# Heat Transfer in Finned or Pinned Surfaces

Uni. Lisboa - Instituto Superior Técnico

Heat Transfer Laboratory I

**Group 60**

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# Content

# 1 Summary

The matter of the experience in question was forced convection heat transfer on extended surfaces, in our case, pinned surfaces. Succinctly, the procedure consisted in measuring several values of temperature and of heating power at different sections with two different air flow speeds, in order to calculate the convection coefficient and efficiency of the pins. The convection correlations were used posteriorly so we could compare and criticize the theoretical results to the ones we achieved. After comparing the results we concluded that there were some significant differences that will be explained later on the report. Finally, we point out that the experimental results showed that a higher flow velocity leads to a larger convection heat transfer, because it increases the average convection coefficient of the heated surface.

The method of the experience consisted in forcing an air flow throughout a rectangular shaped channel. The channel itself was composed by pins where it occurred heat transfer by forced convection with the air. The extended surfaces utility lies in increasing the contact between the rectangular surface (solid) and fluid, therefore improving heat transfer rate. With the experience we aimed to not only estimate the theoretical heating power and compare its value with the experimental one through an energy balance but also to establish a suitable correlation for the convection coefficient. Despite the mentioned above, we can also find the fins efficiency and temperature distribution from the experimental results comparing them to the corresponding model as well as present conclusions and recommendations that arose from the analysis of the data.

# 2 Theoretical and Experimental Calculations

To determine the heat transfer rate associated with a pin, we obtain the temperature distribution along the pin. Considering the extended surface of Figure 2.1, we perform an energy balance on an appropriate differential element where we assume one-dimensional conditions in the longitudinal direction, even though conduction within the pin is actually two-dimensional, and the temperature is uniform across the fin thickness.

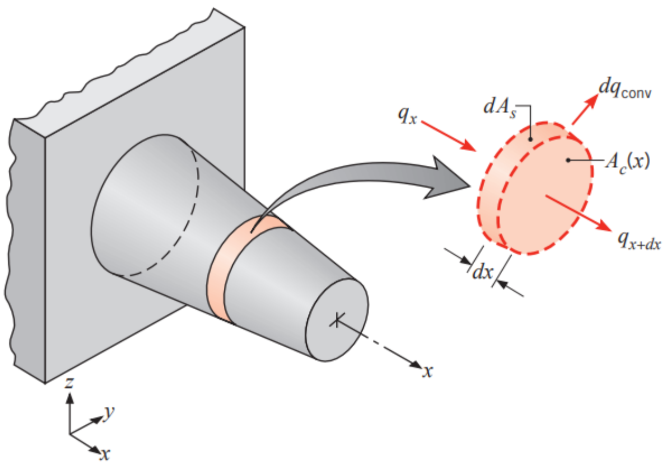



Figura 2.1: Energy Balance for a extended surface

Considering steady-state conditions and assuming that thermal conductivity is constant, we apply the conservation of energy requirement:

$$q_x = q_{x+dx} + dq_{conv} \tag{2.1}$$

Considering the Fourier's law we know that $q_x = -kA_C\frac{dt}{dx}$ and the convection heat transfer rate that is expressed as $dq_{conv} = hdA_s(T - T_\infty)$ we obtain the next equation:

$$\frac{d^2T}{dx^2} + \left(\frac{1}{A_C}\frac{dA_C}{dx}\right)\frac{dT}{dx} - \left(\frac{1}{A_C}\frac{h}{k}\frac{dA_x}{dx}\right)(T - T_\infty) = 0 \tag{2.2}$$

In our experience the pins have a uniform cross-sectional area, so $A_c$ is constant and $A_S = Px$, where $A_S$ is the surface area measured from the base to $x$ and $P$ is the pin's perimeter. With this we can write $\frac{dA_C}{dx} = 0$ and $\frac{dA_S}{dx} = P$. Accordingly with this facts the equation (2.2) reduces to:

$$\frac{d^2T}{dx^2} - \frac{hP}{kA_C}(T - T_\infty) = 0 \tag{2.3}$$

To simplify the form of this equation, we transform the dependent variable by defining an excess temperature $\theta$ as $\theta(x) = T(x) - T_\infty$. Substituting this equation into equation (2.3) we obtain:

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \tag{2.4}$$

Because the previous equation is linear, homogeneous, second-order differential equation with constant coefficients, its general solution has the following form:

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx} \tag{2.5}$$

To evaluate the constants $C_1$ and $C_2$ of equation , we admit two boundary conditions. One of them is specified in terms of the temperature at the base of the pin ($x = 0$):

$$\theta(0) = T_b - T_\infty \equiv \theta_b \tag{2.6}$$

The second condition, specified at the pin tip ($x = L$), considers convection heat transfer from the pin tip. Applying an energy balance to a control surface about a cross section along the pin, we obtain:

$$hA_C[T(L) - T_\infty] = -kA_C \left.\frac{dT}{dx}\right|_{x=L} \tag{2.7}$$

In our experimental case, the value $h$ of the previous equation is a average value, $\overline{h}$, as we explain further ahead in this chapter. With this boundary conditions and solving for $C_1$ and $C_2$, after some manipulation, we obtain this temperature distribution:

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL} \tag{2.8}$$

, where $m = \sqrt{\frac{h\rho}{k_{al} * A_c}}$

The theoretical efficiency of the pin is given by:

$$\eta_f = \frac{tanh(m * L_c)}{m * L_c} \tag{2.9}$$

To calculate the averaged theoretical convection coefficient, we follow the next few steps. Since the maximum velocity of the air flow occurs in the transverse plane ( $S_D > \frac{S_T+D}{2}$ as shown in Annex 7.2.2) the equation used to calculate the values of the maximum velocity of the flow across the pins is:

$$V_{max} = \frac{S_T V}{S_T - D} \tag{2.10}$$

The maximum velocity for the first and second trials are respectively $1.87 m/s$ and $3.74 m/s.$

Then, we proceeded to find the maximum Reynolds number for each test, to asses how should we treat the flow around the pins and determine their Nusselt numbers, using:

$$Re_{D,max} = \frac{V_{max} D}{\nu} \tag{2.11}$$

The respective values of the Reynolds number are: $Re = 1.530 * 10^3$ for the first test and $Re = 3.06 * 10^3$

for the second one.

Table 7.5 [Annex 7.3] suggested an approximation by a single cylinder since the values were near $Re = 10^3$ and from that point, the averaged Nusselt number was easily calculated according to the Churchill and Bernstein equation:

$$\overline{Nu_D} = 0.3 + \frac{0.62Re_D^{1/2}Pr^{1/3}}{[1+(\frac{0.4}{Pr})^{2/3}]^{1/4}}[1+(\frac{Re}{282000})^{5/8}]^{4/5} \tag{2.12}$$

It's important to underline that in this equation $Re_D$ is calculated not with the maximum velocity of the flow, but its entering velocity.

The averaged theoretical convection coefficient can now be calculated, with:

$$\overline{h} = \frac{\overline{Nu_D}k_{air}}{D} \tag{2.13}$$

To calculate the heat exchange by convection that actually occurs between the pins and the air we need to make a balance of the internal energy of the air:

$$q = \dot{m} * A_{conv} * (\overline{T_o} - T_i) \tag{2.14}$$

Then to calculate the experimental convection coefficient we use:

$$\overline{h} = \frac{q}{A_{conv} * (\overline{T(x)} - T_i)} \tag{2.15}$$

,where $\overline{T(x)}$ is the weighted average of temperature on the convection surface. We think that this way of averaging the temperatures is the proper way to do it because it takes into account that most of the surface is at the temperature of the base of the pins (the highest temperature) and only a small percentage of the convection area is at the minimum temperature (the pin's tip). The formula used was:

$$\overline{T(x,y)} = \frac{1}{A_{conv}}\int_0^A T(x,y)\partial A = \frac{1}{A_{conv}}\sum_1^4 T_i * A_i \tag{2.16}$$

The pin's efficiency is given by the ratio of the experimental heat transfer power and the one acquired if the temperature that crosses all pins was not only equal to the temperature at the pin base but also if it was uniform. The experimental fins efficiency is translated by:

$$\eta = \frac{q}{\overline{h} * A_{conv} * (T_s - T_\infty)} \tag{2.17}$$

The effect of variable properties could be taken in account by evaluating all properties at the film temperature, $T_f = 300K$, with the purpose of obtaining the properties of the materials used. By coincidence we reached the same film temperature in both tests, so, we used the same properties throughout the whole report (all the properties are displayed in the annexes). Notice that we calculated $T_f$ by :

$$T = \frac{T_i + \overline{T_0}}{2} \tag{2.18}$$

,where $T_i = T_\infty$ represents the temperature of the air flow at the entree and $\overline{T_0}$ the average temperature of air at the exit.

