Universidade de Aveiro

Mestrado em Engenharia Informática

Simulação e Otimização

Lesson 3: Discrete-Time and Continuous-Time Simulation

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1. Using the initialize-observe-update architecture, write a python program that simulates the discrete-time model defined by the following equation:

$$x_t = a.x_{t-1}, \quad x_0 = 1.0$$

2. Now, write a python program to simulate:

$$x_t = a.x_{t-1} + b, \quad x_0 = 1.0$$

for different values of ***a*** and ***b***.

3. Predator-Prey interactions may be modelled by discrete-time models whose structure is identical to the following equations:

$$x_t = 0.5.x_{t-1} + y_{t-1}; \quad y_t = -0.5.x_{t-1} + y_{t-1}; \quad x_0 = 1; \quad y_0 = 1$$

Simulate these equations in python.

4. Consider the following differential equation:

$$\frac{dx}{dt} = r.x.\left(1 - \frac{x}{K}\right)$$

write a python program that can simulate the trajectory of $\boldsymbol{x}$ when $x(0) = 0.1, r = 0.2$, $k = 1, \Delta t = 0.01$:

4.1. using the Euler forward method

4.2. using the Runge-Kutta method

5. Consider the following differential equation, that may model the behavior of a pendulum:

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}.sin(\theta)$$

Write a python program that may simulate this system and provide the value of $\boldsymbol{\theta}$ and $\boldsymbol{\omega}$ (angular speed; first derivate of $\boldsymbol{\theta}$):

5.1. using the Euler forward method

5.2. using the Runge-Kutta method

Bibliography

[1] “Introduction to the Modeling and Analysis of Complex Systems”, Hiroki Sayama, Open SUNY Textbooks