

COMPLEXITY

USEI17

```
/**  
 * Builds the PERT/CPM graph.  
 */  
public void buildPERT_CPM() {  
    // Add the "START" node  
    activitiesPERT_CPM.clear();  
    ActivitiesMapRepository activitiesMapRepository =  
Instances.getInstance().getActivitiesMapRepository();  
    activities = activitiesMapRepository.getActivitiesMapRepository();  
    pert_CPM.addVertex("START");  
    activitiesPERT_CPM.put("START", new Activity("START", "START", 0, 0, new  
ArrayList<>()));  
  
    for (Activity activity : activities.values()) {  
        // Create the node label with duration  
        String nodeLabel = activity.getActId() + " (" + activity.getDurationWithUnit() +  
")";  
        pert_CPM.addVertex(nodeLabel);  
  
        // Connect "START" to activities without predecessors  
        if (activity.getPrevActIds().isEmpty()) {  
            pert_CPM.addEdge("START", nodeLabel, "0");  
        }  
  
        // Connect activities to their predecessors  
        for (String dependencyId : activity.getPrevActIds()) {  
            Activity dependencyActivity = activities.get(dependencyId);  
            if (dependencyActivity != null) {  
                String dependencyLabel = dependencyId + " (" +  
dependencyActivity.getDurationWithUnit() + ")";  
                pert_CPM.addEdge(dependencyLabel, nodeLabel, "0");  
            }  
        }  
  
        activitiesPERT_CPM.put(activity.getActId(), activity);  
  
        if (activity.getPrevActIds().isEmpty()) {  
            List<String> modifiablePrevActIds = new ArrayList<>(activity.getPrevActIds());  
            modifiablePrevActIds.add("START");  
            activity.setPrevActIds(modifiablePrevActIds);  
        }  
    }  
  
    // Add the "END" node  
    pert_CPM.addVertex("END");  
    activitiesPERT_CPM.put("END", new Activity("END", "END", 0, 0, new ArrayList<>()));  
  
    // Connect all activities without successors to the "END" node  
    for (String vertex : pert_CPM.vertices()) {  
        String actId = vertex.split("-")[0];  
        boolean hasSuccessors = false;  
        // A node without successors has an out-degree of 0  
        if (pert_CPM.outDegree(vertex) == 0 && !vertex.equals("END") &&  
!vertex.equals("START")) {  
            // Connect to "END"  
            pert_CPM.addEdge(vertex, "END", "0");  
            if (activitiesPERT_CPM.containsKey(actId)) {  
  
activitiesPERT_CPM.get("END").getPrevActIds().add(activitiesPERT_CPM.get(actId).getActId())  
        }  
    }  
}
```

```

;
        }
    }
    for (String actId2 : activities.keySet()) {
        Activity activity = activities.get(actId2);
        if (activity.getPrevActIds().contains(actId)) {
            hasSuccessors = true;
        }
    }
    if (!hasSuccessors && !vertex.equals("END") && !vertex.equals("START") &&
activitiesPERT_CPM.containsKey(actId) &&
!activitiesPERT_CPM.get("END").getPrevActIds().contains(actId)) {
        // Connect to "END"

activitiesPERT_CPM.get("END").getPrevActIds().add(activitiesPERT_CPM.get(actId).getActId())
;
        }
    }
}

```

Time Complexity `buildPERT_CPM()`: **O(V^2 + V * P)**

- The outer loop iterates over all activities, leading to a complexity of **O(V)**, where V is the number of activities.
- For each activity, the predecessors are processed, leading to **O(P)** operations per activity, where P is the average number of predecessors.
- The method then iterates over all vertices to connect to the "END" node, contributing an additional **O(V)** complexity.
- Finally, it iterates over all activities again to check for successors, which adds another **O(V)**.
- Combining all, the overall time complexity is **O(V^2 + V * P)**.

