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PENGANTAR STATISTIKA

The t Test for Two Independent Samples

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LEARNING OBJECTIVES

- Understand structure of research study appropriate for independent-measures t hypothesis test
- Test difference between two populations or two treatments using independent-measures *t* statistic
- Evaluate magnitude of the observed mean difference (effect size) using Cohen's *d*, r^2, and/or a confidence interval
- Understand how to evaluate the assumptions underlying this test and how to adjust calculations when needed

Introduction to the Independent-Measures Design

- Most research studies compare two (or more) sets of data
  - Data from two completely different, independent participant groups (an *independent-measures* or *between-subjects* design)
  - Data from the same or related participant group(s) (a *within-subjects* or *repeated-measures* design)
- Computational procedures are considerably different for the two designs
- Each design has different strengths and weaknesses
- Consequently, only between-subjects designs are considered in this chapter

Independent- Measures Research Design

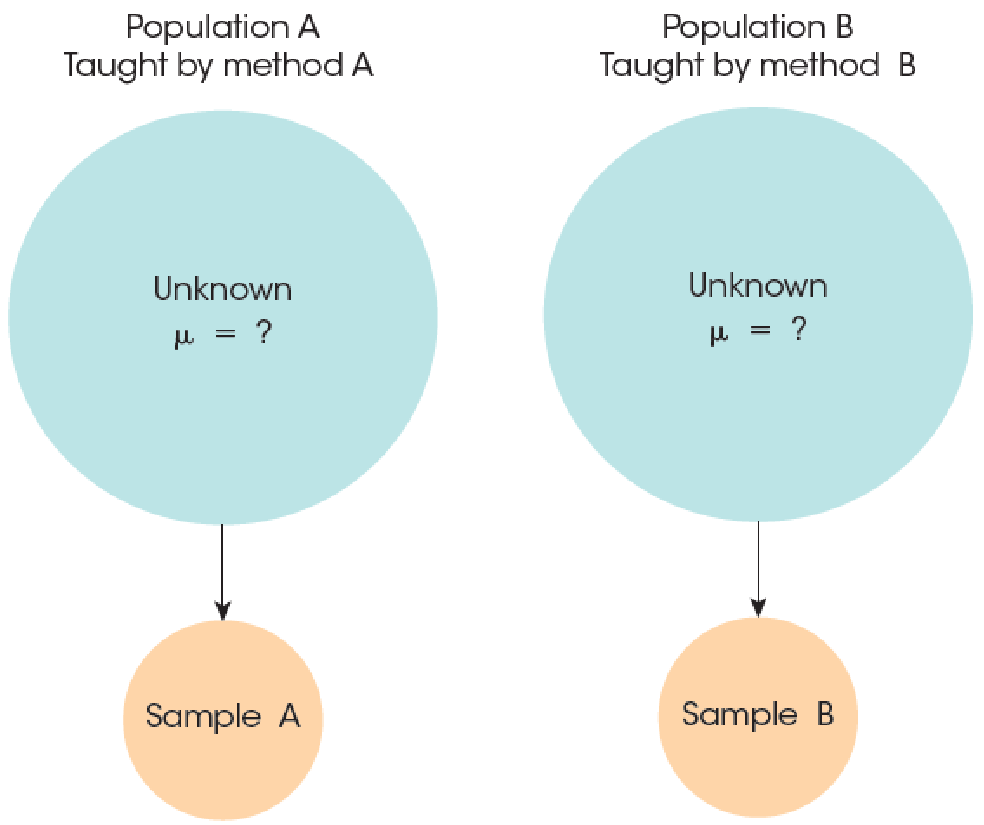


FIGURE 10.1
The structure of an independent-measures research study. Two separate samples are used to obtain information about two unknown populations or treatment conditions.

Independent-Measures Design *t* Statistic

- Null hypothesis for independent-measures test

$$H_0 : \mu_1 - \mu_2 = 0$$

- Alternative hypothesis for the independent- measures test

$$H_1 : \mu_1 - \mu_2 \neq 0$$

Independent-Measures Hypothesis Test Formulas

- Basic structure of the t statistic

$$t = \frac{(M_1 - M_2) - (\mu_1 - \mu_2)}{s_{(M_1 - M_2)}}$$

- $t =$ [(sample statistic) – (hypothesized population parameter)] divided by the estimated standard error
- Independent-measures t test

Estimated standard error

- Measure of standard or average distance between sample statistic (M_1-M_2) and the population parameter
- How much difference it is reasonable to expect between two sample means if the null hypothesis is true

$$s_{(M_1 - M_2)} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

- However, equation above shows standard error concept but is unbiased only if $n_1 = n_2$

Variability of Difference Scores

- What if the value of n_1 is different from n_2?
- Why add sample measurement errors (squared deviations from mean) but subtract sample means to calculate a difference score?

