SNR evolution project report

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Contents

1 Introduction

In this exercise, we simulate the evolution of a Supernova Remnant (SNR) following a supernova explosion that injects an energy $E_0 = 10^{51}$ erg into the surrounding interstellar medium (ISM).

This generates a strong shock wave propagating outward, described by the Sedov-Taylor solution, where the shock radius R_{shock} increases with time t as $R_{\text{shock}} \propto t^{2/5}$.

Our task is to follow the evolution of the SNR over time, tracking the shock radius, temperature, and other physical parameters.

2 Code overview

2.1 Grid Setup

We set up a uniformly spaced spherical coordinate grid extending from 0 to approximately 70 parsecs, with 500 grid points. The cell size is defined as:

$$\Delta x = \frac{R_{\text{max}}}{N_{\text{grid}}}$$

where R_{max} is the maximum radius of the grid and N_{grid} is the number of grid points.

The grid points are defined such that the first point is negative (used only for boundary conditions), with the second point corresponding to zero.

2.2 Initial Conditions

We assume the SN explodes in an ionized Galactic ISM with a density $\rho_0 = 2{\times}10^{-24}$ g/cm$^{\mathbf{3}}$ and a temperature $T_0 = 10^4$ K. The energy density $e(i)$ and pressure $p(i)$ are derived using the relations:

$$e(i) = c_v \rho(i) T(i)$$

$$p(i) = (\gamma - 1) e(i)$$

where $c_v \approx 2 \times 10^8$ in cgs units and $\gamma = 5/3$ for ionized gas.

2.3 Supernova Energy Injection

The SN injects 10^{51} erg of thermal energy into the first two active grid points within a small volume. The energy density in grid points 2 and 3 is updated to:

$$e(2) = e(3) = \frac{E_0}{\frac{4}{3}\pi (xa(4))^3}$$

The pressure and temperature in these points are recalculated accordingly.

2.4 Boundary Conditions

The boundary conditions are set to ensure stability and accuracy in the numerical simulation, with special handling for the first (negative) grid point.

3 Results

3.1 Evolution with no COOLING until $t = 10^5$ years

The code seems to work pretty well when no cooling is applied. The shock radius increases with time as expected. The **density** behind the shock decreases with time, as the **temperature**

