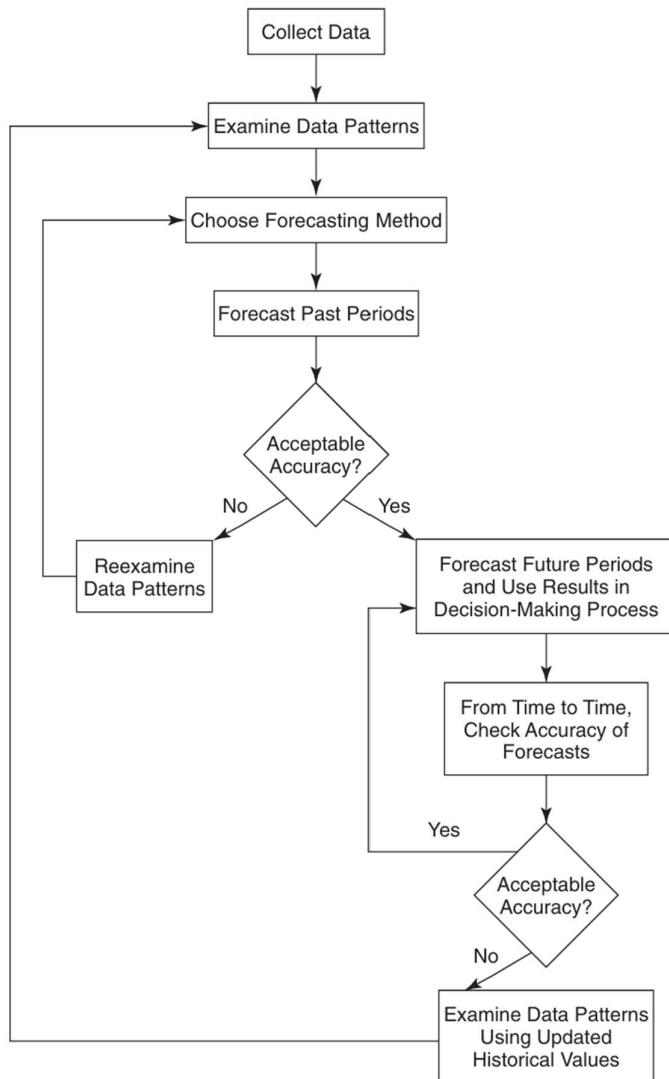


*Managing the Forecasting Process*



**FIGURE 1** The Operational Phase of the Forecasting Process

## Collect Data

1. Data should be reliable and accurate. Proper care must be taken that data are collected from a reliable source with proper attention given to accuracy.
2. Data should be relevant. The data must be representative of the circumstances for which they are being used.
3. Data should be consistent. When definitions concerning data collection change, adjustments need to be made to retain consistency in historical patterns. This can be a problem, for example, when government agencies change the mix or "market basket" used in determining a cost-of-living index. Years ago personal computers were not part of the mix of products being purchased by consumers; now they are.
4. Data should be timely. Data collected, summarized, and published on a timely basis will be of greatest value to the forecaster. There can be too little data (not enough history on which to base future outcomes) or too much data (data from irrelevant historical periods far in the past).

**Cross-sectional** data are observations collected at a single point in time.

**Time series** consists of data that are collected, recorded, or observed over successive increments of time.

**Dependent variable (Y)** → the thing you are trying to explain or predict.

**Independent variables (X)** → the factors you think explain changes in Y.

## Examine Data Patterns

### Data Patterns – Time Series

four general types of patterns

#### 1. Horizontal

When data collected over time fluctuate around a constant level or mean, a

Horizontal pattern exists.

- stationary

#### 2. Trend

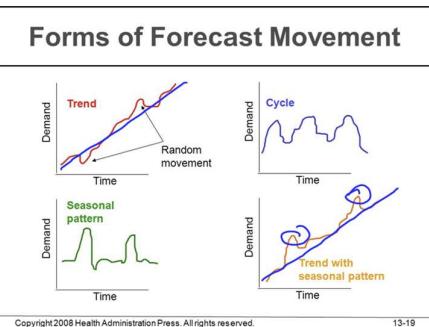
The trend is the long-term component that represents the growth or decline in the time series over an extended period of time.

#### 3. Seasonal

The seasonal component is a pattern of change that repeats itself year after year.

#### 4. Cyclical

The cyclical component is the wavelike fluctuation around the trend.



### Stationarity basics

A stationary time series has:

- Constant mean over time
- Constant variance over time
- Constant autocorrelation structure over time

So — the statistical properties of the series do not change as time passes.

### Seasonality and stationarity

- Seasonal data can be stationary, but only if the seasonal pattern is constant (i.e., repeats with the same amplitude and frequency over time).
- However, most real seasonal series are not stationary, because their mean changes periodically (e.g., sales are higher every December).

Example of stationary seasonal data:

A sine wave that repeats perfectly with the same amplitude and period forever.

Example of non-stationary seasonal data:

Sales that increase every year but also have seasonal peaks (trend + seasonality).

### Trend and stationarity

- If a series has a trend (upward or downward), it cannot be stationary, because the mean changes over time.
- A trending series violates the constant-mean condition.

Trend → Non-stationary.

After differencing (removing the trend), it can become stationary.

## Process for time Series Forecasting

### Convert data to a time series

	<pre>ts(data, start = c(year, period), frequency = n)</pre>
data	your numeric vector (e.g., sales, temperature, etc.)
start	(e.g., c(2020, 1))
	This tells R
	Data starts in January 2020
	This becomes important when:
	<ul style="list-style-type: none"><li>• Plotting (x-axis will show years properly)</li><li>• Using time-based models (like SARIMA, STL decomposition, forecasting)</li><li>• Doing seasonal differencing (<code>diff(y, lag = 12)</code>)</li></ul>
End	end is optional and rarely needed — R calculates it automatically from the length of your data and the start/frequency. You'd only specify end if: <ul style="list-style-type: none"><li>• Your data doesn't cover a full year or period</li><li>• You want to set a specific final date manually</li></ul>
End	(e.g., c(2023, 6)) this defines data to June 2023, even if the number of observations might suggest otherwise.
Frequency	Frequency = 1 Use when: No seasonality (annual data). Examples: <ul style="list-style-type: none"><li>• Yearly population</li><li>• Annual GDP</li><li>• Annual rainfall totals</li></ul> One data point per cycle → no repeating pattern.
	frequency = 4 Use when: Quarterly data with yearly seasonality. Examples: <ul style="list-style-type: none"><li>• Quarterly GDP</li><li>• Quarterly revenue</li><li>• Quarterly expenses</li></ul> 4 quarters per year.
	frequency = 12 Use when: Monthly data with yearly seasonality. Examples: <ul style="list-style-type: none"><li>• Monthly sales</li><li>• Monthly temperature</li><li>• Monthly expenses</li></ul> 12 months per year.

frequency = 52

Use when: Weekly data with yearly seasonality.

