8. Mostre que as funções $f = \text{for } id\ i$ e $g = \text{for } \underline{i}\ i$ são a mesma função. (Qual?)

Resolução

Sabendo que $for\ succ\ a = (\!|[\underline{a}, succ]|\!)$ temos:

$f = for\ id\ i = (\!|[\underline{i}, id]|\!)$

$g = for\ \underline{i}\ i = (\!|[\underline{i}, \underline{i}]|\!)$

{ pela propriedade universal $(F4)$ }

$f = (\!|[\underline{i}, id]|\!) \equiv f.\, in = [\underline{i}, id].\,(id + f)$

$g = (\!|[\underline{i}, \underline{i}]|\!) \equiv g.\, in = [\underline{i}, \underline{i}].\,(id + g)$

{ def. $in = [\underline{0}, succ]$; fusão-$+$, **lei (20)** }

$f = (\!|[\underline{i}, id]|\!) \equiv [f.\,\underline{0}, f.\, succ] = [\underline{i}, id].\,(id + f)$

$g = (\!|[\underline{i}, \underline{i}]|\!) \equiv [g.\,\underline{0}, g.\, succ] = [\underline{i}, \underline{i}].\,(id + g)$

{ absorção-$+$, **lei (22)** }

$f = (\!|[\underline{i}, id]|\!) \equiv [f.\,\underline{0}, f.\, succ] = [\underline{i}.\, id, id.\, f]$

$g = (\!|[\underline{i}, \underline{i}]|\!) \equiv [g.\,\underline{0}, g.\, succ] = [\underline{i}.\, id, \underline{i}.\, g]$

{ eq-$+$, **lei (27)**, natural-id, **lei (1)** }

$f.\,\underline{0} = \underline{i}; f.\, succ = f$

$g.\,\underline{0} = \underline{i}; g.\, succ = \underline{i}.\, g$

{ natural-const, **lei (3)** }

$f.\,\underline{0} = \underline{i}; f.\, succ = f$

$g.\,\underline{0} = \underline{i}; g.\, succ = \underline{i}$

{ igualdade extensional, **lei (71)** }

$(f.\,\underline{0})\ n = \underline{i}\ n; (f.\, succ)\ n = f\ n$

$(g.\,\underline{0})\ n = \underline{i}\ n; (g.\, succ)\ n = \underline{i}\ n$

{ def-comp, **lei (72)**; def-const, **lei (74)** }

$f\ 0 = i\ ; f\ (n+1) = fn$

$g\ 0 = i\ ; g\ (n+1) = i$

{ $\forall n.\ fn = i; \forall n.\ gn = i$ }

$f = g = \underline{i}$

Haskell

In [4]:

```haskell
for b i 0 = i
for b i n = b (for b i (n-1))

--

for id 1 2
for (const 1) 1 2

for id "k" 2
for (const "k") "k" 2
```

```
1
1
"k"
"k"
```