Exploratory Data Analysis in R (edar)

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[2018-05-25 Fri 14:01]

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1 Introduction

The package Exploratory Data Analysis in R (edar) allows efficient exploratory data analyses with few lines of code. It contains some functions to:

- overview and summarise the data set
- check balance of covariates among control and treatment groups
- create organized and ready to export (to latex, html, etc) tables with results of model estimation
- easily create plots with fitted values comparing one or more models under different treatment conditions
- create plots with point estimates and their intervals (dotwisker plots)
- conduct robustness checks of the model results (multiple imputation, post-stratification, etc).
 Quantitative researchers conduct those tasks repeatedly. The package provides functions to do them more efficiency and with minimum code.
- 1. Check the numerical and categorical variables of the data set
 - Look for outliers and missing values

- Check distribution of the variables
- 2. Fit a multivariate regression model
- 3. Display and check the results
- 4. Do multiple imputation and post-stratification (in surveys)
- 5. Repeat 2 and 3
- 6. Recode some Variables or change model specifications
- 7. Repeat

Execpt for item 6, the package edar can spead up all those tasks. For instance, suppose data contains the data set. Those tasks can be performed with few lines of code:

```
1
    # summary tables
   data %>% summarise_alln(.) # summarise all numerical variables of the data in a table
   data %>% summarise_allc(.) # summarise all categorical variables of the data in a table
    data %>% summarise_allcbundle(.) # summarise all categorical variables of the data in a table
    data %>% ebalance(., treatmentVar="treat") ## summary of numerical variables for different
        levels of "treat"
 6
 7
    # summary plots
    s %>% gge_describe(.) ## marginal distribution of all variables
 8
    s %>% gge_density(.) ## marginal distribution of numerical variables only
10
    s %>% gge_histogram(.)## marginal distribution of numerical variables only using histograms
    s %>% gge_barplot(.) ## marginal distribution of non-numerical variables
12
13
    # afeter fitting the models model1, model2, etc ...
14
    tidye(modell, hc=T) ## summarise put summary in a tidy data.frame (using robust std.errors)
15
    tidye(list(model1, model2)) ## same, but summarise both models at once
16
17
    # plots
    gge_coef(model1) ## dotwisker plot (plot with coefficients and std. errors)
18
19
    model %>% gge_fit(., data, "y", "x1") ## plot with fitted values as function of covariate x1
20
21
   # multiple imputation and post-stratification
   emultimputation(data, formula, dep.vars = c(...), ind.vars=c(...))
22
23
    epoststrat(data, population.proportion, strata = ~ stratification.variable1 +
        stratification.variable2...)
```

2 Workflow

Here is an example of workflow with edar. We will use the data set edar_survey that comes with the package:

```
library(magrittr)
library(edar)

data(edar_survey)
help(edar_survey)
```

```
6
    data = edar_survey
 1
    A National Survey from Brazil
 2
    Description:
 3
 4
         The data set is a subset of a national suvery conducted in Brazil
 5
         in 2013. The survey measures preferences of individuals for % \left( 1\right) =\left( 1\right) \left( 1\right) 
 6
         interpersonal and interregional redistribution of income as well
 7
         as preferences for centralization of political authority.
 8
9
    Usage:
10
11
         data(edar_survey)
12
13
    Format:
14
15
         A data frame with 700 rows and 16 columns:
16
17
         gender factor with "men" and "woman"
18
19
         educ factor with "high" if the individual completed high school or
20
              more, and "low" otherwise
21
22
         age integer with age in years
23
24
         yi numeric variable with household income per capita
25
26
         yi.iht inverse hyperbolic transformation of yi
27
28
         state factor with the state in which the individual lives
29
30
         region factor with macroregion
31
32
         ys.mean average household percapita income in the state, computed
33
               using the 2013 Brazilian National Household Survey (PNAD)
34
35
          trust factor, "high" or "low" trust in the federal government
36
37
          treat numeric, 0 for control group or 1 for treatment group. It is
               a randomly generated variable for used for ilustration of the
38
39
               examples and vignettes only
40
41
         ys.gini numeric, Gini coefficient of the state computed using the
42
               2013 Brazilian National Household Survey (PNAD)
43
          racial.frag.ratio numeric, racial fractionalization at the state
44
45
               over racial fractionalization at the national level
```

reduce.income.gap factor, "A"=Agree, "A+"=Strongly Agree,

rich and poor"

"D"=Disagree, "D+"=Strongly Disagree, "N"=Neither Agree or

Disagree that "Government should reduce income gap between

```
52
         transfer.state.tax factor, "A"=Agree, "A+"=Strongly Agree,
53
               "D"=Disagree, "D+"=Strongly Disagree, "N"=Neither Agree or
54
              Disagree that the "Government should redistribute resources
55
              from rich to poor states"
56
57
         minimum.wage factor, captures the answer to "Who should decide
58
              about the minimum wage policy?". The levels are "Each city
              should decide", "Each state should decide", "Should be the
59
              same accros the country"
60
61
62
         unemployment.policy factor, captures the answer to "Who should
63
               decide about the unemployment policy?". The levels are "Each
64
               city should decide", "Each state should decide", "Should be
65
              the same accros the country"
66
         red.to.poor factor, captures the answer to "Who should decide
67
68
              about policies to redistribute income to poor?". The levels
              are "Each city should decide", "Each state should decide",
69
70
              "Should be the same accros the country"
71
72
    Source:
73
         <URL: http://web.fflch.usp.br/centrodametropole/>
```

2.1 Summarise data

First, we can have a quick overview of the data set using the functions <code>summarise_alln</code> and <code>summarise_allc</code> provided by <code>edar</code> package. They show the summary of numerical and categorical variables in the data set, respectively:

```
data %>% summarise_alln(., digits=2)
```

```
# A tibble: 10 x 7
   var
                               NAs Categories Frequency
                                                                           Table Categories.Labels
   <chr>
                       <dbl> <int>
                                         <int> <chr>
                                                                           t> <chr>
 1 educ
                         700
                                             2 high (39.71 %), low
                                                                       (6âĂę <dataâĂę high, low
                                  0
 2 gender
                         700
                                  0
                                             2 man
                                                     (41.29 %), woman (5âĂę <dataâĂę man, woman
 3 minimum.wage
                         695
                                  5
                                             4 Each (8.63 %), Each
                                                                      (16âĂe <dataâĂe Each city should
                                             4 Each (11.78 %), Each (1âĂę <dataâĂę Each city should
 4 red.to.poor
                         679
                                 2.1
 5 reduce.income.gap
                         700
                                             5 A
                                                     (72.43 %), A+
                                                                       (lâĂę <dataâĂę A, A+, D, D+, N
                                  0
 6 region
                         700
                                  0
                                             5 CO
                                                     (6.14 %), NE
                                                                      (47âĂę <dataâĂę CO, NE, NO, SE,
 7 state
                         700
                                  0
                                            27 AC
                                                     (0.29 %), AL
                                                                      (0.âĂę <dataâĂę AC, AL, AM, AP,
 8 transfer.state.tax
                                                     (70.14 %), A+
                                                                       (lâĂe <dataâĂe A, A+, D, D+, N
                         700
                                  0
                                             5 A
 9 trust
                         694
                                  6
                                             3 high (56.92 %), low
                                                                       (4âĂę <dataâĂę high, low
10 unemployment.policy
                                                    (9.59 %), Each (17âĂę <dataâĂę Each city should
                         699
                                  1
                                             4 Each
```

¹ data %>% summarise_allc(.)