Examples:

- Weekly shipping volume
- Weekly demand
- Weekly energy generation

52 weeks per year.

frequency = 7

Use when: Daily data showing weekly seasonality.

Examples:

- Daily website visits (weekday vs weekend)
- Daily sales with a weekly cycle
- Daily foot traffic to stores

Weekly cycle = 7 days.

frequency = 365 (or 365.25)

Use when: Daily data showing yearly seasonality.

Examples:

- Temperature
- Rainfall
- Daylight hours
- Annual biological/medical cycles

Yearly cycle = 365 days.

frequency = 5

Use when: Daily data recorded only on weekdays (business-day data).

Examples:

- Stock market prices
- Stock trading volume
- Business-day call center volume
- Manufacturing output (Mon–Fri)
- Operational metrics without weekends

Weekly business cycle = 5 days.



make time series.R

## **Find out what the data is:**

### Autocorrelation Analysis

„Autocorrelation is the correlation between a variable lagged one or more time periods and itself.”

#### a) Plotting the Data

```
ts.plot(data)
```

Look for:

- Trend → general upward/downward movement
- Seasonality → repeating cycles (e.g., yearly peaks)
- Stationarity → fluctuations around a constant mean/variance

#### b) ACF and PACF

```
acf(data)
```

```
pacf(data)
```

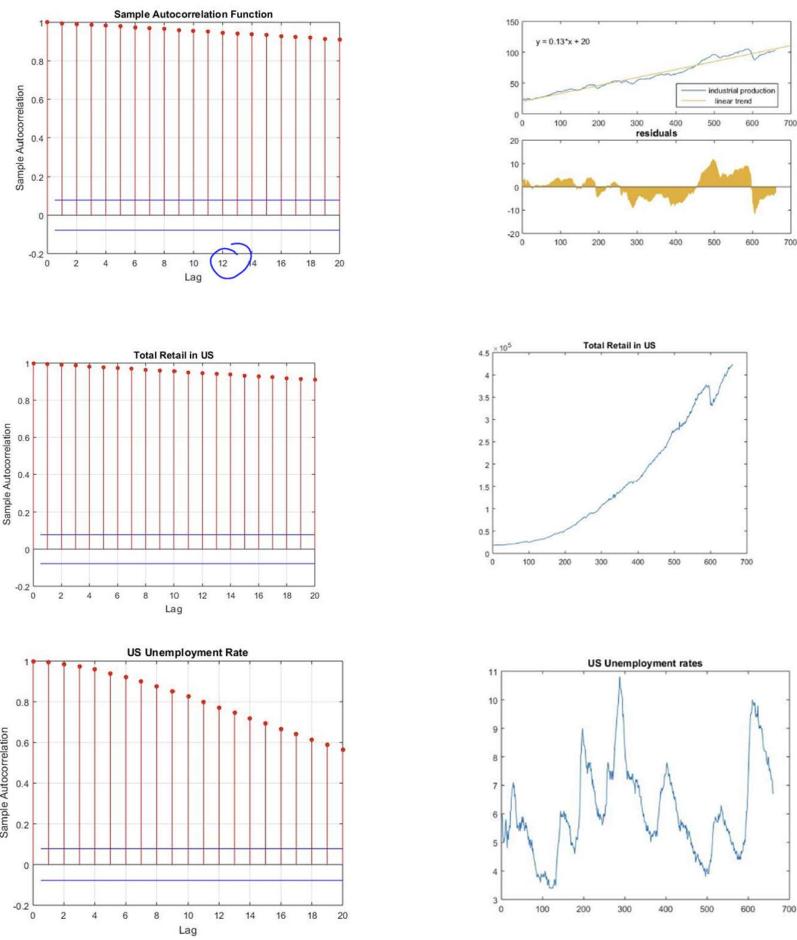
or together

```
library(ggplot2)
```

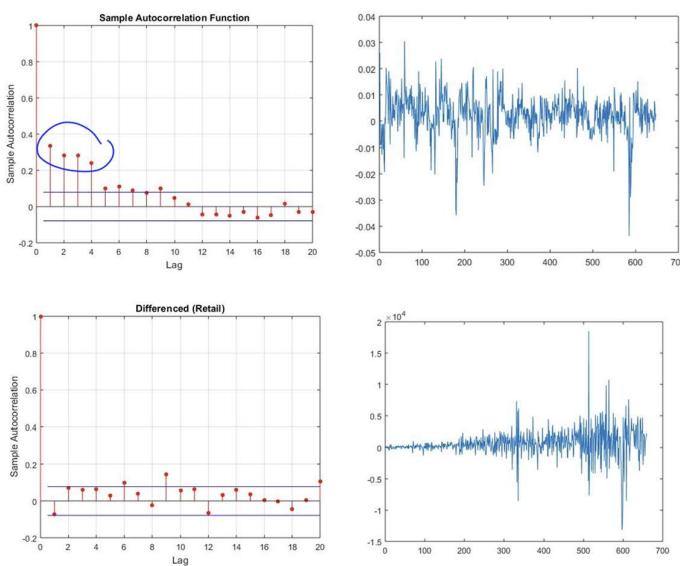
```
tsdisplay(x)
```

- Stationary Series
  - ACF behavior: Autocorrelations drop quickly toward zero as lag increases.
  - Interpretation: The series has no long-term trend; shocks are temporary.
  - Visual cue: Bars in the correlogram fall within the significance bounds after a few lags.
  - Example: White noise or detrended series.
- Trending (Nonstationary) Series
  - ACF behavior: Autocorrelations decay slowly or remain very high even at large lags.
  - Interpretation: The series has a trend or unit root; shocks have lasting effects.
  - Visual cue: Bars stay above the significance bounds for many lags.
  - Example: Raw GDP or cumulative sales data.
- Quick Rule of Thumb
  - Fast drop → stationary.
  - Slow decay / persistent high autocorrelation → trending / nonstationary.

