Iron diffusion in ICM project

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Contents

1 Problem background

We are studying the diffusion of Fe in the ICM of *Perseus Cluster*. Clusters are virialized system made of dark matter, baryonic matter and hot gas with temperature ranging from 10^7 to 10^8 K.

The dark matter is the most abundant component, with a mass of $10^{14} - 10^{15} M_\odot$.

In the central part of the cluster is located a Big Central Galaxy(BCG), which dominates the luminosity of the cluster. The BCG is surrounded by a halo of hot gas, which is the most massive baryonic component of the cluster.

1.1 Grid Construction

The grid is constructed in the radial direction assuming a constant cell size. Another grid is constructed shifted with respect to the first one, in order to define quantities at the cell edges or centers. Considering a maximum radius of 3 Mpc, with a total of 5000 cells, we have a cell size of 0.6 kpc.

The code for the grid is the following:

```
DO i = 1, jmax
    r(i) = r_min + (i-1)*r_max/(jmax-1)
END DO

DO i = 1, jmax-1
    rr(i)=r(i)+0.5*(r(i+1)-r(i))
    WRITE(20,FORMAT) r(i)/kpc, rr(i)/kpc
END DO

```

```
rr(jmax)=rr(jmax-1)+(rr(jmax-1)-rr(jmax-2))
WRITE(20,FORMAT) r(jmax)/kpc, rr(jmax)/kpc
```

Where r is primary grid and rr is the secondary grid. The secondary grid is shifted by half a cell size with respect to the primary grid.

1.2 Dark matter and gas density profiles

The gas in the galaxy cluster follows the gravitational potential of the dark matter halo. We then need to find the density profile of the dark matter halo.

The dark matter density profile is given by an NFW profile:

$$\rho_{dm}(r) = \frac{\rho_{dm,0}}{\left(\frac{r}{r_s}\right)\left(1+\frac{r}{r_s}\right)^2} \tag{1}$$

where $\rho_{dm,0} = 7.35 \times 10^{-26}\text{g/cm}^3$ and $r_s = 435.7$ kpc. Now that we have the density profile of the dark matter, we can calculate the mass of the dark matter inside a spherical shell of radius r. We can do this by integrating the density profile over the volume of the shell:

$$M_{dm}(r) = \int_0^r \rho_{dm}(r)\,\mathrm{d}V = \int_0^r 4\pi r^2 \rho_{dm}(r)\,\mathrm{d}r \tag{2}$$

We can easily solve this integral analytically but for the sake of the project we will solve it numerically. Care must be taken when defining new quantities in the code, as some quantities will be defined at the cell edges, or speherical shell, while others, like densities, will be defined at the cell centers.

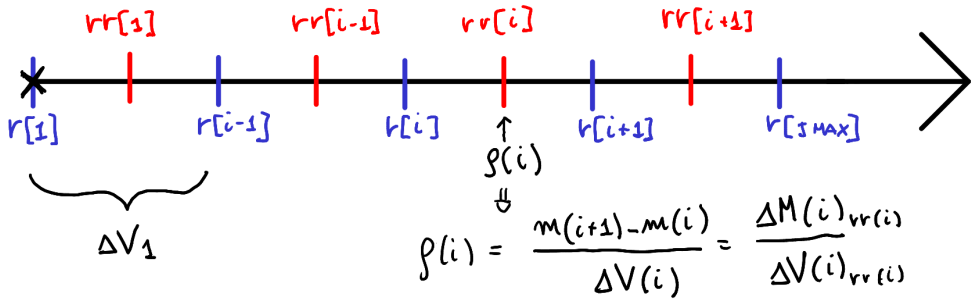


Figure 1: The grid as defined in the code. $r(i)$ is the primary grid, while $rr(i)$ is the secondary grid.

To find the dark matter densities profile, we define $\rho_{dm}(i)$ at the cell center, and multiply it by the volume between $r(i)$ and $r(i+1)$ to find the mass of the dark matter in the shell. Practically, we have:

$$V_i = \frac{4\pi}{3}\left(r_i^3 - r_{i-1}^3\right) \tag{3}$$

$$M_{dm}(i) = \rho_{dm}(i) V_i \tag{4}$$

$$\tag{5}$$

then recursively summing the masses of the shells, we find the total mass of the dark matter inside a sphere of radius $r(i)$. in the code, we have:

```
    DO i = 1, jmax
   x = rr(i)/r_s
   rho_dm(i) = rho_dm0 / (x*(1.+x)**2) ! DM Density defined on the rr(i) grid
END DO

```

```
vol_r(1) = 4./3.*pi*r(1)**3
DO i = 2, jmax-1
   vol_r(i) = 4./3.*pi*(r(i+1)**3-r(i)**3) ! Volume between r(i+1) and r(i)
END DO
vol_r(jmax) = 4./3.*pi*(r(jmax)**3-r(jmax-1)**3)

! DM MASS INTEGRATION !
M_DM(1) = rho_dm0*vol_r(1)
M_DM_EXACT(1) = rho_dm0*vol_r(1)
x=rr(1)/r_s
DO i = 2, jmax
x = rr(i)/r_s
M_DM(i) = M_DM(i-1) + rho_dm(i) * vol_r(i)
M_DM_EXACT(i) = 4.*pi*rho_dm0*r_s**3*(log(1.+rr(i)/r_s)-rr(i)/(rr(i)+r_s))
END DO
```

M_DM_EXACT is the exact mass of the dark matter inside a sphere of radius $r(i)$ found using the analytical formulae:

$$M_{dm}(r) = 4\pi\rho_{dm,0}r_s^3\left[\log\left(1+\frac{r}{r_s}\right)-\frac{r}{r+r_s}\right] \tag{6}$$

We can compare this with the mass we find numerically, and we see that the two are in good agreement.

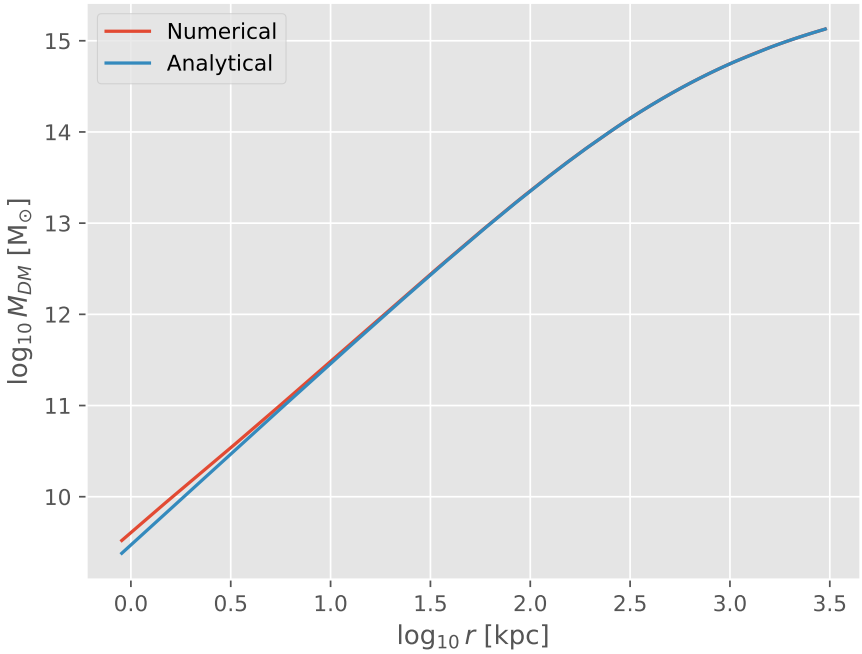


Figure 2: Comparison between numerical and analytical mass of the dark matter inside a sphere of radius r.