[#] A tibble: 10 x 7

	var	N	NAs	Categories	Freque	ency	T∂	ıble Cat€	egories.Label:	.S
	<chr></chr>	<dbl></dbl>	<int></int>	<int></int>	<chr></chr>		<]	list> <chr< td=""><td><u>^</u>></td><td></td></chr<>	<u>^</u> >	
1	educ	700	0	2	high	(39.71 %), low	(6âĂę	<dataâăę< td=""><td>high, low</td><td>1</td></dataâăę<>	high, low	1
2	gender	700	0	2	man	(41.29 %), woman	ı (5âĂę	<dataâăę< td=""><td>man, woman</td><td>•</td></dataâăę<>	man, woman	•
3	minimum.wage	695	5	4	Each	(8.63 %), Each	(16âĂę	<dataâăę< td=""><td>Each city sho</td><td>ould</td></dataâăę<>	Each city sho	ould
4	red.to.poor	679	21	4	Each	(11.78 %), Each	(1âĂę	<dataâăę< td=""><td>Each city sho</td><td>ould</td></dataâăę<>	Each city sho	ould
5	reduce.income.gap	700	0	5	A	(72.43 %), A+	(1âĂę	<dataâăę< td=""><td>A, A+, D, D+</td><td>, N</td></dataâăę<>	A, A+, D, D+	, N
6	region	700	0	5	CO	(6.14 %), NE	(47âĂę	<dataâăę< td=""><td>CO, NE, NO,</td><td>SE,</td></dataâăę<>	CO, NE, NO,	SE,
7	state	700	0	27	AC	(0.29 %), AL	(0.âĂę	<dataâăę< td=""><td>AC, AL, AM,</td><td>AP,</td></dataâăę<>	AC, AL, AM,	AP,
8	transfer.state.tax	700	0	5	A	(70.14 %), A+	(1âĂę	<dataâăę< td=""><td>A, A+, D, D+</td><td>, N</td></dataâăę<>	A, A+, D, D+	, N
9	trust	694	6	3	high	(56.92 %), low	(4âĂę	<dataâăę< td=""><td>high, low</td><td>ļ</td></dataâăę<>	high, low	ļ
10	unemployment.policy	699	1	4	Each	(9.59 %), Each	(17âĂę	<dataâăę< td=""><td>Each city sho</td><td>ould</td></dataâăę<>	Each city sho	ould

The summary of categorical variables produced by summarise_allc contains a column named Table, which contains a table with the counts for each category value of the variable.

```
1 tab = data %>% summarise_allc(.)
2 tab$Table[[6]]
```

It is common to have data sets in which many categorical variables have the same categories. The function <code>summarise_allcbundle</code> provides a summary of all categorical variables of the data set and aggregate those with same categories. The output contain columns named <code>Table</code>, <code>Tablep</code>, and <code>Tablel</code>. <code>Table</code> contains a table with counts of the categories of the variables. <code>Tablep</code> presents the same information, but in percentage. <code>Tablel</code> presents both the counts and percentage, which can be exported directly for reports and articles. The column <code>Variables</code> in the output contains the name of all the variables that have the same <code>Category.Labels</code>

```
data %>% summarise_allcbundle(.)
# A tibble: 6 x 6
  N. Variables Variables Categories. Labels
                                                                        Table
                                                                                    Tablep
                                                                                                Tablel
        <int> <list>
                          <chr>
                                                                                    t>
                                                                        <list>
                                                                                                st>
             2 <chr [2] > A, A+, D, D+, N
                                                                        <data.fraâĂę <data.fraâĂę <data.:
             1 <chr [1]> AC, AL, AM, AP, BA, CE, DF, ES, GO, MA, MâĂę <data.fraâĂę <data.fraâĂę <data.fraâAę
2
             1 <chr [1] > CO, NE, NO, SE, SU
                                                                        <data.fraâĂę <data.fraâĂę <data.:
3
             3 <chr [3]> Each city should decide, Each state shoulâĂę <data.fraâĂę <data.fraâĂę <data.fraâĂę
                                                                        <data.fraâĂę <data.fraâĂę <data.
5
             2 <chr [2] > high, low
                                                                        <data.fraâĂę <data.fraâĂę <data.:
6
             1 <chr [1] > man, woman
tab = data %>% summarise_allcbundle(.)
```

Variable	high	low	NA
educ	278	422	0
trust	395	299	6

tab\$Table[[5]]

1 tab\$Tablep[[5]]

Variable	high	low	NA
educ	39.71	60.29	0
trust	56.43	42.71	0.86

1 tab\$Table1[[5]]

Variable	high	low	NA
educ	39.71 % (N=278)	60.29 % (N=422)	0 % (N=0)
trust	56.43 % (N=395)	42.71 % (N=299)	0.86 % (N=6)

2.2 Checking balance of covariates

We can easily check the distribution of covariates among two factor levels. Consider the variable treat, which represents the treatment condition (1=treatment, 0=control). We can describe the distribution of covariates using ebalance(). The table follows recomendations in Imbens and Rubin (2015).

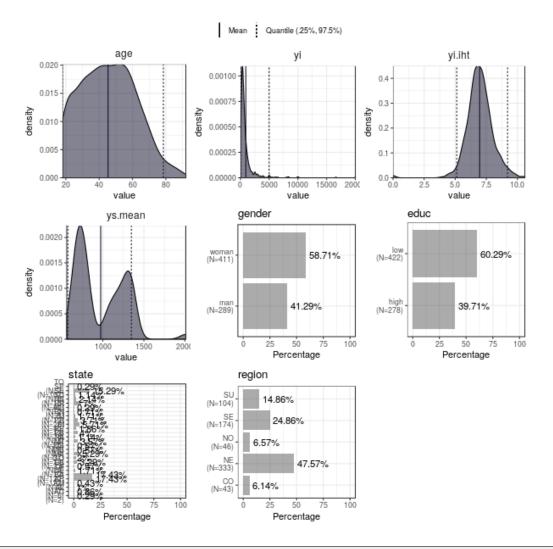
```
data %>% ebalance(., treatmentVar='treat') %>% print(., digits=2)
```