## Trending Series



## Stationary Series



### c) ADF Test

Hypothesis	Meaning
$H_0$ (null)	Series has a unit root → non-stationary
$H_1$ (alternative)	Series does not have a unit root → stationary

- p-value < 0.05 → reject  $H_0$  → series is stationary
- p-value ≥ 0.05 → fail to reject  $H_0$  → series is non-stationary

Install and load package

```
install.packages("tseries")
library(tseries)
```

Run ADF test

```
adf_result <- adf.test(data_ts)
```

View result

```
adf_result
```

### d) KPSS Test (Kwiatkowski–Phillips–Schmidt–Shin)

$H_0$ (null)	The series is stationary
$H_1$ (alternative)	The series is non-stationary (has a unit root)

- $p < 0.05$  → the series is NOT stationary
- $p \geq 0.05$  → the series is stationary

Install and load package

```
install.packages("urca")
library(urca)
```

Run KPSS test

```
kpss_result <- ur.kpss(data_ts)
```

Get summary

```
summary(kpss_result)
```

### e) Decomposition

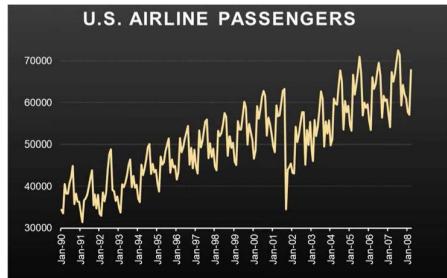
Method	When to use	Pros / Cons
Classical ( <code>decompose()</code> )	Regular series, fixed seasonal pattern, additive or multiplicative	Simple, easy to visualize; cannot handle changing seasonal patterns
STL ( <code>stl()</code> )	Flexible, can handle changing seasonal amplitude, robust trend	Handles outliers, seasonal component can vary over time; more flexible than classical
X-11 / SEATS / TBATS	Complex series: multiple seasonalities (daily + weekly + yearly), or long series with irregular cycles	TBATS can model multiple seasonal periods, non-integer frequencies; SEATS/X-11 used for official economic statistics

Classical

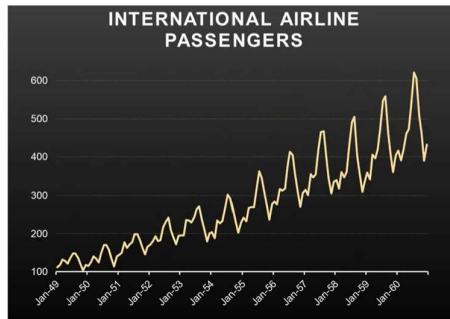
```
decompose(data_ts, type = c("additive", "multiplicative"), filter = NULL)
```

- Series has stable and regular seasonality.
- `decompose()` requires the time series to have a frequency > 1. Otherwise, seasonality cannot be computed.
- Data is not noisy and trend/seasonal patterns are simple.
- Quick and simple method for understanding data structure.

- Additive
  - Use when **seasonal fluctuations are roughly constant**, regardless of level



- Multiplicative
  - Use when seasonal fluctuations grow with the level of the series



## STL decomposition

```
library(forecast)
stl_decomp <- stl(data_ts, s.window="periodic")
```

Fixed seasonal pattern

```
stl_fixed <- stl(data_ts, s.window="periodic")
```

Flexible seasonal pattern

```
stl_flexible <- stl(data_ts, s.window=13)
```

- smooth over 13 months  
*plot(stl\_decomp)*
- More flexible, can handle changing seasonality
- Allows robust trend estimation
  - "periodic" → if the seasonal pattern does not change over time
  - Numeric → if the seasonal effect changes gradually (e.g., increasing holiday sales over years)

## SEATS decomposition

```
install.packages("seasonal")
library(seasonal)
```

Fit the X-11/SEATS model

```
x11_fit <- seas(data_ts, x11 = "")
```

- *seas()* is the main function.
- *x11 = ""* → tells the function to use the **X-11 decomposition method**.

View the results

```
summary(x11_fit)
plot(x11_fit)
```

- Plots the original series, trend, seasonal component, and remainder.

### Split the data into Train/Test

- Test set must be same length as forecast Horizon to evaluate later!!!
- Common rules of thumb:
  - 70–90% for training, 10–30% for testing
  - For example, if you have 2010–2025 data:
    - Training: 2010–2022
    - Testing: 2023–2025
- window() ensures the time continuity is preserved
- No random sampling!

Example: 2010–2025 monthly

```
sales_ts <- ts(sales, start=c(2010,1), frequency=12)
Train: 2010-2022
train_ts <- window(sales_ts, end=c(2022,12))
Test: 2023-2025
test_ts <- window(sales_ts, start=c(2023,1))
```

Example Yearly data

- Frequency = 1 (one observation per year)
  - Example: 1995–2005
    - # yearly time series
- ```
yearly_ts <- ts(data$Sales, start=1995, frequency=1)
# Split
train_ts <- window(yearly_ts, end=2002)
test_ts <- window(yearly_ts, start=2003)
```

- end and start are just years, no months/quarters.

