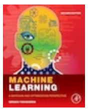
UNIVERSIDADE DO ALGARVE
FACULDADE DE CIÊNCIAS E TECNOLOGIA

Inductive Learning

Readings: (:chapt. 3)

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Pure inductive learning

- Let *f* be an *unknown* target function

Problem: find a hypothesis *h*
such that $h \approx f$
given a training set of examples

An example is a pair (x, $f(x)$)

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Pure inductive learning

- The previous problem is an *ill-defined problem*

  In general, data are not enough to identiy an unique hypotesis.

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Pure inductive learning

- A binary function of two binary variables (x1, x2)

x_1	x_2	h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	h_9	h_{10}	h_{11}	h_{12}	h_{13}	h_{14}	h_{15}	h_{16}
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

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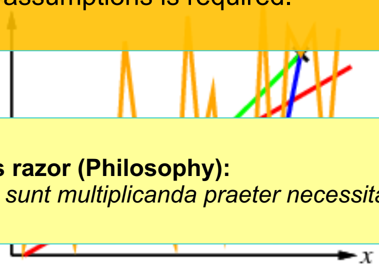
Pure inductive learning

- How to choose among diferent hypothesis (models)?

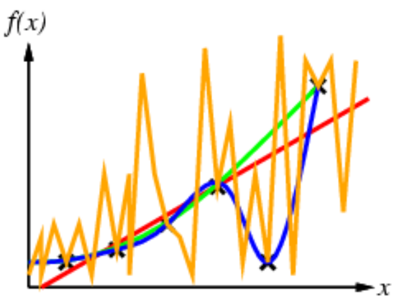
As the problem is ill-defined, an *inductive bias*, i.e., a set of à priori assumptions is required.



Ockham's razor (Philosophy):
"entia non sunt multiplicanda praeter necessitatem"

Ockham's razor – Why?

- Advantages
  - There are less simple hypothesis than complex ones.
  - highly probable that a sufficiently complex hypothesis will fit the data
  - An simple hypothesis that fits data is less probable to be a statistical coincidence.



Inductive learning hypothesis

- Inductive learning hypothesis: Any hypothesis found to approximate the target function well over a sufficient large set of training examples will also approximate the target function well over other unobserved examples.

Inductive Learning

- Generalization: What's the model's performance on new data?
- Overfitting: $\mathcal{H}$ is more complex than f
- Underfitting: $\mathcal{H}$ is less complex than f

To take away

- Inductive learning
  - Pure vs. biased inductive learning
  - Ockham's razor
  - Inductive Learning hypothesis

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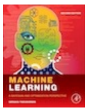
Learning in parametric modeling (Simple linear regression)

Readings: (

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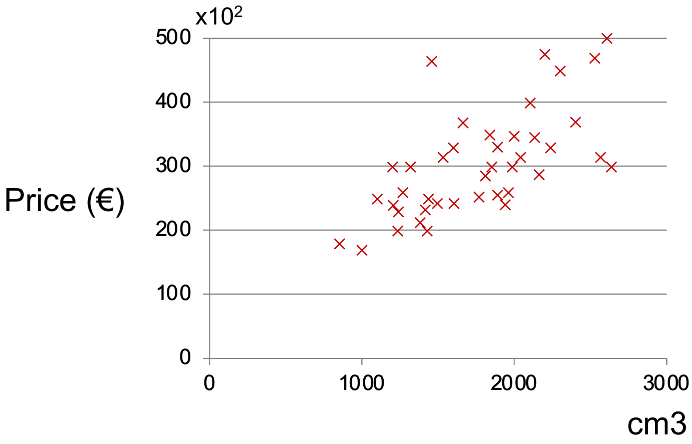
Working example:
Car's price as a function of engine volume



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Some notation

Cm3 (x)	Price in € (y)
2104	460
852	178
…	…

D={($x^{(i)}$, $y^{(i)}$) | i=1,…, m} – Data set

m - Number of examples (elements) in the data set
$x^{(i)}$ - input variable (feature) of the i-th example
$y^{(i)}$ – desired output variable of the i-th example