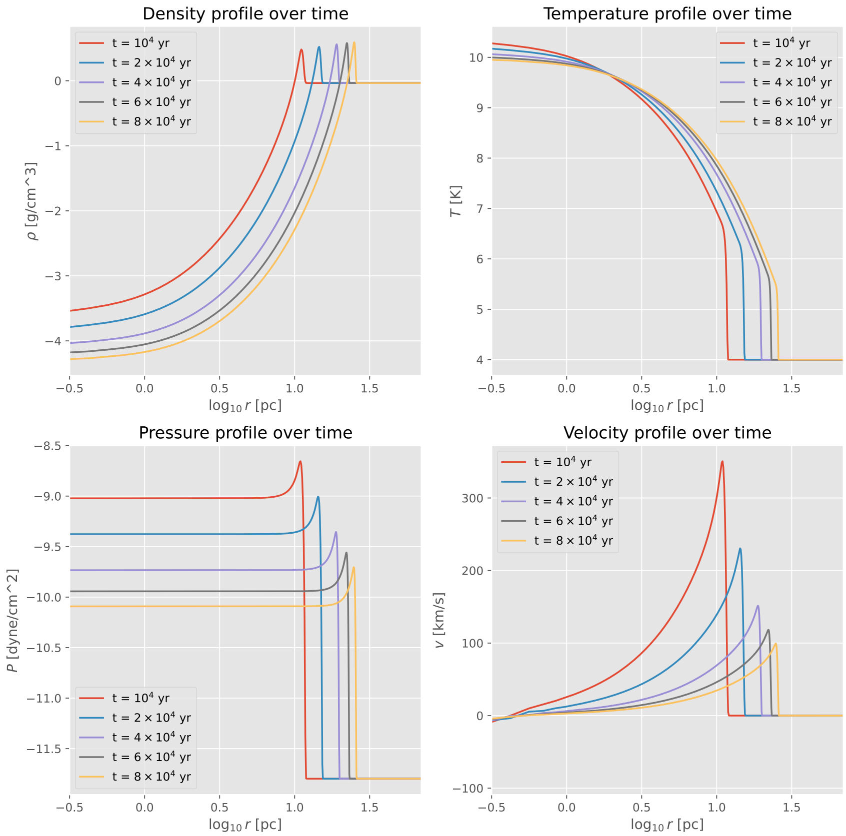


Figure 1: Evolution of the SNR without cooling until $t = 10^5$ years

3.2 Evolution with NO COOLING until $t = 5 \times 10^5$ years

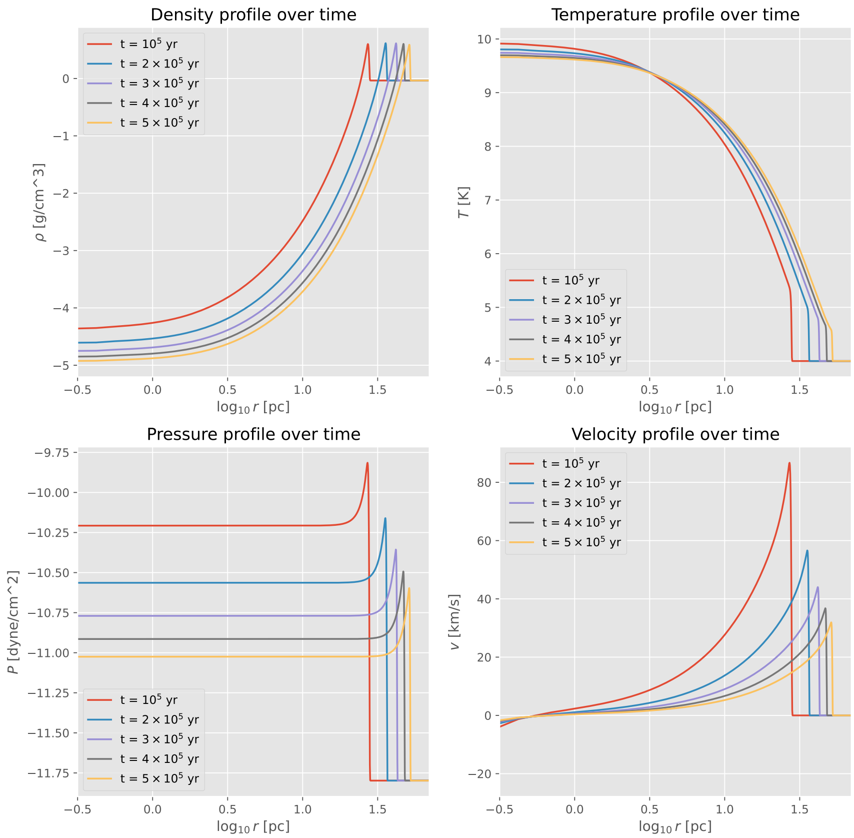


Figure 2: Evolution of the SNR without cooling until $t = 5 \times 10^5$ years

3.3 R_{shock} evolution in time. Sedov solution

To check if the simulation results agree with Sedov solution, we'll need to:

- Extract the position of the shock front at different times
- Plot the position of the shock front as a function of time
- Compare the results with the Sedov solution

The Sedov solution is given by:

$$R_{\text{shock}} \approx \left(\frac{E_0}{\rho_0}\right)^{1/5} t^{2/5}$$

To find the position of the shock front at different times we have used the following code:

```
DO i=2, jmax
IF(d(i) == maxval(d)) THEN
   write(*,*) "SHOCK POSITION", xa(i)/pc
```

```
      write(*,*) "SHOCK VELOCITY", v(i)/100000.0  !in km/s
      write(37,*) t/yr, xb(i)/pc, v(i)/100000.0
   END IF
END DO
```

Which basically finds the maximum density in the grid and prints the position of the shock front. It works fine for not too large times until the shock front approaches the boundary of the grid, where it starts to oscillate.

The results are plotted in the following figure:

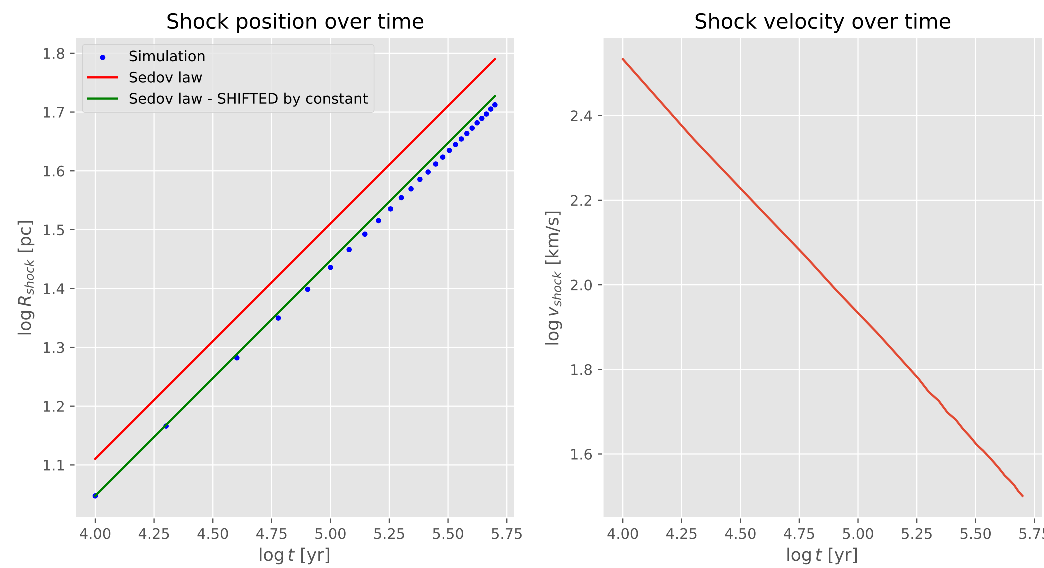


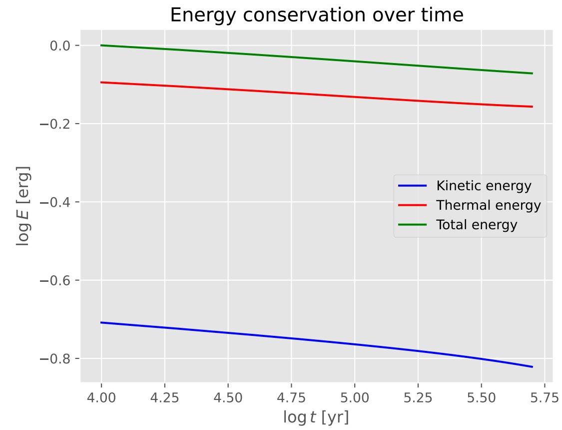
Figure 3: Sedov solution vs simulation results. The green line is the Sedov solution translated to match the first point with the first point of the simulation.