Example Quarterly data

- Frequency = 4 (4 quarters per year)
  - Example: Q1 1995 → Q4 2000
    - # quarterly time series
- ```
quarterly_ts <- ts(data$Sales, start=c(1995,1), frequency=4)
# Split
train_ts <- window(quarterly_ts, end=c(2000,3)) # Q3 2000
test_ts <- window(quarterly_ts, start=c(2000,4)) # Q4 2000
```

Example Monthly Data

- Frequency = 12 (12 months per year)
  - Example: Jan 1995 → Dec 2000
    - # monthly time series
- ```
monthly_ts <- ts(data$Sales, start=c(1995,1), frequency=12)
# Split
```

```
train_ts <- window(monthly_ts, end=c(2000,10)) # Oct 2000  
test_ts <- window(monthly_ts, start=c(2000,11)) # Nov 2000
```

#### Example Daily data

- Frequency = 365 (or 7 for weekly if needed)
- Example: Jan 1, 1995 → Dec 31, 1997
  - # daily time series
  - daily\_ts <- ts(data\$Sales, start=c(1995,1), frequency=365)
  - # Split
  - train\_ts <- window(daily\_ts, end=c(1997,120)) # day 120 of 1997
  - test\_ts <- window(daily\_ts, start=c(1997,121)) # day 121 of 1997
- Daily data uses c(year, day\_number) → tricky, because window() counts periods in the year.
- For daily data, often it's easier to use row indices or tsibble/zoo objects.



First steps find out  
stationary or not and

## Choose Forecasting Method

TABLE 6 Choosing a Forecasting Technique

| Method                          | Pattern of Data | Time Horizon | Type of Model | Minimal Data Requirements |          |
|---------------------------------|-----------------|--------------|---------------|---------------------------|----------|
|                                 |                 |              |               | Nonseasonal               | Seasonal |
| Naive                           | ST, T, S        | S            | TS            | 1                         |          |
| Simple averages                 | ST              | S            | TS            | 30                        |          |
| Moving averages                 | ST              | S            | TS            | 4-20                      |          |
| Exponential smoothing           | ST              | S            | TS            | 2                         |          |
| Linear exponential smoothing    | T               | S            | TS            | 3                         |          |
| Quadratic exponential smoothing | T               | S            | TS            | 4                         |          |
| Seasonal exponential smoothing  | S               | S            | TS            |                           | 2 × s    |
| Adaptive filtering              | S               | S            | TS            |                           | 5 × s    |
| Simple regression               | T               | I            | C             | 10                        |          |
| Multiple regression             | C, S            | I            | C             | 10 × V                    |          |
| Classical decomposition         | S               | S            | TS            |                           | 5 × s    |
| Exponential trend models        | T               | I, L         | TS            | 10                        |          |
| S-curve fitting                 | T               | I, L         | TS            | 10                        |          |
| Gompertz models                 | T               | I, L         | TS            | 10                        |          |
| Growth curves                   | T               | I, L         | TS            | 10                        |          |
| Census X-12                     | S               | S            | TS            |                           | 6 × s    |
| Box-Jenkins                     | ST, T, C, S     | S            | TS            | 24                        | 3 × s    |
| Leading indicators              | C               | S            | C             | 24                        |          |
| Econometric models              | C               | S            | C             | 30                        |          |
| Time series multiple regression | T, S            | I, L         | C             |                           | 6 × s    |

Pattern of the data: ST: stationary; T: trending; S: seasonal; C: cyclical

Time horizon: S: short term (less than three months); I, intermediate term; L, long term

Type of model: TS: time series; C: causal

Seasonal s: length of seasonality

Variable: V: number of variables

Table: A Guide to Selecting an Appropriate Forecasting Method \*

| Forecasting Method    | Data Pattern                                            | Quantity of Historical Data (Number of Observations)            | Forecast Horizon     |
|-----------------------|---------------------------------------------------------|-----------------------------------------------------------------|----------------------|
| Naïve                 | Stationary                                              | 1 or 2                                                          | Very short           |
| Moving averages       | Stationary                                              | Number equal to the periods in the moving average               | Very short           |
| Exponential smoothing |                                                         |                                                                 |                      |
| Simple                | Stationary                                              | 5 to 10                                                         | Short                |
| Adaptive response     | Stationary                                              | 10 to 15                                                        | Short                |
| Holt's                | Linear trend                                            | 10 to 15                                                        | Short to medium      |
| Winters'              | Trend and seasonality                                   | At least 4 or 5 per season                                      | Short to medium      |
| Bass model            | S-curve                                                 | Small, 3 to 10                                                  | Medium to long       |
| Regression-based      |                                                         |                                                                 |                      |
| Trend                 | Linear and non-linear trend with or without seasonality | Minimum of 10 with 4 or 5 per season if seasonality is included | Short to medium      |
| Causal                | Can handle nearly all data patterns                     | Minimum of 10 per independent variable                          | Short, medium & long |
| TS decomposition      | Can handle trend, seasonal and cyclical patterns        | Enough to see two peaks and troughs in the cycle                | Short, medium & long |
| ARIMA                 | Stationary or transformed to stationary                 | Minimum 50                                                      | Short, medium & long |

