Prova 3

1-a) $T(1,0,0)=(2,0,0)$, $T(0,1,0)=(2,1,2)$ e $T(0,0,1)=(3,0,1)$.

$T(x,y,z)=x(2,0,0)+y(2,1,2)+z(3,0,1)$
$T(x,y,z)=(2x+2y+3z,\ y\ ,\ 2y+z)$

$$[T]=\begin{bmatrix}2&2&3\\0&1&0\\0&2&1\end{bmatrix}$$

$$\begin{vmatrix}2-\lambda&2&3\\0&1-\lambda&0\\0&2&1-\lambda\end{vmatrix}\begin{matrix}2-\lambda&2\\0&1-\lambda\\0&2\end{matrix}=0$$

$(2-\lambda)(1-\lambda)(1-\lambda)=0$
$\lambda_1=2 \quad \lambda_2=\lambda_3=1$

Para $\lambda_1=2$

$$\begin{bmatrix}2&2&3\\0&1&0\\0&2&1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=2\begin{bmatrix}x\\y\\z\end{bmatrix}$$

$2x+2y+3z=2x \Rightarrow x=x$
$y=2y \Rightarrow y=0$
$2y+z=2z \Rightarrow z=0$

O autovetor associado a $\lambda_1=2 \Rightarrow (x,0,0) \Rightarrow [(1,0,0)]$

Para $\lambda_2=1$

$$\begin{bmatrix}2&2&3\\0&1&0\\0&2&1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}x\\y\\z\end{bmatrix}$$

$2x + 2y + 3z = x \Rightarrow x = -3z$

$y = y$

$2y + z = z \Rightarrow y = 0$

O autovetor é associado é $(-3z, 0, z) \Rightarrow [(-3,0,1)]$

b) $T(x,y,z) = (x+2y-z, 3y-z, 4z)$

$$[T] = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & -1 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\left|\begin{matrix} 1-\lambda & 2 & -1 \\ 0 & 3-\lambda & -1 \\ 0 & 0 & 4-\lambda \end{matrix}\right| \begin{matrix} 1-\lambda & 2 \\ 0 & 3-\lambda \\ 0 & 0 \end{matrix} = 0$$

$(1-\lambda)(3-\lambda)(4-\lambda) = 0$

$\lambda_1 = 1, \lambda_2 = 3, \lambda_3 = 4$

Para $\lambda_1 = 1$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & -1 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$x + 2y - z = x \Rightarrow x = x$

$3y - z = y \Rightarrow y = 0$

$4z = z \Rightarrow z = 0$

O autovetor associado é $(x,0,0)$

Para $\lambda_2 = 3$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & -1 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 3 \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$x + 2y - z = 3x \Rightarrow 2x = 2y \Rightarrow x = y$

$3y - z = 3y \Rightarrow y = y$

$4z = 3z \Rightarrow z = 0$

O autovetor associado $(x, x, 0) \Rightarrow [(1,1,0)]$

Para $\lambda_3 = 4$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & -1 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 4 \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$x + 2y - z = 4x \Rightarrow 3x = -3z \Rightarrow x = -z$

$3y - z = 4y \Rightarrow y = -z$

$4z = 4z \Rightarrow z = z$

O autovetor associado é $(-z, -z, z) \Rightarrow [(-1,-1,1)]$

2- $T(x,y,z) = (x+y, y, z)$

$$[T] = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left| \begin{matrix} 1-\lambda & 1 & 0 \\ 0 & 1-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{matrix} \right| \begin{matrix} 1-\lambda & 0 \\ 0 & 1-\lambda \\ 0 & 0 \end{matrix} = 0$$

$(1-\lambda)^3 = 0$

$\lambda = 1$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$x + y = x \Rightarrow y = 0 \Rightarrow x = x$

$y = y$

$z = z$

$[(1,0,0),(0,0,1)]$

Não pode formar uma base do $\mathbb{R}^3$ pois só tem dois autovetores. Logo T não é diagonalizável