USEI18

```

/**
 * Checks if there are circular dependencies in the graph.
 *
 * @return true if a cycle is detected, false otherwise.
 */
public boolean hasCircularDependencies() {
    Set<String> visited = new HashSet<>();
    Set<String> recursionStack = new HashSet<>();

    for (String vertex : pert_CPM.vertices()) {
        if (detectCycle(vertex, visited, recursionStack)) {
            return true; // Cycle detected
        }
    }
    return false; // No cycle detected
}

```

Time Complexity `hasCircularDependencies()`: $O(V * (V + E))$

- The method iterates over all vertices in the graph, contributing $O(V)$ complexity.
- For each vertex, `detectCycle()` is called, which has a time complexity of $O(V + E)$, where E is the number of edges.
- Combining both, the worst-case time complexity is $O(V * (V + E))$.

```
private boolean detectCycle(String vertex, Set<String> visited, Set<String> recursionStack)
{
    if (recursionStack.contains(vertex)) {
        return true; // Cycle detected
    }

    if (visited.contains(vertex)) {
        return false; // Already processed, no cycle
    }

    visited.add(vertex);
    recursionStack.add(vertex);

    // Check all adjacent vertices
    for (String neighbor : pert_CPM.adjVertices(vertex)) {
        if (detectCycle(neighbor, visited, recursionStack)) {
            return true; // Cycle detected in subgraph
        }
    }
    recursionStack.remove(vertex);
    return false; // No cycle detected
}
```

Time Complexity `detectCycle()`: $O(V + E)$

- The method performs a recursive DFS, visiting each vertex once, contributing $O(V)$.
- For each vertex, it checks all adjacent vertices, which results in $O(E)$ complexity.
- Combining both, the time complexity is $O(V + E)$.

```

/**
 * Validates the graph.
 * @return true if the graph is valid, false otherwise.
 */
public boolean validateGraph() {
    boolean hasCircular=false;
    if (hasCircularDependencies()) {
        hasCircular=true;
    }
    else {
        hasCircular=false;
    }
    return hasCircular;
}

```

Time Complexity validateGraph(): **O(V * (V + E))**

- The method calls `hasCircularDependencies()`, which has a time complexity of **$O(V * (V + E))$** .
- Therefore, the overall time complexity is **$O(V * (V + E))$** .

USEI19

```

public List<String> topologicalSort() {
    if (hasCircularDependencies()) {
        throw new IllegalStateException("The graph has circular dependencies.");
    }
    Stack<String> stack = new Stack<>(); // Stack to store the sorted order
    Set<String> visited = new HashSet<>(); // Set to track visited nodes
    // Iterate over all vertices in the graph
    for (String vertex : pert_CPM.vertices()) {
        if (!visited.contains(vertex)) {
            topologicalSortUtil(vertex, visited, stack);
        }
    }
    // Convert the stack into the result list
    List<String> sortedOrder = new ArrayList<>();
    while (!stack.isEmpty()) {
        sortedOrder.add(stack.pop());
    }
    return sortedOrder;
}

```

Time Complexity topologicalSort(): **O(V * (V + E))**

- The method first checks for circular dependencies, which takes **$O(V * (V + E))$** time.

- It then iterates over all vertices, contributing $O(V)$.
- For each unvisited vertex, `topologicalSortUtil()` is called, which also adds $O(V)$.
- Finally, converting the stack to a list adds $O(V)$.
- Combining all, the overall time complexity is $O(V * (V + E))$.

```
private void topologicalSortUtil(String vertex, Set<String> visited, Stack<String> stack) {
    // Mark the current vertex as visited
    visited.add(vertex);

    // Recur for all adjacent vertices
    for (String neighbor : pert_CPM.adjVertices(vertex)) {
        if (!visited.contains(neighbor)) {
            topologicalSortUtil(neighbor, visited, stack);
        }
    }

    // Push the current vertex to the stack
    stack.push(vertex);
}
```

Time Complexity `topologicalSortUtil()`: $O(V + E)$

- The method performs a recursive DFS, visiting each vertex once, contributing $O(V)$.
- For each vertex, it explores all adjacent vertices, resulting in $O(E)$.
- Therefore, the time complexity is $O(V + E)$.