### 1. Time-Series Forecasting (Univariate)

| Method                                     | Data Pattern                                   | Time Horizon      | Minimum Historical Data     | Notes                                   |
|--------------------------------------------|------------------------------------------------|-------------------|-----------------------------|-----------------------------------------|
| Naïve                                      | Stationary, trending, seasonal                 | Short (<3 months) | 1 point                     | Last value = forecast.                  |
| Simple Averages                            | Stationary                                     | Short             | 5-10 points                 | Mean of last N observations.            |
| Moving Averages                            | Stationary                                     | Short             | Window length               | Smooths noise.                          |
| Simple Exponential Smoothing (SES)         | Stationary                                     | Short             | 5-10 points                 | Weighted decaying average.              |
| Adaptive Response ES                       | Stationary                                     | Short             | 10-15 points                | Dynamic smoothing.                      |
| Adaptive Filtering                         | Seasonal                                       | Short             | 2 × (24 months for monthly) | Seasonal smoothing.                     |
| Holt's Linear                              | Trend                                          | Short-Medium      | 10-15 points                | Level + trend.                          |
| Holt-Winters (Winters)                     | Trend + Seasonal                               | Short-Medium      | 4-5 per season              | Additive or multiplicative seasonality. |
| Quadratic Exponential Smoothing            | Quadratic trend                                | Short             | ≥4 points                   | Captures curvature.                     |
| Seasonal ES (Holt-Winters variant)         | Seasonal                                       | Short             | 2 × s                       | Seasonal exponential smoothing.         |
| Bass Diffusion                             | S-curve                                        | Medium-Long       | 3-10 points                 | Product adoption.                       |
| Classical Decomposition                    | Seasonal                                       | Short             | 5 × s (60 monthly)          | Requires long stable seasonality.       |
| TS Decomposition (e.g., STL)               | Trend + Seasonal + Cyclical                    | Short-Long        | Enough for 2 peaks/troughs  | Resible decomposition.                  |
| Exponential Trend Models                   | Trend                                          | Medium-Long       | ≥10 points                  | Exponential growth patterns.            |
| S-Curve Fitting / Gompertz / Growth Curves | Trend                                          | Medium-Long       | ≥10 points                  | Logistic/S-curve growth.                |
| Box-Jenkins (ARIMA Framework)              | Stationary, trend (via differencing), seasonal | Short             | ≥24 points                  | Identification-oriented.                |
| ARMA                                       | Stationary with autocorrelation                | Short-Medium      | ≥50 points                  | Autoregressive + moving average.        |
| ARIMA                                      | Stationary (or transformed)                    | Short-Long        | ≥50 points                  | Trend handled via differencing.         |
| SARIMA                                     | Trend + Seasonality + autocorrelation          | Short-Medium      | ≥50 points                  | Seasonal ARIMA.                         |

## Classic decomposition and TS decomposition not for forecasting

### 2. Dynamic Time-Series Models (TS + Explanatory Variables)

| Method                               | Data Pattern                    | Time Horizon | Minimum Historical Data | Notes                                         |
|--------------------------------------|---------------------------------|--------------|-------------------------|-----------------------------------------------|
| ADL (Autoregressive Distributed Lag) | Stationary (or differenced)     | Short-Medium | $\geq 30$ observations  | Lags of Y and X included.                     |
| ARDL                                 | Mix of I(0)/I(1); cointegration | Short-Medium | 30-50 points            | Short-run + long-run effects.                 |
| VAR                                  | Multivariate stationary         | Short-Medium | 50-100+ points          | All variables depend on each other's lags.    |
| IRF                                  | Same as VAR                     | Diagnostics  | Same as VAR             | Shock-response tool, not a forecasting model. |

### 3. Regression-Based Forecasting (Causal + Time-Series Regression)

(This section is now fully integrated to reflect that regression can be used directly for time-series forecasting.)

| Regression Method               | Data Pattern                                        | Time Horizon | Min Historical Data                   | Time-Series Suitability                                         |
|---------------------------------|-----------------------------------------------------|--------------|---------------------------------------|-----------------------------------------------------------------|
| Simple Regression               | Trend                                               | Short-Long   | $\sim 10$ points                      | Often used to detrend or model structural trend.                |
| Multiple Regression             | Seasonal + Cyclical                                 | Short-Long   | $10 \times$ number of variables       | Supports seasonality via dummies; AIC/BIC selection.            |
| Time-Series Multiple Regression | Trend + Seasonal                                    | Medium-Long  | $6 \times s$ (e.g. 72 months monthly) | Deterministic seasonality + structural drivers.                 |
| Trend-Based Regression          | Linear/nonlinear trend, with or without seasonality | Short-Medium | $\geq 10 + 4-5$ per season            | Flexible trend modeling.                                        |
| Causal Regression (General)     | Nearly all types                                    | Short-Long   | $\geq 10$ per variable                | Broad causal/structural modeling framework.                     |
| Regression with Lags (ADL form) | Trends, seasonality, autocorrelation                | Short-Medium | $\geq 30$ points                      | Bridge between regression and time-series (dynamic regression). |
| Regression with ARIMA Errors    | Trend + Seasonal + stochastic residuals             | Short-Long   | $\geq 50$ points                      | Handles remaining autocorrelation after regression.             |
| Leading Indicator Models        | Cyclical predictors                                 | Short-Long   | $\geq 24$ points                      | Example: composite leading indicators (CLI).                    |
| Econometric Structural Models   | Cyclical, structural                                | Short-Long   | $\geq 30$ points                      | Theory-driven forecasts.                                        |

## Time series forecasting 1 Variable:

### Naive

- Stationary, Trending, Seasonal
- Time Horizon forecast: Short, less than 3 months
- Number of past observation: You only need one past data point (e.g., last month's value).
- Time Series



Naive Forecast.R

### Simple Averages

- Stationary
- Time Horizon forecast: Short, less than 3 months
- Time Series
- Number of past observation: 5 - 10 Observations (Example: Average of the last 6 months of sales used as forecast)



simple moving  
average in excel.xlsx

### Moving Averages

- Stationary
- Time Horizon forecast: Short, less than 3 months
- Time Series
- Number of past observation: equal to the periods in the moving average  
(Example: 3-month moving average → use last 3 months' values)
- How to know which Moving Average to use?

| Data Frequency | Example Data                      | Typical Seasonal Cycle | Suggested MA Windows | Forecast Horizon (h) | Explanation                                                             |
|----------------|-----------------------------------|------------------------|----------------------|----------------------|-------------------------------------------------------------------------|
| Daily          | Website visits (3 months)         | Weekly (7 days)        | 3, 5, 7              | 1–7 days             | MA3 → reacts quickly; MA5 → smooths ~workweek; MA7 → full weekly cycle  |
| Weekly         | Retail store weekly sales         | Monthly (~4 weeks)     | 2, 4, 6              | 1–4 weeks            | MA2 → fast reaction; MA4 → smooths 1 month; MA6 → slightly longer trend |
| Monthly        | Electricity consumption (3 years) | Yearly (12 months)     | 3, 6, 12             | 1–12 months          | MA3 → 3-month smoothing; MA6 → half-year; MA12 → full year              |
| Quarterly      | GDP over 5 years                  | Yearly (4 quarters)    | 2, 4, 6              | 1–4 quarters         | MA2 → half-year; MA4 → full-year; MA6 → 1.5 years                       |
| Yearly         | Population growth over 50 years   | Multi-year             | 2, 3, 5              | 1–5 years            | MA2 → 2-year smoothing; MA3 → medium; MA5 → long-term trend             |