Variable	mut	st	muc	SC	NorDiff	InRatioSdtDev	pit	pic	
age	45.83	16.61	44.9	16.27	0.06	0.02	0.03	0.06	
yi	946.36	1671.84	916.04	1418.32	0.02	0.16	0.03	0.07	
yi.iht	6.95	1.07	6.98	1.08	-0.03	-0.01	0.03	0.07	
ys.mean	981.54	297.97	966.04	302.6	0.05	-0.02	0.04	0.02	
ys.gini	0.52	0.03	0.53	0.03	-0.16	-0.1	0.03	0.05	
racial.frag.ratio	0.87	0.13	0.87	0.14	0.02	-0.07	0	0.05	
${\sf Mahalanobis Dist}$	nil	nil	nil	nil	0.22	nil	nil	nil	
pscore	0.5	0.5	0.46	0.5	0.07	0	0.02	0.04	
LinPscore	-0.09	26.61	-1.92	26.54	0.07	0	0.04	0.07	
N	337	nil	363	nil	nil	nil	nil	nil	

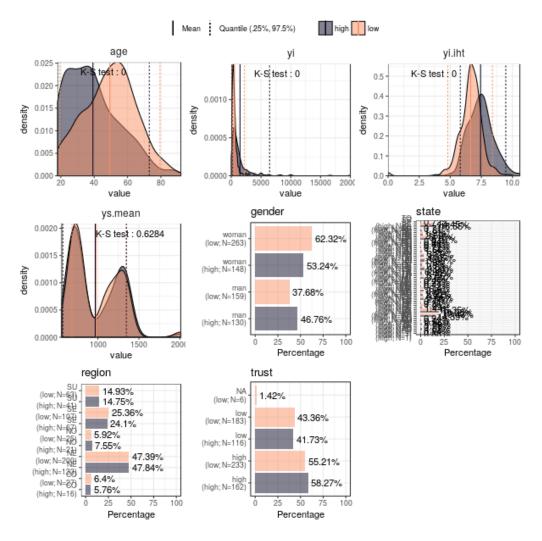
2.3 Summary plots

The package also provides some functions to easily visualise the marginal distribution of many variables at once. The marginal densities can be grouped by factors using the parameter group. When the marginal densities are presented by group, the plot include the p-value of the Kolmogorov-Smirnov distance.

```
1  g = data[,1:8] %>% gge_describe(.)
2  print(g)
```



1 g = data[,1:9] %>% gge_describe(., group='educ')
2 print(g)



Other similar functions provided by the package are:

- gge_barplot()
- gge_density()
- gge_histogram()
- gge_barplot()

2.4 Analyzing output of model estimation

Researchers commonly:

- 1. Estimate models
- 2. Produce summary output
- 3. Create plots with fitted curve

The package edar make it easy to display results of estimation. It can be achieved with minimum code. Suppose we estimated five different models:

```
1
    set.seed(77)
 2
    data = tibble::data_frame(n = 300,
 3
                              x1 = rnorm(n, 3, 1),
                              x2 = rexp(n),
 5
                              cat1 = sample(c(0,1), n, replace=T),
 6
                              cat2 = sample(letters[1:4], n, replace=T),
                              y = -10*x1*cat1 + 10*x2*(3*(cat2=='a') -3*(cat2=='b') +1*(cat2=='c')
 7
         -1*(cat2=='d')) +
 8
                                  rnorm(n,0,10),
 9
                              y.bin = ifelse(y < mean(y), 0, 1),
10
                              y.mul = 1+ifelse( - x1 - x2 + rnorm(n, sd=10) < 0, 0,
11
                                       ifelse( -2*x2 + rnorm(n, sd=10) < 0, 1, 2)),
12
13
14
    formula1 = y \sim x1
    formula2 = y \sim x1 + x2
15
              = y ~ x1*cat1 + x2*cat2
16
    formula3
17
    formula4bin = y.bin \sim x1+x2*cat2
18
    formula4bin1 = y.bin \sim x1+x2
19
    formula4bin2 = y.bin \sim x1*cat1+x2*cat2
    formula5mul = y.mul \sim x1 + x2
20
21
22
   model.g1 = lm(formula1, data)
23
    model.g2 = lm(formula2, data)
24
    model.g3 = lm(formula3, data)
    model.bin = glm(formula4bin, data=data, family='binomial')
25
26
    model.bin1 = glm(formula4bin, data=data, family='binomial')
27
    model.bin2 = glm(formula4bin, data=data, family='binomial')
    model.mul = nnet::multinom(formula5mul, data)
```

2.4.1 **Tables**

We want to vizualize the model estimate. The function tidye creates tidy summary tables with the output. It is a wrap function for <code>broom::tidy()</code>, and it works with list of models. Here are some examples:

```
tidye(model.g3)

tidye(model.g3)

### works with other types of dependent variables

# tidye(model.bin)
# tidye(model.mul)
```

term	estimate	std.error	conf.low	conf.high	statistic	p.value
(Intercept)	3.6042	3.0375	-2.3742	9.5826	1.1866	0.2364
x1	-0.9053	0.8167	-2.5126	0.7021	-1.1085	0.2686
cat1	-2.2011	3.6151	-9.3164	4.9142	-0.6089	0.5431
x2	28.0061	1.3544	25.3403	30.6719	20.6774	0
cat2b	-0.1835	2.3532	-4.8151	4.4481	-0.078	0.9379
cat2c	-0.9414	2.2746	-5.4184	3.5355	-0.4139	0.6793
cat2d	-1.4556	2.4636	-6.3044	3.3932	-0.5909	0.5551
x1:cat1	-9.2755	1.1527	-11.5442	-7.0069	-8.0471	0
x2:cat2b	-58.1667	1.8639	-61.8352	-54.4982	-31.2071	0
x2:cat2c	-17.6127	1.7246	-21.0071	-14.2183	-10.2125	0
x2:cat2d	-38.3783	2.0687	-42.4499	-34.3068	-18.5523	0

We can have robust standard errors, and keep or not information of non-corrected values for comparison.

```
## with robust std.errors
tidye(model.g3, hc=T)
```

term	estimate	std.error	conf.low	conf.high	statistic	p.value
(Intercept)	3.6042	3.2952	-2.8544	10.0628	1.0938	0.275
x1	-0.9053	0.8481	-2.5676	0.7571	-1.0673	0.2867
cat1	-2.2011	3.7761	-9.6023	5.2001	-0.5829	0.5604
x2	28.0061	1.5784	24.9124	31.0998	17.7432	0
cat2b	-0.1835	2.5577	-5.1965	4.8295	-0.0717	0.9429
cat2c	-0.9414	2.4039	-5.6531	3.7703	-0.3916	0.6956
cat2d	-1.4556	2.691	-6.7299	3.8187	-0.5409	0.589
x1:cat1	-9.2755	1.2346	-11.6953	-6.8558	-7.5131	0
x2:cat2b	-58.1667	1.8969	-61.8846	-54.4488	-30.664	0
x2:cat2c	-17.6127	1.8342	-21.2077	-14.0176	-9.6023	0
x2:cat2d	-38.3783	2.3255	-42.9364	-33.8203	-16.5029	0

