Introdução à Álgebra Linear

Revisão – Prova 1

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Lista 1

- 1- Dada as matrizes:

$$A=\begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, B=\begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}, C=\begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}, D=\begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} \text{ e } E=\begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$$

- Calcule se possível:

c) -2(D + 3E), d) A^t + C, h) (2E)*D

Lista 1

- c) -2(D + 3E) $D=\begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}$ e $E=\begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$

$$-2\left(\begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}+3\begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}\right)$$

Lista 1

$$-2\left(\begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} + 3\begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}\right) \quad -2\left(\begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 18 & 3 & 9 \\ -3 & 3 & 6 \\ 12 & 3 & 9 \end{bmatrix}\right)$$

$$-2\begin{bmatrix} 19 & 8 & 11 \\ -4 & 3 & 7 \\ 15 & 5 & 13 \end{bmatrix} = \begin{bmatrix} -38 & -16 & -22 \\ 8 & -6 & -14 \\ -30 & -10 & -26 \end{bmatrix}$$

Lista 1

- d) $A^t + C$

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 4 & 3 & 3 \\ 3 & 3 & 6 \end{bmatrix}$$

Lista 1

- h) (2E)*D

$$D=\begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} \text{ e } E=\begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$$

$$2\begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}.\begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}$$

Lista 1

$$2\begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}.\begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} \qquad \begin{bmatrix} 12 & 2 & 6 \\ -2 & 2 & 4 \\ 8 & 2 & 6 \end{bmatrix}.\begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 12.1+2.(-1)+6.3 & 12.5+2.0+6.2 & 12.2+2.1+6.4 \\ -2.1+2.(-1)+4.3 & -2.5+2.0+4.2 & -2.2+2.1+4.4 \\ 8.1+2.(-1)+6.3 & 8.5+2.0+6.2 & 8.2+2.1+6.4 \end{bmatrix}$$

$$\begin{bmatrix} 28 & 72 & 50 \\ 8 & -2 & 14 \\ 24 & 52 & 42 \end{bmatrix}$$

Lista 1

2- Determine os valores de a, b, c e d:

$$A = \begin{bmatrix} a & 3 \\ -1 & a+b \end{bmatrix} = \begin{bmatrix} 4 & d-2c \\ d+2c & -2 \end{bmatrix}$$

- a = 4 (I)
- d – 2c = 3 → d = 2c + 3 (II)
- d + 2c = -1
- Substituindo (II)
- 2c + 3 + 2c = -1 → 4c = -4 → c = -1 (III)

Lista 1

- Substituindo (III) em (II)
- $d = 2c + 3 \rightarrow d = 2(-1) + 3 = 1$
- $a + b = -2 \rightarrow b = -a - 2$
- Substituindo (I)
- $b = -4 - 2 = -6$

Lista 2

- 3. Resolva, se possível, os seguintes sistemas:
-

d) $\begin{cases} x+y+z=2 \\ 2x-z=-1 \\ 3x+y=1 \end{cases}$

e) $\begin{cases} x+2z=1 \\ 2x-y-2z+w=0 \\ 4x-3y-9z+w=-1 \\ 3x-2y+z-11w=3 \end{cases}$

Lista 2

- d) $\begin{cases} x+y+z=2 \\ 2x-z=-1 \\ 3x+y=1 \end{cases}$

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 0 & -1 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix} L_2 = L_2 - 2L_1 \qquad \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & -2 & -3 & -5 \\ 3 & 1 & 0 & 1 \end{bmatrix} \begin{matrix} L_2 = L_2/(-2) \\ L_3 = L_3 - 3L_1 \\ \end{matrix} \qquad \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 3/2 & 5/2 \\ 0 & -2 & -3 & -5 \end{bmatrix}.$$

Lista 2

- $$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 3/2 & 5/2 \\ 0 & -2 & -3 & -5 \end{bmatrix} L_3 = L_3 + 2L_2 \qquad \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 3/2 & 5/2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
- Tem-se 3 incógnitas e 2 equações ao final
- Múltiplas soluções

Lista 2

- $$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 3/2 & 5/2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
- Considerando $z = t$ e substituindo em L_2
- $y + 3/2\ z = 5/2 \rightarrow y + (3/2)t = 5/2$
- $y = 5/2 - 3/2\ t$

Lista 2

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 3/2 & 5/2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

