定积分的应用

平面图形的面积

- 与x轴围成的面积$\int_\alpha^\beta ydx$
- 平面图形参数方程
  - $$\begin{cases} x = x(t) \\ y = y(t) \end{cases} \quad \text{除端点外无交点}$$
  - $A = |\int_\alpha^\beta y(t)x'(t)dt| = |\int_\alpha^b etax(t)y'(t)dt|$

空间面积

- 旋转曲面积
  - 关于x轴旋转
    - $dS = 2\pi f(x)\sqrt{1 + [f'(x)]^2}\Delta x$
  - 关于y轴旋转的同理
  - 绕斜轴旋转
    - $S = \frac{1}{\sqrt{1+m^2}} \int_p^q [f(x) - mx - b](1 + mf'(x))dx$
  - 使用极坐标
    - $S = 2\pi \int_\alpha^\beta r(\theta)\sin\theta\sqrt{(r(\theta))^2 + (r'(\theta))^2}d\theta$

曲线的弧长

光滑曲线

$$\begin{cases} x = x(t) \\ y = y(t) \end{cases} \quad \alpha \le t \le \beta, x'(t), y'(t)\text{在}[\alpha, \beta]\text{上连续，}$$

$$\text{并且}(x'(t))^2 + (y'(t))^2 \neq 0$$

计算

平面

$$C = \int_\alpha^\beta \sqrt{(x'(t))^2 + (y'(t))^2}dt$$

空间

$$C = \int_\alpha^\beta \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}dt$$

证明：p389

曲率

定义

角度比弧长

$$\tan\theta = \frac{y'(t)}{x'(t)} \Rightarrow \theta = \arctan\frac{y'(t)}{x'(t)}$$

$$\frac{dC}{dt} = \sqrt{(x'(t))^2 + (y'(t))^2}$$

$$\cdots$$

y=y(x)时：

$$\text{曲率：} k = \left|\frac{y''(x)}{(1+(y'(x))^2)^{\frac{3}{2}}}\right|$$