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Hypothesis

$$h_\theta(x) = \theta_0 + \theta_1 x$$

where θ_0, θ_1 are real-valued coefficients or weights

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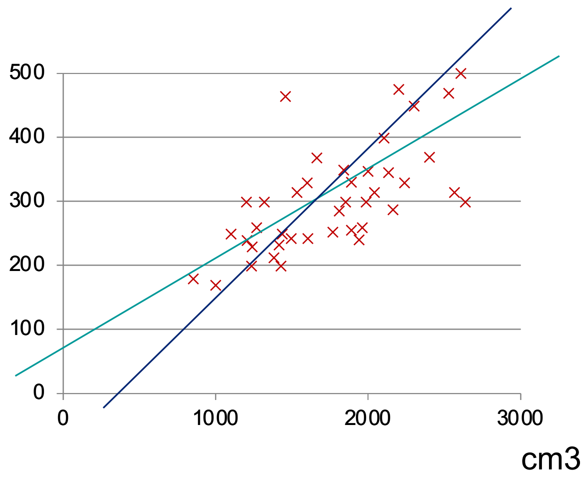
Working example:
Car's price as a function of its engine



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Performance function (or loss function)

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \Big(\underbrace{h_\theta(x^{(i)}) - y^{(i)}}_{\text{resídual}} \Big)^2$$

Objective: $\underset{\theta_0, \theta_1}{\text{minimize}}\ J(\theta_0, \theta_1)$

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Y-intercept and sample slope

- Hypothesis: $h_\theta(x) = \theta_0 + \theta_1 x$
- Y-Intercept: $\theta_0 = \bar{y} - \theta_1 \bar{x}$
- Sample' slope $\theta_1 = \frac{SS_{xy}}{SS_{xx}} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$

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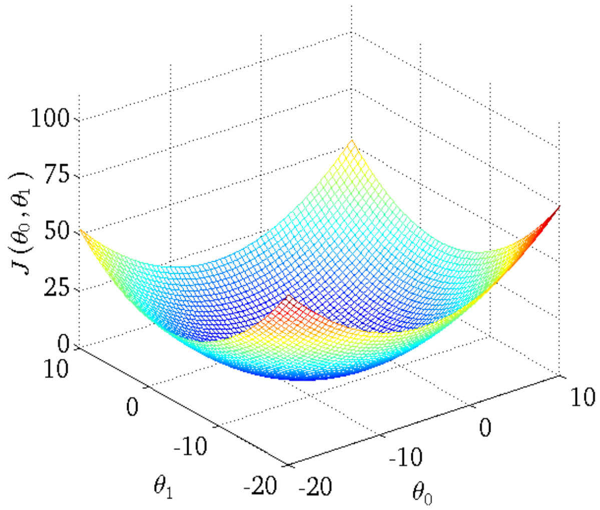
J as function of θ_0 and θ_1



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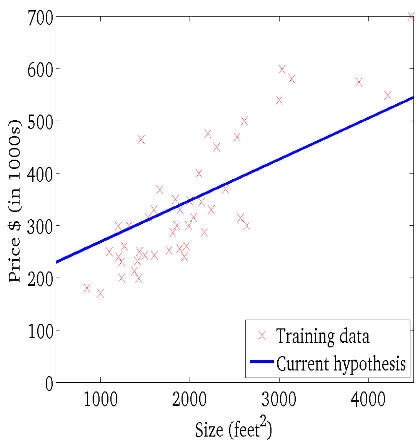
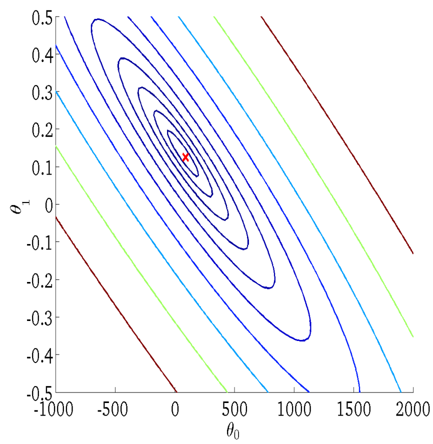
Contour of $J(\theta_0, \theta_1)$