3.3.1 Energy conservation

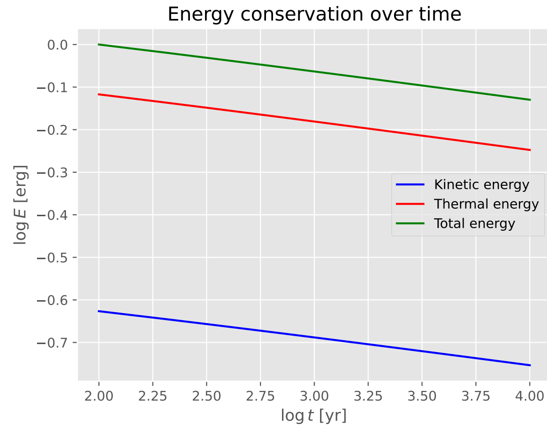
To check if the energy is conserved in the simulation, we have plotted the total energy in the system as a function of time. The total energy is given by the sum of the thermal energy and the kinetic energy of the gas:

$$E_{\text{total}} = \sum_i \left(e(i) + \frac{1}{2}\rho(i)v(i)^2 \right) \Delta x^3$$

The results are shown in the following figure:



(a) Ratio of kinetic and thermal energy wrt total energy. N=500



(b) Ratio of kinetic and thermal energy wrt total energy. N=5000. The time is lower than in the previous plot. (The code is not optimized for large grids and long times)

As we can see the energy is not perfectly conserved, but the error is not very large.

3.3.2 Luminosity in the X-ray band

Gas with temperatures $T \geq 10^6$ K is considered an X-ray emitter. So when the temperature of the gas is above this threshold, the code should calculate the X-ray luminosity of the SNR. To do that we need to integrate the emissivity over the entire volume of the remnant to get the total X-ray luminosity L_x. We used the following code:

```fortran
L_x=0.
DO i=2, jmax
   IF(temp(i) .ge. 1.d6) THEN ! ONLY IF THE TEMPERATURE IS ABOVE 10^6 K
      L_x = L_x + 4./3.*pi*(xb(i)**3-xb(i-1)**3)*Cool(temp(i), d(i), .false.) ! Otherwise no X
      ↪  emission !
   ELSE
      L_x=L_x
   END IF
END DO
```

The results are shown in the following figure:

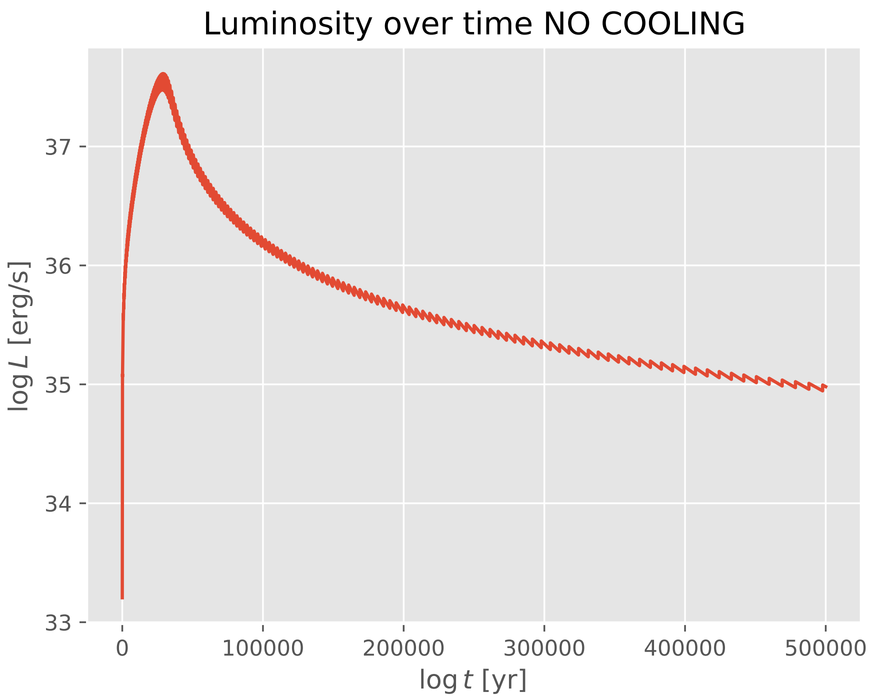


Figure 5: X-ray luminosity of the SNR as a function of time.