- We introduced the **law of large numbers**, which states that statistics obtained from **large samples tend to be better** (more accurate) estimates of population parameters than statistics obtained from small samples.
- This same fact holds for sample variances: **The variance obtained from a large sample** is a more accurate estimate of σ^2 than the variance obtained from a small sample.
- One method for **correcting the bias in the standard error** is to **combine the two sample variances** into a single value called the ***pooled variance*.**
- The pooled variance is **obtained by averaging or "pooling" the two** sample variances using a procedure that **allows the bigger sample to carry more weight in determining the final value.**

Pooled Variance

- Pooled variance (s_p^2 provides an unbiased basis for calculating the standard error

$$s_p^2 = \frac{SS_1 + SS_2}{df_1 + df_2}$$

Degrees of freedom

- Degrees of freedom (*df*) for *t* statistic is *df* for first sample + *df* for second sample

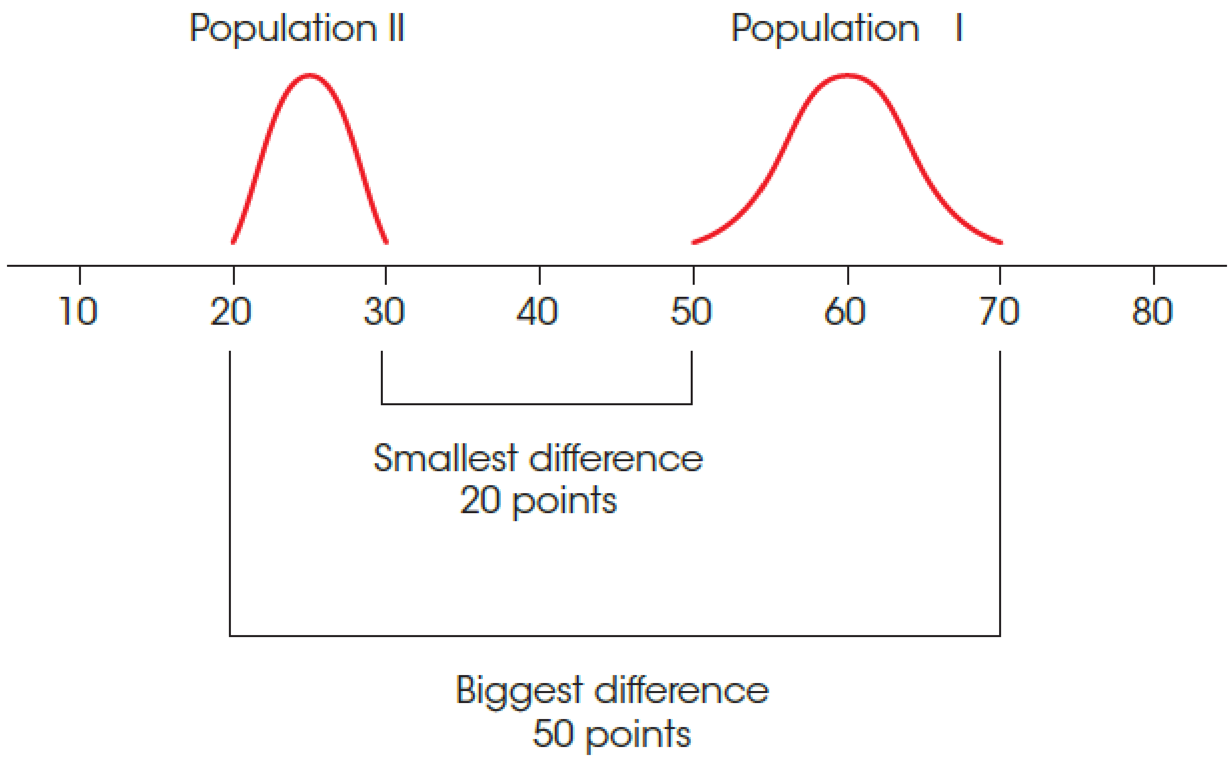
$$df = df_1 + df_2 = (n_1 - 1) + (n_2 - 1)$$

- Note: this term is the same as the denominator of the pooled variance

Two population distributions

FIGURE 10.2

Two population distributions. The scores in Population I range from 50 to 70 (a 20-point spread) and the scores in Population II range from 20 to 30 (a 10-point spread). If you select one score from each of these two populations, the closest two values are $X_1 = 50$ and $X_2 = 30$. The two values that are farthest apart are $X_1 = 70$ and $X_2 = 20$.