The small discrepancy between the two is due to the fact that the numerical integration is done assuming that density is constant between $r(i)$ and $r(i+1)$, while in reality the density is a continuous function of r, defined in the equation 1.

Finding the gas density profile

The next step is to integrate the **hydrostatic equilibrium equation** to find the gas density profile. The hydrostatic equilibrium equation is given by:

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{GM}{r^2}\rho_g \quad \text{where } P = \frac{kT}{\mu m_p}\rho_g \tag{7}$$

Assuming an isothermal gas, we can write the equation as:

$$\frac{\mathrm{d}\ln\rho_g}{\mathrm{d}r} = -\frac{\mu m_p}{kT}\frac{GM}{r^2} \tag{8}$$

which becomes in finite difference form:

$$\rho_{j+1/2} = \rho_{j-1/2} - \Delta r \frac{\mu m_p}{kT}\frac{GM_j}{r_j^2} \tag{9}$$

```
rho_g(1) = exp(log(rho_0))
rho_g_exact(1) = rho_0
WRITE(20, *) "rho_gas ", "rho_gas_exact ", "rho_dm"
WRITE(20, FORMAT) rho_g(1), rho_g_exact(1), rho_dm(1)
M_GS(1) = rho_g(1)*vol_r(1)
DO i = 2, jmax-1
   rho_g(i) = log(rho_g(i-1))-(G*M_DM(i)/rr(i)**2)*mu*mp*(rr(i+1)-rr(i))/(boltz*temp_K)*fc
   rho_g(i) = exp(rho_g(i))
   rho_g_exact(i) = rho_0*exp(-27.*b/2.)*(1.+r(i)/r_s)**(27.*b/(2.*r(i)/r_s))
   M_GS(i) = M_GS(i-1) + rho_g(i)*vol_r(i)
   WRITE(20, FORMAT) rho_g(i), rho_g_exact(i), rho_dm(i)
END DO
rho_g(jmax) =
↪  exp(log(rho_g(jmax-1))-(G*M_DM(jmax)/r(jmax)**2)*mu*mp*(rr(jmax)-rr(jmax-1))/(boltz*temp_K)*fc)
rho_g_exact(jmax) = rho_0*exp(-27*b/2)*(1+rr(jmax)/r_s)**(27*b/(2*rr(jmax)/r_s))
M_GS(jmax) = M_GS(jmax-1) + rho_g(jmax) * vol_r(jmax)
WRITE(20, FORMAT) rho_g(jmax), rho_g_exact(jmax), rho_dm(jmax)
```

Assuming a central gas density of $4 \times 10^{-26}\mathrm{g/cm}^3$, we find the gas density profile:

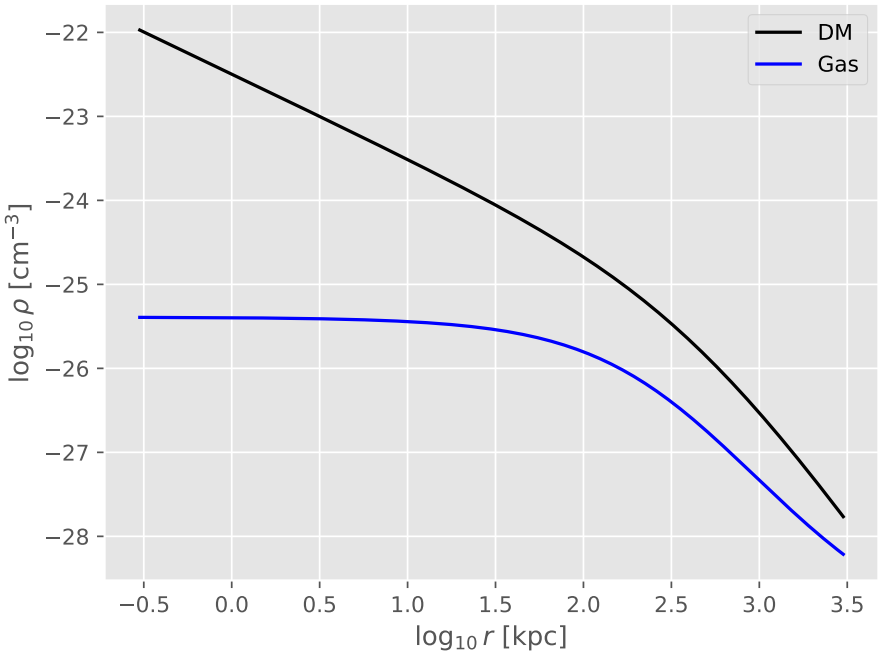


Figure 3: Gas density profile due to dark matter potential only.

The gas density profile is in good agreement with the analytical profile:

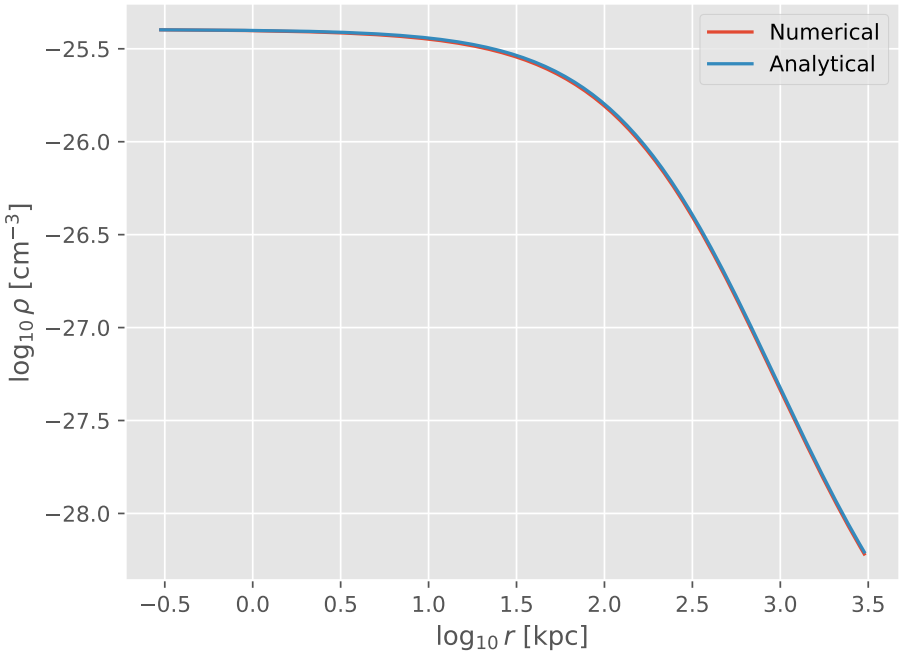


Figure 4: Gas density profile due to dark matter potential only comparison with analytical profile.

Now we impose to have a gas density profile that makes the **baryonic fraction** at the virial radius to be 0.16. The baryonic fraction is defined as:

$$f_b = \frac{M_{gas}}{M_{dm} + M_{gas}} \tag{10}$$

where M_{gas} is the mass of the gas inside a sphere of radius r and M_{dm} is the mass of the dark matter inside the same sphere.