3.4 Evolution with COOLING until $t = 10^5$ years

Radiative losses are included in the simulation by adding a cooling term to the energy equation. The cooling function is given by:

```fortran
Real*8 FUNCTION Cool(Temp1, d1, disable_cooling)
USE DATA
IMPLICIT NONE
Real*8:: Temp1, d1, kev
LOGICAL:: disable_cooling
kev = 1.16d7
IF(disable_cooling) THEN
   cool = 0.
ELSE

   IF(Temp1 > 2.32e5) THEN
      cool = 1.d-22*(8.6*1.e-3*(Temp1/kev)**(-1.7)+0.058*(Temp1/kev)**0.5+0.063)*(d1/2.17d-24)**2

   ELSE IF(Temp1 < 2.32e5 .AND. Temp1 > 0.0017235*kev) THEN

      COOL = (6.72d-22 * (Temp1/kev/0.02)**0.6)*(d1/2.17d-24)**2
   ELSE IF(Temp1 < 0.0017235*kev) THEN

      COOL = (1.544d-22*(Temp1/kev/0.0017235)**6)*(d1/2.17d-24)**2
   ELSE
   cool = 0.
   END IF
END IF
END FUNCTION Cool
```

Basically radiative losses depends on the temperature and density of the gas. The cooling is activated only if the user wants to. This function is called in the main loop of the simulation, where the energy equation is updated:

```
   DO i=2, jmax-1
   e(i) = e(i) - dt_min * Cool(temp(i), d(i), cooling_OFF)
END DO
CALL BCb(e, jmax)

DO i=2, jmax-1
   temp(i)=e(i)/cv/d(i)
   IF(temp(i)<10**4) THEN
      temp(i)=10**4
   END IF
   e(i)=cv*d(i)*temp(i)
END DO
CALL boundary_conditions(e, jmax)
CALL boundary_conditions(temp, jmax)
```

The **IF** ensure that the temperature doesn't drop below 10^4 K.

Now, plotting again the evolution of the SNR with cooling until $t = 10^5$ years we get the following results:

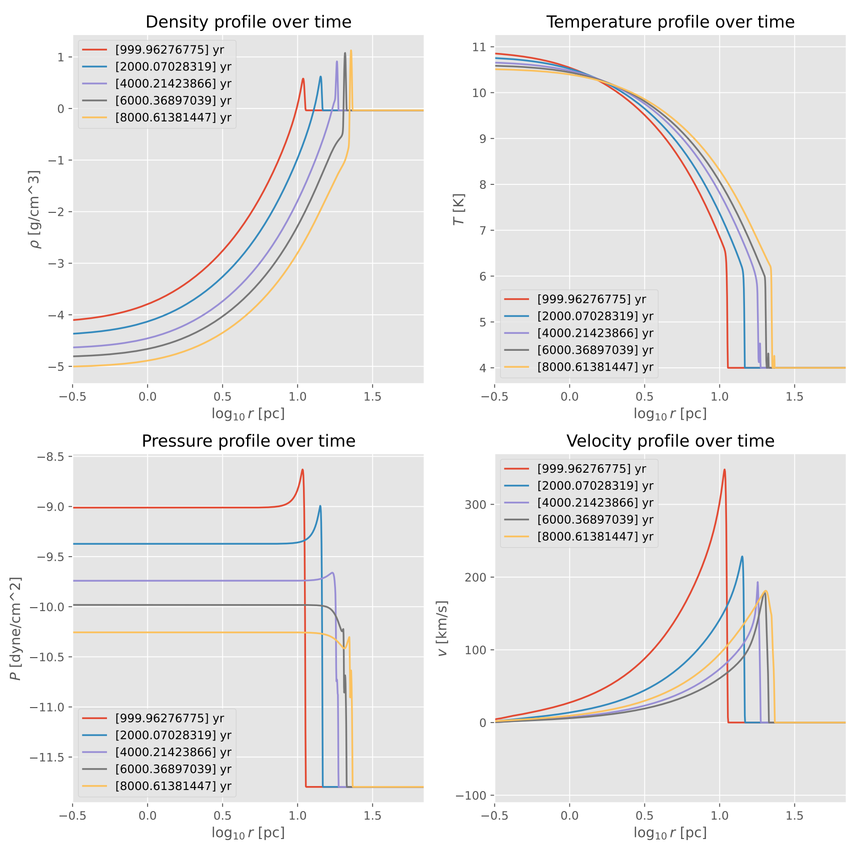


Figure 6: Evolution of the SNR with cooling until $t = 10^5$ years. The grid has 1000 points.

As we can see, after some time, the shock front starts to oscillate. This effect is enhanced when we observe the further evolution of the SNR until $t = 5 \times 10^5$ years:

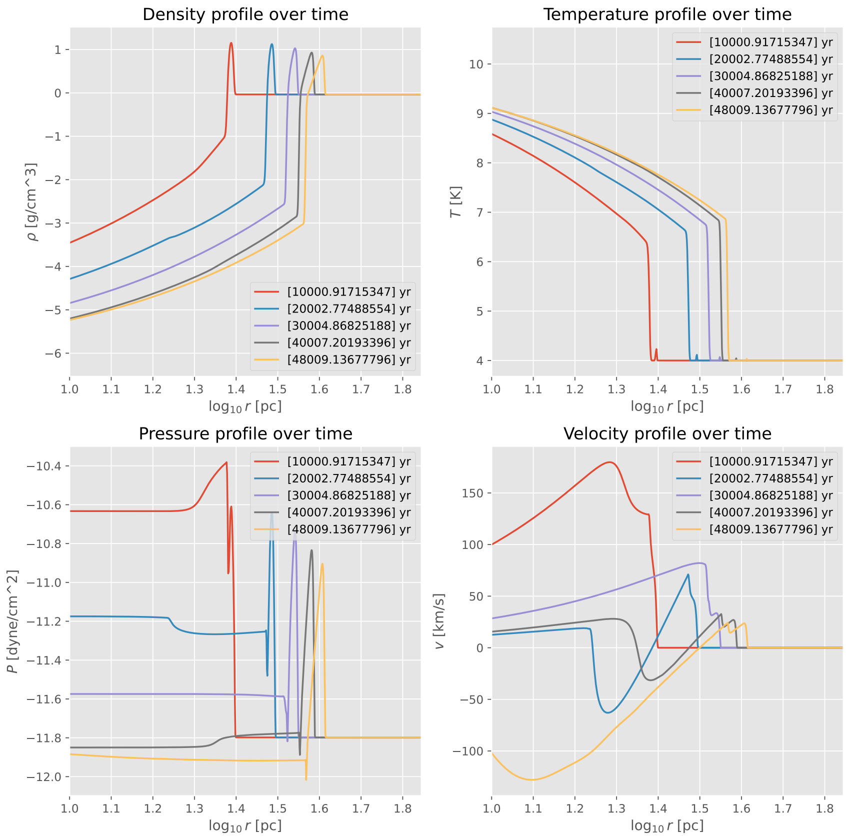


Figure 7: Evolution of the SNR with cooling until $t = 5 \times 10^5$ years. The grid has 1000 points. Only the outer part of the grid is shown.

In fact, if we plot the shock front evolution:

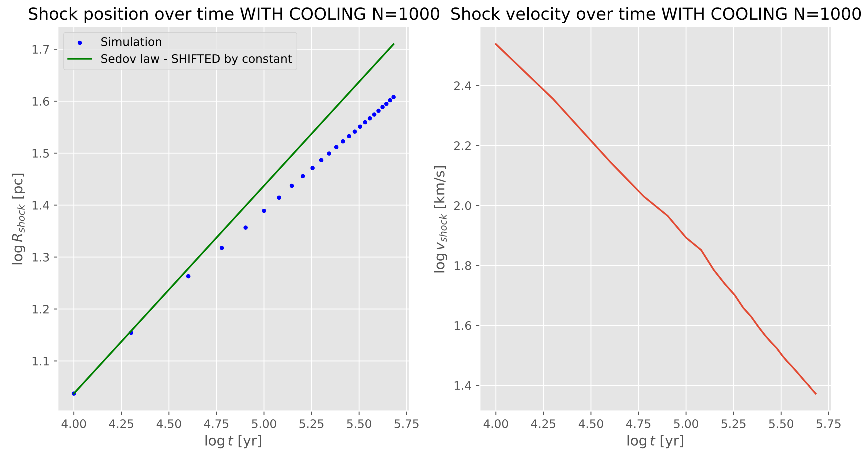


Figure 8: Shock front evolution with cooling. The green line is the Sedov solution translated to match the first point with the first point of the simulation.

As we can see, at around $\log(t) \approx 4.5$ the simulation stops following the Sedov solution. This is due to the fact that the radiative phase begun, so the energy losses are becoming important. In fact, the energy conservation is not very good in this case:

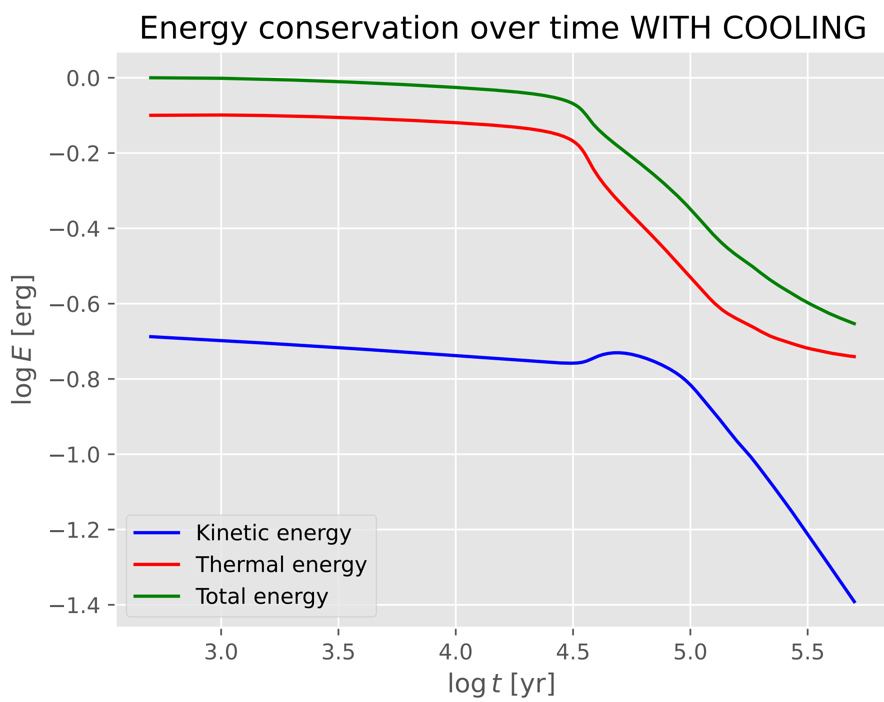


Figure 9: Energy conservation with cooling. The grid has 1000 points.