# 3 Experimental results

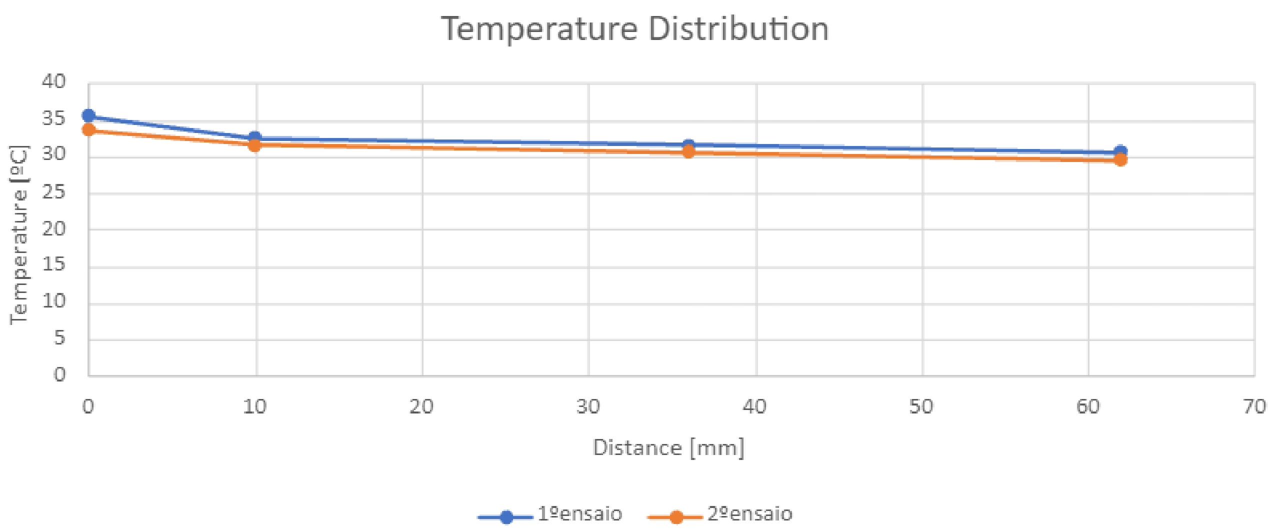
For each test a heating power of 70W was selected and two sets of data were collected, corresponding to the use of the surface with pins for two speeds with values close to 1 m/s and 2 m/s.

The temperature data was measured by leaning a probe against the surface of the plate, and against the surface of the pins in three different positions, x = 10mm, x = 36mm and x = 62mm. The outlet air temperature was measured at the same distances. For each value obtained, we waited some amount of time so the temperature read by the probe could stabilize.

The following table shows in the first two columns the values measured in the laboratory, and in the next two the values with the corrections suggested by the experience guide.

| | 1º Test | 2º Test | 1º Test* | 2º Test* |
|---|---|---|---|---|
| Power imposed at the base [W] | 70 | 70 | - | - |
| Air speed [m/s] | 1 | 2 | - | - |
| Interior plate temperature [ºC] | 56 | 51 | 54,1 | 49,1 |
| Exterior plate temperature [ºC] | 38 | 36 | 35,5 | 33,5 |
| Inlet air temperature [ºC] | 26 | 27 | 23,5 | 24,5 |
| Fin/pin surface temperature x= 10 mm [ºC] | 35 | 34 | 32,5 | 31,5 |
| Fin/pin surface temperature em x= 36 mm [ºC] | 34 | 33 | 31,5 | 30,5 |
| Fin/pin surface temperature em x= 62 mm [ºC] | 33 | 32 | 30,5 | 29,5 |
| Outlet air temperature x= 10 mm [ºC] | 33 | 33 | 30,5 | 30,5 |
| Outlet air temperature x= 36 mm [ºC] | 32 | 31 | 29,5 | 28,5 |
| Outlet air temperature x= 62 mm [ºC] | 31 | 29 | 28,5 | 26,5 |

The temperature distribution in the pins for each test is shown in the graph bellow:




# 4 Experimental Results and Theoretical Analysis

## 4.1 Energy Heat Balance and Comparison with the Heating Power

At first sight the heating power (70W) and the energy balance of the air seem to not be related. In the first test what might be happening for the energy balance of the air to be lower is a mixture of losses by conduction to the outside of the duct and also despite the fact that the aluminum, of which the pins are made of, has a high conduction coefficient, it still has some resistance. This resistance paired with the fact that in the first test the velocity is low (1m/s), might promote the heating up of the pins (increase of internal energy) because there isn't enough mass of air to "pick up" that energy by convection.