To find the right value for the **central gas density** we used the **bisection method**. We defined a function that calculates the baryonic fraction at the virial radius, and we used the bisection method to find the central gas density that makes the baryonic fraction equal to 0.16.

```
!!! Bisection method to find the right density for the gas that makes the baryonic fraction to be
↪  0.16

eps_bar_frac = 0.00001 ! epsilon for stopping the bisection method
COND_1 = .FALSE.
rho_0_control = 4.e-26
SPL = 1.E-22 ! Left guess for gas density, "very big"
SPR = 1.E-28 ! Right guess for gas density, "very small"



END DO ! End of the bisection method, now we have the right value for the gas density
```

Then we need to add the contribution due to the **BCG** to the gas density profile. We have computed before the mass due to stars.

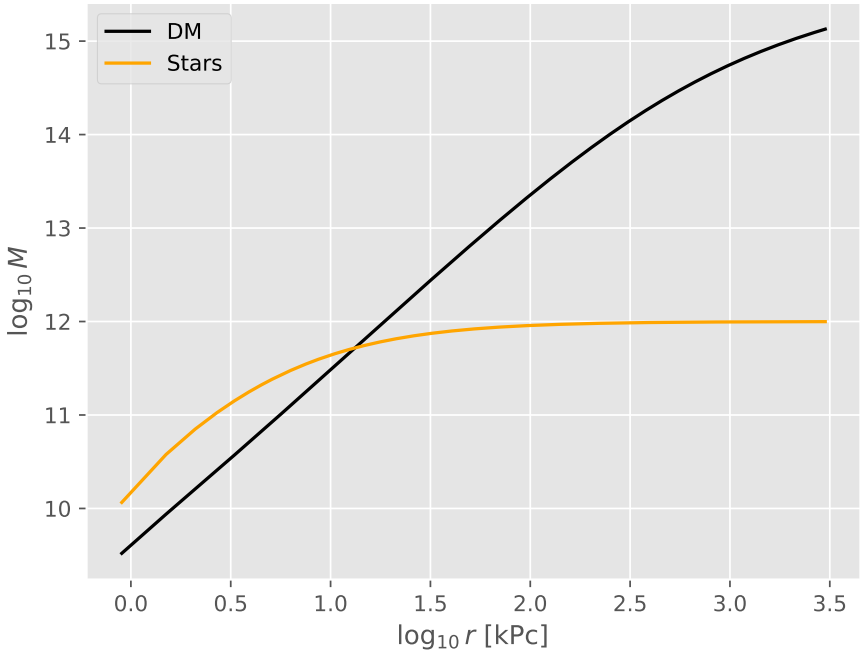


Figure 5: Cumulative mass distribution of the cluster.

```
mBCG = 10.**12 * msol ! Total mass of the central galaxy
a = 12.*kpc/(1.+sqrt(2.))
DO i = 1, jmax
   M_S(i) = mBCG*(r(i))**2/((r(i)+a)**2) ! Stars mass defined on the rr(i) grid
END DO
```

We just need to find ρ_j again with the new mass distribution:

$$\rho_{j+1/2} = \rho_{j-1/2} - \Delta r \frac{\mu m_p}{kT} \frac{GM_j}{r_j^2} \tag{11}$$

where $M_j = M_{dm}(j) + M_{stars}(j)$.

$$\rho_{j+1/2} = \rho_{j-1/2} - \Delta r \frac{\mu m_p}{kT} \frac{G[M_{dm}(j) + M_{stars}(j)]}{r_j^2} \tag{12}$$

Just as before we then use bisection to find the right value for central density.

Then we need to do the same for the gas density profile, but with a variable **temperature profile**. This profile was measured from xray emission of the cluster. The temperature profile $t_r(i)$ is given by:

$$T_r(i) = T_{mg}(i) \times 1.35 \times \left(\frac{\mathrm{yy}^{1.9} + 0.45}{\mathrm{yy}^{1.9} + 1} \right) \times \frac{1}{\left(1 + \left(\frac{y}{0.6}\right)^2\right)^{0.45}}$$

```
DO i = 1, jmax
y=rr(i)/r500
yy=y/0.045
x500 = rr(i)/(1.4*1000*kpc)
t_r(i) = temp_k*1.35*(yy**1.9+0.45)/(yy**1.9+1.)* &    !! this is for Perseus !!
         1./(1.+(y/0.6)**2)**0.45
WRITE(20,"(e15.7)") t_r(i)
END DO
```

The results are plotted in the figure below.

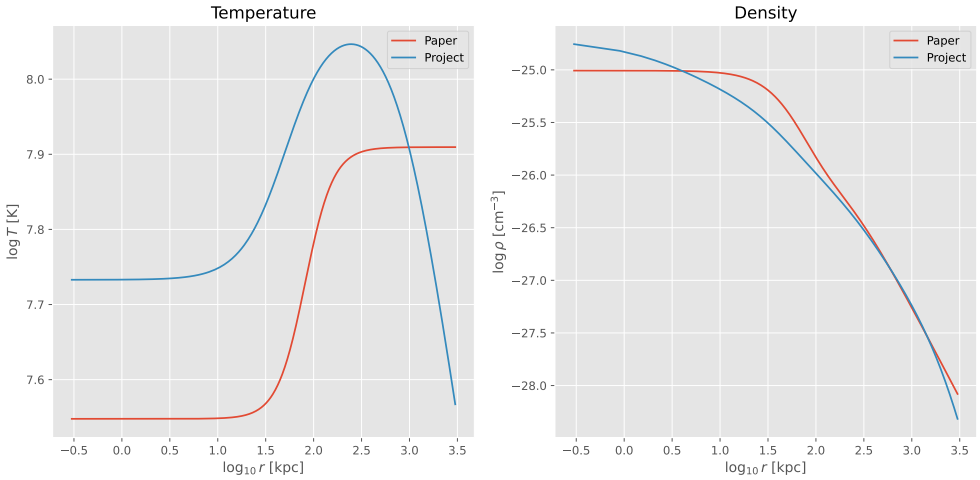
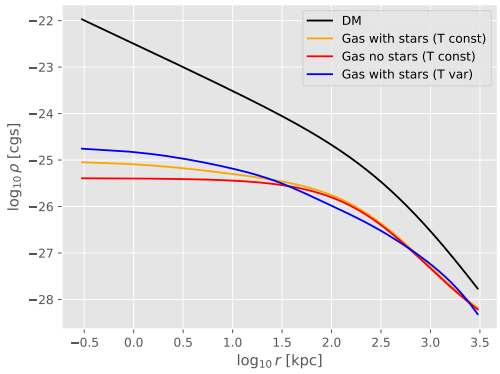


Figure 6: Temperature and density profiles of the gas in the ICM.

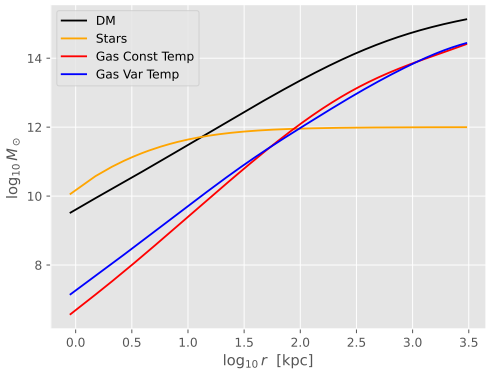
The temperature profile of the project is a little bit higher but the general shape is the same. The density of the gas is in good agreement with the one used by Rebusco et al.