- Substituindo $y = 5/2 - (3/2)t$ e $z = t$ em L_1
- $x + y + z = 2 \rightarrow x = 2 - y - z \rightarrow x = 2 - 5/2 + 3/2\, t - t$
- $x = (½)\, t - 1/2$

Lista 2

e) $\begin{cases} x+2z=1 \\ 2x-y-2z+w=0 \\ 4x-3y-9z+w=-1 \\ 3x-2y+z-11w=3 \end{cases}$

$$\begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 2 & -1 & -2 & 1 & 0 \\ 4 & -3 & -9 & 1 & -1 \\ 3 & -2 & 1 & -11 & 3 \end{bmatrix} L_2 = L_2 - 2L_1 \qquad \begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & -1 & -6 & 1 & -2 \\ 4 & -3 & -9 & 1 & -1 \\ 3 & -2 & 1 & -11 & 3 \end{bmatrix}$$

Lista 2

$$\begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & -1 & -6 & 1 & -2 \\ 4 & -3 & -9 & 1 & -1 \\ 3 & -2 & 1 & -11 & 3 \end{bmatrix} L_3 = L_3 - 4L_1$$

$$\begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & -1 & -6 & 1 & -2 \\ 0 & -3 & -17 & 1 & -5 \\ 3 & -2 & 1 & -11 & 3 \end{bmatrix} L_4 = L_4 - 3L_1$$

$$\begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & -1 & -6 & 1 & -2 \\ 0 & -3 & -17 & 1 & -5 \\ 0 & -2 & -5 & -11 & 0 \end{bmatrix} L_2 = -L_2 \begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 6 & -1 & 2 \\ 0 & -3 & -17 & 1 & -5 \\ 0 & -2 & -5 & -11 & 0 \end{bmatrix}$$

Lista 2

$$\begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 6 & -1 & 2 \\ 0 & -3 & -17 & 1 & -5 \\ 0 & -2 & -5 & -11 & 0 \end{bmatrix} L_3 = L_3 + 3L_2$$

$$\begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 6 & -1 & 2 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & -2 & -5 & -11 & 0 \end{bmatrix} L_4 = L_4 + 2L_2 \quad \begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 6 & -1 & 2 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 7 & -13 & 4 \end{bmatrix} L_4 = L_4 - 7L_3 \quad \begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 6 & -1 & 2 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 & -3 \end{bmatrix}$$

Lista 2

- $W = -3$ (I)
- Substituindo (I) em L_3
- $z – 2w = 1 \rightarrow z = 1 + 2w \rightarrow z = 1 + 2.(-3) \rightarrow$ $z = -5$ (II)
- Substituindo (I) e (II) em L_2
- $y + 6z - w = 2 \rightarrow y = 2 – 6z + w \rightarrow y = 2 – 6(-5) – 3 \rightarrow y = 29$ (III)
- Substituindo (II) em L_1
- $x + z = 1 \rightarrow x = 1 – z \rightarrow 1 - (-5) = 6$

Lista 2

4. Discuta, em função de m, os seguintes sistemas:

a) $\begin{cases} x+y=3 \\ 2x+my=6 \end{cases}$ $\begin{bmatrix} 1 & 1 & 3 \\ 2 & m & 6 \end{bmatrix} L_2=L_2-2L_1$ $\begin{bmatrix} 1 & 1 & 3 \\ 0 & m-2 & 0 \end{bmatrix}$

(m-2)y = 0 → y = 0 ou m-2 = 0

Se y = 0 → m ≠ 2

Se m-2 = 0 → m = 2

Lista 2

- Se $m = 2$, y pode assumir vários valores e x também – Sistema com múltiplas soluções
- Se $m \neq 2$, $y = 0$ e $x = 3$ $\rightarrow$ Sistema com uma solução

Lista 3

- Verifique se as matrizes abaixo são invertíveis, e se for, encontre sua inversa:
- $A=\begin{bmatrix}1 & 0 & 1\\ 1 & 1 & 0\\ 0 & 1 & 1\end{bmatrix}$
- $$A=\begin{bmatrix}1 & 0 & 1\\ 1 & 1 & 0\\ 0 & 1 & 1\end{bmatrix}\begin{matrix}1 & 0\\ 1 & 1\\ 0 & 1\end{matrix}=1\,x\,1\,x\,1+0\,x\,0\,x\,0+1\,x\,1\,x\,1-(1\,x\,1\,x\,0+1\,x\,0\,x\,1+0\,x\,1\,x\,1)=2\neq 0$$