Moving Averages.R

## Exponential smoothing

### Simple

- Stationary
- Time Horizon forecast: Short, less than 3 months
- Number of past observation: 5 - 10 historical Observations (example monthly data: Forecast for October uses weighted average of last 5–10 months)
- Time Series
- Beta true or False:

| Parameter                 | Meaning                                                                            | When to use                                                                                                                                       |
|---------------------------|------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>beta = FALSE</code> | No trend is included. Only the <b>level</b> of the series is smoothed.             | Use when your time series is <b>stationary</b> (no clear upward or downward trend). Example: monthly sales that fluctuate around a constant mean. |
| <code>beta = TRUE</code>  | Include a <b>trend component</b> . The model smooths both the level and the trend. | Use when your series shows a <b>linear or approximately linear trend</b> over time. Example: monthly revenue steadily increasing each month.      |



Simple Exponential  
smoothing.R

### Adaptive Response

- Stationary
- Time Horizon forecast: Short, less than 3 months
- Number of past observation: 10 - 15 historical Observations (example monthly data: Forecast for October uses weighted last 12 months)
- Time Series
- Which model ANN; AAN; AAA?

| Model | Level    | Trend    | Seasonality | Notes                                                                                                                                                         |
|-------|----------|----------|-------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------|
| ANN   | Additive | None     | None        | Only smooths the level of the series. Equivalent to <b>simple exponential smoothing (SES)</b> . Use when there is <b>no trend or seasonality</b> .            |
| AAN   | Additive | Additive | None        | Smooths level and trend. Equivalent to <b>Holt's linear method</b> . Use when series shows a <b>linear trend but no seasonality</b> .                         |
| AAA   | Additive | Additive | Additive    | Smooths level, trend, and seasonal component. Equivalent to <b>Holt-Winters additive</b> . Use when series shows both <b>trend and additive seasonality</b> . |



SES adaptive  
response.R

### Adaptive filtering

- Seasonality
- Time Horizon forecast: Short, less than 3 months
- Number of past observation:  $2 \times s$  Example if monthly ( $s = 12$ ) -  $2 \times 12 = 24$  months (2 years)
- Time Series



SES adaptive  
filtering.R

### Holts (linear exponential smoothing )

- Linear and Trend
- Time Horizon forecast: Short, less than 3 months to medium
- Number of past observation: 10 - 15 historical Observations (example monthly data: Forecast linear trend using last 12 months)
- Time Series



Holt's Linear Trend.R

### Quadratic exponential smoothing (Holt extended to quadratic trend)

- Trend
- Time Horizon forecast: Short, less than 3 months
- Number of past observation: At least 4 points to capture curvature. (example monthly data: Last 4 months enough to capture curvature)
- Time Series



Quadratic  
Exponential Smoother

### Seasonal exponential smoothing (Holt-Winters)

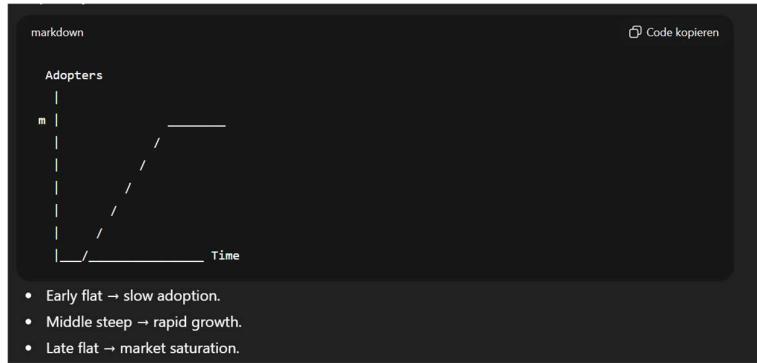
- Seasonal
- Time Horizon forecast: Short, less than 3 months
- Number of past observation:  $2 \times s$  Example if monthly ( $s = 12$ ) -  $2 \times 12 = 24$  months (2 years)
- Time Series



Seasonal exponential  
smoothing (Holt-Wint)

### Bass

- The Bass model is a specific type of S-curve designed for new product adoption.
- S curve
  - An **S-curve** is a **cumulative growth pattern** often seen in:
    - Product adoption
    - Technology diffusion
    - Market penetration
  - **Characteristics:**
    - **Slow start:** Early adopters → innovators begin adoption.
    - **Rapid growth:** Imitators accelerate adoption as more people adopt.
    - **Saturation:** Growth slows as market approaches maximum potential (m in Bass model).



- Time Horizon forecast: Medium to long
- Number of past observation: 3 - 10 historical Observations (example monthly data: Use first 6 months of product adoption to forecast next month)
- Time Series

## Exponential Trend models

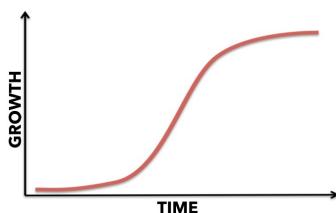
- Trend
- time series forecasting techniques that account for data that is growing or decaying at an accelerating rate
- Assumes the data grows/decays multiplicatively  
Example: 10, 12, 14.4, 17.28 → each period grows by **20% of the current value.**
- Time Horizon forecast: Medium to long
- Number of past observation: 10
- Time Series



Exponential Trend  
Forecast.R

## S-curve fitting

- Trend
- Time Horizon forecast: Medium to long
- An S-curve is a general term for any curve with a sigmoid (S-shaped) growth pattern.



