6. Considere o isomorfismo

$$\begin{tikzcd} A^{B+C} \arrow[rr, bend left, "\mathsf{unjoin}"] \arrow[rr, phantom, "\cong"] & & A^B \times A^C \arrow[ll, bend left, "\mathsf{join}"] \end{tikzcd} \tag{F3}$$

onde

$$\begin{aligned} &\mathsf{join}\ (f, g) = [f\ , g] \\ &\mathsf{unjoin}\ k = (k \cdot i_1, k \cdot i_2) \end{aligned}$$

Mostre que join $\cdot$ unjoin $= id$ e que unjoin $\cdot$ join $= id$.

Resolução

Vamos primeiro mostrar que

$join.\, unjoin = id$

{ pointwise, **lei (71)** }

$\equiv (join.\, unjoin)\ k = id\ k$

{ def-comp, **lei (72)** }

$\equiv join(unjoin\ k) = id\ k$

{ def. unjoin }

$\equiv join(k.\, i_1, k.\, i_2) = id\ k$

{ def. join }

$\equiv [k.\, i_1, k.\, i_2] = id\ k$

{ fusão-$+$, **lei (20)** }

$\equiv k.\, [i_1, i_2] = id\ k$

{ reflexão-$+$, **lei (19)** }

$\equiv k.\, id = id\ k$

{ natural-id, **lei (1)**; def-id, **lei (73)** }

$\equiv k = k$

{ propriedade reflexiva da igualdade }

True

Resolução

Vamos agora mostrar que

$unjoin\ .\ join = id$

{ pointwise, **lei (71)** }

$$\equiv (unjoin\ .\ join)\ (f, g) = id\ (f, g)$$

{ def-comp, **lei (72)** }

$$\equiv unjoin\ (join\ (f, g)) = id\ (f, g)$$

{ def. join }

$$\equiv unjoin\ ([f, g]) = id\ (f, g)$$

{ def. unjoin }

$$\equiv ([f, g].\ i_1, [f, g].\ i_2) = id\ (f, g)$$

{ cancelamento-$+$, **lei (18)** }

$$\equiv (f, g) = id\ (f, g)$$

{ def-id, **lei (73)** }

$$\equiv (f, g) = (f, g)$$

{ propriedade reflexiva da igualdade }

True