Recap of density profiles

After we have found the gas density profile with the BCG and the variable temperature profile, we can compare it with the other profiles we have found before. The results are plotted in the figure below.



(a) Density profiles with different physical conditions.



(b) Matter distribution with stars and dark matter.

Potential due to	Temperature Profile	Baryonic Fraction	ρ_{gas} (in 10^{-26} g/cm^3)
Only DM	Constant	0.160	4.047
DM + Stars	Constant	0.170	8.975
DM + Stars	Variable/Observed	0.170	17.548

Table 1: Central density of the gas in the ICM for different physical conditions.

So as we can see the central density is higher and the profile is different in the central part of the cluster due to the presence of the BCG and the variable temperature profile.

2 Second part: Diffusion of Fe in the ICM

2.1 Code overwiew

The code will ask the user to input some parameters:

```
random_approach_sn = 0
WRITE(*,*) "For how many years you want to evolve the system: [Gyr] (timestep =", dt/years,
↪  "years, or: ", dt/gyr, " Gyr"
READ(*,*) time_wanted
time_wanted = time_wanted * 3.1536d16
timesteps = INT(time_wanted / dt)
! ALLOCATE(rho_fe_2D(timesteps+1, jmax))
! ALLOCATE(M_Fe(timesteps+1, jmax))
WRITE(*,*) "How often in time you want to output the data to file?:  [Gyr] "
READ(*,*) delta_t_debug
delta_t_debug = delta_t_debug * 3.1536d16

k_time_step = 1

WRITE(*,*) "Do you want to start with NO IRON in the ICM? [1 = TRUE, 0 = FALSE]"
READ(*,*) start_with_zero_iron


```

```
IF(start_with_zero_iron .eq. 1) THEN
WRITE(*,*) "How many supernovae per century you want to inject?  [N_SN/century]"
READ(*,*) SN_unit
END IF
! WRITE(*,*) "Do you want to inject supernovae with a random approach? [1 = TRUE, 0 = FALSE]"
! READ(*,*) random_approach_sn

if (time_wanted .LE. 13.8*gyr) then
   initial_time = 13.8*gyr - time_wanted
ELSE
   initial_time = 8.0*gyr  ! Needed if we want to evolve the system to a time greater than 13.8
   ↪  Gyr, considering that literature
                           ! suggest that Perseus cluster is 5-6 Gyr old
END IF

IF(start_with_zero_iron .eq. 1) THEN
   Z_Fe = 0.
   rho_fe = 0.
END IF
```

The most important one are the total time we want to evolve the system, and if we want to start with the observed iron abundance, which will activate just the diffusion, or if we want to start with zero iron, which will activate the supernovae injection from a past time to the present.

If the total simulation time is larger than the age of the universe, we will start the simulation at 8 Gyr, as the Perseus cluster is 5-6 Gyr old.

2.2 Diffusion of iron in the ICM

Now that we have the distribution of the gas in the ICM we have to study the diffusion of iron. To do that we need to solve a diffusion equation:

$$\frac{\partial n_{Fe}}{\partial t} = D\nabla^2 n_{Fe} \tag{13}$$

where n_{Fe} is the number density of iron and D is the diffusion coefficient. In general the diffusion coefficient is a function of the temperature and the density of the gas, but for the sake of simplicity we will assume it to be constant. We will also define the iron abundance

$$Z_{Fe} = \frac{1}{1.4}\frac{\rho_{Fe}}{\rho_{gas}}$$

.

$$\frac{\partial \rho_{Fe}}{\partial t} = \frac{1}{1.4}\frac{1}{r^2}\frac{\partial (r^2 D\rho \frac{\partial Z_{Fe}}{\partial r})}{\partial r} + S_{Fe}(r,t) \tag{14}$$

Where $S_{Fe}(r,t)$ is the source term for the iron abundance, and $\frac{\partial Z_{Fe}}{\partial r}$ is the gradient of the iron abundance. We can write the gradient in finite difference form:

$$\text{gradzfe}_j^n = \frac{Z^n_{Fe,j+1/2} - Z^n_{Fe,j-1/2}}{r_{j+1/2} - r_{j-1/2}} \tag{15}$$

At the end, we have that

$$\rho^{n+1}_{Fe,j+1/2} = \rho^n_{Fe,j+1/2} + \frac{\Delta t}{1.4}\frac{\left[r^2 D\rho \cdot \text{gradzfe}^n_{j+1} - r^2 D\rho \cdot \text{gradzfe}^n_j\right]}{(r^3_{j+1} - r^3_j)/3} \tag{16}$$

Then again $Z_{Fe} = 1.4\frac{\rho_{Fe}}{\rho}$.

For **stability** reasons we must have that:

$$\Delta t \leq \frac{\Delta x^2}{2D} \tag{17}$$

This value should be calculated at every time step, as the diffusion coefficient is a function of the gas density and temperature, and we must choose the minimum value as:

$$\Delta t = C \cdot \min\left(\frac{\Delta x^2}{2D}\right) \quad \text{With } C \leq 1.0. \tag{18}$$

But considering that we have a constant diffusion coefficient and a fixed grid, we can calculate it once and use it for the whole simulation.

3 Start with observed iron abundance at the present time

3.1 a. Fe conservation in the ICM

```
DO i = 1, jmax
    Z_Fe(i) = 1.4*0.3*((2.2+(rr(i)/kpc/80.)**3)/(1.+(rr(i)/kpc/80.)**3))*zfesol
    Z_Fe(i) = Z_Fe(i) - zfeout
    rho_fe(i) = Z_Fe(i)*rho_g_mean(i)/1.4
END DO

DO i=2, jmax-1
    grad_zfe(i) = (Z_Fe(i)-Z_Fe(i-1))/(rr(i)-rr(i-1))
ENDDO
grad_zfe(1) = 0.
grad_zfe(jmax) = 0.
```

This part of the code sets the observed value of Z_{Fe} in the ICM, and then calculates the iron density in the ICM, and the gradient. We want to make sure that the total iron mass in the ICM is conserved while the iron **before 100 Kpc** is reduced, which will confirm that Fe is diffused. To do that we will just evolve the system from this point on, without any supernovae injection.

```
dt=(r(2)-r(1))**2/(2*param_D(1))*C_param
```

The **delta t** as said before is calculated at the beginning of the simulation, and then used for the whole simulation. Considering that the diffusion coefficient and the grid are constant, we can do this.

```
    M_Fe = 0.
M_Fe(1) = rho_fe(1)*(vol_r(1))
DO i=2, jmax
   M_Fe(i) = (rho_fe(i) * vol_r(i)) ! Here volume is really a delta_V
ENDDO
M_Fe_initial = sum(M_Fe)

DO i=1, index_100kpc
   M_Fe_100kpc_initial = M_Fe_100kpc_initial + M_Fe(i)
END DO