- Number of past observation: 10
- Time Series



S-Curve - Logistic  
Growth Forecast.R

## Gompertz Models

- Trend
- It works well only for data that has an S-curve pattern.
- Time Horizon forecast: Medium to long
- Number of past observation: 10
- Time Series

```
How to estimate starting values from your dataset

1. a (asymptote / max value)
   • Usually slightly higher than the maximum of your observed data.
   • Example:
      r
      a_start <- max(train_ts) * 1.05 # 5% above max
      Code kopieren

2. b (displacement)
   • Determines how "shifted" the curve is.
   • If your data starts near zero: b = 1 is a reasonable default.
   • If your data starts already high, increase b.
      r
      b_start <- 1 # displacement
      Code kopieren

3. c (growth rate)
   • Higher values -- faster rise to the asymptote.
   • Lower values -- slower growth.
   • A good rule of thumb:
      r
      c_start <- 0.1 # slow growth
      c_start <- 0.5 # faster growth
      Code kopieren

   • Look at how fast your data increases; adjust accordingly.
```



Gompertz Forecast.R

## Box Jenkinson

- Stationary, Trend, Seasonality
- Time Horizon forecast: Short, less than 3 months
- Number of past observation: 24
- Time Series



Manual ARIMA or  
Box-Jenkins ARIMA.R

## ARMA

- Stationary, autocorrelation (AR + MA)
- If series is non-stationary, you must difference it (then it becomes ARIMA instead of pure ARMA)
- Time Horizon forecast: Short to medium
- Number of past observation: min 50.
- Only for stationary series; non-stationary → difference → ARIMA
- Time Series



ARMA.R

## ARIMA

- Stationary or transformed to stationary

- Time Horizon forecast: short, medium, long
- Number of past observation: min 50.
- Time Series

| When to use which                         |          |               |                                               |
|-------------------------------------------|----------|---------------|-----------------------------------------------|
| Scenario                                  | stepwise | approximation | Notes                                         |
| Small/medium dataset (<~500 observations) | FALSE    | FALSE         | Full, accurate search.                        |
| Large dataset (>~1000 observations)       | TRUE     | TRUE          | Faster search, may sacrifice slight accuracy. |
| Large dataset, want more accurate model   | FALSE    | TRUE          | Full search with approximate likelihoods.     |



Auto ARIMA.R

## SARIMA

- SARIMA only works if your ts() object has a seasonal frequency (frequency > 1)
- Trend, Seasonality, Autocorrelation (AR + MA + seasonal)
- Time Horizon forecast: Short to medium
- Number of past observation: min 50.
- Extends ARIMA to include seasonal effects; series may need differencing
- Time Series

| When to use which                         |          |               |                                               |
|-------------------------------------------|----------|---------------|-----------------------------------------------|
| Scenario                                  | stepwise | approximation | Notes                                         |
| Small/medium dataset (<~500 observations) | FALSE    | FALSE         | Full, accurate search.                        |
| Large dataset (>~1000 observations)       | TRUE     | TRUE          | Faster search, may sacrifice slight accuracy. |
| Large dataset, want more accurate model   | FALSE    | TRUE          | Full search with approximate likelihoods.     |



SARIMA.R

## 2 Variables:

First check if both variables are stationary or not

If BOTH variables are stationary ( $I(0)$ ).

This is the BEST case because ALL models are allowed.

If ONE variable is stationary ( $I(0)$ ) and the other is non-stationary

You MUST use ARDL/ADL models (BEST CHOICE)

ARDL was created exactly for situations where variables have mixed integration orders.

BOTH variables non-stationary ( $I(1)$ )

Test if variables are cointegrated

Phillips–Ouliaris Test (Engle–Granger)  
 Or Johansen Test (for  $>1$  x)  
 Allowed forecasting models when NOT cointegrated  
 Difference both, then use ADL/VAR/Regression  
**COINTEGRATED**  
 ECM  
 VECM  
 Use Static Regression on levels (Run a normal regression using the raw (non-differenced) values of Y and X.)



two variables, check  
if cointegrated and do

### Causal/ Static Regression Models

These models focus on the relationships between variables at a single point in time and do not require data to be a time series

#### Simple Regression

- Trend
- can use it to detrend or deseasonalise the data.
- For model selection, we minimize one of the following: Akaike information criterion (AIC), Bayesian information criterion (BIC)
- Number of past observation: 10



Simple Regression.R

#### Multiple Regression

- Cyclical and seasonal
- can use it to detrend or deseasonalise the data.
- The multiple regression setup works for time series data.
- For model selection, we minimize one of the following: Akaike information criterion (AIC), Bayesian information criterion (BIC)
- Number of past observation:  $10 \times V$



Multiple  
Regression.R

#### Dynamic Forecasting

Past: test importance → statistical evidence

#### ADL

- All variables must be stationary or made stationary

- Lagged dependent and independent variables (dynamic effects)
- Dependent variable: Usually one main variable that you are trying to forecast or explain.
  - Example: Monthly sales.
- Independent variables: Can be one or more predictors.
  - Example: Advertising spend, temperature, promotions.
- When to use it:
  - You have one main variable to forecast (univariate) or a few predictors (small multivariate).
  - You want to capture short-term dynamics — e.g., how past sales or marketing spend affects current sales.
  - Significant test

The outcome of the ADL model:

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 24.93273   7.67834   3.247 0.01234 **  
L(pass, 1)  -0.63624   0.03565 17.567 < 2e-16 ***  
L(pass, 2)  -0.08686   0.03479  2.497 0.012807 *    
L(pass, 3)  -0.09824   0.03505  2.803 0.005233 **  
L(pass, 4)  -0.12612   0.03466  3.639 0.000298 ***  
L(pass, 5)  -0.12286   0.03443  -3.568 0.000390 ***  
L(pass, 6)  -0.09465   0.03449  -0.986 0.321491    
L(pass, 7)  -0.64200   0.03499  -1.839 < 2e-16 ***  
L(pass, 8)  -0.49252   0.03513 -14.019 < 2e-16 ***  
s1          -0.59551   0.43060  -1.383 0.167218    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

If X has **no significant lags**, then X does not help explain Y, in that case do not use ADL

- Preferably stationary series (or you difference first).
- Time horizon: Short to medium.
- Data needed: At least 30 observations to estimate the lags reliably.
- Example: Forecasting monthly electricity demand using past electricity demand and past temperature.



ADLR

## ARDL

- If stationary: ARDL works like ADL.
- What it is: A generalization of ADL. Can handle variables that are a mix of stationary ( $I(0)$ ) and non-stationary ( $I(1)$ ).