3- $T(x,y,z) = (x+y+z, 2y+z, 3z)$

$$[T] = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\begin{vmatrix} 1-\lambda & 1 & 1 \\ 0 & 2-\lambda & 1 \\ 0 & 0 & 3-\lambda \end{vmatrix} \begin{matrix} 1-\lambda & 1 \\ 0 & 2-\lambda \\ 0 & 0 \end{matrix} = 0$$

$(1-\lambda)(2-\lambda)(3-\lambda) = 0$

$$p(A) = \left(\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \left(\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \right)$$

$$\left(\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \right)$$

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$$p(A) = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} =$$

$$= \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} -2 & 1 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

É polinômio minimal

4- $\langle u, v \rangle = 2ax + by + 4cz$ $\quad w = (\alpha, \beta, \gamma)$

i) $\langle u, u \rangle = 2aa + bb + 4cc = 2a^2 + b^2 + 4c^2 \geq 0$

$\langle u, u \rangle = 0$ sse $u = 0$

ii) $\langle \alpha u, v \rangle = 2\alpha ax + \alpha by + 4\alpha cz = \alpha \langle u, v \rangle$

iii) $\langle u + v, w \rangle = 2(a+x)\alpha + (b+y)\beta + 4(c+z)\gamma =$
$2a\alpha + 2x\alpha + b\beta + y\beta + 4c\gamma + 4z\gamma = \langle u, w \rangle + \langle v, w \rangle$

iv) $\langle u, v \rangle = 2ay + by + 4cz = 2ya + yb + 4zc =$
$\langle v, u \rangle$

5- a) $\langle u, v \rangle = 2.2.3 + 5.6 + 4.8.9 = 12 + 30 + 288 = 330$

b) $\|u\| = \sqrt{2.2.2 + 5.5 + 4.8.8} = \sqrt{8 + 25 + 256} = \sqrt{289}$

c) $\|v\| = \sqrt{2.3.3 + 6.6 + 4.9.9} = \sqrt{18 + 36 + 324} = \sqrt{378}$

6- $v_1' = (1,1,0)$

$v_2' = (1,0,1) - ((1+0+0)/(1+1+0)).(1,1,0) =$
$= (1,0,1) - (1/2, 1/2, 0) = (1/2, -1/2, 1)$

$v_3' = (0,2,2) - ((0-1+2)/(1/4+1/4+1))(1/2,-1/2,1) - ((0+2+0)/(1+1+0))(1,1,0) =$
$= (0,2,2) - (1/3, -1/3, 2/3) - (1,1,0) = (-4/3, 4/3, 4/3)$

$u_1 = (1,1,0)/\sqrt{2} = (1/\sqrt{2}, 1/\sqrt{2}, 0)$

$u_2' = (1/2,-1/2,1)/\sqrt{1/4+1/4+4/4} = (1/2,-1/2,1)/\sqrt{6/4} = (1,-1,2)/\sqrt{6}$

$u_3' = (-4/3,4/3,4/3)/\sqrt{16/9+16/9+16/9} = 3(-4/3, 4/3, 4/3)/4 = (-1,1,1)$

7- a) $\langle (x,y,z),(1,0,1)\rangle = 0$
$x+z=0 \Rightarrow x=-z \Rightarrow z=-x$

$\langle (x,y,z),(1,1,0)\rangle = 0$
$x+y=0 \Rightarrow x=-y \Rightarrow y=-x$

$\langle (x,y,z),(2,1,1)\rangle = 0$
$2x+y+z=0 \Rightarrow 0=0$

$S^\perp = (x,-x,-x) \Rightarrow [(1,-1,-1)]$

b) $\langle (x,y,z),(1,1,0)\rangle = 0$
$x+y=0 \quad x=-y$
$\langle (x,y,z),(1,1,1)\rangle = 0$
$y+x+z=0 \Rightarrow z=0$
$S^\perp = (x,-x,0) \Rightarrow [(1,-1,0)]$