$h_\theta(x)$



$J(\theta_0, \theta_1)$



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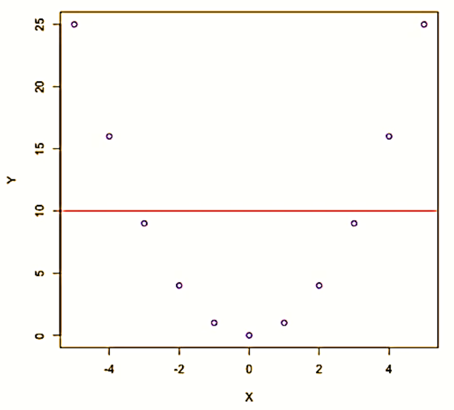
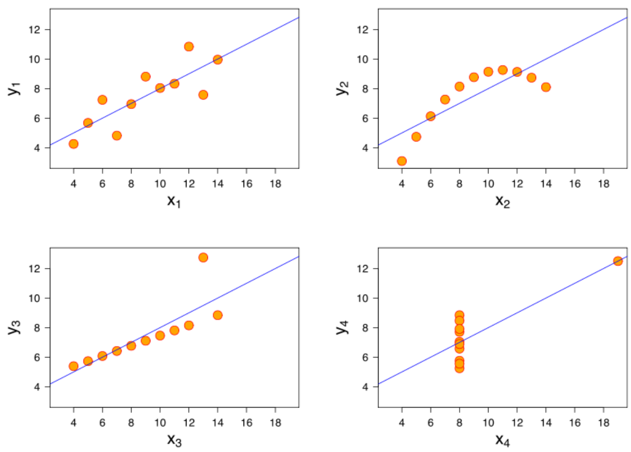
Linear regression of a quadratic function



F. J. Anscombe's quartet (1973)



Residuals

- Residuals are the difference between the observed (y) and predicted responses (h).
- For the normal error regression model, we assume that the error term is normally distributed with null mean and constant variance.
- If the model is appropriate for the data, this should be reflected in the residuals. In particular, the residual vs. input graphic should be centered in zero and show no tendency.

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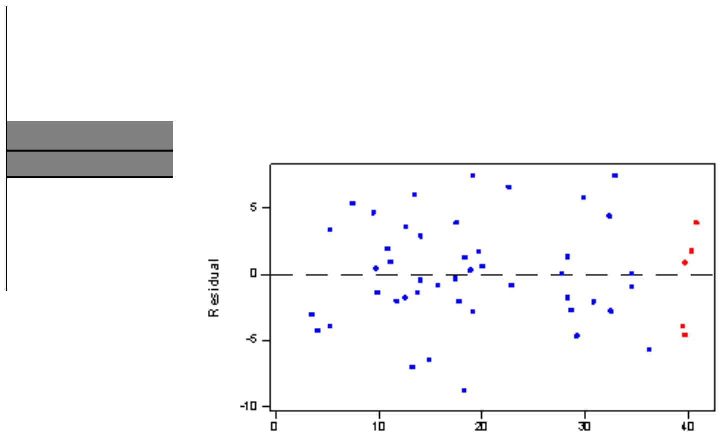
Residuals, the pattern we would like to see



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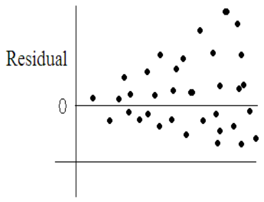
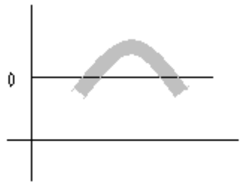
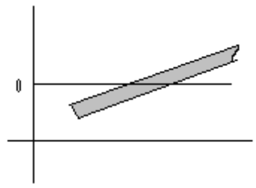
Residuals with some kind of anomalies







To take away

- Inductive learning
  - Pure vs. biased inductive learning
  - Ockham's razor
  - Inductive Learning hypothesis
- Simple Linear regression
  - Notation
  - Performance or total loss function
  - Y-Intercept, slope, and residuals
  - Limitations

