6. Recorra à lei Eq-+ (entre várias outras) para mostrar que a definição que conhece da função factorial,

$$fac\ 0 = 1$$
$$fac\ (n+1) = (n+1) * fac\ n$$

é equivalente à equação seguinte

$$fac \cdot [\underline{0}, \mathsf{succ}] = [\underline{1}, \mathsf{mul} \cdot \langle \mathsf{succ}, fac \rangle].$$

onde succ $n = n + 1$ e mul $(a, b) = a * b$.

Resolução

$$fac \cdot [\underline{0}, succ] = [\underline{1}, mul \cdot < succ, fac >]$$

{ ***fusão-+, lei (20)*** *}*

$$[fac \cdot \underline{0}, fac \cdot succ] = [\underline{1}, mul \cdot < succ, fac >]$$

{ ***eq-+, lei (27)*** *}*

$$\begin{aligned} fac \cdot \underline{0} &= \underline{1} \\ fac \cdot succ &= mul \cdot < succ, fac > \end{aligned}$$

{ ***lei (71), lei (72)*** *}*

$$\begin{aligned} fac\ (\underline{0}\ x) &= \underline{1}\ x \\ fac\ (succ\ x) &= mul\ (< succ, fac >\ x) \end{aligned}$$

{ ***lei (74)*** *}*

$$\begin{aligned} fac\ 0 &= 1 \\ fac\ (succ\ x) &= mul\ (< succ, fac >\ x) \end{aligned}$$

{ ***def.*** $succ$, ***def.*** mul, ***def.*** $< f, g >$ *}*

$$\begin{aligned} fac\ 0 &= 1 \\ fac\ (x+1) &= (x+1) * fac\ x \end{aligned}$$

In [1]:

```
succ n = n + 1
mul (a,b) = a * b
split f g x = (f x, g x)

fac 0 = 1
fac x = mul . split succ fac $ (x-1)
```

In [2]:

```
-- type checking

:t fac
```

```
fac :: forall p. (Eq p, Num p) => p -> p
```

In [3]:

```
-- testing with 9

fac 9
```

```
362880
```