Estimated Standard Error for the Difference Between Two Means

- The estimated standard error of $M_1 - M_2$ is then calculated using the pooled variance estimate

$$s_{(M_1 - M_2)} = \sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}$$

Comparison (single sample vs 2 independent samples)

- Compare the development of the formula from previous chapter (chp9) to accommodate two samples:

	Sample Data	Hypothesized Population Parameter	Estimated Standard Error	Sample Variance
Single-sample t statistic	M	μ	$\sqrt{\frac{s^2}{n}}$	$s^2 = \frac{SS}{df}$
Independent-measures t statistic	$(M_1 - M_2)$	$(\mu_1 - \mu_2)$	$\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}$	$s_p^2 = \frac{SS_1 + SS_2}{df_1 + df_2}$

Hypothesis tests on t-statistic 2 independent samples

- Remember the 4-steps
  - Step 1: state hypothesis and select alpha level
  - Step 2: determine critical region
  - Step 3: obtain data and compute t-statistic
  - Step 4: make decision on H_0

Example

- esearch has shown that people are more likely to show dishonest and self-interested behaviors in darkness than in a well-lit environment (Zhong, Bohns, & Gino, 2010).
- In one experiment, participants were given a set of 20 puzzles and were paid $0.50 for each one solved in a 5-minute period. However, the participants reported their own performance and there was no obvious method for checking their honesty. Thus, the task provided a clear opportunity to cheat and receive undeserved money. One group of participants was tested in a room with dimmed lighting and a second group was tested in a well-lit room. The reported number of solved puzzles was recorded for each individual. The following data represent results similar to those obtained in the study.

Number of Solved Puzzles			
Well-Lit Room		Dimly Lit Room	
11	6	7	9
9	7	13	11
4	12	14	15
5	10	16	11
$n = 8$		$n = 8$	
$M = 8$		$M = 12$	
$SS = 60$		$SS = 66$	

Question:
The researcher would like to know whether the students who were tested in a dimly lit room would have higher or lower scores.

- Step 1: state hypothesis and select alpha level

$$H_0: \mu_1 - \mu_2 = 0 \qquad \text{(No difference.)}$$
$$H_1: \mu_1 - \mu_2 \neq 0 \qquad \text{(There is a difference.)}$$

We will set $\alpha = .05$.

Directional hypotheses could be used and would specify whether the students who were tested in a dimly lit room should have higher or lower scores.

- Step 2: determine critical region

$$\begin{aligned} df &= df_1 + df_2 \\ &= (n_1 - 1) + (n_2 - 1) \\ &= 7 + 7 \\ &= 14 \end{aligned}$$

The *t* distribution for *df* = 14 is presented in Figure 10.3. For $\alpha = .05$, the critical region consists of the extreme 5% of the distribution and has boundaries of $t = +2.145$ and $t = -2.145$.

- Step 3: obtain data and compute t-statistic–
  - the calculations be divided into three parts: pool variance s^2_p → estimated std error $s_{(M1-M2)}$ → t-statistic

$$s_p^2 = \frac{SS_1 + SS_2}{df_1 + df_2} = \frac{60 + 66}{7 + 7} = \frac{126}{14} = 9$$

