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Exercício R.1) $\sum_{i=1}^{n} i = 1 + 2 + \cdots + n$

Exercício R.2) $\sum_{0}^{n-2} (n-i-1)$

Exercício R.3) $\sum_{1}^{4} n^2 = 1^2 + 2^2 + 3^2 + 4^2$

Exercício R.4) $\sum_{1}^{4} 3i = 3 + 6 + 9 + 12 = 30$

Exercício R.5) $\sum_{1}^{4} (3-2i) =$

$(3-2\cdot1) + (3-2\cdot2) + (3-2\cdot3) + (3-2\cdot4)$

$1 + (-1) - 3 - 5 = -8$

Exercício R.6) $\sum_{i}^{3} (2i + x) =$

$\sum_{i}^{3} 2i + \sum_{i}^{3} x = 2\cdot(1+2+3) + 3x$

$= 12 + 3x$

Exercício R.7) $\sum_{0}^{5} i\cdot(i-1)(5-i)$

$= \sum_{2}^{4} i(i-1)(i-5) = 2(1)(-3) +$

$3(2)(-2) + 4(3)(1) = -6 + (-12) + 12$

$= 30$

Exercício R.8) Sim.

Exercício R.9) $4 + 25 + 64 + 121$

$= \sum_{0}^{3} (3i+2)^2$

Exercício R.10) $\sum_{1}^{6} 8K - 6m = ?$

$= \sum_{1}^{6} 8K - \sum_{1}^{6} 6m = 8\sum_{1}^{6} K - 6\sum_{1}^{6} m$

$= 8\cdot(6K) - 6(1+2+\cdots+6)$

$= 48K - 21(6) = 48K - 126$

Exercício R.11) $\sum_{}^{n} a_i + \sum_{1}^{n} b_i$

$= b_1 + b_2 + \sum_{3}^{n} a_i + b_i$

Exercício R.12)

a) (V) $\sum_{K=0}^{200} K^3 = \sum_{K=1}^{200} K^3$, pois $K=0$ é zero

b) (F) $\sum_{p=0}^{1000} (3+p) = 3 + \sum_{p=0}^{1000} p$, pois não segue a associatividade

Exercício R.12

c) (V) $\sum_{l=1}^{n} (3l) = 3\sum_{l=1}^{n} l$, pois segue a distributividade

d) (F) $\sum_{K=0}^{12} K^p = \left(\sum_{K=0}^{12} K\right)^p$, pois estará elevando o somo e não

e) (V) $\sum_{t=6}^{32} (3+t) = 75 + \sum_{t=6}^{32} t$, pois somo de 3 é 75

Exercício R.13) $\sum_{0}^{4} (3+2i) = \sum_{0}^{4} (3+(4-i))$

$\sum_{0}^{4} (3+2\cdot(4-i)) = (3+2\cdot(4)) + (3+2\cdot(3)) +$
$(3+2(2)) + (3+2(1)) + (3+2(0))$

$= 11 + 9 + 7 + 5 + 3 = 35$

Está somando ao contrário

Exercício R.14) $1, 4, 7, 10, 13, \ldots$

$1 + 3\cdot i \quad a = 1, \; b = 3$

Exercício R. 15)

$$S_n = \sum_{0}^{n} a + b \cdot i = \sum_{0}^{n} a + b(n-i)$$

$$= \sum_{0}^{n} a + bn - bi$$

$$2S = \sum_{0}^{n} a + bi + \sum_{0}^{n} a + bn - bi$$

$$2S = \sum_{0}^{n} [2a + bn] = \sum_{0}^{n} 2a + bn$$

$$2S = (2a + bn) \cdot \sum_{0}^{n} 1 = (2a + bn)(n+1)$$

$$\boxed{S = \frac{(2a + bn)(n+1)}{2}}$$

Exercício R. 16) $0+1+2+3+\dots+n = \sum_{0}^{n} 1$

$$S = \frac{(2(0) + 1n)\,n+1}{2} = \frac{(n)(n+1)}{2} = \boxed{\frac{n^2+n}{2}}$$