This accumulated energy from the first test was being transferred to the faster air in the second test, because if we only supply 70W to the pins there's no other way to justify having an air energy balance of

78.6W. Most likely we didn't wait enough time for the system to reach the stationary state, and because the air was faster, we had double the mass of air that was absorbing more heat thus the accumulation of heat in the pins was not increasing but the contrary was succeeding.

Despite all these conflicting results, we can clearly conclude that with higher air velocity we have higher heat transfer by convection to the air.

| | 1º Test | 2º Test |
|---|---|---|
| Air energy balanced calculated power [m/s] | 58,97 | 78,6 |

## 4.2 Convection Coefficients

The theoretical values calculated of average convection coefficient were a lot lower than the ones we obtained experimentally, and that fact might be explained by a premature transition of the flow to turbulent that results in a spike of the coefficients. The Reynolds number of both tests suggests that it is laminar but maybe because of the roughness of the surface or the closeness of the pins to one another (that might result in the boundary layers around the cylinders to mix and transition) this apparent suggestion becomes less plausible. This change in the regime of the flow makes the empirical correlation used to calculate the Nusselt number inaccurate, thus explaining the discrepancy in the values.

| | 1º Test | 2º Test |
|---|---|---|
| Experimental $h_{med}(W/m^2K)$ | 110,72 | 195,05 |
| Theoretical $h_{med}(W/m^2K)$ | 29,24 | 41,55 |

## 4.3 Fins Efficiency

The theoretical values are larger than the ones we found. The reason for this difference is the inaccurate theoretical prediction of the convection coefficient, because of the transition to turbulence that wasn't taken into account.

If the theoretical prevision of the convection coefficient was correct, meaning that the theoretical values would perfectly match the experimental ones, the efficiency of the pins would be a lot closer (0.81567 for the first test and 0.727174 for the second test). The efficiency would actually be higher in the experimental results than the theoretical ones but that is just because of the fact that stationary state was not reached before measuring the temperature of the pins, as explained in the analysis of heating power and the energy balance of the air

| | 1º Test | 2º Test |
|---|---|---|
| Experimental $\eta$ | 0,7875 | 0,7944 |
| Theoretical $\eta$ | 0,9499 | 0,9198 |

## 4.4 Temperature Distribution

The inaccuracies made before while calculating the average theoretical coefficient of convection explain the lower decrease in temperature throughout the length of the pin for the theoretical profiles that use that value. So with the increase of the length of the pin x=10mm, x= 36mm and x=62mm we get for the first test errors of 23%, 28% and 36%, respectively, and for the second test 20%, 26% and 37%. The substantial increase in the error on the tip of the pin can be explained by the fact that the $\frac{\theta}{\theta_b}$ profile (equation 2.8) that we deduced assumes that the heat transfer through the tip of the pin was negligible. Also the accuracy of this errors is greatly influenced by the precision of the temperature probe, since the smallest division it had was to the unit of the Celsius, and for such a small variation in temperature (2ºC along the pin for both tests), at least one decimal place in the temperature readings would substantially improve this errors.

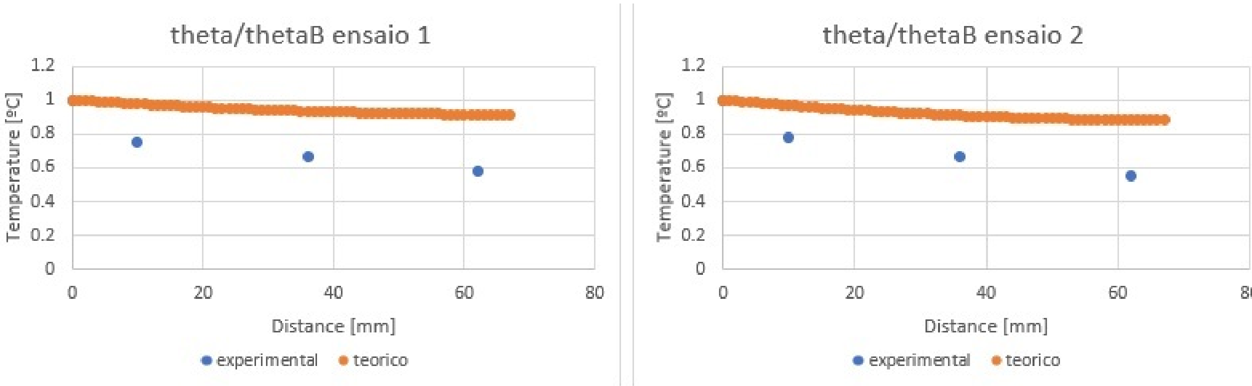



Figura 4.1: Graphics with theoretical profile of $\frac{\theta}{\theta_b}$ (using theoretical $\overline{h}$ values) and the experimental results