USEI20

`calculateTimes()`

- **Method Calls:**
 - `clearTimes(activities)`: $O(n)$
 - `calculateESEF(activities)`: $O(n^3)$
 - `calculateLSLF(activities)`: $O(n^3)$
 - `calculateSlack(activities)`: $O(n)$
- **Complexity:** The complexity is determined by the sum of the complexities of these individual methods.
- Combining all, the overall time complexity is $O(n^3)$

```

public void calculateTimes() {
    pert_cpm = Instances.getInstance().getPERT_CPM();
    LinkedHashMap<String, Activity> activities =
pert_cpm.getActivitiesPERT_CPM();
    clearTimes(activities);
    calculateESEF(activities);
    calculateLSLF(activities);
    calculateSlack(activities);
}

```

clearTimes(LinkedHashMap<String, Activity> activities)

- **Operation:** Iterates over all the activities in the LinkedHashMap and clears the values to their default 0.
- **Complexity:** $O(n)$, where n is the number of activities.

```

private void clearTimes(LinkedHashMap<String, Activity> activities) {
    for (Activity activity : activities.values()) {
        activity.clearTimes();
    }
}

```

calculateESEF(LinkedHashMap<String, Activity> activities)

- **Operation:** Get the start activity so you can calculate from the start as the ES and EF calculations start from the beginning for the calculation we will use a recursive method.
- **Calls calculateESEF(LinkedHashMap<String, Activity> activities)** that has an complexity of $O(n^3)$.
- **Complexity:** $O(n^3)$

```

private static void calculateESEF(LinkedHashMap<String, Activity>
activities) {
    Activity start = activities.get("START");
    start.calculateESEF(activities);
}

```

calculateESEF(LinkedHashMap<String, Activity> activities)

Logic Breakdown:

1. Handles a special case for the "START" node ($O(1)$).
2. Iterates over the predecessor activities (prevActIds), performing:
 - A lookup in the LinkedHashMap ($O(1)$).
 - A comparison and update operation ($O(1)$).

3. Computes earliestStart as earliestStart equal to the biggest earliest finish (**O(1)**).
4. Computes earliestFinish as earliestStart + duration (**O(1)**).

Complexity:

- Iterates over **n** predecessors: **O(n)**.
- Calls the function calculateESEFRec(LinkedHashMap<String, Activity> activities) **n** times that has a complexity of **O(n²)** so:
- Total: **O(n³)**

```
public void calculateESEF(LinkedHashMap<String, Activity> activities) {
    if (actId.equalsIgnoreCase("START")) {
        earliestStart = 0.0;
        earliestFinish = 0.0;
        List<Activity> nextActIds = new ArrayList<>();
        for (Activity activity : activities.values()) {
            if (activity.getPrevActIds().contains(actId)) {
                nextActIds.add(activity);
            }
        }
        for (Activity activity : nextActIds) {
            activity.calculateESEFRec(activities);
        }
    }
}
```

calculateESEFRec(LinkedHashMap<String, Activity> activities)

- **Logic:**
 - **For Loop:** Iterates over prevActIds (predecessor activities), performing constant-time operations. **O(p)** where p ≤ n.
 - **Find Successors:** Loops through all activities to identify successors. **O(n)**.
 - **Recursive Call:** Propagates to successors.
- **Complexity:** **O(n²)** in the worst case due to nested loops and recursion across the activity graph.

```
private void calculateESEFRec(LinkedHashMap<String, Activity> activities) {
    if (actId.equalsIgnoreCase("END")) {
        for (String prevActId : prevActIds) {
            double prevActFinish =
activities.get(prevActId).getEarliestFinish();
            if (prevActFinish > earliestStart) {
                earliestStart = prevActFinish;
            }
        }
        earliestFinish = earliestStart + duration;
```

```

        return;
    }
    for (String prevActId : prevActIds) {
        double prevActFinish =
activities.get(prevActId).getEarliestFinish();
        if (prevActFinish > earliestStart) {
            earliestStart = prevActFinish;
        }
    }
    earliestFinish = earliestStart + duration;
List<Activity> nextActIds = new ArrayList<>();
for (Activity activity : activities.values()) {
    if (activity.getPrevActIds().contains(actId)) {
        nextActIds.add(activity);
    }
}
for (Activity activity : nextActIds) {
    activity.calculateESEFRec(activities);
}
}

```

calculateLSLF(LinkedHashMap<String, Activity> activities)

