9. Sabendo que for $f\ i = (\!|[\underline{i}\ ,f]|\!)$ para $\mathsf{F}\ f = id + f$ (naturais), recorra à lei de fusão-cata para demonstrar a propriedade:[3]

$$f \cdot (\text{for}\ f\ i) = \text{for}\ f\ (f\ i) \qquad \text{(F5)}$$

Resolução

Mostrar que $f\ .\ (\text{for} f\ i) = \text{for} f\ (f\ i)$ é o mesmo que mostrar que $f\ .\ (\!|[\underline{i}, f]|\!) = (\!|[\underline{f\ i}, f]|\!)$

$f\ .\ (\!|[\underline{i}, f]|\!) = (\!|[\underline{f\ i}, f]|\!)$

{ universal-cata, **lei (45)**, fazendo $k = f\ .\ (\!|[\underline{i}, f]|\!)$ }

$\equiv (f\ .\ (\!|[\underline{i}, f]|\!))\ .\ in = [\underline{f\ i}, f]\ .\ (id + f\ .\ (\!|[\underline{i}, f]|\!))$

{ fusão-cata, **lei (48)** }

$\Rightarrow (f\ .\ (\!|[\underline{i}, f]|\!))\ .\ (\!|in|\!) = (\!|[\underline{f\ i}, f]|\!)$

{ reflexão-cata, **lei (47)** }

$\Rightarrow (f\ .\ (\!|[\underline{i}, f]|\!))\ .\ id = (\!|[\underline{f\ i}, f]|\!)$

{ natural-id, **lei (1)** }

$\Rightarrow f\ .\ (\!|[\underline{i}, f]|\!) = (\!|[\underline{f\ i}, f]|\!)$

{ def. $for f i = f\ .\ (\!|[\underline{i}, f]|\!)$ }

$f\ .\ (\text{for} f\ i) = \text{for} f\ (f\ i)$

In [21]:

```
:load ../src/Nat.hs
:load ../src/Nat.hs

-- Ex#1
succ . for succ 4 $ 2
for succ (succ 4) 2

-- C language
-- suc(forloop1(suc,4,2)) = forloop1(suc,suc(4),2)

-- Ex#2
(+10) . for (+4) 0 $ 2
for (+4) ((+10)(0)) 2

-- C language
-- add(10,forloop2(add,4,0,2)) = forloop2(add,4,add(10,0),2)

-- Ex#3
(*5) . for (*3) 1 $ 4
for (*3) ((*5)(1)) 4

-- C language
-- mul(5,forloop2(mul,3,1,4)) = forloop2(mul,3,mul(5,1),4)
```

7
18
18
405
405