If we used the values of the averaged convection coefficient obtained experimentally instead of the one obtained theoretical the $\frac{\theta}{\theta_b}$ profiles would be a lot closer to the values obtained with the probes in the experience. The errors obtained doing this change for x=10mm, x=36mm and x=62mm are for the first test 19% , 15% and 20% and for the second test 12% 2% and 7%. Despite the substantial decrease of the error because of the change in the convection coefficient, we can still observe its increase close to the tip of the pin.

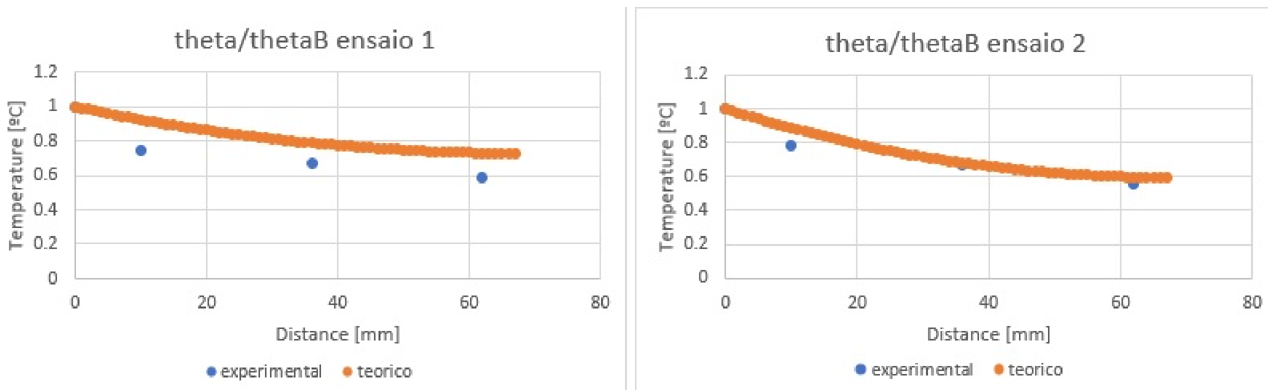



Figura 4.2: Graphics with theorical profile of $\frac{\theta}{\theta_b}$ (using experimental $\overline{h}$ values) and the experimental results

## 5 Conclusion

The major strain of this experiment was determining the adequate correlation to obtain the coefficient of convection. Furthermore, the assumption that we can consider an averaged convection coefficient is valid since the errors, if we use the experimental averaged convection coefficient, are very low and increase close to the tip of the pin because we assumed there was no convection through it and probably exists in a very small amount. In relation to the results obtained, we concluded that higher velocity pushes higher convection heat transfer, as the convection coefficient depends on the flow velocity.

There are also other deductions that we can get from the discrepancies between the experimental and theoretical results. The Reynolds number that we calculate supposedly indicated that we were in the presence of a laminar flow, however, the significantly higher experimental convection coefficient value implies that ,maybe because of the roughness of the pins surface, we transitioned to turbulent flow and enhanced greatly this coefficient. The difference in efficiencies can also be explained by the unaccounted for transition to turbulent flow.

In the matter of the heating power states, as said before, are caused due to some of the heat being absorbed by the fins- which implies that we were not at steady state. We also did not consider heat losses to the environment, when they always exist. To conclude, we would like to present some suggestions in order to improve the experiment itself and minimize the errors that occurred: the equipment of measurement provided could be a lot more precise (at least one decimal place) and the surface of the pins could be previously polished, to decrease the roughness and consequently avoid the transition of the flow to turbulent regime and therefore increasing the validity of the the Churchill and Bernstein equation to calculate $\overline{Nu_D}$.