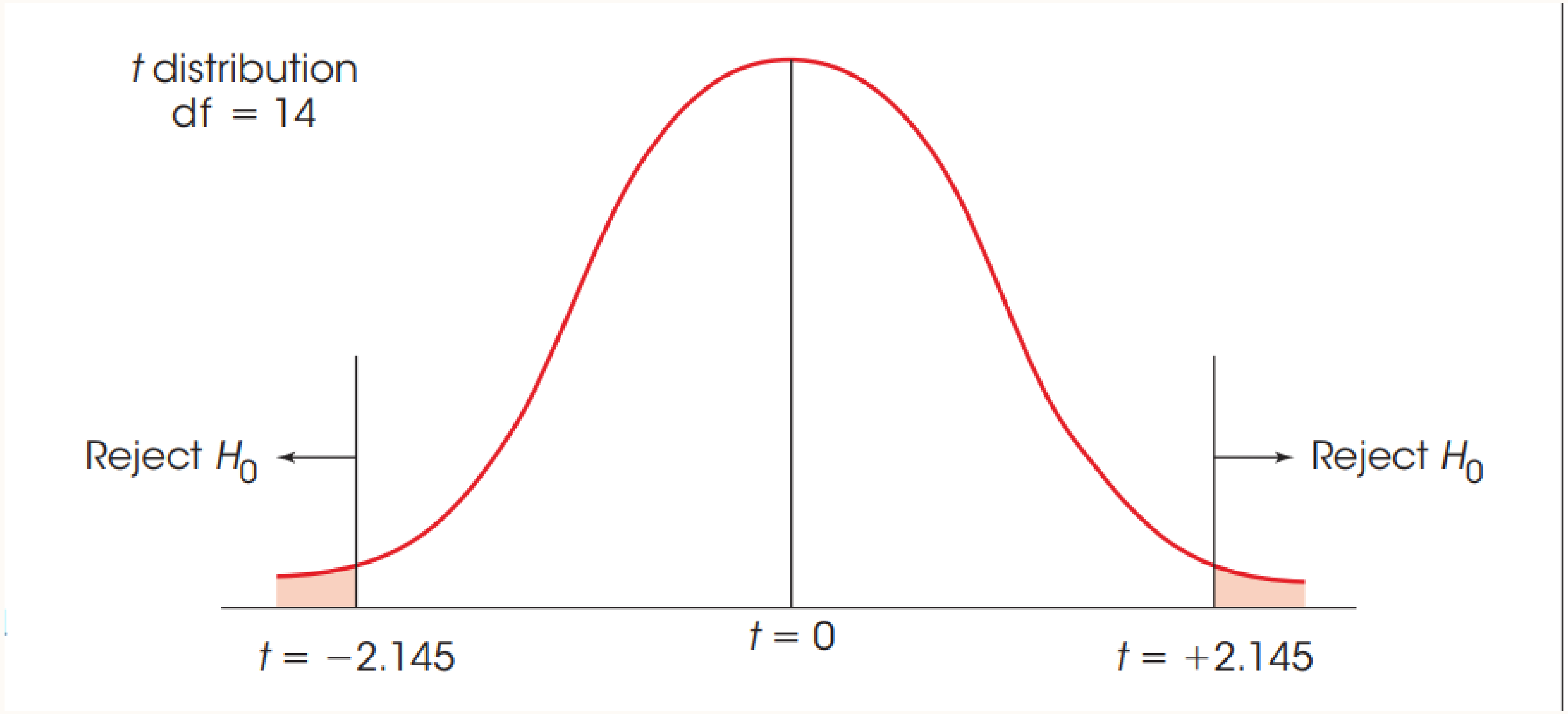
$$s_{(M_1-M_2)} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{9}{8} + \frac{9}{8}} = \sqrt{2.25} = 1.50$$

$$t = \frac{(M_1 - M_2) - (\mu_1 - \mu_2)}{s_{(M_1-M_2)}} = \frac{(8 - 12) - 0}{1.5} = \frac{-4}{1.5} = -2.67$$

- Step 4: make decision on H_0

- the obtained value (t = -2.67) is in the critical region.
- In this example, the obtained sample mean difference is 2.67 times greater than would be expected if there were no difference between the two populations. In other words, this result is **very unlikely if H_0 is true**.
- Therefore, we reject H_0 and **conclude that there is a significant difference between the reported scores in the dimly lit room and the scores in the well-lit room**. Specifically, the students **in the dimly lit room reported significantly higher scores than those in the well-lit room.**

The Critical Region



Learning Check

- Which combination of factors is most likely to produce a significant value for an independent-measures *t* statistic?

A
- a small mean difference and small sample variances

B
- a large mean difference and large sample variances

C
- a small mean difference and large sample variances

D
- a large mean difference and small sample variances

Learning Check - Answer

- Which combination of factors is most likely to produce a significant value for an independent-measures *t* statistic?

A
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C
- a small mean difference and large sample variances

D
- **a large mean difference and small sample variances**

Learning Check

- Decide if each of the following statements is True or False

T/F

- If both samples have n = 10, the independent-measures t statistic will have df = 19

T/F

- For an independent-measures t statistic, the estimated standard error measures how much difference is reasonable to expect between the sample means if there is no treatment effect

Learning Check - Answer

- Decide if each of the following statements is True or False

False

- If both samples have n = 10, the independent-measures t statistic will have df = 19

True

- For an independent-measures t statistic, the estimated standard error measures how much difference is reasonable to expect between the sample means if there is no treatment effect

