Expontial Distribution Simulation

Benjamin Milks

Exponential Distributions

The exponential distribution is modeled by the probability distribution function; $\lambda/e^{x*\lambda}$. Lambda represents the rate of events. In this example the lambda value is 0.2. R allows for us to use the function rexp to produce samples from the exponential distribution. I will get a sample of 40 numbers from the exponential distribution and calculate its mean. This process will be repeated 1000 times to get a distribution of means. This distribution will be compared to 1000 samples from the exponential distribution, a sample of 1000 uniforms, and 1000 means of 40 uniforms.

R Simulations

I want to make many comparisons, one of them being 1000 samples from the exponential distribution to 1000 means of 40 samples from the exponential distribution. This was my code to get the data.

```
expdistributionsamples <- rexp(1000, .2)
expmeans = NULL
for (i in 1:1000) expmeans = c(expmeans, mean(rexp(40,.2)))
```

The exponential function is a unique function. As your sample size increases you can see the mean and standard deviation of what you sampled go to 1/lambda. The mean of sample means should also be approaching 1/lambda.

Mean

```
expmean <- mean(expdistributionsamples)
samplesmean <- mean(expmeans)
theoreticalmean <- 1/.2
```

```
## [1] "The theoretical mean for the exponential distribution for a lambda of 0.2 is 5"
```

```
## [1] "The simulated mean is 5.084 And mean of means of 40 samples is 4.988"
```

Variance

Variance is of sample means is different from the variance of the distribution from which it was sampled, and this is not what happens with means and means of means. The variance of the 1000 samples(not the means of samples), should be the same as the theoretical variance of the population. The difference is the variance in the original exponential distribution is greater than the variance that comes from the means of 40 samples. The variance from means of 40 samples should be smaller than the theoretical variance of the exponential distribution. The standard deviation of the distribution is 1/lambda, so we know the variance is standard deviation squared, or 1^2 / lambda^2. The variance of sample means should be 1^2 / ((lambda^2) * 40), which is much smaller, which makes a higher percentage of the distribution near the mean, and a narrower distribution.

```
theoreticalvariance <- (1*1)/(0.2*0.2)
samplestheovar <- 25/40
samplesinpracticevar <- var(expmeans)
```

```
## [1] "The variance for the exponential distribution for a lamba of 0.2 is 25"
```

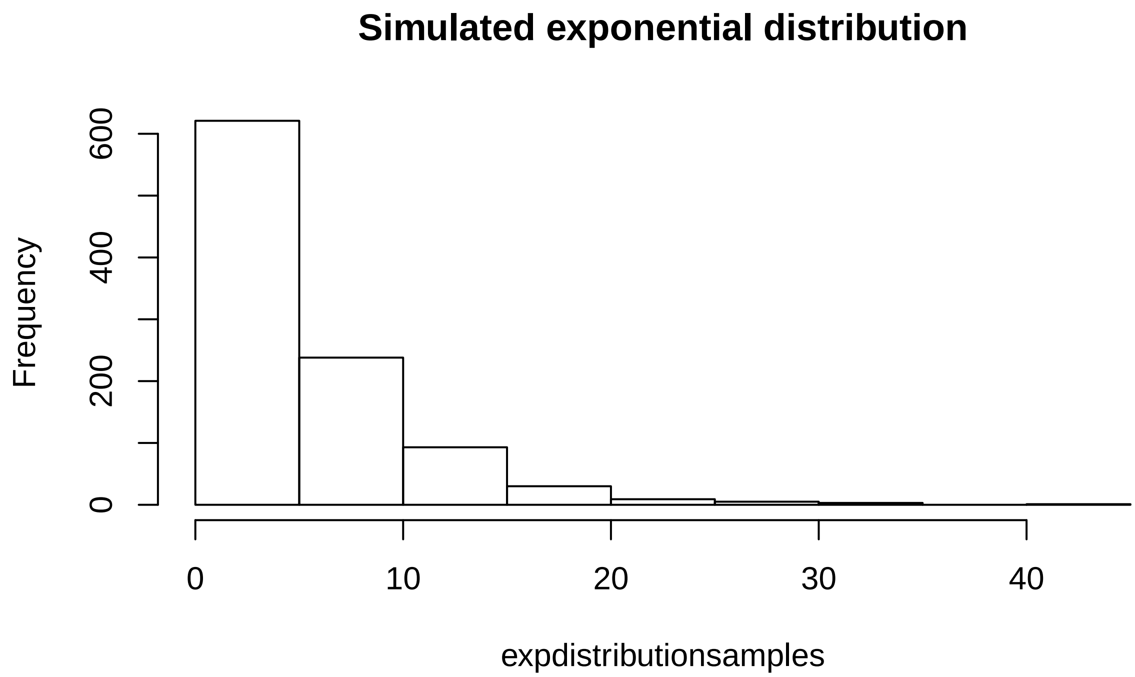
```
## [1] "The variance in our simulation of 1000 samples is 26.457"
```