```

We compute the **initial** mass of iron inside each cell/shell and then we sum it up to find the total mass of iron in the ICM. We also sum the mass of iron inside the first 100 Kpc, to check if the iron is diffused later on.

```
! START OF THE WHILE LOOP FOR THE IRON DIFFUSION
! The loop will stop when the time passed is equal to the time wanted by the user
DO WHILE(end_of_time .eqv. .FALSE.)
CALL CPU_TIME(time_start_debug)

<.. OTHER CODE..>

! DIFFUSION
DO i=2, jmax-1
   rho_fe(i) = rho_fe(i) + (dt/1.4)*(r(i+1)**2*param_D(1)*rho_g_mean(i+1)*grad_zfe(i+1)- &
               r(i)**2*param_D(1)*rho_g_mean(i)*grad_zfe(i)) &
               /((r(i+1)**3-r(i)**3)/3.)
   Z_Fe(i) = 1.4*rho_fe(i)/rho_g_mean(i) ! just updating the value at radius r(i) for the iron
   ↪  density, the diffusion is going on.
END DO
Z_Fe(1) = Z_Fe(2)
Z_Fe(jmax) = Z_Fe(jmax-1)
rho_fe(1)=rho_fe(2)              ! Boundary conditions
rho_fe(jmax) = rho_fe(jmax-1)
k_time_step = k_time_step+1

DO i=2, jmax-1
   grad_zfe(i) = (Z_Fe(i)-Z_Fe(i-1))/(rr(i)-rr(i-1))
ENDDO
grad_zfe(1) = 0.
grad_zfe(jmax) = 0.
! CONDITIONS FOR IRON CONSERVATION

! IRON MASS IN THE ICM

```

We then evolve the system in time, by updating the iron density in each cell, and then calculating the gradient. We then check if the iron mass in the ICM is conserved, and if the iron mass in the first 100 Kpc is reduced.

```
    ! Check and print if the iron mass under some kpc has been halved
IF((M_Fe_100kpc .LE. M_Fe_100kpc_initial/2.) .AND. stop_debug_mass_index .EQ. 0 .AND. SN_unit .EQ.
↪  0) THEN
   WRITE(*,*)
   WRITE(*,*) "------------------------------------------------------------------------ "
   WRITE(*,*) "The iron mass in the ICM under 100 kpc has been halved "
   WRITE(*,*) "M_i: ", M_Fe_100kpc_initial/msol, " M_Fe(t): ", M_Fe_100kpc/msol, "tau: ",
   ↪  TIME_PASSED/3.15d16, "Gyr"
   WRITE(*,*) "------------------------------------------------------------------------ "
   WRITE(*,*)
   tau_diffusion = TIME_PASSED
   stop_debug_mass_index = 1
END IF
```

The code will also print **when** the mass of iron in the first 100 Kpc(or other value) is reduced by a **factor 2**, which will confirm that the iron is diffused in the ICM. This time will be $\tau_{diffusion}$.

3.1.1 Result: a. Fe conservation in the ICM

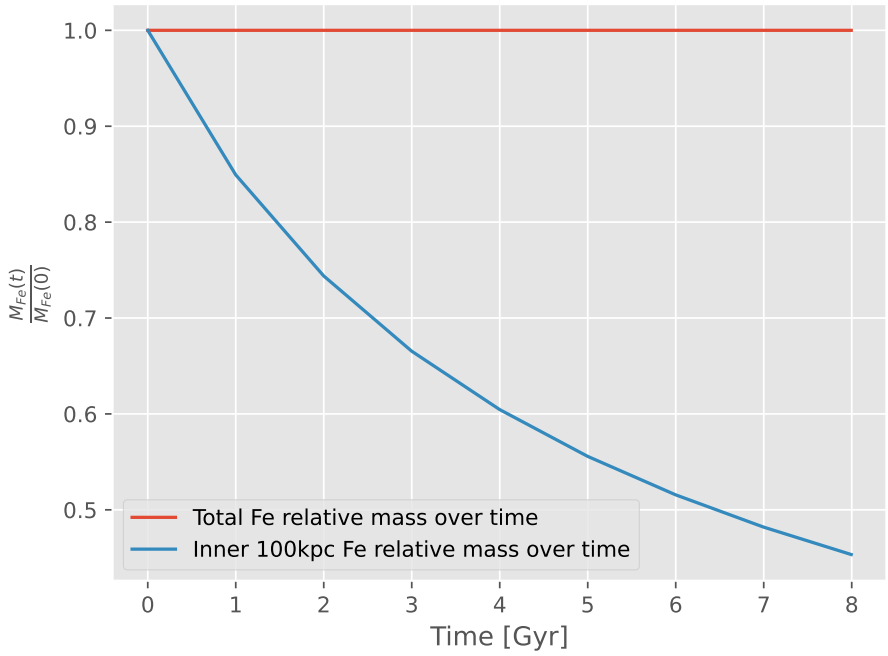


Figure 8: Iron conservation over whole grid and diffusion/reduction in the first 100 Kpc.

Time [Gyr]	$M_{Fe,tot}$	$M_{Fe,<100}$
0.0	$8.69 \times 10^8\, M_\odot$	$3.81 \times 10^8\, M_\odot$
1.0	$8.69 \times 10^8\, M_\odot$	$3.47 \times 10^8\, M_\odot$
2.0	$8.69 \times 10^8\, M_\odot$	$3.17 \times 10^8\, M_\odot$
3.0	$8.69 \times 10^8\, M_\odot$	$2.92 \times 10^8\, M_\odot$
4.0	$8.69 \times 10^8\, M_\odot$	$2.71 \times 10^8\, M_\odot$
5.0	$8.69 \times 10^8\, M_\odot$	$2.53 \times 10^8\, M_\odot$

We can clearly see that the total amount of Iron in the ICM is conserved while the iron in the first 100 Kpc is reduced by a factor of 1/3. This confirms that the iron is diffused in the ICM. **Diffusion coefficient** is set to $D = 1.32 \times 10^{29} \mathrm{cm}^2/\mathrm{s}$.

3.2 b. Evolution of $\mathbf{Z_{Fe}(r)}$ in the ICM over time

We will now evolve the system from the present time to the future, and plot the evolution of the iron abundance in the ICM. For this purpose we will remove the background Fe abundance to see just the diffusion of Fe in the ICM.

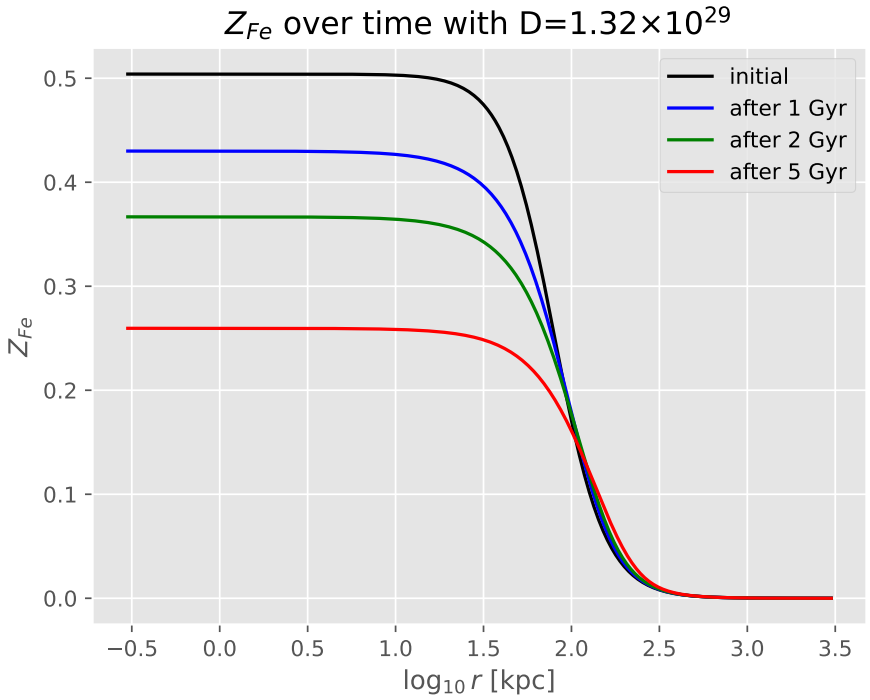


Figure 9: Evolution of the iron abundance in the ICM over time.