Lista 3

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] L_2 = L_2 - L_1 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] L_3 = L_3 - L_2 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 2 & 1 & -1 & 1 \end{array}\right]$$

Lista 3

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 2 & 1 & -1 & 1 \end{array}\right] L_3 = L_3/2 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1/2 & -1/2 & 1/2 \end{array}\right] L_1 = L_1 - L_3 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 1/2 & -1/2 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1/2 & -1/2 & 1/2 \end{array}\right]$$

Lista 3

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 1/2 & -1/2 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1/2 & -1/2 & 1/2 \end{array}\right] L_2 = L_2 + L_3 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 1/2 & -1/2 \\ 0 & 1 & 0 & -1/2 & 1/2 & 1/2 \\ 0 & 0 & 1 & 1/2 & -1/2 & 1/2 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1/2 & 1/2 & -1/2 \\ -1/2 & 1/2 & 1/2 \\ 1/2 & -1/2 & 1/2 \end{bmatrix}$$

Lista 3

- 2. Calcule a determinante das matrizes abaixo

$$A=\begin{bmatrix} 1 & 3 & 2 & 0 \\ -2 & -3 & -4 & 12 \\ 3 & 0 & 4 & -36 \\ -5 & -3 & -8 & 49 \end{bmatrix}$$

- det A = 1.A_{11} + 3.A_{12} + 2.A_{13} + 0.A_{14}

$$A_{11}=(-1)^{1+1}\begin{bmatrix} -3 & -4 & 12 \\ 0 & 4 & -36 \\ -3 & -8 & 49 \end{bmatrix}\begin{matrix} -3 & -4 \\ 0 & 4 \\ -3 & -8 \end{matrix} = 1\,x(-588-432+0)-(-144-864+0)=-1020+1008=-12$$

Lista 3

- $$A_{12}=(-1)^{1+2}\begin{bmatrix}-2 & -4 & 12\\ 3 & 4 & -36\\ -5 & -8 & 49\end{bmatrix}\begin{matrix}-2 & -4\\ 3 & 4\\ -5 & -8\end{matrix}=(-1)x((-392-720-288)-(-240-576-588))=(-1)x(-1400+1404)=(-1)x(4)=-4$$

$$A_{13}=(-1)^{1+3}\begin{bmatrix}-2 & -3 & 12\\ 3 & 0 & -36\\ -5 & -3 & 49\end{bmatrix}\begin{matrix}-2 & -3\\ 3 & 0\\ -5 & -3\end{matrix}=1x(0-540-108)-(0-216-471)=-648+657=9$$

- det A = 1.(-12) + 3.(-4) + 2.9 = -12-12+18=-6

Lista 3

- Encontre os valores desconhecidos que tornam a matriz A invertível (calcule o determinante para verificar)

$$A=\begin{bmatrix} 1 & x^2 & x^4 \\ 0 & x+2 & x^3 \\ 0 & 0 & x-4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & x^2 & x^4 \\ 0 & x+2 & x^3 \\ 0 & 0 & x-4 \end{bmatrix}\begin{matrix} 1 & x^2 \\ 0 & x+2 \\ 0 & 0 \end{matrix}=(x+2)(x-4)=x^2-4x+2x-8=x^2-2x-8$$

Lista 3

- $x^2 - 2x – 8 \neq 0$
- $\Delta = 4 + 32 = 36$
- $x^1 = (2+6)/2 = 4$
- $x^2 = (2-6)/2 = -2$
- $\{x \in R/x\neq 4 \text{ e } x\neq -2\}$

Exercícios

- 1- Obter o produto A.B , sabendo que

$$A = \begin{bmatrix} 4 & 5 \\ -2 & 1 \\ 6 & 0 \end{bmatrix} \text{ e } B = \begin{bmatrix} 4 & 2 & 1 & 5 \\ -5 & 0 & 2 & 2 \end{bmatrix}$$

Exercícios

- 2- Dadas as matrizes:

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 8 \\ 6 \end{bmatrix} e \ \mathbf{C} = \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix}$$

  Obter:

  a) a matriz X tal que A.X=B;

  b) Considerando D a matriz formada por B e X, onde a 1ª coluna é B e a 2ª X encontre a matriz Z tal que A.D + Z=C.