- **Steps:**
 1. Creates a reversed LinkedList of the activities (**O(n)**).
 2. Gets the activity end form the reversed list (**O(1)**).
 3. Iterates over the activities, calling calculateLSLF on the end activity.
 - If calculateLSLF involves looking up predecessor/successor activities in the LinkedHashMap, the cost is per lookup due to LinkedHashMap's constant-time access property.
- Calls **calculateLSLF(LinkedHashMap<String, Activity> activities)** that has an complexity of **O(n^3)**.
- **Complexity: O(n^3)**

```

private static void calculateLSLF(LinkedHashMap<String, Activity>
activities) {
    LinkedList<Activity> reversedActivities = new
LinkedList<>(activities.values());
    Collections.reverse(reversedActivities);
    Activity end = reversedActivities.getFirst();
    end.calculateLSLF(activities);
}

```

calculateLSLF(LinkedHashMap<String, Activity> activities)

- **Operation:** Calls calculateLSLFRRec on the **END** activity. This method recursively propagates through all predecessors.
- **Complexity:** $O(n^3)$ in the worst case because:
 - At each step, it loops over predecessors ($O(p)$),
 - Recursively calls calculateLSLFRRec for each predecessor.
 - Total cost per activity: $O(n) \times O(n) = O(n^2)$.
 - Since recursion may visit every activity: $O(n^3)$.

```
public void calculateLSLF(LinkedHashMap<String, Activity> activities) {
    if (actId.equalsIgnoreCase("END")) {
        latestFinish = earliestFinish;
        latestStart = earliestStart;
    }
    List<Activity> prevActIds = new ArrayList<>();
    for (String prevActId : this.prevActIds) {
        prevActIds.add(activities.get(prevActId));
    }
    for (Activity activity : prevActIds) {
        double activityStart = latestStart;
        if (activity.getLatestFinish() > activityStart && activityStart != 0) {
            activity.setLatestFinish(activityStart);
            activity.setLatestStart(activity.getLatestFinish() - activity.getDuration());
        } else if (activity.getLatestFinish() == 0) {
            activity.setLatestFinish(activityStart);
            activity.setLatestStart(activity.getLatestFinish() - activity.getDuration());
        }
        activity.calculateLSLFRRec(activities);
    }
}
```

calculateLSLFRRec(LinkedHashMap<String, Activity> activities)

- **Logic:**
 - **For Loop:** Iterates over prevActIds (predecessor activities), performing constant-time operations. $O(p)$ where $p \leq n$.
 - **Recursive Call:** Propagates backward through all predecessors.
- **Complexity:** $O(n^3)$ in the worst case due to recursive calls and nested iterations over the graph structure.

```
private void calculateLSLFRRec(LinkedHashMap<String, Activity> activities) {
    if (prevActIds.isEmpty()) {
        latestFinish = 0;
        latestStart = 0;
```

```

        return;
    }
    List<Activity> prevActIds = new ArrayList<>();
    for (String prevActId : this.prevActIds) {
        prevActIds.add(activities.get(prevActId));
    }
    for (Activity activity : prevActIds) {
        double activityStart = latestStart;
        if (activity.getLatestFinish() > activityStart && activityStart != 0) {
            activity.setLatestFinish(activityStart);
            activity.setLatestStart(activity.getLatestFinish() - activity.getDuration());
        } else if (activity.getLatestFinish() == 0) {
            activity.setLatestFinish(activityStart);
            activity.setLatestStart(activity.getLatestFinish() - activity.getDuration());
        }
        activity.calculateLSLFRec(activities);
    }
}

```

calculateSlack(LinkedHashMap<String, Activity> activities)

- **Operation:** Iterates over all the activities in the LinkedHashMap, calling calculateSlack on each activity.
- **Complexity:** O(n)

```

private static void calculateSlack(LinkedHashMap<String, Activity>
activities) {
    for (Activity activity : activities.values()) {
        activity.calculateSlack();
    }
}

```

- **Calls:**

```

public void calculateSlack() {
    slack = latestStart - earliestStart;
}

```

USEI21

```

/**
 * Exports the schedule to a CSV file.
 * @param file File to export the schedule to.
 */
public void exportScheduleToCSV(File file) {

    CalculateTimes calculateTimes = new CalculateTimes();
    calculateTimes.calculateTimes();
    MapGraph<String, String> pert_CPM_without_duration = getPert_CPMWithoutDuration();
}