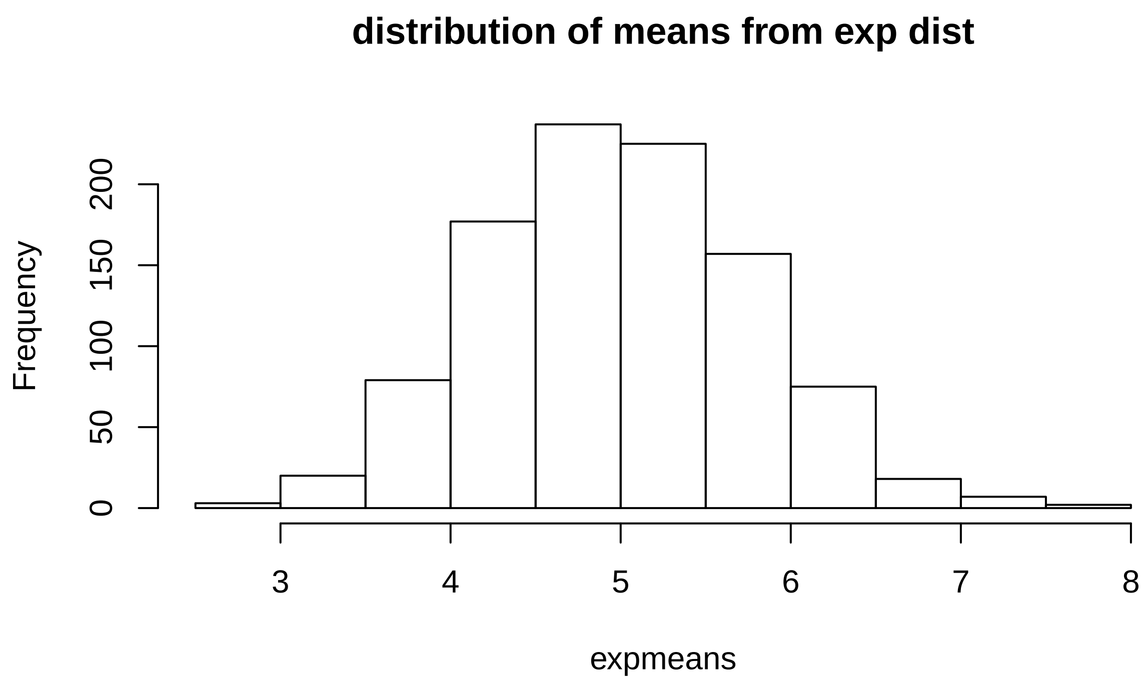
```
## [1] "The variance of sample means from exponential distribution with a lamba of 0.2 i
```

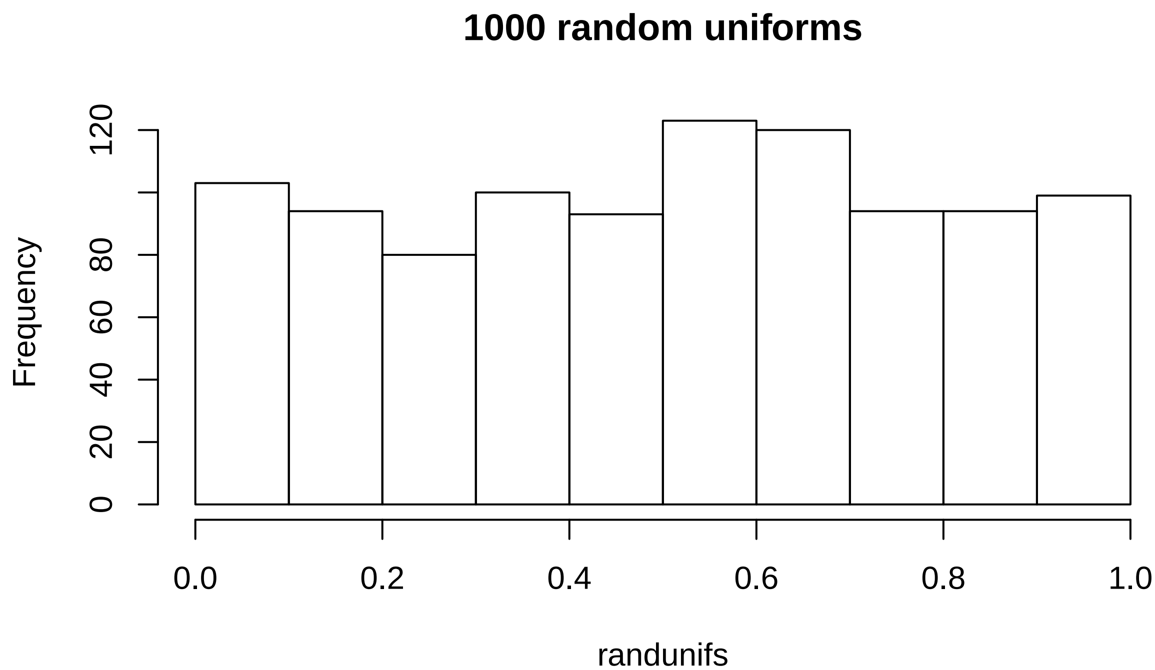
```
## [1] "My simulation of sample means had a variance of 0.6232"
```