¹ tidye(model.g3, hc=T, keep.nohc=T) # keep no heterocedastic corrected std.errors

term	estimate	std.error	conf.low	conf.high	statistic	p.value	std.error.nohc	statistic.nohc	p.valu
(Intercept)	3.6042	3.2952	-2.8544	10.0628	1.0938	0.275	3.0375	1.1866	(
×1	-0.9053	0.8481	-2.5676	0.7571	-1.0673	0.2867	0.8167	-1.1085	(
cat1	-2.2011	3.7761	-9.6023	5.2001	-0.5829	0.5604	3.6151	-0.6089	(
x2	28.0061	1.5784	24.9124	31.0998	17.7432	0	1.3544	20.6774	
cat2b	-0.1835	2.5577	-5.1965	4.8295	-0.0717	0.9429	2.3532	-0.078	(
cat2c	-0.9414	2.4039	-5.6531	3.7703	-0.3916	0.6956	2.2746	-0.4139	(
cat2d	-1.4556	2.691	-6.7299	3.8187	-0.5409	0.589	2.4636	-0.5909	(
x1:cat1	-9.2755	1.2346	-11.6953	-6.8558	-7.5131	0	1.1527	-8.0471	
x2:cat2b	-58.1667	1.8969	-61.8846	-54.4488	-30.664	0	1.8639	-31.2071	
x2:cat2c	-17.6127	1.8342	-21.2077	-14.0176	-9.6023	0	1.7246	-10.2125	
x2:cat2d	-38.3783	2.3255	-42.9364	-33.8203	-16.5029	0	2.0687	-18.5523	

Finally, we can create tables with list of models.

```
1
2 ## list of models
3 tidye(list(Gaussian=model.g3, Binomial=model.bin, Multinomial=model.mul)) %>% print(., n=Inf)
```

y.multin.cat	model	term	estimate	std.error	conf.low	conf.high	statistic	p.value
nil	Gaussian	(Intercept)	3.6042	3.0375	-2.3742	9.5826	1.1866	0.2364
nil	Gaussian	x1	-0.9053	0.8167	-2.5126	0.7021	-1.1085	0.2686
nil	Gaussian	cat1	-2.2011	3.6151	-9.3164	4.9142	-0.6089	0.5431
nil	Gaussian	x2	28.0061	1.3544	25.3403	30.6719	20.6774	0
nil	Gaussian	cat2b	-0.1835	2.3532	-4.8151	4.4481	-0.078	0.9379
nil	Gaussian	cat2c	-0.9414	2.2746	-5.4184	3.5355	-0.4139	0.6793
nil	Gaussian	cat2d	-1.4556	2.4636	-6.3044	3.3932	-0.5909	0.5551
nil	Gaussian	x1:cat1	-9.2755	1.1527	-11.5442	-7.0069	-8.0471	0
nil	Gaussian	x2:cat2b	-58.1667	1.8639	-61.8352	-54.4982	-31.2071	0
nil	Gaussian	x2:cat2c	-17.6127	1.7246	-21.0071	-14.2183	-10.2125	0
nil	Gaussian	x2:cat2d	-38.3783	2.0687	-42.4499	-34.3068	-18.5523	0
nil	Binomial	(Intercept)	1.3429	0.8402	-0.3366	2.9946	1.5982	0.11
nil	Binomial	x1	-0.7064	0.1766	-1.0668	-0.3716	-3.9992	0.0001
nil	Binomial	x2	6.8998	2.4031	3.1397	12.5526	2.8712	0.0041
nil	Binomial	cat2b	0.8125	0.9307	-0.9529	2.7251	0.8731	0.3826
nil	Binomial	cat2c	0.8889	0.7803	-0.5932	2.5013	1.1392	0.2546
nil	Binomial	cat2d	0.3712	0.7899	-1.1366	1.9951	0.47	0.6384
nil	Binomial	x2:cat2b	-10.5099	2.8835	-17.0461	-5.7408	-3.6449	0.0003
nil	Binomial	x2:cat2c	-6.0388	2.4337	-11.7324	-2.172	-2.4813	0.0131
nil	Binomial	x2:cat2d	-7.2537	2.4283	-12.9408	-3.4126	-2.9872	0.0028
2	Multinomial	(Intercept)	-0.9266	0.4976	-1.9018	0.0487	-1.8621	0.0626
2	Multinomial	x1	-0.0099	0.1505	-0.3048	0.285	-0.0657	0.9477
2	Multinomial	x2	-0.2114	0.1776	-0.5596	0.1367	-1.1902	0.234
3	Multinomial	(Intercept)	-0.5612	0.5229	-1.586	0.4636	-1.0734	0.2831
3	Multinomial	x1	-0.2168	0.1646	-0.5393	0.1058	-1.3173	0.1877
3	Multinomial	x2	-0.24	0.1995	-0.6311	0.151	-1.203	0.229

It can easily be exported to standard publication format using the package kable or the function etab() provided by edar

```
tidye(list(Gaussian=model.g3, Binomial=model.bin, Multinomial=model.mul)) %>%
   kableExtra::kable(., "html", booktabs = T ) %>%
   kableExtra::kable_styling(bootstrap_options = c("striped", "hover", "condensed"))

# tidye(list(Gaussian=model.g3, Binomial=model.bin, Multinomial=model.mul)) %>%
# kableExtra::kable(., "html", booktabs = T ) %>%
# kableExtra::kable_styling(latex_options = c("scale_down"))
```