Exercício R. 17) $\sum_{0}^{n-1} i+1 = 1+2+3+\dots+n$

$$S = \frac{(2a + bn)(n+1)}{2} = \frac{(2(1) + 1n)(n+1)}{2}$$

$$= \frac{(2(0) + n - 1)(n+1)}{2} = \frac{n^2 + n - n - 1}{2}$$

Exercício Resolvido (18)

$$\sum_{0}^{n-2} (n - i - 1) = \sum_{0}^{n-2} n - \sum_{0}^{n-2} i - \sum_{0}^{n-2} 1$$

$$= (n-2)n - \frac{(0+n)(n+1)}{2} - \sum_{0}^{n-2} 1$$

$$= n^2 - 2n - \frac{(n^2 + n)}{2} - (n-1)$$

$$= \frac{2n^2 - 4n - n^2 - n - 2n + 1}{2}$$

$$= \boxed{\frac{n^2}{2} - \frac{n}{2} = \Theta(n^2)}$$

0+1+2+3+4

Exercício R. 19)

$$\sum_{1}^{n} i = \sum_{0}^{n} i = (a_0) + \sum_{1}^{n} i$$

$a_0 = 0$

portanto: $\sum_{1}^{n} i = \sum_{1}^{n} i$

Exercício R. 20)

$$\sum_{1}^{n} a_i \neq \sum_{0}^{n} a_i \rightarrow a_1 \neq a_0$$

perderia-se 1 termo

Exercício R. 21)

$$\sum_{1}^{n} a_i = \sum_{0}^{n-1} a_{i+1}$$

pela comutatividade

$$\sum_{0+1}^{n-1+1} a_{i-1+1} = \sum_{1}^{n} a_i \text{ , ou}$$

pode-se observar que o resultado é sempre $i+1$

Exercício R. 22)

$$\sum_{i=2}^{n-1} i \cdot (i-1) \cdot (n-i) = \sum_{i=0}^{n} i \cdot (i-1) \cdot (n-$$

$n(0) = 0$, $n(1) = 0$

Exercício R. 23)

$$S_n = \sum_{0}^{n} a \cdot x^i \qquad a_i = a \cdot x^i$$

$$S_n + a \cdot x^{n+1} = a \cdot x^0 + \sum_{0}^{n} a \cdot x^{i+1}$$

$$S_n + a \cdot x^{n+1} = x \sum_{0}^{n} a \cdot x^i + a$$

$$S_n + a \cdot x^{n+1} = x \cdot S_n + a$$

$$S_n - x \cdot S_n = a - a \cdot x^{n+1}$$

$$S_n (1 - x) = a - a \cdot x^{n+1}$$

$$\boxed{S_n = \frac{a - a \cdot x^{n+1}}{1 - x}}$$

Exercício R. 24)

$S_n = \sum_{0}^{n} i \cdot 2^i$ $a_i = i \cdot 2^i$

$S_n + a_{n+1} = a_0 + \sum_{0}^{n} a_{i+1}$

$S_n + (n+1)2^{n+1} = 0 \cdot 2^0 + \sum_{0}^{n} (i+1)2^{i+1}$

$S_n + 2(n+1)2^n = 2\sum_{0}^{n} (i+1)2^i$

$S_n + 2(n+1)2^n = 2 \cdot \left[\sum_{0}^{n} 2^i \cdot i + 2^i\right]$

$S_n + 2(n+1)2^n = 2\left[\sum_{0}^{n} 2^i \cdot i + \sum_{0}^{n} 2^i\right]$

$S_n + 2(n+1)2^n = 2S_n + 2\sum_{0}^{n} 2^i$

$S_n + 2(n+1)2^n = 2S_n + 2 \cdot (2^{n+1} - 1)$

$S_n = (n-1) \cdot 2^{n+1} + 2$

Exercício R. 25)

$\sum_{0}^{n} (3+i) = \sum_{0}^{n} 3 + \sum_{0}^{n} i$

$= 3(n+1) + \frac{n(n+1)}{2}$

$= \frac{6(n+1) + n(n+1)}{2}$

$= \frac{6n+6+n^2+n}{2} = \frac{n^2+7n+6}{2}$

1º caso base:

$\frac{0^2+7(0)+6}{2} = 3$ (correto)

2º indução: $S_n = S_{n-1} + a_n$

$S_n = \frac{(n-1)^2 + 7(n-1) + 6}{2} + (3+n)$

$S_n = \frac{n^2 - 2n + 1 + 7n - 7 + 6}{2} + (3+n)$

$S_n = \frac{n^2+5n}{2} + 3 + n = \frac{n^2+7n+6}{2}$

(correto)

Exercício (26) 12.