**1 ARDL Stationarity Requirements**

- When people say:  
"ARDL: Can be I(0) or I(1), Cannot be I(2)"

This is about integration order:

| Term | Meaning                                           |
|------|---------------------------------------------------|
| I(0) | Stationary at level (no differencing needed)      |
| I(1) | Becomes stationary after first differencing       |
| I(2) | Becomes stationary only after second differencing |

- When to use it:
  - You want to model both short-run and long-run relationships.
  - Some variables are non-stationary but you still want to use levels rather than differencing everything.
- Often used in economics for cointegration analysis.
- Time horizon: Short to medium.
- Data needed: 30–50 observations minimum.
- Example: Modeling GDP growth with interest rates and investment, where some variables are trending over time



ARDL.R

## VAR

All variables must be stationary

- A multivariate time series model where every variable depends on its own past and the past of all other variables.
- When to use it:
  - You have multiple interdependent variables.
  - You want to capture how shocks or changes in one variable affect the others.
  - Series should be stationary (or differenced if non-stationary).
- Time horizon: Short to medium.
- Data needed: ≥50–100 observations (more if more variables or lags).
- Example: Forecasting GDP, inflation, and interest rates together; each depends on its own past and the past of the other two.



VAR.R

For dynamic forecasting and regression models use this Test

## ★ 1. Breusch–Pagan Test (BP test)

### ★ What it tests

Whether residuals have constant variance (homoscedasticity).

Null hypothesis:

- $H_0$ : Residuals are homoscedastic (constant variance)
- $H_1$ : They are heteroscedastic (variance changes)

### ★ Interpretation

- $p > 0.05 \rightarrow$  GOOD  
Residuals have constant variance  $\rightarrow$  OK
- $p < 0.05 \rightarrow$  BAD  
Heteroscedasticity  $\rightarrow$  not ideal but NOT fatal for forecasting.

### ★ When to proceed?

You can still use the model even if BP is significant.

Forecasting is robust to heteroscedasticity.

### ★ When to stop?

Only if variance changes are extreme or if you do structural analysis, not forecasting.

Conclusion:

BP test failing is not a model killer in ADL forecasting.

## ★ 2. Durbin–Watson Test / Breusch–Godfrey (Autocorrelation)

### ★ What it tests

Whether residuals are autocorrelated.

Null hypothesis:

- $H_0$ : No autocorrelation
- $H_1$ : Autocorrelation exists

### ★ Interpretation

- $p > 0.05 \rightarrow$  GOOD  
No autocorrelation  $\rightarrow$  OK
- $p < 0.05 \rightarrow$  VERY BAD  
There is autocorrelation  $\rightarrow$  model is invalid.

### ★ Why is autocorrelation so bad?

Because it means:

- the model misses lags
- the dynamic structure is wrong
- you did not include enough AR lags
- or X is missing important dynamics

This destroys the validity of ADL.

### ★ When to proceed?

Only when residual autocorrelation is NOT significant.

### ★ When to stop?

If autocorrelation exists  $\rightarrow$  STOP: Reject ADL.

Try:

- increase p\_range
- include more Y-lags
- or switch to ARIMA or VAR

Conclusion:

Autocorrelation is the most important diagnostic.

If it fails  $\rightarrow$  ADL is not acceptable.

## ★ 3. Jarque–Bera Normality Test

### ↗ What it tests

Whether residuals are normally distributed.

Null hypothesis:

- $H_0$ : Residuals are normal
- $H_1$ : Residuals are non-normal

### ↗ Interpretation

- $p > 0.05 \rightarrow \text{GOOD}$   
Residuals normal  $\rightarrow$  fine
- $p < 0.05 \rightarrow \text{NOT IDEAL}$   
Residuals non-normal  $\rightarrow$  BUT forecasting is still allowed.

### ↗ When to proceed?

→ You can still use the model for forecasting.

Normality is NOT required for time series forecasting.

(Only required for confidence intervals and hypothesis testing.)

### ↗ When to stop?

→ You do NOT stop because of JB test alone.

Conclusion:

JB failing is **not a reason** to reject the model for forecasting.

Forecast Combination:



combining forecast  
models.R

## Gliederung:

1. Introduction (Purpose of study, scope of study, sources and methods)
2. Analysis and findings
  - 2.1 -> is the data y variable or not, seasonal or trending explain how this was found out
  - 2.2 Decomposition -> y variable and explain what is seen
  - 2.3 Model 1 -> time series model explain what was done and how
  - 2.4 Model 2 -> time series model explain how it was done and outcome (ARIMA)
    - 2.4.1 Manuell ARIMA
    - 2.4.2 Auto ARIMA
  - 2.5 Model 3 -> Exponantial smoothing Method (Simple, HOlt, Holt winters etc.)
  - 2.6 explain variable x is stationary or not? transformation?  
Have a look at variable x  
if we have two variables.  
First check if both variables are stationary or not  
If BOTH variables are stationary ( $I(0)$ ).  
This is the BEST case because ALL models are allowed.  
if ONE variable is stationary ( $I(0)$ ) and the other is non-stationary  
You MUST use ARDL/ADL models (BEST CHOICE)  
ARDL was created exactly for situations where variables have mixed integration orders.  
BOTH variables non-stationary ( $I(1)$ )  
Test if variables are cointegrated  
Phillips–Ouliaris Test (Engle–Granger)  
Or Johansen Test (for  $>1$  x)  
Allowed forecasting models when NOT cointegrated  
Difference both, then use ADL/VAR/Regression  
COINTEGRATED  
ECM  
VECM  
Use Static Regression on levels-> Test Breusch-Pagan,  
Durbin-Watson and Jarque-Bera tests -> concluded that there are big problems with this regression and that the results of it are not trustworthy
  - 2.7 Model 4 -> Causal/ Static Regression Models ->
  - 2.8 Model 5 -> Dynamic Forecasting (VAR and ADL)

- 2.8 Comparasion of the models and combination models -> compare all model RMSE etc. -> combination of models equal weights and Granger-Ramathan method  
-> compare RMSE etc.
3. Conclusions -> show final model and plot
4. Recommendations