(Intercept) 3.6042 3.0375 -2.3742 9.5826 1.1866 0.2364 NA Gaussian <td style="text-align:left;"> x1 -0.9053align:right;"> 0.8167 -2.5126 0.7021 < td > td style="text-align:right;"> -1.1085 < td > td style="text-align:right;"> 0.2686</td> </tr> <tr> <td style="text-align:left;"> NA </td> <td style="text-align:left;"> Gaussian cat1 -2.2011 <td style="text-align:right;">3.6151 -9.3164align:right;"> 4.9142 -0.6089 0.5431 NA Gaussian x2 28.0061 <td style="text-align:right;"> 1.3544 </td> <td style="text-align:right;"> 25.3403 </td> <td style="text-align:right;"> 25.3403 </td> align:right;"> 30.6719 20.6774 0.0000 NA ctd style="text-align:left;"> text-align:left;"> text 2.3532 -4.8151 4.4481 -0.0780 -0.0780 align:right;">0.9379 NA NAalign:left;"> Gaussian cat2c -0.9414 2.2746 -5.4184 3.5355 -0.4139 0.6793 NA Gaussian cat2d cat2d cat2d align:right;"> -1.4556 2.4636-6.3044 3.3932 -0.5909 0.5551 NA Gaussian x1:cat1 <td style="text-align:right;">-9.2755 1.1527align:right;"> -11.5442 -7.0069 $-8.0471 < \text{td} > \text{td style} = \text{``text-align:right;''} > 0.0000 < \text{/td} > \text{/tr} > \text{td style} = \text{``text-align:left;''} > 0.0000 < \text{/td} > \text{/tr} > \text{td style} = \text{``text-align:left;''} > 0.0000 < \text{/td} > \text{/td} > \text{td style} = \text{``text-align:left;''} > 0.0000 < \text{/td} > \text{td style} = \text{``text-align:left;''} > 0.0000 < \text{/td} > \text{td style} = \text{``text-align:left;''} > 0.0000 < \text{'td} > \text{td style} = \text{``text-align:left;''} > 0.0000 < \text{'td} > \text{td style} = \text{``text-align:left;''} > 0.0000 < \text{'td} > \text{td style} = \text{``text-align:left;''} > 0.0000 < \text{'td} > \text{td style} = \text{``text-align:left;''} > 0.0000 < \text{'td} > \text{td style} = \text{``text-align:left;''} > 0.0000 < \text{'td} > \text{td style} = \text{``text-align:left;''} > 0.0000 < \text{'td} > \text{td style} = \text{``text-align:left;''} > 0.0000 < \text{'td} > \text{td style} = \text{``text-align:left;''} > 0.0000 < \text{'td} > \text{td style} = \text{``text-align:left;''} > 0.0000 < \text{'td} > \text{td style} = \text{``text-align:left;''} > 0.0000 < \text{'td} > \text{td style} = \text{'`text-align:left;''} > 0.00000 < \text{'td} > \text{'$ NA td = "text-align:left;" > Gaussian td > tyle="text-align:left;" > x2:cat2b > tyle="text-align:left;" > x2:cat2b < tyle="text-align:left;" > x -58.1667 1.8639 <td style="text-align:right;"> -61.8352 -54.4982 -54.4982 align:left;"> NA Gaussian x2:cat2c -17.6127 1.7246 -21.0071 -14.2183 -10.2125 0.0000 <tr> <td style="text-align:left;"> NA </td> <td style="text-align:left;"> Gaussian </td> <td style="text-align:left;"> x2:cat2d -38.3783 -38.3783align:right;"> 2.0687 -42.4499

-34.3068 -18.5523 0.0000 NA Binomial (Intercept) 1.3429 0.8402 -0.3366 -0.3366 align:right;"> 2.9946 1.5982 2.9946 0.1100 NA Binomial x1 -0.7064 <td style="text-align:right;"> 0.1766 </td> <td style="text-align:right;"> -1.0668 </td> <td style="text-align:right;"> -1.0668 </td> align:right;"> -0.3716 -3.9992 0.0001 NA Binomial x2 6.8998 2.4031 3.1397 3.4031 align:right;"> 12.5526 2.8712 2.8712 0.0041 NA NABinomial cat2b 0.8125 0.9307 -0.9529 <td style="text-align:right;"> 2.7251 </td> <td style="text-align:right;"> 0.8731 </td> <td style="text-align:right;"> 0.8731 </td> align:right;"> 0.3826 NA NA align:left;"> Binomial cat2c 0.8889 < td > ctd style="text-align:right;"> 0.7803 < td > ctd style="text-align:right;"> -0.5932 2.5013 1.1392 <td style="text-align:left;"> Binomial cat2d cat2d align:right;"> 0.3712 0.7899 0.7899 -1.1366 1.9951 0.4700 0.6384 NA Binomial x2:cat2b <td style="text-align:right;"> -10.5099 </td> <td style="text-align:right;"> 2.8835 </td> <td style="text-align:right;"> -17.0461 -5.7408 -5.7408 align:right;">-3.6449 0.0003 -3.6449align:left;"> NA Binomial x2:cat2c -6.0388 2.4337 -11.7324 -2.1720 -2.4813 0.0131 <tr> <td style="text-align:left;"> NA </td> <td style="text-align:left;"> Binomial </td> <td style="text-align:left;"> x2:cat2d -7.2537 -7.2537align:right;"> 2.4283 -12.9408 -3.4126 -2.9872 0.0028 style="text-align:left;"> Category 2 ctyle="text-align:left;"> ct Multinomial (Intercept) -0.9266 0.4976 -1.9018 0.0487 -1.8621 0.0626 Category 2