```

```

try (FileWriter writer = new FileWriter(file)) {
    // CSV header
    writer.write("act_id;cost;duration;es;ls;ef;lf;prev_act_id1;...;prev_act_idN\n");

    // Write the data for each activity
    for (String actId : activities.keySet()) {
        Activity activity = activities.get(actId);

        // Get the dependencies of the activity
        Collection<Edge<String, String>> dependencies =
pert_CPM_without_duration.incomingEdges(actId);

        // Convert the dependencies to a string
        String dependenciesStr = (dependencies == null || dependencies.isEmpty())
            ? "-"
            : dependencies.stream()
                .map(Edge::getVOrig) // Get the IDs of the dependencies
                .reduce((a, b) -> a + ", " + b) // Concatenate the IDs
                .orElse("-");

        // Write the activity data to the file
        writer.write(String.format("%s;%d;%d;%f;%f;%f;%s\n",
            activity.getActId(),
            activity.getCost(),
            activity.getDuration(),
            activity.getEarliestStart(),
            activity.getLatestStart(),
            activity.getEarliestFinish(),
            activity.getLatestFinish(),
            dependenciesStr
        ));
    }

    System.out.println("Schedule exported successfully to: " + file.getAbsolutePath());
}

} catch (IOException e) {
    System.out.println("Error while exporting schedule: " + e.getMessage());
}
}

```

Time Complexity `exportScheduleToCSV()`: **O(V + E)**

- It calls `getPert_CPMWithoutDuration()`, which has a time complexity of **O(V + E)**.
- The outer loop over activities has a time complexity of **O(V)**.
- For each activity, in the worst case, `incomingEdges` iterates over all vertices and performs a lookup for each vertex. This results in **O(V)** for each call. Since `incomingEdges` is called once for each activity, this step contributes **O(V^2)**.
- The reduce operation on dependencies has a time complexity of **O(P)**, where P is the number of dependencies per activity, related to E.
- Writing the data to the file involves **O(V)** operations.
- Combining all these, the final complexity is **O(V + E)**.

```

/**
 * Builds the PERT/CPM graph without duration.
 */
public MapGraph<String, String> getPert_CPMWithoutDuration() {
    MapGraph<String, String> pert_CPM_without_duration = new MapGraph<>(true);
    for (String vertex : pert_CPM.vertices()) {
        pert_CPM_without_duration.addVertex(vertex.split("■")[0]);
    }
    for (String vertex : pert_CPM.vertices()) {
        for (Edge<String, String> edge : pert_CPM.outgoingEdges(vertex)) {
            pert_CPM_without_duration.addEdge(vertex.split("■")[0], edge.getVDest().split("■")[0], "0");
        }
    }
    return pert_CPM_without_duration;
}

```

Time Complexity `getPert_CPMWithoutDuration()`: $O(V + E)$

- The first loop iterates through all vertices in `pert_CPM`, which takes $O(V)$, where V is the number of vertices.
- The second loop iterates through all vertices again, and for each vertex, it iterates through its outgoing edges. The complexity of this part is $O(E)$, where E is the number of edges.
- The split operation and the `addVertex` and `addEdge` methods each run in constant time $O(1)$.
- Combining these steps, the overall time complexity is $O(V + E)$, where V is the number of vertices and E is the number of edges in `pert_CPM`.

USEI22

`findCriticalPaths()`

- **Method Calls:**
 - Iterates over all predecessors of the "END" activity: $O(n)$, where n is the number of activities.
 - Calls `findCriticalPathsRec(activity, path, criticalPaths)` recursively: $O(m+c \cdot l)$, where m is the number of edges, c is the number of critical paths, and l is the average length of a critical path.
 - Reverses all critical paths: $O(c \cdot l)$.
- **Complexity:** The total complexity is determined by the recursive calls and the reversal of critical paths.
Combining everything, the overall complexity is $O(n+m+c \cdot l)$ so it has an overall complexity of $O(c \cdot l)$.

```

public LinkedHashMap<Integer, List<Activity>> findCriticalPaths() {
    LinkedHashMap<Integer, List<Activity>> criticalPaths = new