Recalling the gradient method

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Problem formulation

- Find the x* such that minimizes

$$f : R^n \to R$$

where f is a smooth C^1 function

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The gradient

$$f : R^n \to R$$

$$\nabla f(x_1,\dots,x_n) := \left(\frac{\partial f}{\partial x_1},\dots, \quad \frac{\partial f}{\partial x_n} \right)^{\mathrm{T}}$$

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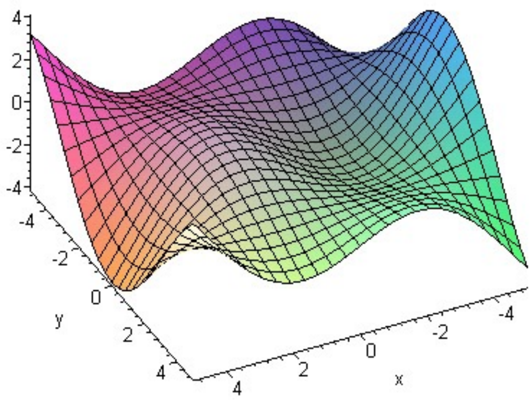
Application example



$$f := (x, y) \to \cos\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}y\right)x$$

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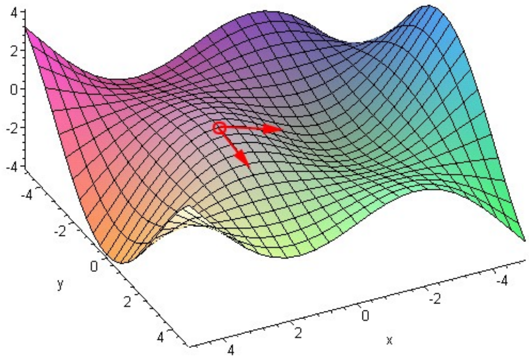
Directional derivatives

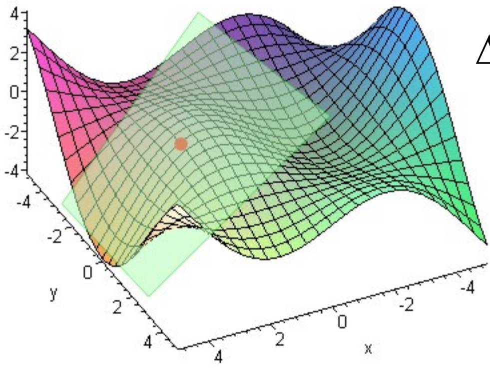
$$\frac{\partial f(x,y)}{\partial y}$$

$$\frac{\partial f(x,y)}{\partial x}$$



The gradient properties

- The gradient defines a (hyper) plane approximating the function infinitesimally



$$\Delta z = \frac{\partial f}{\partial x} \cdot \Delta x + \frac{\partial f}{\partial y} \cdot \Delta y$$

The gradient properties

- **Proposition**: let $f : R^n \to R$ be a smooth C^1 function around p, if f has local minimum (or maximum) at p then,