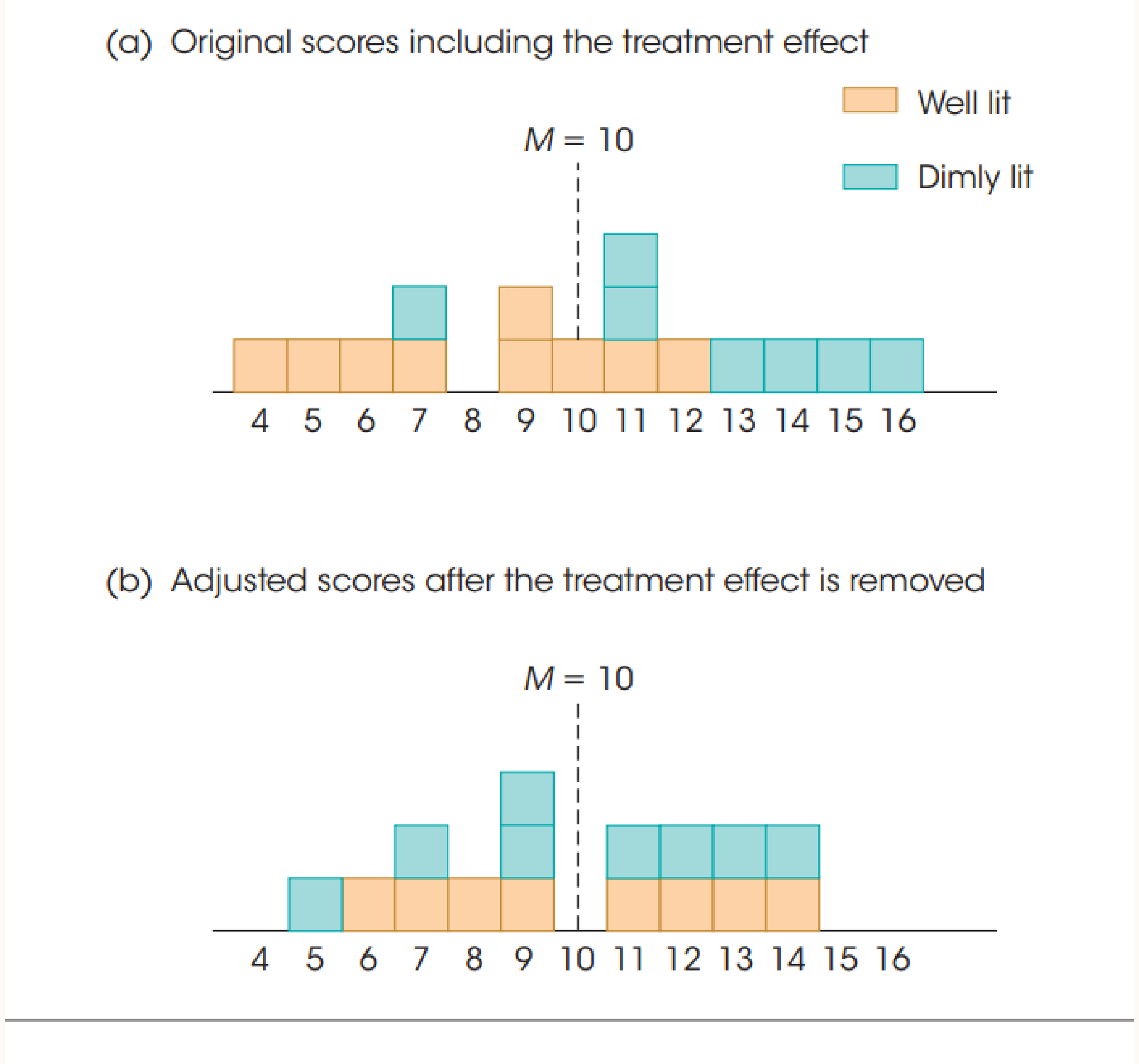
Measuring Effect Size

- If the null hypothesis is rejected, the size of the effect should be determined using *either*
- Cohen's *d*

$$\text{estimated } d = \frac{\text{estimated mean difference}}{\text{estimated standard deviation}} = \frac{M_1 - M_2}{\sqrt{s_p^2}}$$

- *or* Percentage of variance explained $r^2 = \dfrac{t^2}{t^2 + df}$

Two Groups of Scores



Confidence Intervals for Estimating $\mu_1 - \mu_2$

- Difference $M_1 - M_2$ is used to estimate the population mean difference
- t equation is solved for unknown $(\mu_1 - \mu_2)$

$$\mu_1 - \mu_2 = M_1 - M_2 \pm t s_{(M_1 - M_2)}$$

Confidence Intervals and Hypothesis Tests

- Estimation can provide an indication of the size of the treatment effect
- Estimation can provide an indication of the significance of the effect
- If the interval contain zero, then it is not a significant effect
- If the interval does NOT contain zero, the treatment effect was significant