## 6 References

Fins Laboratory Guide

[1] Fundamentals of Heat and Mass Transfer: F.P. Incropera, D.P. de Witt, T.L Bergman e A.S. Lavine 2006 John Wiley  Sons, 6th Edition

# 7 Annexes

## 7.1 Surface Geometry

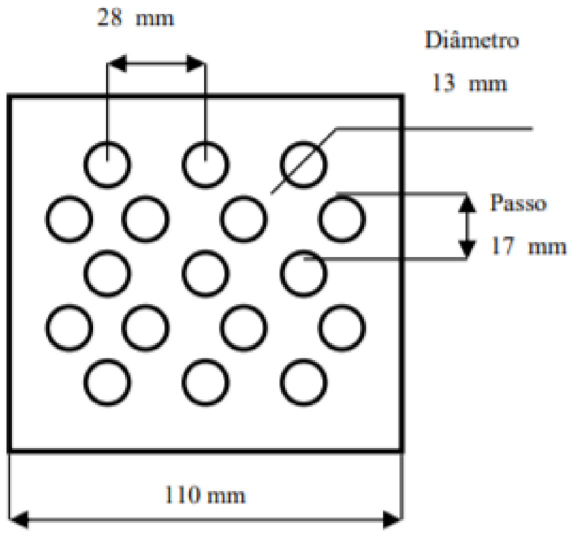



Figura 7.1: Layout of the surface used

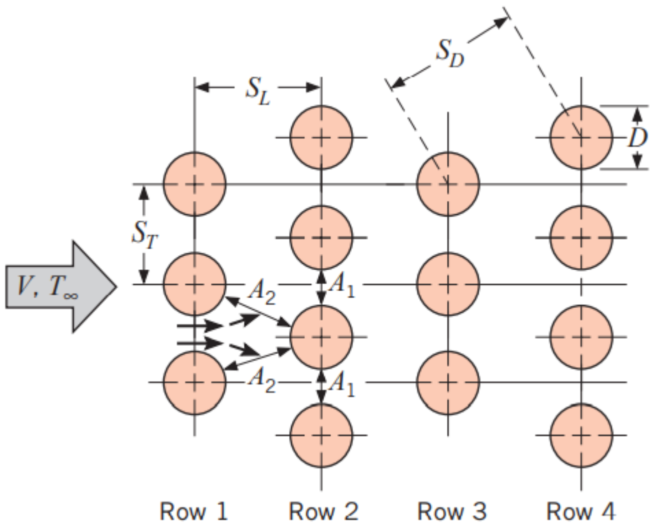



Figura 7.2: Scheme of the surface used with SD

## 7.2 Properties and Experimental Results

### 7.2.1 Properties

$$\rho = 1,1614 Kg/m^3$$

$$c_p = 1007 J/Kg.K$$

$$\mu = 184,6 * 10^{-7} N.s/m^2$$

$$\nu = 15,89 * 10^{-6} m^2/s$$

$$\alpha = 22,5 * 10^{-6} m^2/s$$

$$\mathrm{Pr} = 0{,}707$$

$$K_{air} = 26,3 * 10^{-3} W/m.K$$

$$K_{aluminium} = 237 W/m.K$$

### 7.2.2 Experimental analysis

***Maximum velocity of the flow across the pins:***

$$V_{max1} = \frac{28}{28-13} = 1,87 m/s$$
$$V_{max2} = 3,74 m/s$$

***SD calculation (Figure 7.2):***

$$S_D = [S_L^2 + (\frac{S_T}{2})^2]^{1/2} = 22,02 mm > \frac{S_T + D}{2} = 20,5 mm$$

***Maximum Reynolds Number for each test:***

$$\mathrm{Re}_{D,max1} = \frac{V_{max1}*D}{\nu} = 1,530*10^3$$

$$\mathrm{Re}_{D,max2} = \frac{V_{max2}*D}{\nu} = 3,06*10^3$$

$$\mathrm{Re}_{D_1} = \frac{V_1*D}{\nu} = 818,12$$

$$\mathrm{Re}_{D_2} = \frac{V_2*D}{\nu} = 1,636*10^3$$

***Nusselt Numbers:***

$$\overline{Nu_{D1}} = 14,455$$

$$\overline{Nu_{D2}} = 20,538$$

***Mass Flow:***

$$\mathrm{m}_1 = 1,1614*0,120,07*1 = 0,00976Kg/s$$
$$\mathrm{m}_2 = 1,1614*0,120,07*2 = 0,01951Kg/s$$