$$(\nabla f)_p = \bar{0}$$

Gradient (steepest descent) algorithm

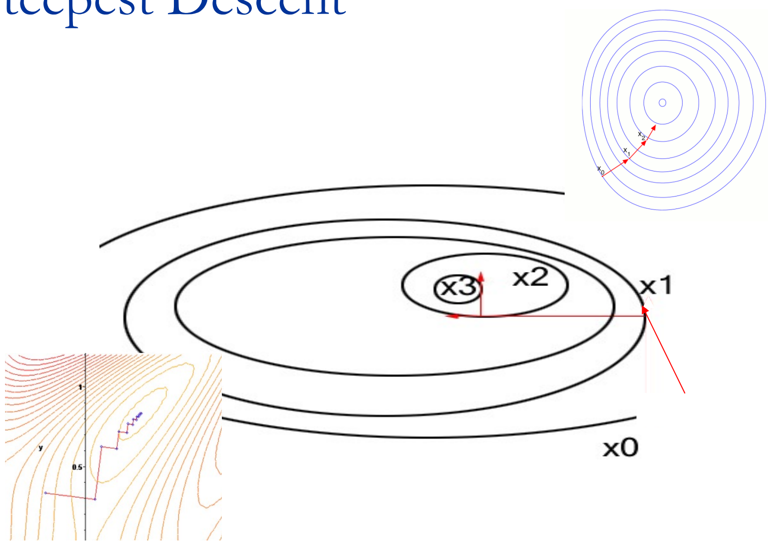
i=0

$x_0 \in R^n$

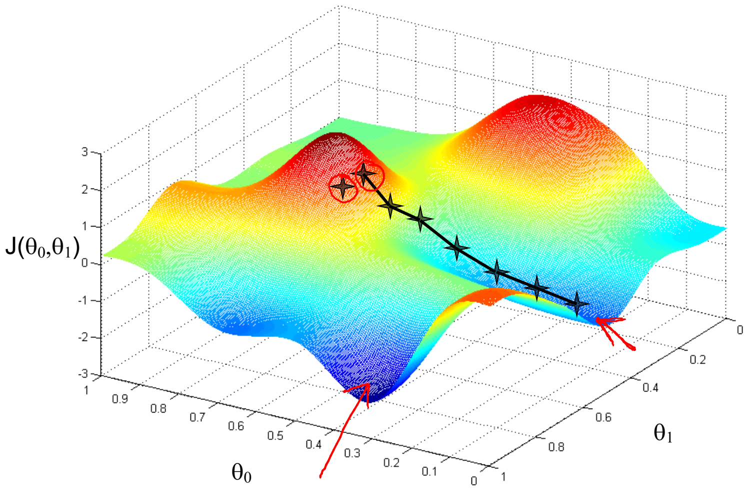
Repeat until $\nabla f(x_i) = 0$

1. compute *search direction* $h_i = -\nabla f(x_i)$
2. update $x_{i+1} = x_i + \eta \cdot h_i$

Steepest Descent



The effect of initial conditions

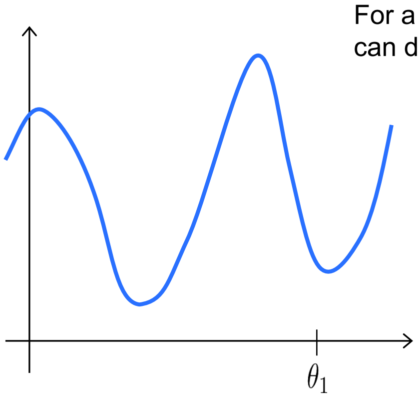


Local minima and weight initialization

- The stepest decent gradient finds a minimum, not necessarily a global minimum,
- Run the algorithm *N* times with small different random initial values for the weights.

Learning step size parameter, η

For a too high η, gradient can diverge



For a too small η convergence will be too slow

Gradient and regression in brief

Gradient

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)$$

$$(\text{for } j = 1 \text{ and } j = 0)$$

}

Linear regression model

$$h_\theta(x) = \theta_0 + \theta_1 x$$

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

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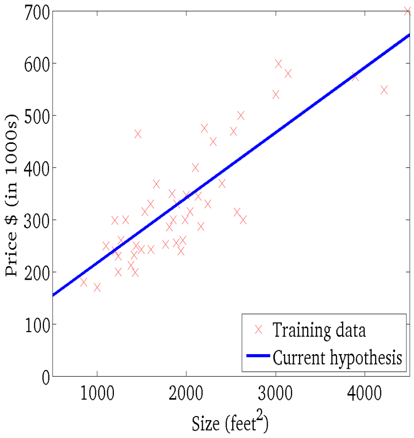
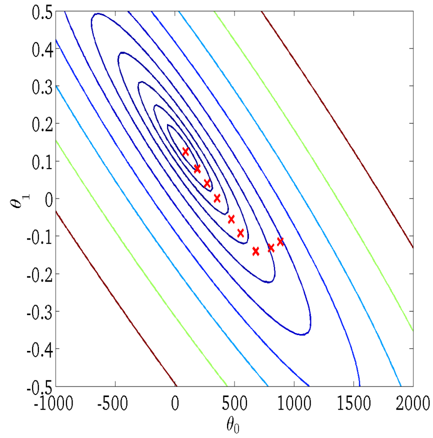
Evolution of J(θ_0, θ_1)





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To take away today