx?bassnum=MS0001&ckck=1>. [Accessed 10 04 2023].