3. Considere o isomorfismo de ordem superior *flip* definido pela composição de isomorfismos seguinte:

$$\begin{array}{ccccccc} (C^B)^A & \cong & C^{A\times B} & \cong & C^{B\times A} & \cong & (C^A)^B \\ f & \mapsto & \widehat{f} & \mapsto & \widehat{f}.\mathsf{swap} & \mapsto & \overline{\widehat{f}\cdot\mathsf{swap}} = \mathit{flip}\ f \end{array}$$

- Mostre que *flip*, acima definida por $\mathit{flip}\ f = \overline{\widehat{f}\cdot\mathsf{swap}}$, é um isomorfismo por ser a sua própria inversa, isto é, por

$$\mathit{flip}\ (\mathit{flip}\ f) = f \tag{F2}$$

se verificar.

- Mostre ainda que $\mathit{flip}\ f\ x\ y = f\ y\ x$.

Haskell

In [1]:

```
-- loading Cp.hs

:load ../src/Cp.hs

--- checking with add x y

add x y = x + length y

:t add
:t uncurry add
:t (uncurry add) . swap
:t curry ((uncurry add) . swap )
```

```
add :: forall (t :: * -> *) a. Foldable t => Int -> t a -> Int
uncurry add :: forall (t :: * -> *) a. Foldable t => (Int, t a) -> Int
(uncurry add) . swap :: forall (t :: * -> *) a. Foldable t => (t a, Int) -> Int
curry ((uncurry add) . swap ) :: forall (t :: * -> *) a. Foldable t => t a -> Int -> Int
```

Resolução (F2)

Vamos então mostrar que $flip\ (flip\ f) = f$.

$flip\ (flip\ f) = f$

{ def. $flip\ f = \overline{\widehat{f}\,.\,swap}$ }

$\equiv flip\ (\overline{\widehat{f}\,.\,swap}) = f$

{ def. $flip\ f = \overline{\widehat{f}\,.\,swap}$ }

$\equiv \overline{(\widehat{\overline{\widehat{f}\,.\,swap}}).\,swap} = f$

{ uncurry . curry = id }

$$\equiv \overline{(\widehat{f} \,.\, swap) .\, swap} = f$$

{ assoc-comp, **lei (2)** }

$$\equiv \overline{\widehat{f} \,.\, (swap .\, swap)} = f$$

{ swap . swap = id }

$$\equiv \overline{\widehat{f}} = f$$

{ curry . uncurry = id }

$$\equiv f = f$$

{ propriedade reflexiva da igualdade }

True

Haskell

In [2]:

```haskell
flip f = curry (uncurry f . swap)

-- type checking

:t flip

-- checking with \x y -> x + 3 * y

flip (\x y -> x + 3*y) 2 3
```

```
flip :: forall a b c. (a -> b -> c) -> b -> a -> c
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```

Resolução

$$flip\ f\ x\ y\ =\ f\ y\ x$$

{ def. $flip\ f = \overline{\widehat{f} \,.\, swap}$ }

$$\equiv \overline{(\widehat{f} \,.\, swap)}\ x\ y\ =\ f\ y\ x$$

{ def. curry, **lei (83)** }

$$\equiv (\widehat{f} \,.\, swap)\ (x, y)\ =\ f\ y\ x$$

{ def-comp, **lei (72)** }

$$\equiv \widehat{f}\,(swap\ (x, y))\ =\ f\ y\ x$$

{ def. swap (x,y) = (y,x) }

$$\equiv \widehat{f}\,(y, x)\ =\ f\ y\ x$$

{ def. uncurry, **lei (84)** }

$\equiv \; f\,y\,x = \; f\,y\,x$

{ igualdade extensional, **lei (71)** }

$\equiv \; f = \; f$

{ propriedade reflexiva da igualdade }

True