</td> <td style="text-align:left;"> Multinomial </td> <td style="text-align:left;"> x1 </td> <td style="text-align:right;"> -0.0099 0.1505 0.1505 align:right;"> -0.3048 0.2850 -0.0657 0.9477 0.9477 0.9477 0.9477 0.9477 0.9477 0.9477 0.9477 0.9477 0.9477 0.9477 0.9477 0.9477Category 2 Multinomial x2 -0.2114 0.1776 -0.5596 0.1367 0.1367 align:right;">-1.1902 0.2340 -1.1902align:left;"> Category 3 Multinomial <td style="textalign:left;"> (Intercept) -0.5612 0.5229 -1.5860 0.4636 -1.0734 0.2831 Category 3 Multinomial </td> <td style="text-align:left;"> \times 1 </td> <td style="text-align:right;"> -0.2168 </td> <td style="text-align:right;">0.1646 -0.5393align:right;"> 0.1058 -1.3173 $0.1877 < /\mathsf{td} > </\mathsf{tr} > <\mathsf{td} \; \mathsf{style} = \text{``text-align:left;''} > \mathsf{Category} \; 3 < /\mathsf{td} > <\mathsf{td} \; \mathsf{style} = \text{``text-align:left;''} > \mathsf{Category} \; 3 < /\mathsf{td} > <\mathsf{td} \; \mathsf{style} = \text{``text-align:left;''} > \mathsf{Category} \; 3 < /\mathsf{td} > <\mathsf{td} \; \mathsf{style} = \text{``text-align:left;''} > \mathsf{Category} \; 3 < /\mathsf{td} > <\mathsf{td} \; \mathsf{style} = \text{``text-align:left;''} > \mathsf{Category} \; 3 < /\mathsf{td} > <\mathsf{td} \; \mathsf{style} = \text{``text-align:left;''} > \mathsf{Category} \; 3 < /\mathsf{td} > <\mathsf{td} \; \mathsf{style} = \text{``text-align:left;''} > \mathsf{Category} \; 3 < /\mathsf{td} > <\mathsf{td} \; \mathsf{style} = \mathsf{``text-align:left;''} > \mathsf{Category} \; 3 < /\mathsf{td} > <\mathsf{td} \; \mathsf{style} = \mathsf{``text-align:left;''} > \mathsf{Category} \; 3 < /\mathsf{td} > <\mathsf{td} \; \mathsf{style} = \mathsf{``text-align:left;''} > \mathsf{Category} \; 3 < /\mathsf{td} > <\mathsf{td} \; \mathsf{style} = \mathsf{``text-align:left;''} > \mathsf{Category} \; 3 < /\mathsf{td} > <\mathsf{td} \; \mathsf{style} = \mathsf{``text-align:left;''} > \mathsf{Category} \; 3 < /\mathsf{td} > <\mathsf{td} \; \mathsf{style} = \mathsf{``text-align:left;''} > \mathsf{Category} \; 3 < /\mathsf{td} > <\mathsf{td} \; \mathsf{style} = \mathsf{``text-align:left;''} > \mathsf{Category} \; 3 < /\mathsf{td} > <\mathsf{td} \; \mathsf{style} = \mathsf{``text-align:left;''} > \mathsf{Category} \; 3 < /\mathsf{td} > <\mathsf{td} \; \mathsf{style} = \mathsf{``text-align:left;''} > \mathsf{Category} \; 3 < /\mathsf{td} > <\mathsf{td} \; \mathsf{style} = \mathsf{``text-align:left;''} > \mathsf{Category} \; 3 < /\mathsf{td} > <\mathsf{td} \; \mathsf{style} = \mathsf{``text-align:left;''} > \mathsf{Category} \; 3 < /\mathsf{td} > <\mathsf{td} \; \mathsf{style} = \mathsf{``text-align:left;''} > \mathsf{Category} \; 3 < /\mathsf{td} > <\mathsf{td} \; \mathsf{style} = \mathsf{``text-align:left;''} > \mathsf{Category} \; 3 < /\mathsf{td} > <\mathsf{td} \; \mathsf{style} = \mathsf{``text-align:left;''} > \mathsf{category} \; 3 < /\mathsf{td} > <\mathsf{td} \; \mathsf{style} = \mathsf{``text-align:left;''} > \mathsf{category} \; 3 < /\mathsf{td} > <\mathsf{td} \; \mathsf{style} = \mathsf{``text-align:left;''} > \mathsf{category} \; 3 < /\mathsf{td} > <\mathsf{td} \; \mathsf{style} = \mathsf{``text-align:left;''} > \mathsf{category} \; 3 < /\mathsf{td} > <\mathsf{td} \; \mathsf{style} = \mathsf{``text-align:left;''} > \mathsf{category} \; 3 < /\mathsf{td} > \mathsf{category} \; 3 < \mathsf{td} > \mathsf{category} \; 3 < \mathsf{td} > \mathsf{category} \; 3 < \mathsf{td} > \mathsf{td}$ Multinomial x2 -0.2400 0.1995 -0.6311 <td style="text-align:right;"> 0.1510 </td> <td style="text-align:right;"> -1.2030 </td> <td style="text-align:right;"> -1.2030 </td> align:right;"> 0.2290

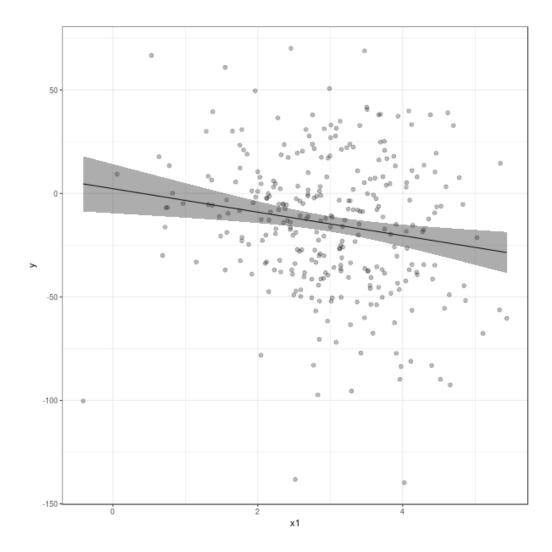
1 list(Binomial=model.bin, Multinomial=model.mul,Gaussian=model.g3) %>%
2 etab

Covariate	Binomial	Gaussian	Multinomial Category 2	Multinomial Category 3
(Intercept)	1.3429	3.6042	-0.9266	-0.5612
	(-0.3366, 2.9946)	(-2.3742, 9.5826)	(-1.9018, 0.0487)	(-1.586, 0.4636)
x1	-0.7064	-0.9053	-0.0099	-0.2168
	(-1.0668, -0.3716)	(-2.5126, 0.7021)	(-0.3048, 0.285)	(-0.5393, 0.1058)
x2	6.8998	28.0061	-0.2114	-0.24
	(3.1397, 12.5526)	(25.3403, 30.6719)	(-0.5596, 0.1367)	(-0.6311, 0.151)
cat1		-2.2011		
		(-9.3164, 4.9142)		
cat2b	0.8125	-0.1835		
	(-0.9529, 2.7251)	(-4.8151, 4.4481)		
cat2c	0.8889	-0.9414		
	(-0.5932, 2.5013)	(-5.4184, 3.5355)		
cat2d	0.3712	-1.4556		
	(-1.1366, 1.9951)	(-6.3044, 3.3932)		
x1:cat1		-9.2755		
		(-11.5442, -7.0069)		
x2:cat2b	-10.5099	-58.1667		
	(-17.0461, -5.7408)	(-61.8352, -54.4982)		
x2:cat2c	-6.0388	-17.6127		
	(-11.7324, -2.172)	(-21.0071, -14.2183)		
x2:cat2d	-7.2537	-38.3783		
	(-12.9408, -3.4126)	(-42.4499, -34.3068)		

2.4.2 Plot fitted values

After the estimation a good way to visualize and present marginal effects are plots with fitted values. It is easy to do with edar package.

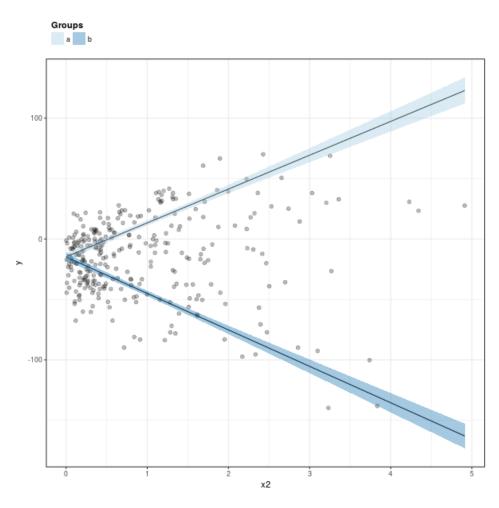
```
1 model.g1 %>% gge_fit(., data, 'y', "x1")
```