***Theoretical Averaged Convection Coefficient:***

$$\overline{h_1} = 29,244W/m^2.K$$

$$\overline{h_2} = 42,550W/m^2.K$$

***Heat Exchange:***

$$\mathrm{q}_1 = 0,00976*1007*(29,5-23,5) = 58,97W$$

$$\mathrm{q}_2 = 0,01951*1007*(28,5-24,5) = 78,6W$$

***Convection Area:***

$$\mathrm{A}_{conv} = 0,11^2 + 17*(-\frac{\pi*0,013^2}{4} + 0,067*\pi*0,013) = 0,05636m^2$$

***First Test:***

$$\mathrm{T}_1 = T_5 = 35,5^oC; A_1 = 0,11^2 + 17*(-A_c + 0,14925A_{lp}) = 0,01679m^2$$

$$\mathrm{T}_2 = 32,5^oC; A_2 = 17*0,3881A_{lp} = 0,01805m^2$$

$$\mathrm{T}_3 = 32,5^oC; A_3 = A_2 = 0,01805m^2$$

$$\mathrm{T}_4 = 32,5^oC; A_4 = 0,00347m^2$$

***Second Test:***

$$\mathrm{T}_1 = T_5 = 33,5^oC; A_1 = 0,11^2 + 17*(-A_c + 0,14925A_{lp}) = 0,01679m^2$$

$$\mathrm{T}_2 = 31,5^oC; A_2 = 17*0,3881A_{lp} = 0,01805m^2$$

$$\mathrm{T}_3 = 30,5^oC; A_3 = A_2 = 0,01805m^2$$

$$\mathrm{T}_4 = 29,5^oC; A_4 = 0,00347m^2$$

***Weighted Temperature Average:***

$$\overline{T(x,y)_1} = \frac{1}{0,05636}*(35,5*0,01679 + 32,5*0,01805 + 31,5*0,01805 + 30,5*0,00347) = 32,95^oC$$

$$\overline{T(x,y)_2} = \frac{1}{0,05636}*(33,5*0,01679 + 31,5*0,01805 + 30,5*0,01805 + 29,5*0,00347) = 31,65^oC$$

***Experimental Averaged Convection Coefficient:***

$$\overline{h_1} = \frac{q_1}{A_{conv}*(\overline{T(x)_1} - T_i)} = 110,72W/m^2.K$$

$$\overline{h_2} = \frac{q_2}{A_{conv}*(\overline{T(x)_2} - T_i)} = 195,05W/m^2.K$$

***Experimental Fins Efficiency:***

$$\eta_1 = \frac{58,97}{110,72*0,05636*(35,5-23,5)} = 0,7875$$

$$\eta_2 = \frac{78,6}{195,05*0,05636*(33,5-24,5)} = 0,7944$$

***Constant m:***

$$\mathrm{m}_1 = 6,1617$$

$$\mathrm{m}_2 = 7,3446$$

***Theoretical Fins Efficiency:***

$$\eta_1 = \frac{tanh(6,1617*0,07025}{6,16173*0,07025} = 0,94189$$

$$\eta_2 = 0,91979$$

## 7.3 Table 7.5 of reference [1]

**TABLE 7.5** Constants of Equation 7.58 for the tube bank in cross flow [16]

| Configuratio | $Re_{D,max}$ | $C_1$ | $m$ |
|---|---|---|---|
| Aligned | $10$–$10^2$ | 0.80 | 0.40 |
| Staggered | $10$–$10^2$ | 0.90 | 0.40 |
| Aligned | $10^2$–$10^3$ | Approximate as a single (isolated) cylinder | |
| Staggered | $10^2$–$10^3$ | | |
| Aligned $(S_T/S_L > 0.7)^a$ | $10^3$–$2 \times 10^5$ | 0.27 | 0.63 |
| Staggered $(S_T/S_L < 2)$ | $10^3$–$2 \times 10^5$ | $0.35(S_T/S_L)^{1/5}$ | 0.60 |
| Staggered $(S_T/S_L > 2)$ | $10^3$–$2 \times 10^5$ | 0.40 | 0.60 |
| Aligned | $2 \times 10^5$–$2 \times 10^6$ | 0.021 | 0.84 |
| Staggered | $2 \times 10^5$–$2 \times 10^6$ | 0.022 | 0.84 |

[a] For $S_T/S_L < 0.7$, heat transfer is inefficient and aligned tubes should not be used.