And we can see that the X-ray luminosity is very different after the activation of the cooling:

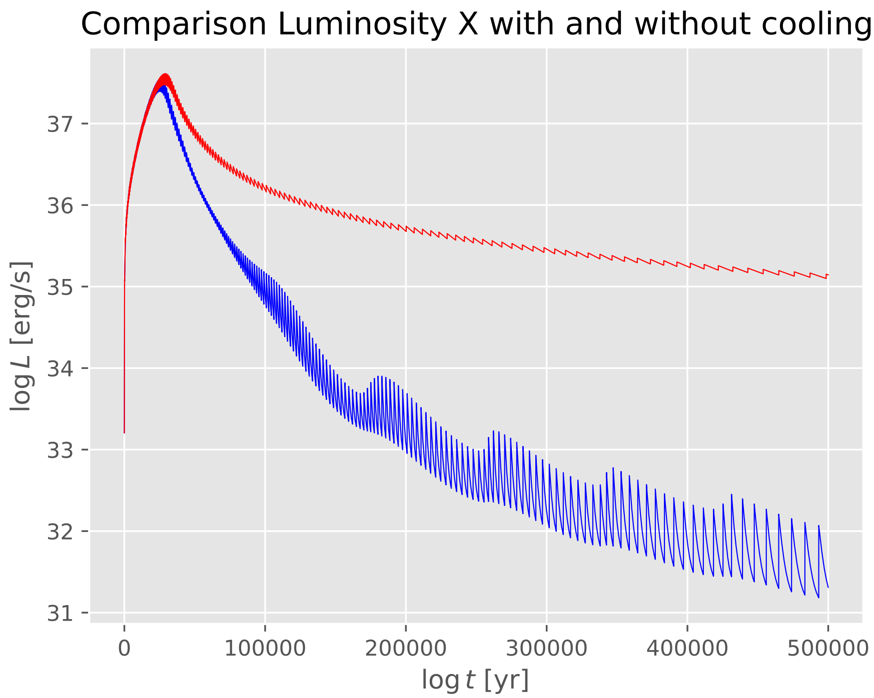


Figure 10: Comparison of X-ray luminosity with and without cooling.

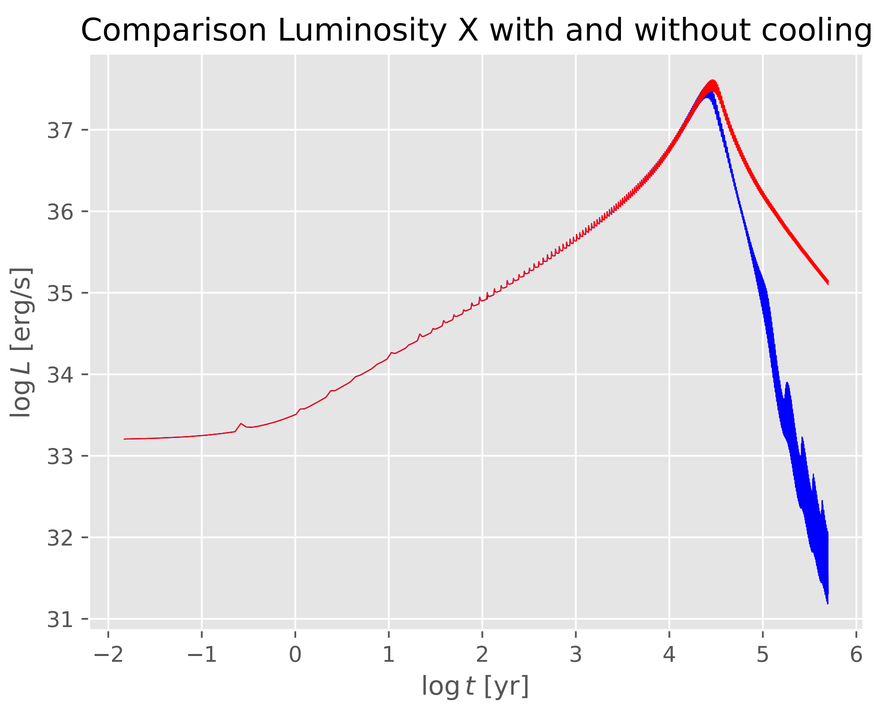


Figure 11: Comparison of the X-ray luminosity with and without cooling. The plot is in logarithmic scale. The peak is located at $\log(t) \approx 4.5$ as the sudden drop in thermal energy

We can see that the peak of the x-ray emission corresponds to the moment when the thermal energy is at its maximum. After that, the cooling starts to dominate and the X-ray luminosity decreases.

3.5 Evolution with different initial conditions

3.5.1 Lower initial temperature, higher initial density

Setting the initial temperature to $T_0 = 10^2$ K and the initial density to $\rho_0 = 10^{-22}$ g/cm3, and $N = 500$ we get the following results:

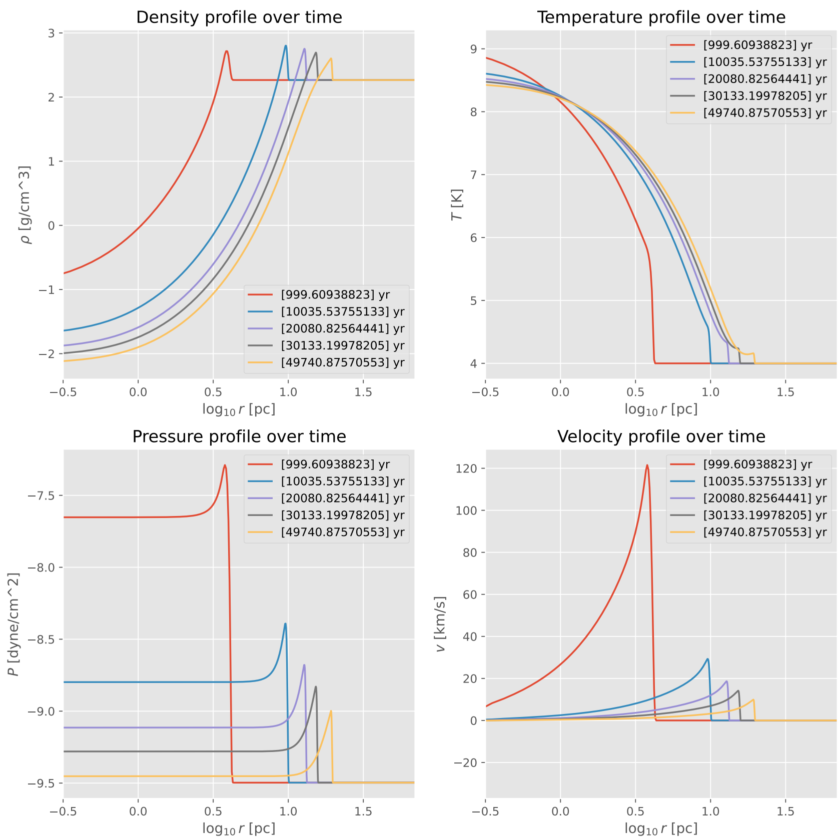


Figure 12: Evolution of the SNR without cooling until $t = 10^5$.

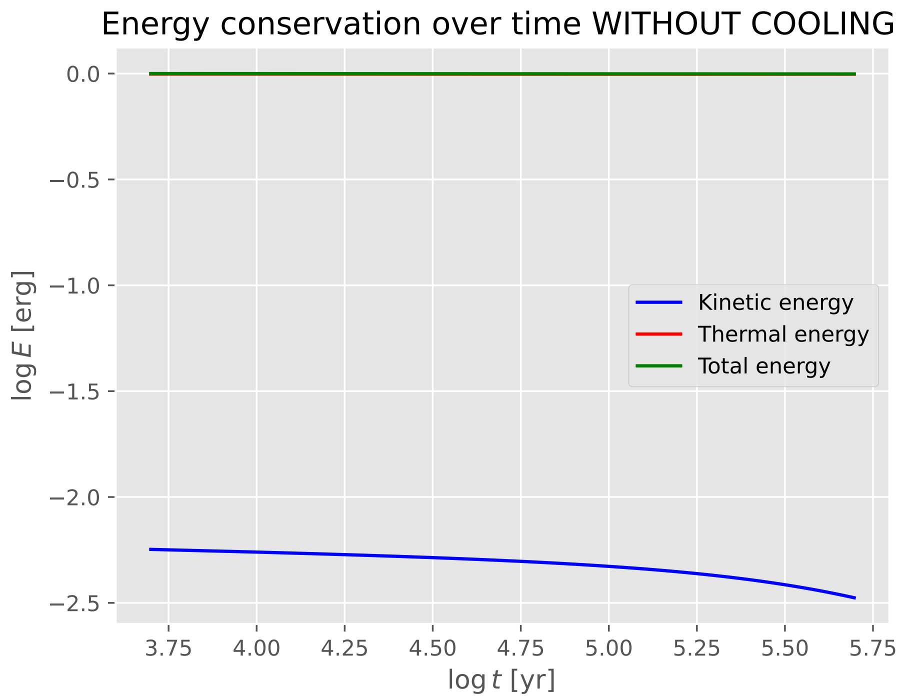


Figure 13: Evolution of the SNR without cooling until $t = 5 \times 10^5$. Practically all the energy is in the thermal form.

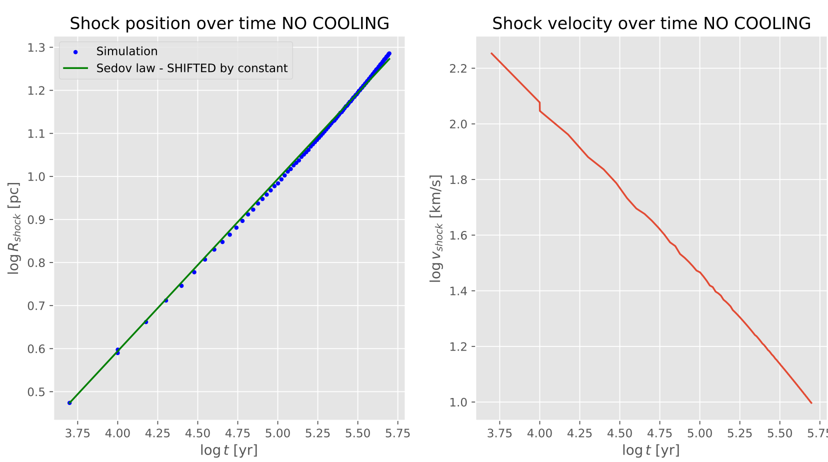


Figure 14: X-ray luminosity of the SNR as a function of time.