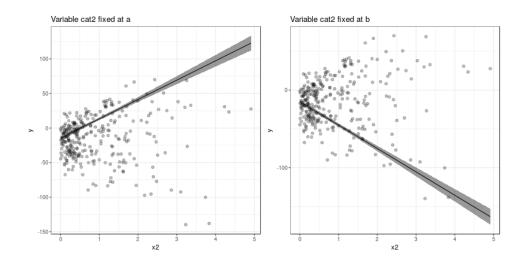


There are many options avaiable with the $gge_fit()$ function. We can at once:

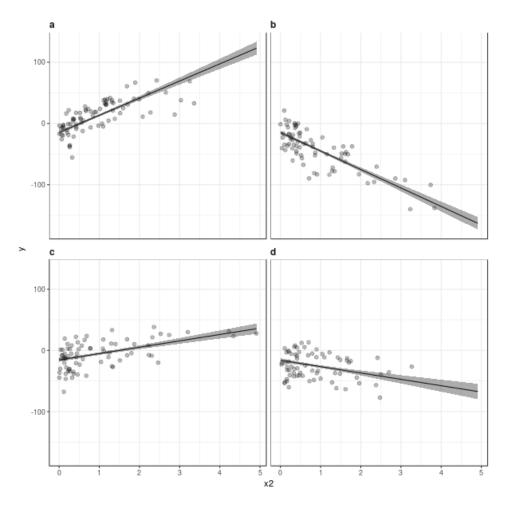
- Compare fitted values for different groups
- Compare fitted values for different model specifications, given a list of models
- Create a grid of plots with fitted values for different groups and model specifications
- 1. Fitted values for different groups

```
1 model.g3 %>% gge_fit(., data, 'y', "x2", cat.values=list(cat2=c('a', "b")))
```





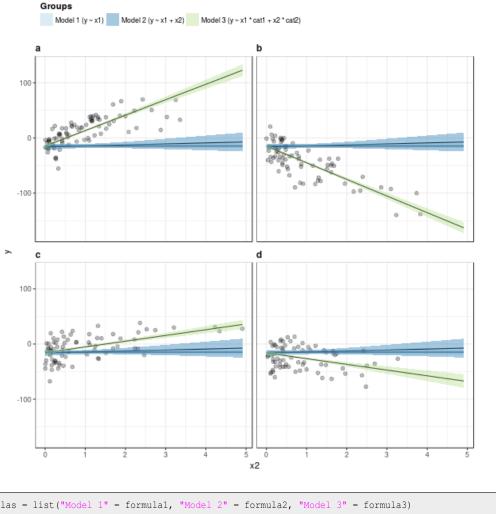
```
1 model.g3 %>% gge_fit(., data, 'y', "x2", facets='cat2')
```



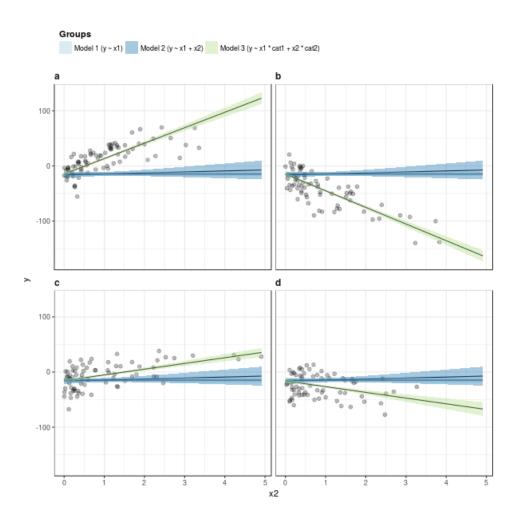
Categories 0 0 1 D 100 50 -50 -100 -50 -100

We can also compare a list of models

```
formulas = list("Model 1" = formula1, "Model 2" = formula2, "Model 3" = formula3)
models = list("Model 1" = model.g1, "Model 2" = model.g2, "Model 3" = model.g3)
models %>% gge_fit(., data, "y", "x2", formulas)
```



```
formulas = list("Model 1" = formula1, "Model 2" = formula2, "Model 3" = formula3)
models = list("Model 1" = model.g1, "Model 2" = model.g2, "Model 3" = model.g3)
models %>% gge_fit(., data, "y", "x2", formulas, legend.ncol.fill=3, facets='cat2')
```



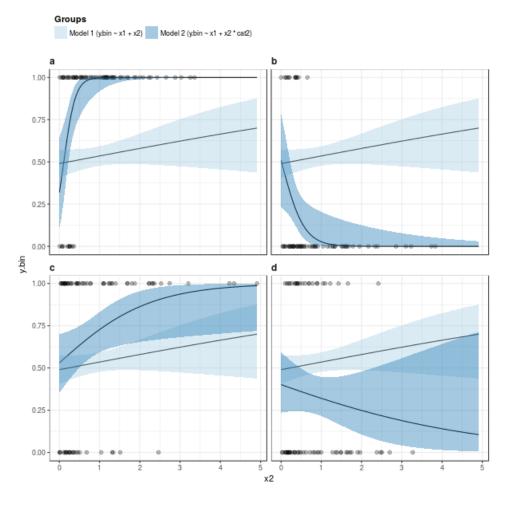
The same applies for logistic regressions.

```
formula.bin1 = y.bin ~ x1+x2
formula.bin2 = y.bin ~ x1+x2*cat2
model.bin1 = glm(formula.bin1, data=data, family='binomial')

model.bin2 = glm(formula.bin2, data=data, family='binomial')

formulas = list("Model 1" = formula.bin1, "Model 2" = formula.bin2)
models = list("Model 1" = model.bin1, "Model 2" = model.bin2)

models %>% gge_fit(., data, "y.bin", "x1", formulas)
```



models %>% gge_fit(., data, "y.bin", "x2", formulas, facets='cat2')

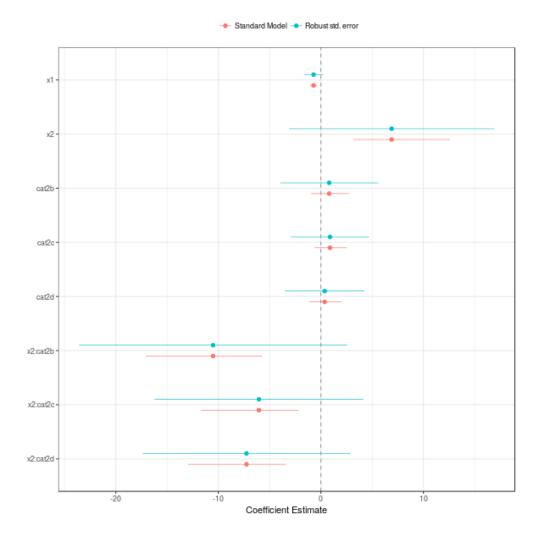
object 'models' not found

2.4.3 Plot with coefficients (dotwisker)

The edar package also provides a wrap function for the <code>dotwisker()</code> plot from the package with same name. As before, the function accepts list of models or tidy summaries of the estimation. There are also options to use robust standard errors in the plot.

```
models=tidye(list('Standard Model'=model.bin2)) %>%

dplyr::bind_rows(tidye(list('Robust std. error'=model.bin2), hc=T))
gge_coef(models, model.id='model')
```



2.5 Multiple-imputation and post-stratification

Multiple imputation and post-stratification are easy to conduct. The options are limited. The package survey and the package mice contain more options.