Example

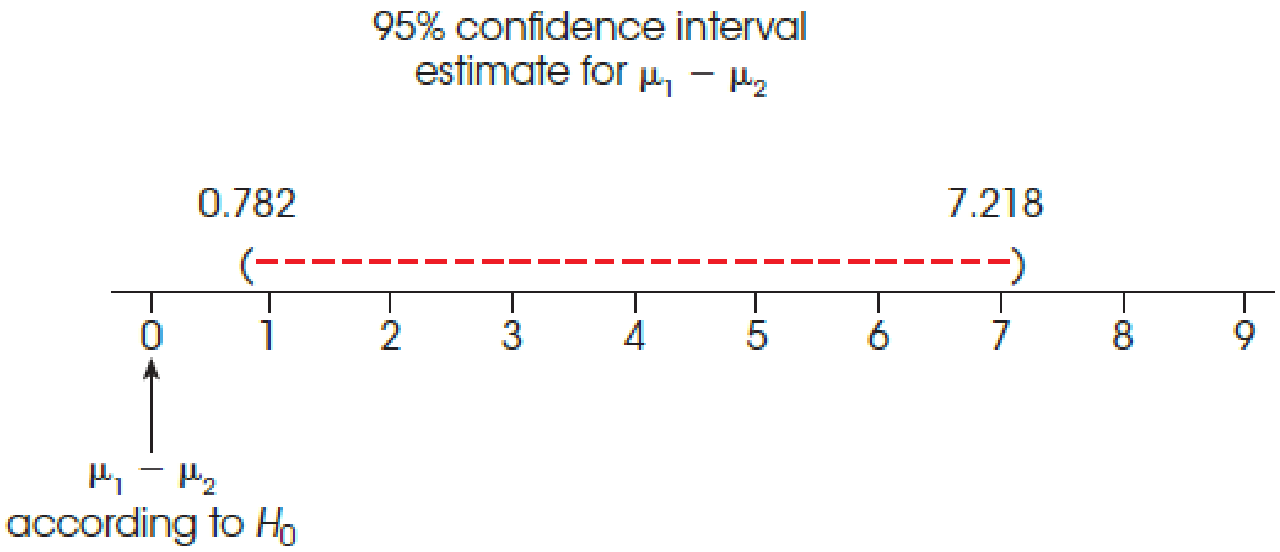
- The data from the study produced a mean grade of $M = 12$ for the group in the dimly lit room and a mean of $M = 8$ for the group in the well-lit room The estimated standard error for
- The mean difference was $s_{M1-M2} = 1.5$. With $n = 8$ scores in each sample, the independentmeasures t statistic has $df = 14$. To have 95% confidence, we simply estimate that the t statistic for the sample mean difference is located somewhere in the middle 95% of all the possible t values. According to the t distribution table, with $df = 14$, 95% of the t values are located between $t = +2.145$ and $t = -2.145$. Using these values in the estimation equation, we obtain

$$
\begin{aligned}
\mu_1 - \mu_2 &= M_1 - M_2 \pm t s_{(M_1 - M_2)} \\
&= 12 - 8 \pm 2.145(1.5) \\
&= 4 \pm 3.218
\end{aligned}
$$

- This produces an interval of values ranging from **4 - 3.218 = 0.782 to 4 + 3.218 = 7.218.**
- Thus, our conclusion is that students who were tested in the dimly lit room had higher scores that those who were tested in a well-lit room, and the mean difference between the two populations is somewhere between 0.782 points and 7.218 points.
- Furthermore, we are 95% confident that the true mean difference is in this interval because the only value estimated during the calculations was the *t* statistic, and we are 95% confident that the *t* value is located in the middle 95% of the distribution. Finally, note that the confidence interval is constructed around the sample mean difference. As a result, the sample mean difference,
- ***M*** **1 -** ***M*****2 = 12 - 8 = 4 points, is located exactly in the center of the interval.**

95% Confidence Interval

FIGURE 10.4
The 95% confidence interval for the population mean difference ($\mu_1 - \mu_2$) from Example 10.6. Note that $\mu_1 - \mu_2 = 0$ is excluded from the confidence interval, indicating that a zero difference is not an acceptable value (H_0 would be rejected in a hypothesis test with $\alpha = .05$).