As we can see, iron in the central region reduce with time, while the iron in the outer region increases.

Diffusion time estimate

An estimate of the order of magnitude of **Diffusion time** can be:

$$\tau_{diff} \approx \frac{L^2}{D} \approx \frac{L^2}{1.3 \times 10^{29}} \tag{19}$$

We can try to estimate the diffusion time as the time needed to reduce by a factor of 2 the iron mass in the central region.

This is just an assumption, as this central region is arbitrary chosen. But we can assume that if the time needed to reduce by a factor of 2(or 3 or e) the iron mass inside a region of length L, is greater than the age of the universe, then the diffusion is not efficient enough. The age of the cluster is around 5-6 Gyr, so we can try to find some values of L that will give us a diffusion time of around 5-6 Gyr. Or, we can try to find the diffusion coefficient that will give us a diffusion time of 5-6 Gyr for a given L.

L is the distance from the center of the cluster where the iron peak is located. An estimate of this value, based on the idea that the iron peak is located at the point where the variation (aka "gradient") of the iron abundance

$$|Z_{Fe}(i) - Z_{Fe}(i-1)|$$

is maximum, is around 64 kpc.

Then one can also find the "gradient of the gradient" of the iron abundance, which will give us some left and right limits around the center of the peak.

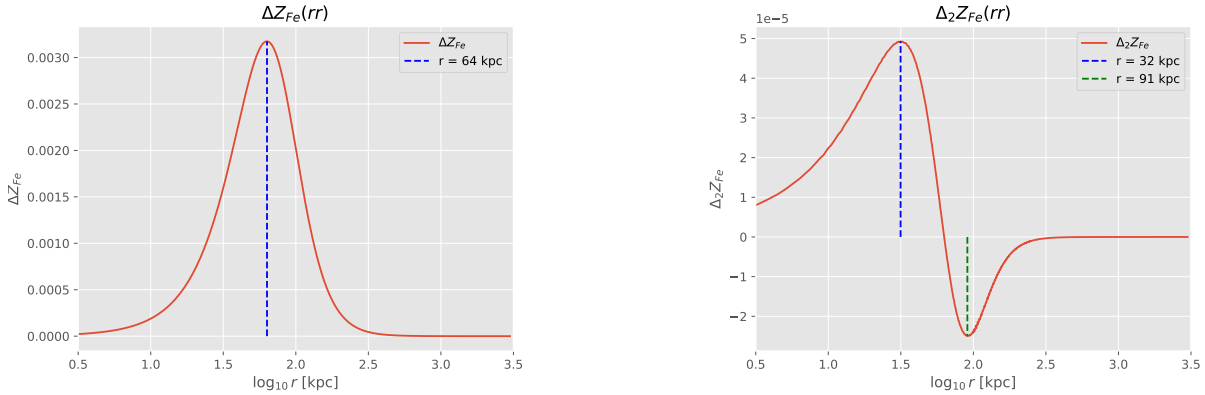


Figure 10: Left: ΔZ_{Fe}. Right: $\Delta(\Delta Z_{Fe})$.

We resume the findings in the table below:

Diffusion [cm^2/s]	L [kpc]	τ_{diff} [Gyr]
1.32 $\times 10^{29}$	32	5.39
1.32 $\times 10^{29}$	64	6.46
1.32 $\times 10^{29}$	91	8.91

L[kpc]	$\tau_{diff}[Gyr]$	D[cm^2/s]	$M_{Fe,tot,i}[M_\odot]$	$M_{Fe,tot,f}[M_\odot]$	$M_{Fe}^{initial}(< L)[M_\odot]$	$M_{Fe}^{final}(< L)[M_\odot]$
32	5.391	1.32	8.693e+08	8.693e+08	5.287e+07	1.916e+07
32	4.744	1.5	8.693e+08	8.693e+08	5.287e+07	1.779e+07
32	3.558	2	8.693e+08	8.693e+08	5.287e+07	1.49e+07
32	2.372	3	8.693e+08	8.693e+08	5.287e+07	1.139e+07
64	6.539	1.32	8.693e+08	8.693e+08	2.069e+08	9.386e+07
64	5.754	1.5	8.693e+08	8.693e+08	2.069e+08	8.793e+07
64	4.316	2	8.693e+08	8.693e+08	2.069e+08	7.511e+07
64	2.877	3	8.693e+08	8.693e+08	2.069e+08	5.875e+07
91	8.917	1.32	8.693e+08	8.693e+08	3.42e+08	1.617e+08
91	7.847	1.5	8.693e+08	8.693e+08	3.42e+08	1.515e+08
91	5.885	2	8.693e+08	8.693e+08	3.42e+08	1.294e+08
91	3.923	3	8.693e+08	8.693e+08	3.42e+08	1.012e+08
100	10.02	1.32	8.693e+08	8.693e+08	3.814e+08	1.672e+08
100	8.821	1.5	8.693e+08	8.693e+08	3.814e+08	1.56e+08
100	6.616	2	8.693e+08	8.693e+08	3.814e+08	1.538e+08
100	4.411	3	8.693e+08	8.693e+08	3.814e+08	1.209e+08

I've highlighted the values that give us a diffusion time of around 5-6 Gyr. We can see that in all cases the total iron mass in the ICM is conserved, while the iron mass in the first 100 Kpc is reduced by some amount proportional to the time passed. In general a value of D = 2×10^{29}cm^2/s can give us a diffusion time of around 5-6 Gyr for a distance of 64 kpc, while maintaing the difussion time reasonable also for 32 and 100 kpc.

4 Source term only analysis

Using only the source term, without diffusion, lead to a very peaked iron abundance in the central region, which is not observed. This is due to the fact that the source term is not diffused in the ICM, and it remains

in the central region.