Here is an example of multiple imputation for two models with different output variables.

```
1
    data = tibble::data_frame(x1 = rnorm(200,3,1),
 2
                              x2 = rexp(200),
 3
                              cat.var = sample(c(0,1), 200, replace=T),
 4
                              cat.var2 = sample(letters[1:4], 200, replace=T),
 5
                              y1 = 10*x1*cat.var+rnorm(200,0,10) +
 6
                                   3*x2*(6*(cat.var2=='a') -3*(cat.var2=='b') +
 7
                                        1*(cat.var2=='c') +1*(cat.var2=='d')),
 8
                              y2 = -10*x1*cat.var+rnorm(200,0,10) +
 9
                                   10*x2*(3*(cat.var2=='a') -3*(cat.var2=='b') +
10
                                         1*(cat.var2=='c') -1*(cat.var2=='d'))
11
                              ) %>%
12
        dplyr::mutate(cat.var=as.factor(cat.var))
13
    data$x1[sample(1:nrow(data), 10)] = NA
14
```

```
15
16
   formula = "x1*cat.var+x2*cat.var2"
17
   imp = emultimputation(data, formula, dep.vars = c("y1", "y2"), ind.vars=c("x1", "x2", "cat.var",
        "cat.var2"))
18
   imp
   $y1
                                               df p.value low.95 high.95 nmis
              term estimate
                                se
                                          t
                                                                                 fmi lambda
                     1.2196 3.4872
                                     0.3497 183.9 0.7269 -5.660
                                                                   8.100
                                                                           NA 0.0218 0.0112
   1
       (Intercept)
   2
                     0.3293 0.8412 0.3914 182.8 0.6959 -1.331
                                                                  1.989
                                                                            10 0.0245 0.0139
   3
          cat.var2 -4.9541 4.5860 -1.0802 158.3 0.2817 -14.012
                                                                  4.104
                                                                            0 0.0638 0.0521
                x2 17.2509 1.3802 12.4993 181.6 0.0000 14.528 19.974
                                                                           0 0.0274 0.0167
   4
   5
         cat.var2b
                   0.5389 3.0200
                                   0.1784 176.4 0.8586 -5.421
                                                                  6.499
                                                                           NA 0.0375 0.0266
   6
         cat.var2c -3.7201 3.0468 -1.2210 179.1 0.2237 -9.732
                                                                   2.292
                                                                           NA 0.0325 0.0218
   7
         cat.var2d -2.1013 3.0617 -0.6863 177.7 0.4934 -8.143
                                                                   3.941
                                                                           NA 0.0351 0.0243
       x1:cat.var2 10.5961 1.4690
                                   7.2130 155.1 0.0000
                                                           7.694 13.498
                                                                           NA 0.0680 0.0560
   9 x2:cat.var2b -26.7177 2.0414 -13.0880 185.4 0.0000 -30.745 -22.690
                                                                           NA 0.0173 0.0068
    [ reached getOption("max.print") -- omitted 2 rows ]
   $y2
                                          t
                                               df p.value
                                                          low.95 high.95 nmis
                                                                                   fmi lambda
              term estimate
                                se
       (Intercept)
                     7.0397 3.4878
                                     2.0184 122.8 0.0457
                                                            0.1357 13.9437
                                                                             NA 0.1089 0.0946
   2
                x1 -0.4107 0.8368 -0.4908 127.6 0.6244 -2.0664
                                                                    1.2450
                                                                             10 0.1028 0.0888
   3
          cat.var2 2.9983 4.5419
                                   0.6601 104.7 0.5106 -6.0077 12.0043
                                                                              0 0.1344 0.1181
   4
                x2 27.6086 1.3153 20.9904 185.2 0.0000 25.0137 30.2035
                                                                             0 0.0178 0.0072
   5
         cat.var2b -5.6630 2.9412 -1.9254 152.0 0.0560 -11.4740
                                                                    0.1479
                                                                             NA 0.0719 0.0598
```

Post-stratification for simple probabilistic sample is also straightforward.

[reached getOption("max.print") -- omitted 2 rows]

cat.var2d -2.4124 3.0181 -0.7993 134.9 0.4255 -8.3812

cat.var2c -6.4962 3.0161 -2.1538 129.7 0.0331 -12.4634 -0.5290

x1:cat.var2 -10.6894 1.4361 -7.4433 111.9 0.0000 -13.5349 -7.8439

9 x2:cat.var2b -58.4786 1.9527 -29.9479 186.0 0.0000 -62.3309 -54.6264

\$weights

6

7

```
[1] 0.6250 0.5357 0.3061 0.5357 0.6250 0.3061 0.5357 0.5319 0.5357 0.6250 0.5357 0.3061 0.5319 0.5319 0.5319 0.5357 0.3061 0.5357 0.5319 0.6250 0.5357 0.5357 0.6250 0.5319 0.5319 0.5319 0.5357 0.5319 0.5357 0.5319 0.5357 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.5319 0.531
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NA 0.1000 0.0862

NA 0.0933 0.0800

NA 0.1240 0.1085

NA 0.0150 0.0045

3.5564

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[49] 0.3061 0.6250 0.3061 0.5319 0.5357 0.6250 0.5319 0.3061 0.5319 0.6250 0.3061 0.6250 0.3061 0.5
[65] 0.3061 0.5357 0.3061 0.6250 0.6250 0.3061 0.5357 0.6250 0.5319 0.6250 0.6250 0.3061 0.3061 0.3
[81] 0.5319 0.3061 0.3061 0.5357 0.5357 0.6250 0.5357 0.5357 0.6250 0.3061 0.6250 0.6250 0.6250 0.6250 0.6250 0.6250 0.6250 0.6250 0.6250 0.6250 0.6250 0.6250 0.6250 0.6250 0.6250 0.6250 0.6250 0.6250 0.6250 0.6250 0.6250 0.6250 0.6250 0.6250 0.6250 0.6250 0.6250 0.6250 0.6250 0.6250 0.6250 0.6250 0.6250 0.6250 0.6250 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5319 0.5357 0.5357 0.5319 0.5357 0.5357 0.5319 0.5357 0.5319 0.5357 0.5319 0.3061 0.5357 0.5357 0.5319 0.5357 0.5319 0.5357 0.5319 0.5357 0.5319 0.5357 0.5319 0.5357 0.5319 0.5357 0.5319 0.5357 0.5357 0.5319 0.5357 0.5319 0.5357 0.5319 0.5357 0.5357 0.5357 0.5319 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357 0.5357
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