In the Literature

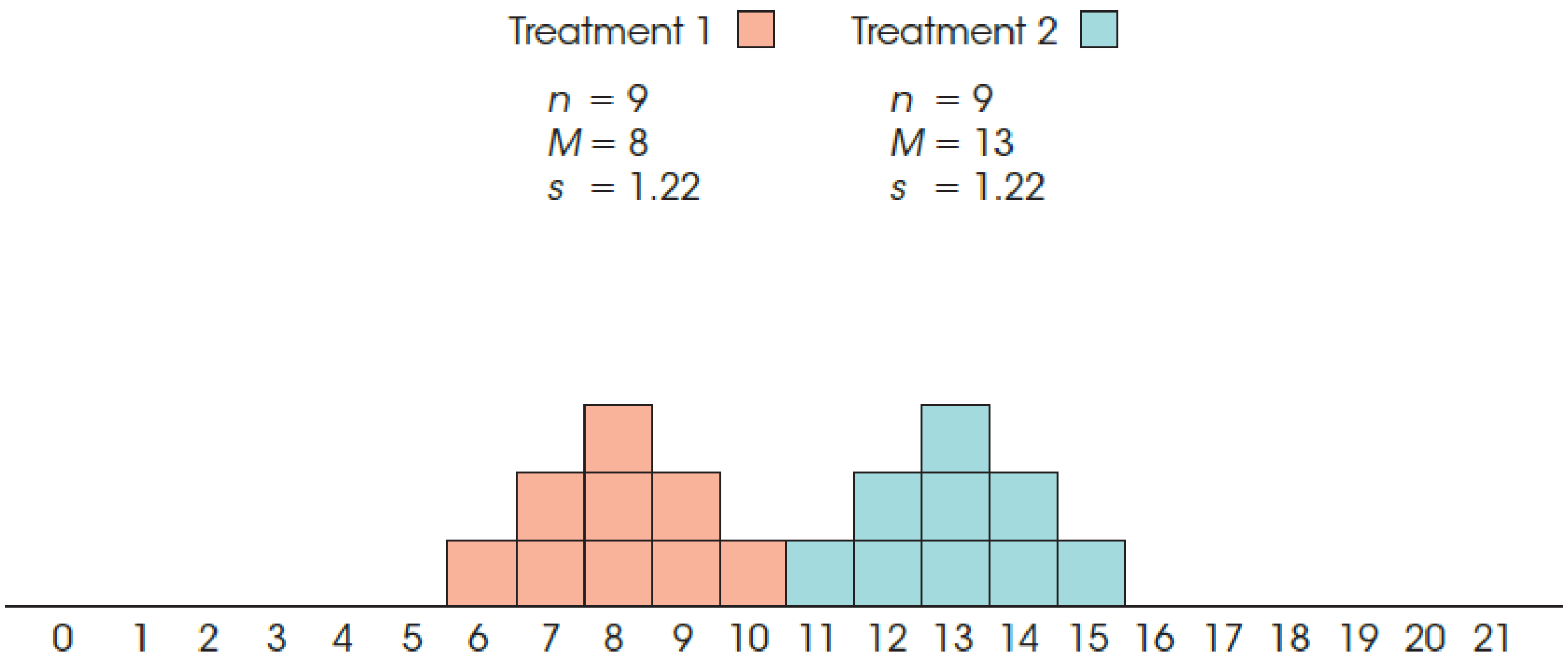
- Report whether the difference between the two groups was significant or not
- Report descriptive statistics (*M* and *SD*) for each group
- Report *t* statistic and *df*
- Report *p*-value
- Report CI immediately after *t*, e.g., 90% CI [0.782, 7.218].

Directional Hypotheses and One-tailed Tests

- Use directional test only when predicting a specific direction of the difference is justified
- Locate critical region in the appropriate tail
- Report use of one-tailed test explicitly in the research report

Two Sample Distributions

FIGURE 10.5
Two sample distributions representing two different treatments. These data show a significant difference between treatments, $t(16) = 8.62$, $p < .01$, and both measures of effect size indicate a very large treatment effect, $d = 4.10$ and $r^2 = 0.82$.