3.5.2 Higher initial temperature, lower initial density

Setting the initial temperature to $T_0 = 10^6$ K and the initial density to $\rho_0 = 10^{-26}$ g/cm$^{\mathbf{3}}$, we get the following results:

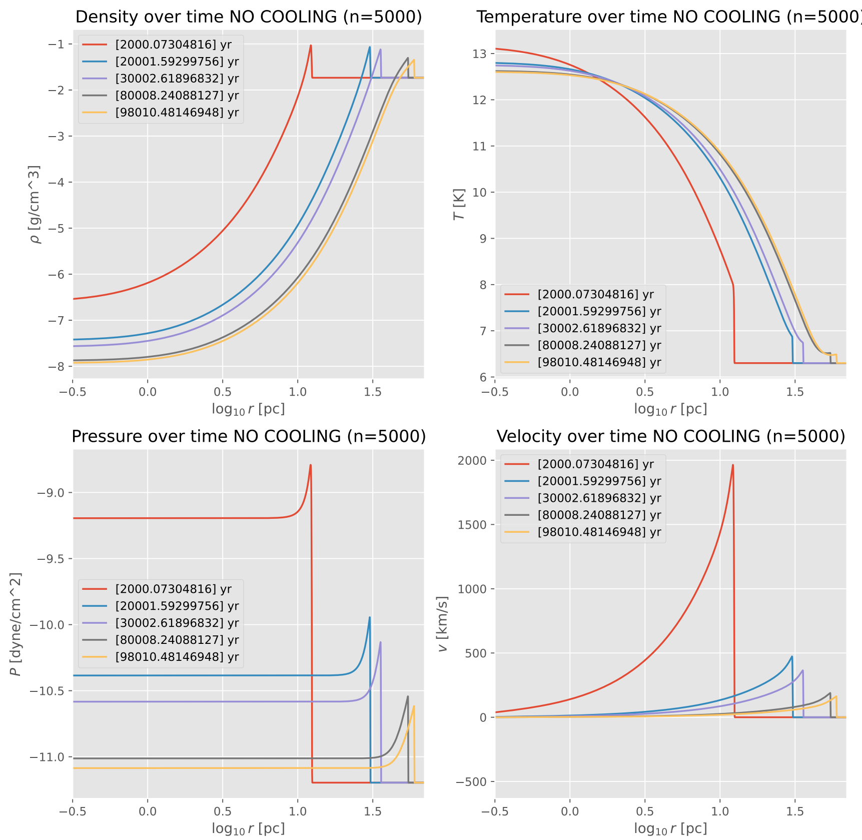


Figure 15: Evolution of the SNR without cooling until $t = 10^5 yr$.

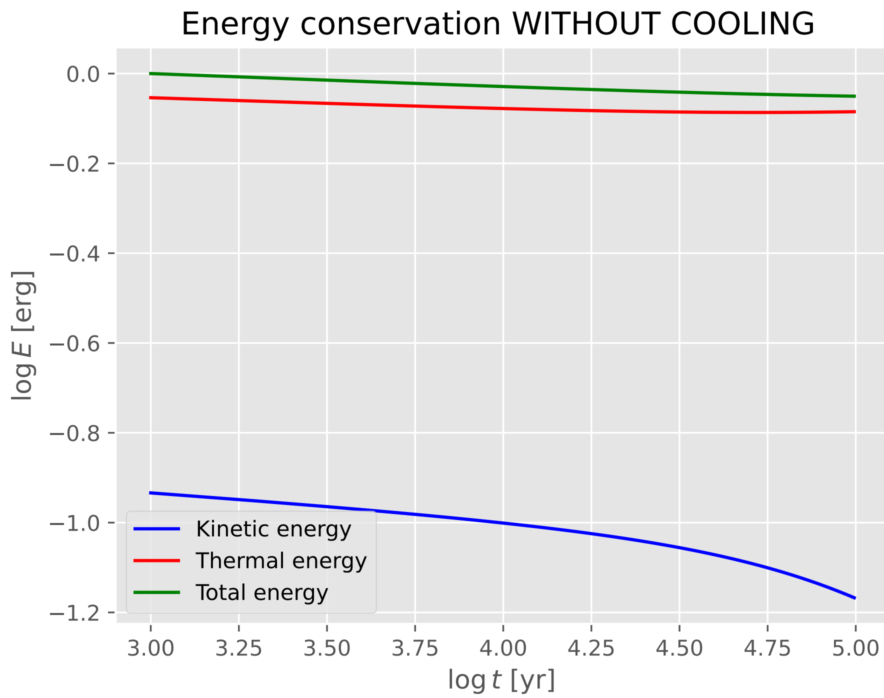


Figure 16: Evolution of the SNR without cooling until $t = 10^5 yr$.

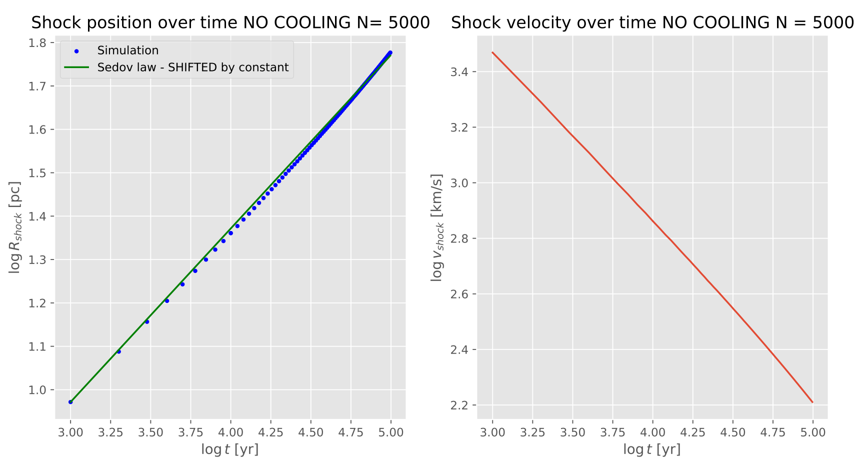


Figure 17: X-ray luminosity of the SNR as a function of time.