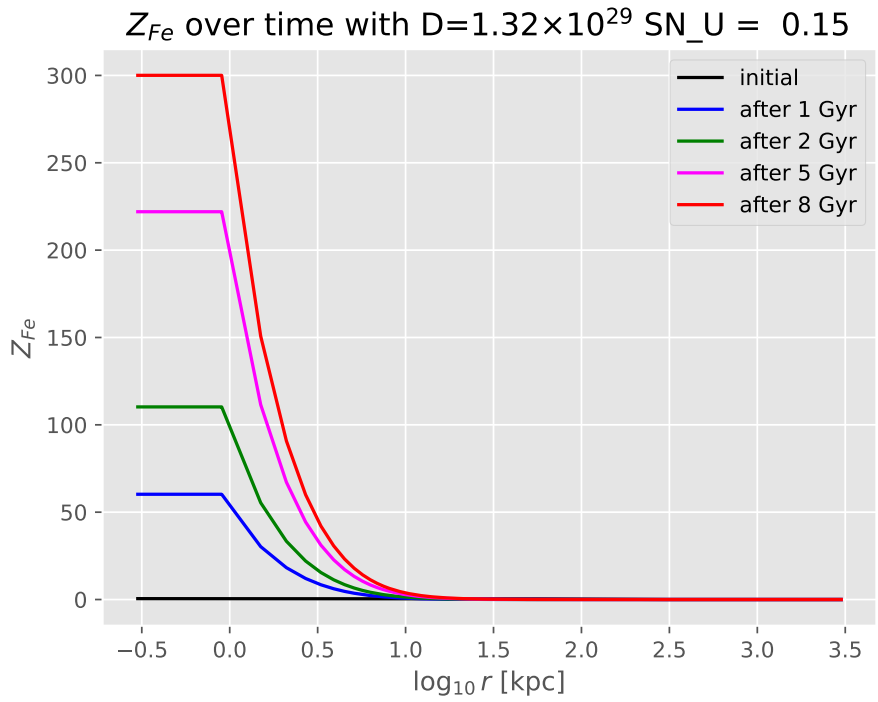


Figure 11: Iron abundance in the ICM using only the source term.

5 Source term and diffusion analysis

We have run several simulations with different diffusion coefficient and different SNu per century in order to reproduce the observed iron abundance in the ICM. Each simulation is run for 8 Gyr. The width of the peak is taken where the Z_{Fe} is at least 0.1 $Z_{Fe}\odot$.

$D[\mathrm{cm}^2\mathrm{s}^{-1}]$	SN_{unit}	$M_{Fe}^{initial}[M_\odot]$	$M_{Fe}^{final}[M_\odot]$	$M_{Fe}^{initial}(<100)[M_\odot]$	$M_{Fe}^{final}(<100)[M_\odot]$
1.32	0.15	0.00E+00	2.04E+08	0.00E+00	1.29E+08
1.32	0.20	0.00E+00	2.72E+08	0.00E+00	1.72E+08
1.32	0.40	0.00E+00	5.44E+08	0.00E+00	3.44E+08
1.32	0.50	0.00E+00	6.79E+08	0.00E+00	4.30E+08
1.32	0.70	0.00E+00	9.51E+08	0.00E+00	6.03E+08
1.50	0.15	0.00E+00	2.03E+08	0.00E+00	1.22E+08
1.50	0.20	0.00E+00	2.71E+08	0.00E+00	1.63E+08
1.50	0.40	0.00E+00	5.42E+08	0.00E+00	3.25E+08
1.50	0.50	0.00E+00	6.77E+08	0.00E+00	4.06E+08
1.50	0.70	0.00E+00	9.48E+08	0.00E+00	5.69E+08
1.50	2.00	0.00E+00	2.71E+09	0.00E+00	1.63E+09
2.00	0.15	0.00E+00	2.02E+08	0.00E+00	1.06E+08
2.00	0.20	0.00E+00	2.69E+08	0.00E+00	1.41E+08
2.00	0.40	0.00E+00	5.38E+08	0.00E+00	2.83E+08
2.00	0.50	0.00E+00	6.72E+08	0.00E+00	3.53E+08
2.00	0.70	0.00E+00	9.41E+08	0.00E+00	4.95E+08
3.00	0.15	0.00E+00	2.00E+08	0.00E+00	8.50E+07
3.00	0.20	0.00E+00	2.67E+08	0.00E+00	1.13E+08
3.00	0.40	0.00E+00	5.34E+08	0.00E+00	2.26E+08
3.00	0.50	0.00E+00	6.67E+08	0.00E+00	2.83E+08
3.00	0.70	0.00E+00	9.34E+08	0.00E+00	3.96E+08
5.00	0.70	0.00E+00	9.28E+08	0.00E+00	2.90E+08

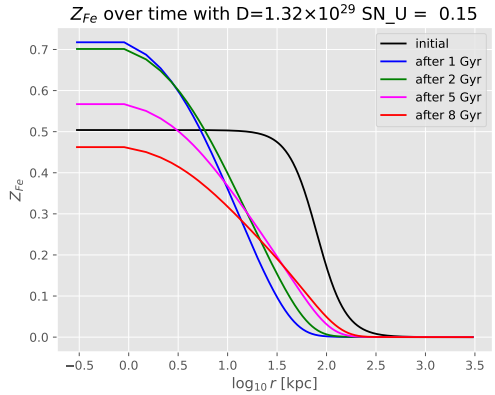
Above, the green rows are the one that can reach the observed iron abundance in the ICM with a "standard" 0.15 Snu, while the yellow one are the one that can reach the observed iron abundance but needs much more SN per century. (2x,3x,..)

The plot of the Z_{Fe} in the ICM for the selected simulations is shown below.

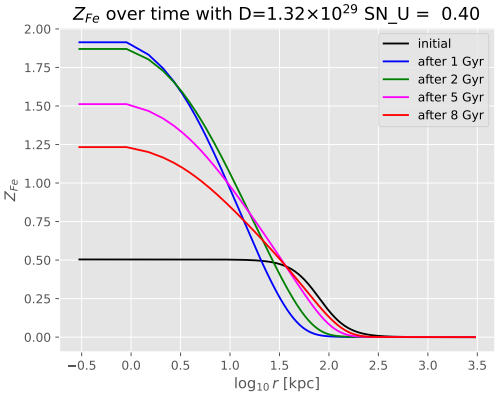
$$Z_{Fe}^{obs}(tot) \approx 8.69 \times 10^8 M_\odot$$

$$Z_{Fe}^{obs}(<100kpc) \approx 3.81 \times 10^8 M_\odot$$

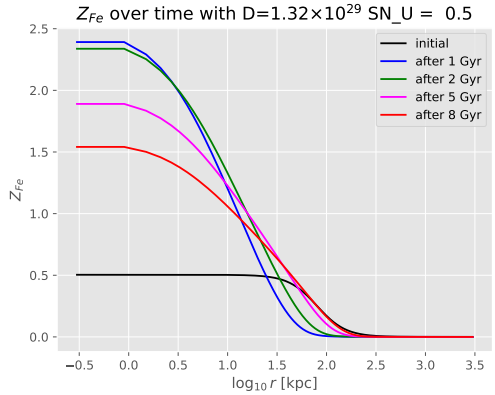
ID	$D[\mathrm{cm}^2\mathrm{s}^{-1}]$	SN_{unit}	$M_{Fe}^{initial}[M_\odot]$	$M_{Fe}^{final}[M_\odot]$	$M_{Fe}^{initial}(<100)[M_\odot]$	$M_{Fe}^{final}(<100)[M_\odot]$
1	1.32	0.15	0.00E+00	2.04E+08	0.00E+00	1.29E+08
2	1.32	0.40	0.00E+00	5.44E+08	0.00E+00	3.44E+08
3	1.32	0.50	0.00E+00	6.79E+08	0.00E+00	4.30E+08
4	1.50	0.15	0.00E+00	2.03E+08	0.00E+00	1.22E+08
5	1.50	0.40	0.00E+00	5.42E+08	0.00E+00	3.25E+08
6	2.00	0.40	0.00E+00	5.38E+08	0.00E+00	2.83E+08
7	2.00	0.50	0.00E+00	6.72E+08	0.00E+00	3.53E+08
8	2.00	0.70	0.00E+00	9.41E+08	0.00E+00	4.95E+08
9	3.00	0.15	0.00E+00	2.00E+08	0.00E+00	8.50E+07
10	3.00	0.70	0.00E+00	9.34E+08	0.00E+00	3.96E+08