```

```

LinkedHashMap<>();
List<Activity> path = new ArrayList<>();
// The method will start from the "END" node
Activity endActivity = activitiesPERT_CPM.get("END");
// We will need to get the predecessors of the "END" node
List<String> endPredecessors =
activitiesPERT_CPM.get("END").getPrevActIds();
for (String endPredecessor : endPredecessors) {
    Activity activity = activitiesPERT_CPM.get(endPredecessor);
    path.add(endActivity);
    path.add(activity);
    findCriticalPathsRec(activity, path, criticalPaths);
    path.clear();
}
// The list will be inverted so it goes from start to end
for (List<Activity> criticalPath : criticalPaths.values()) {
    Collections.reverse(criticalPath);
}
return criticalPaths;
}

```

findCriticalPathsRec(Activity activity, List<Activity> path, LinkedHashMap<Integer, List<Activity>> criticalPaths)

- **Steps:**

- **Check Slack:** If slack is not zero, return immediately. $O(1)$.
- **Base Case ("START" Node):** Copy the path and add it to criticalPaths. Copying the path is $O(l)$, where l is the path length.
- **Recursive Calls for Predecessors:**
 - Iterates over the predecessor activities of the current activity: $O(n)$.
 - For each predecessor:
 - Adds it to path: $O(1)$.
 - Calls findCriticalPathsRec: Recursive cost $O(m)$.
 - Removes the predecessor from path: $O(1)$.

- **Complexity:**

- Each edge in the graph is processed once during recursion, so traversing all edges contributes $O(m)$.
- Path copying for ccc critical paths contributes $O(c \cdot l)$.

- **Overall Complexity: $O(m+c \cdot l)$**

```

private void findCriticalPathsRec(Activity activity, List<Activity> path,
LinkedHashMap<Integer, List<Activity>> criticalPaths) {

```

```

    if (activity.getSlack() != 0) {
        return;
    }
    if (activity.getActId().equals("START")) {
        List<Activity> pathCopy = new ArrayList<>(path);
        criticalPaths.put(criticalPaths.size() + 1, pathCopy);
        return;
    }
    List<String> predecessors = activity.getPrevActIds();
    for (String predecessor : predecessors) {
        Activity predecessorActivity = activitiesPERT_CPM.get(predecessor);
        path.add(predecessorActivity);
        findCriticalPathsRec(predecessorActivity, path, criticalPaths);
        path.remove(predecessorActivity);
    }
}

```

USEI23

```

public List<Activity> getBottleneckActivities() {
    Map<Activity, Integer> dependencyCount = new HashMap<>();
    Map<Activity, Integer> pathCount = new HashMap<>();
    // Initialize counts
    for (Activity activity : activities.values()) {
        dependencyCount.put(activity, 0);
        pathCount.put(activity, 0);
    }

    // Count dependencies
    for (Activity activity : activities.values()) {
        for (String prevActId : activity.getPrevActIds()) {
            Activity prevActivity = activities.get(prevActId);
            if (prevActivity != null) {
                dependencyCount.put(prevActivity, dependencyCount.get(prevActivity) + 1);
            }
        }
    }

    // Count paths
    LinkedHashMap<Integer, List<Activity>> criticalPaths = findCriticalPaths();
    for (List<Activity> path : criticalPaths.values()) {
        for (Activity activity : path) {
            pathCount.put(activity, pathCount.getOrDefault(activity, 0) + 1);
        }
    }

    // Find maximum counts
    int maxDependencies = Collections.max(dependencyCount.values());
    int maxPaths = Collections.max(pathCount.values());

    // Collect bottleneck activities

```

```

List<Activity> bottleneckActivities = new ArrayList<>();
for (Activity activity : activities.values()) {
    if (!activity.getActId().equals("START") && !activity.getActId().equals("END")){
        if ((dependencyCount.get(activity) > 1 && dependencyCount.get(activity) == maxDependencies) ||
(pathCount.get(activity) == maxPaths && dependencyCount.get(activity) > 1)) {
            bottleneckActivities.add(activity);
        }
    }
}

// Identify predecessors of the "END" node
List<String> endPredecessors = activitiesPERT_CPM.get("END").getPrevActIds();
Activity mostCriticalPredecessor = null;
int maxPredecessorPaths = 0;

for (String endPredecessor : endPredecessors){
    Activity activity = activitiesPERT_CPM.get(endPredecessor);
    int pathsThroughPredecessor = pathCount.getOrDefault(activity, 0);
    if (pathsThroughPredecessor > maxPredecessorPaths) {
        maxPredecessorPaths = pathsThroughPredecessor;
        mostCriticalPredecessor = activity;
    }
}

// Add the most critical predecessor to the bottleneck activities list
if (mostCriticalPredecessor != null && !bottleneckActivities.contains(mostCriticalPredecessor)){
    bottleneckActivities.add(mostCriticalPredecessor);
}

return bottleneckActivities;
}