$\sum_{1}^{n} [(2i+1)^2 - (2i)^2] =$

$\sum_{1}^{n} [4i^2 + 4i + 1 - 4i^2] = \sum_{1}^{n} [4i+1]$

$4\sum_{1}^{n} i + \sum_{1}^{n} 1 = 4\frac{n(n+1)}{2} + n = 2n^2+2n+n$

$= 2n^2 + 3n$

1º caso base:

$2(1) + 3(1) = 5$

2º indução: $S_n = S_{n-1} + a_n$

$S_n = 2(n-1)^2 + 3(n-1) + (4n+1)$

$S_n = 2(n^2 - 2n + 1) + 3n - 3 + 4n + 1$

$S_n = 2n^2 - \cancel{4n} + \cancel{2} + 3n - \cancel{3} + 4n + 1$

$S_n = 2n^2 + 3n$

Exercício R. 27)

$\sum_{1}^{n} [(5i+1)^2 - (5i-1)^2] =$

$\sum_{1}^{n} [(25i^2 + 10i + 1) - (25i^2 - 10i + 1)]$

$\sum_{1}^{n} [20i] = 20\sum_{1}^{n} i = 20\left(\frac{n(n+1)}{2}\right)$

$= \underline{40(n^2+n)} = \underline{40n^2 + 40n} =$

$S_n = 10n^2 + 10n$

1º Passo Base: $10(1) + 10(1) = 20$ (V)

2º indução: $S_n = S_{n-1} + a_n$

$S_n = 10(n-1)^2 + 10(n-1) + 20n$

$S_n = 10(n^2 - 2n + 1) + 10n - 10 + 20n$

$S_n = 10n^2 - \cancel{20n} + \cancel{10} + 10n - \cancel{10} + \cancel{20n}$

$S_n = 10n^2 + 10n$

Exercício R. ... $(n-1)\cdot 2^{n+1}+2$

$S_n=\sum_0^n i\cdot 2^i=(n-1)\cdot 2^{n+1}+2$

$S_n=S_{n-1}+a_n$

$S_n=((n-1)-1)\cdot 2^{(n-1)+1}+2+n2^n$

$S_n=(n-2)\cdot 2^n+2+n2^n$

$S_n=n2^n-2^n+2+n2^n$

$S_n=(n-1)2^n2+2$

$S_n=(n-1)2^{n+1}+2$

Exercício R. 29)

$S_n=\sum_0^n i^2 \quad S_{n+1}=S_n+a_{n+1}=a_0+\sum_0^n a_{i+1}$

$a_i=i^2$

$S_n+(n+1)^2=i^0+\sum_0^n(i+1)^2$

$S_n+n^2+2n+1=0+\sum_0^n i^2+2i+1$

$S_n+n^2+2n+1=\sum_0^n i^2+2\sum_0^n i+\sum_0^n 1$

$S_n+n^2+2n+1=S_n+n(n+1)+n+1$

$S_n=\sum_0^n i^3 \quad S_{n+1}=S_n+a_{n+1}=a_0+\sum_0^n a_{i+1}$

$a_i=i^3$

$S_n+(n+1)^3=0+\sum_0^n(i+1)^3$

$S_n+(n^2+2n+1)(n+1)=\sum_0^n(i^2+2i+1)(i+1)$

$S_n+n^3+n^2+2n^2+2n+n+1=\sum_0^n(i^3+i^2+2i^2+2i+i+1)$

$S_n+n^3+3n^2+3n+1=\sum_0^n(i^3+3i^2+3i+1)$

$S_n+n^3+3n^2+3n+1=\sum_0^n(i^3)+\sum_0^n 3i^2+\sum_0^n 3i+\sum_0^n 1$

$\cancel{S_n}+n^3+3n^2+3n+1=\cancel{S_n}+3S_2+\underline{3n(n+1)}+n+1$

$4n^2 \quad 4n$

$2n^3+\cancel{6}n^2+\cancel{6}n+\cancel{2}=6S_n+\cancel{3n^2}+\cancel{3n}+\cancel{2n}+\cancel{2}$

$2n^3+4n^2=6S_n$

$S_n=\dfrac{n^3+2n^2}{3}$