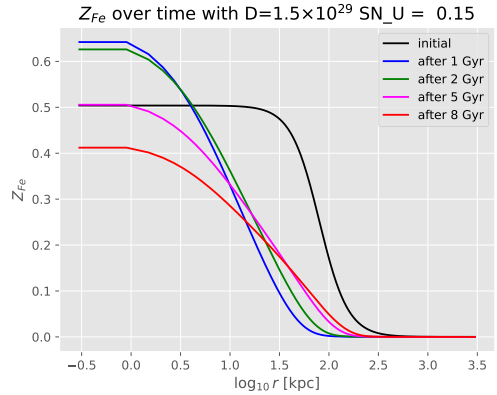
(a) ID = 1. Not enough diffusion.



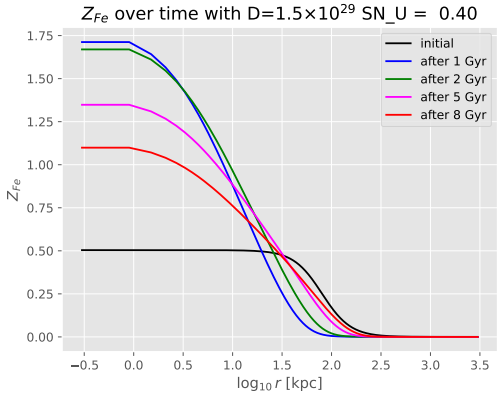
(b) ID = 2. At 8 Gyr iron peak is around 100 Kpc.



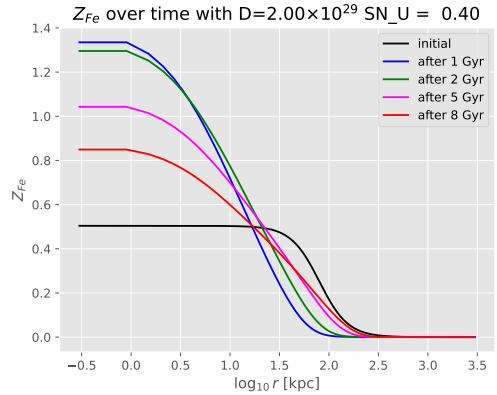
(a) ID = 3. Iron peak at 5-8 Gyr is around 100 Kpc. But we are injecting three times more SN.



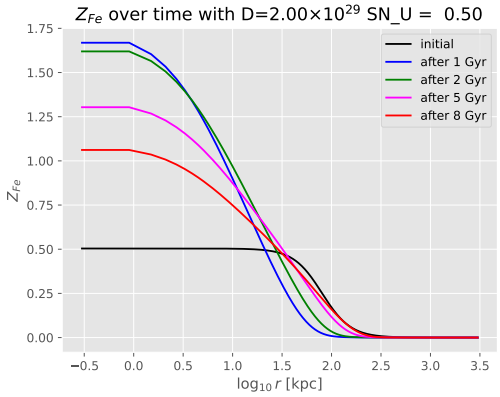
(b) ID = 4. Not diffused enough. But at 5Gyr central iron is correct.



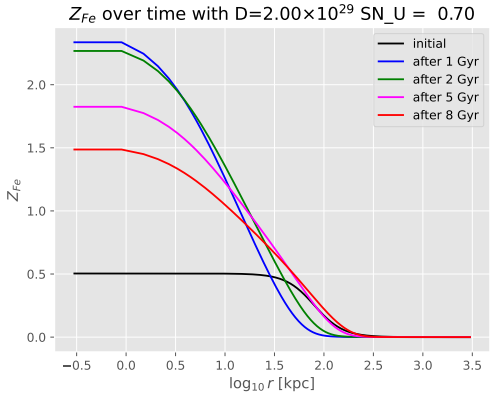
(a) ID = 5. Iron peack at 5-8Gyr is around 80-100kpc. But we are injecting approx three times more SN.



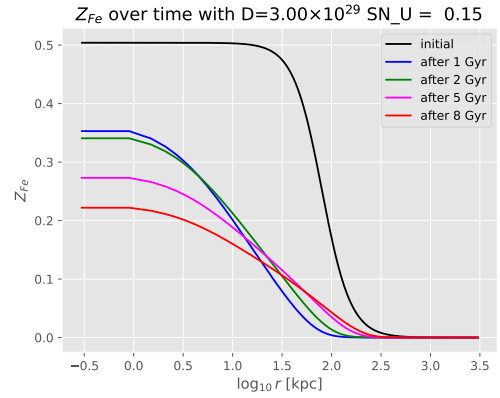
(b) ID = 6. Not so good agreement with peak morphology



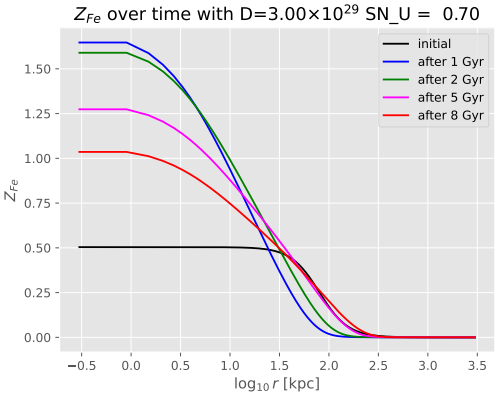
(a) ID = 7



(b) ID = 8. Best match with observed iron peak morphology, total Fe mass and observed mass of Fe at 100kpc. But we are injecting a lot of SNe!



(a) ID = 9. Not enough iron inside the central region.



(b) ID = 10. As in ID = 8, but with a higher diffusion coefficient which.

6 Comments

For the last part of the project we have used an injection of SNe that varies in time with a fixed diffusion coefficient.

```
source_term_rho(i) = (3.13d-21 * rho_stellar(i) *
↪  SN_unit)*(((initial_time+time_passed)/(13.8*gyr))**(-1.1)) &
+ (4.7d-20*rho_stellar(i)*Z_Fe(i)/1.4 * ((initial_time+time_passed)/(13.8*gyr))**(-1.26))
```

$$S(r,t) = 3.13\times10^{-21}\rho_{stellar}(r)SN_{unit}\left(\frac{t}{13.8\,\mathrm{Gyr}}\right)^{-1.1} + 4.7\times10^{-20}\rho_{stellar}(r)Z_{Fe}(r)\left(\frac{t}{13.8\,\mathrm{Gyr}}\right)^{-1.26} \quad (20)$$

We have found that the best match with the observed iron abundance in the ICM is given by a diffusion coefficient of $D = 2.00 \times 10^{29}\mathrm{cm}^2/\mathrm{s}$ and a SNu of 0.70. This is the best match with the observed iron abundance in the ICM, but we are injecting a lot of SNe.

We could use a varying diffusion coefficient, but this would make the simulation more complex.

The main problem is that we need some other mechanism to reduce the iron in the central region faster than in the outer region.