```

getBottleneckActivities():

- **Steps:**
 - Initializing dependencyCount and pathCount: **O(n)** if there are n activities.
 - Counting dependencies: If each activity has m predecessors, the complexity of the loop is **O(n*m)**, where m is the average number of predecessors per activity, in the worst case m equals n so the complexity will be **O(n²)**.
 - Finding critical paths: **O(c*l)**, as explained before, but in the worst case, there will be **O(n)** paths, each of length **O(n)** so the complexity will be **O(n²)**.
 - Finding maximum dependency and path counts: Both uses of Collections.max have a complexity of O(n) so the complexity will be O(2n) which equals **O(n)**.

- Collecting the bottleneck activities: Loops over the activities.values() which is $O(n)$ and performs constant-time checks and conditions for each activity, so this part has a time complexity of **$O(n)$** .
 - Identifying predecessors of the “End” node: Loop over the endPredecessors list, the complexity is $O(k)$, k being the number of predecessors of the node “End”, since k will likely be much smaller than n it can be considered **$O(1)$** .
 - Adding the Most Critical Predecessor: Checking if the mostCriticalPredecessor is already in the bottleneckActivities list and adding it if it's not. This step has a time complexity of **$O(1)$** .
- **Complexity:**
 - The final complexity of the whole method in the worst case scenario will be **$O(n^2)+O(n^2)+O(n)=O(n^2)$** .

USEI24

```

public void simulateDelaysAndRecalculate(LinkedHashMap<String, Integer> delays) {
    ActivitiesMapRepository activitiesMapRepository = Instances.getInstance().getActivitiesMapRepository();
    // Apply delays to specified activities
    for (String actId : delays.keySet()) {
        Activity activity_PERT_CPM = activitiesPERT_CPM.get(actId);
        Activity activity = activities.get(actId);
        if (activity != null) {
            int newDuration = activity.getDuration() + delays.get(actId);
            if (newDuration > 0){
                activity.setDuration(newDuration);
                activity_PERT_CPM.setDuration(newDuration);
            }else if(newDuration < 0){
                throw new IllegalArgumentException("Activity duration must be positive or zero.\n" +
                    "Activity ID: " + actId + "\n" + "Current Duration: " + activity.getDuration() + "\n");
            }
        }
    }

    // Recalculate times
    CalculateTimes calculateTimes = new CalculateTimes();
    calculateTimes.calculateTimes();

    // Recalculate Slack times
    for (Activity activity : activities.values()) {
        activity.calculateSlack();
    }
    activitiesMapRepository.setActivitiesMapRepository(activities);
}

// Method to calculate total project duration

```

```

public String calculateTotalProjectDuration() {
    double maxFinishTime = 0.0;
    String unit = activities.values().iterator().next().getDurationUnit();
    for (Activity activity : activities.values()) {
        if (activity.getEarliestFinish() > maxFinishTime) {
            maxFinishTime = activity.getEarliestFinish();
        }
    }

    if (maxFinishTime > 1 && !unit.endsWith("s")) {
        unit += "s";
    }

    return maxFinishTime + " " + unit;
}

```

simulateDelaysAndRecalculate(LinkedHashMap<String, Integer> delays)

- Steps:
 - Apply Delays to Specified Activities: The method iterates over the keys in the delays map (which has d keys, where d is the number of delayed activities). For each delayed activity, it adjusts the duration. Looping through d activities gives $O(d)$.
 - Recalculate times: As explained before the complexity of calculateTimes() is $O(n^3)$.
 - Recalculate Slack Times: Iterating over all activities to recalculate slack times takes $O(n)$.
- **Complexity:** $O(d)+O(n^3)+O(n)+O(n)$, as $O(n^3)$ dominates over the rest the complexity will be $O(n^3)$

CalculateProjectDuration()

- Iterating over all activities to find the maximum finish time takes $O(n)$