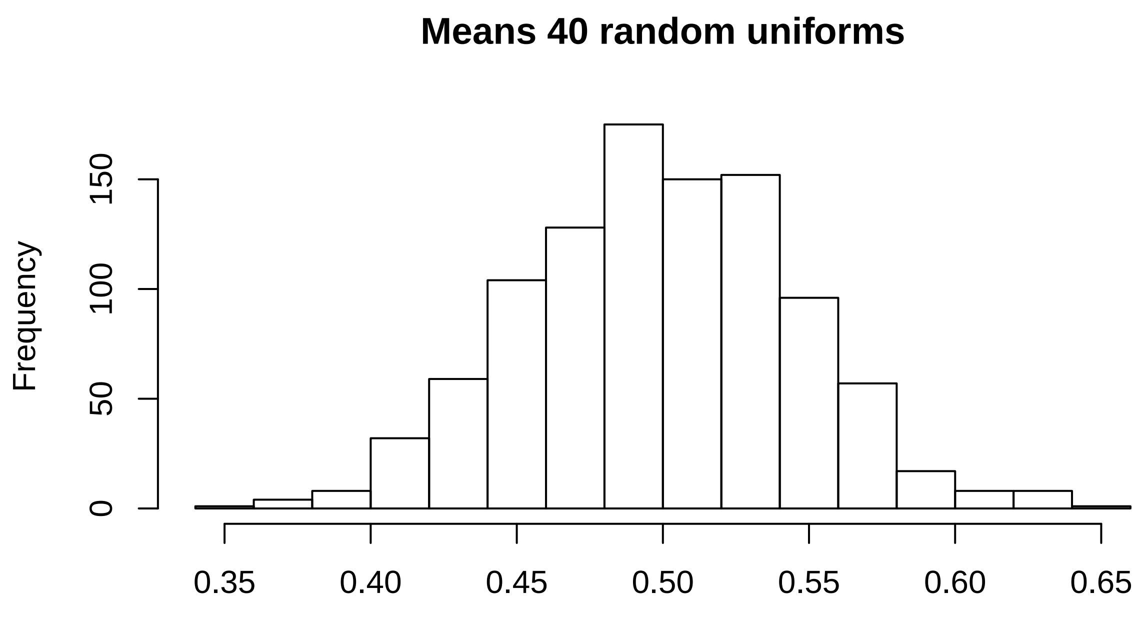
Now if we show histograms of different distributions we can compare their shapes. The first histogram is showing my simulation of the exponential distribution. The 1000 random uniforms, which is the third histogram, has a pretty similar frequency for all of the values between zero and one. When we look at the histogram of means for exponential and uniform, the second and fourth respectively, we see that they have a peak in the middle, and it goes down on both sides with roughly equal amounts on both sides. Both distributions of samples means look very different from the distribution they took from, but they look like they have the same shape as each other. We can see that distribution of the means of 40 exponentials is roughly normal, and that is how it should be in theory. The simulation was pretty close to the actual one.

```
meansofuniform = NULL
for (i in 1:1000) meansofuniform=c(meansofuniform,mean(runif(40)))
randunifs <- runif(1000)
```


Simulated exponential distribution
Frequency
0
200
400
600
0
10
20
30
40
expdistributionsamples


distribution of means from exp dist
Frequency
0
50
100
150
200
3
4
5
6
7
8
expmeans


1000 random uniforms
Frequency
0
20
40
60
80
120
0.0
0.2
0.4
0.6
0.8
1.0
randunifs


Means 40 random uniforms
Frequency
0
50
100
150
0.35
0.40
0.45
0.50
0.55
0.60
0.65
meansofuniform

``‘