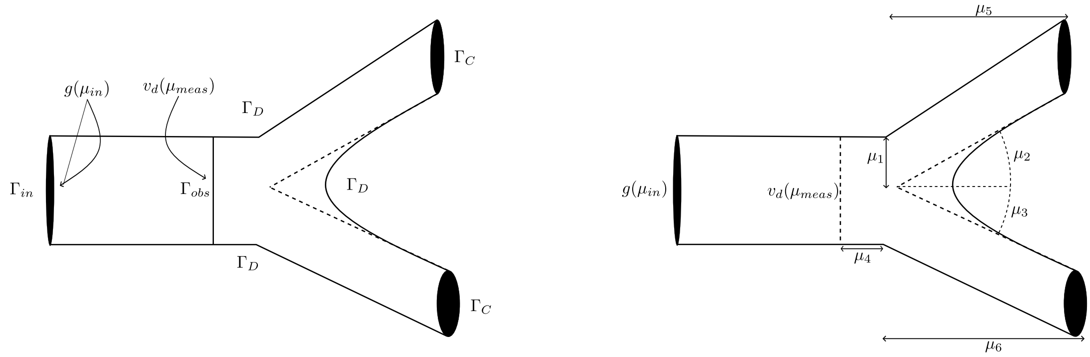
We want to solve the optimal control problem given by

$$\min J(v,p,u) = \frac{1}{2}\int_{\Gamma_{obs}} |v - v_d|^2 ds + \frac{\alpha_1}{2}\int_{\Gamma_C} |\nabla_t u|^2 ds + \frac{\alpha_2}{2}\int_{\Gamma_C} |u|^2 ds$$

s.t.

$$\begin{cases} -\nu\Delta v + \nabla p = f & \text{in } \Omega \\ \text{div} v = 0 & \text{in } \Omega \\ v = g & \text{on } \Gamma_{in} \\ v = 0 & \text{on } \Gamma_w \\ pn - \nu\partial_n v = u & \text{on } \Gamma_C \end{cases} . \tag{1}$$

We want to solve this configuration by considering a 2D realistic representation of an arterial bifurcation, parametrized as shown in Fig. **??**. We consider an inverse problem in hemodynamics, focusing on a simplified model of an arterial bifurcation. The computational domain is parametrized, with an inflow boundary denoted as Γ_{in}, two outflow boundaries Γ_C, and the physical vessel wall represented by Γ_D. The primary variables of interest are the velocity $\vec{v}$ and pressure p, which are assumed to satisfy (1).



(a) Boundary conditions: no-slip conditions on $\Gamma_D(\mu)$, Poiseuille velocity profile $g(\mu_{in})$ on Γ_{in}, unknown Neumann flux on the outflow sections.

(b) Parametrization of the original domain

Figure 1: 2D representation of the arterial bifurcation considered for this implementation.

We assume that the velocity profile is known along a segment of the inflow boundary Γ_{in}, but no direct measurements of the Neumann flux at the outflow boundaries Γ_C are available. The control variable in this problem is the unknown Neumann flux at Γ_C. The goal is to deduce this control variable using the velocity data along Γ_{in}, and consequently, to recover the velocity and pressure fields across the entire domain.

The problem involves several parameters, including geometric parameters μ_{geom}, which describe the dimensions of the bifurcation (e.g., the length of each branch, the angle of the bifurcation), a parametrized velocity profile μ_{meas}, and a parametrized inflow velocity profile $g(\mu_{\text{in}})$. These parameters together influence the flow field that we aim to reconstruct. This formulation represents a simplified version of the more complex hemodynamic flow problem, assuming two-dimensional geometry, steady-state conditions, and simplified constitutive laws. We further assume that the measured velocity profile can be approximated by a simple analytical function, parametrized for the data assimilation process.