Two Samples from Different Treatment Populations

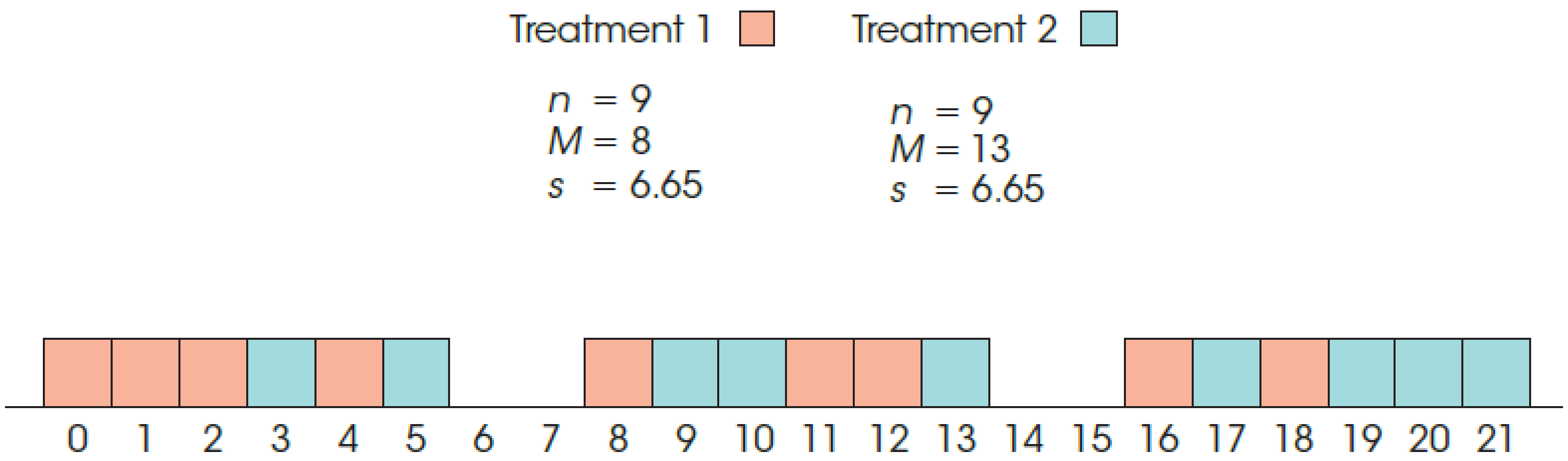


FIGURE 10.6

Two sample distributions representing two different treatments. These data show exactly the same mean difference as the scores in Figure 10.5; however the variance has been greatly increased. With the increased variance, there is no longer a significant difference between treatments, $t(16) = 1.59$, $p > .05$, and both measures of effect size are substantially reduced, $d = 0.75$ and $r^2 = 0.14$.

Assumptions for the Independent-Measures *t*-Test

There are three assumptions that should be satisfied before you use the independent measures *t* formula for hypothesis testing:

1. The **observations within each sample must be independent**
2. The **two populations** from which **the samples are selected must be normal**
3. The two populations from which the samples are selected must have equal variances: ***homogeneity of variance*** (states that the two populations being compared must have the same variance).

Hartley's *F*-max test

- Test for homogeneity of variance

$$F - \max = \frac{s^2(\text{largest})}{s^2(\text{smallest})}$$

  - Large value indicates large difference between sample variance
  - Small value (near 1.00) indicates similar sample variances

Example

- Suppose, for example, that two independent samples each have n = 10 with sample variances of 12.34 and 9.15. For these data,

$$F\text{-max} = \frac{s^2(\text{largest})}{s^2(\text{smallest})} = \frac{12.34}{9.15} = 1.35$$

With a alpha .05, k = 2, and $df = n - 1 = 9$, **the critical value from the table (check the F-max table for k=2, df=9) is 4.03**. Because the obtained F-max **is smaller than this critical value**, you conclude that the **data do not provide evidence that the homogeneity of variance assumption has been violated. In this case, you may proceed with the independent-measures *t* test using pooled variance.**

Pooled Variance Alternative

- If sample information suggests violation of homogeneity of variance assumption:
- Calculate standard error
- Adjust *df* for the *t* test as given:
- **For chapter 12**

$$df = \frac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\dfrac{\left(\dfrac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \dfrac{\left(\dfrac{s_2^2}{n_2}\right)^2}{n_2 - 1}}$$

Learning Check

- For an independent-measures research study, the value of Cohen's *d* or r^2 helps to describe

A
- the risk of a Type I error

B
- the risk of a Type II error

C
- how much difference there is between the two treatments

D
- whether the difference between the two treatments is likely to have occurred by chance

Learning Check - Answer

- For an independent-measures research study, the value of Cohen's *d* or r^2 helps to describe

A	the risk of a Type I error
B	the risk of a Type II error
C	**how much difference there is between the two treatments**
D	whether the difference between the two treatments is likely to have occurred by chance

Learning Check

- Decide if each of the following statements is True or False

T/F

- The homogeneity assumption requires the two sample variances to be equal

T/F

- If a researcher reports that $t(6) = 1.98$, $p > .05$, then H_0 was rejected

Learning Check - Answer

- Decide if each of the following statements is True or False

False

- The homogeneity assumption requires the two sample variances to be equal

False

- If a researcher reports that $t(6) = 1.98$, $p > .05$, then H_0 was rejected

SPSS Output for the Independent-Measures Test

Group Statistics

	VAR00002	N	Mean	Std. Deviation	Std. Error Mean
VAR00001	1.00	10	93.0000	4.71405	1.49071
	2.00	10	85.0000	4.21637	1.33333

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
VAR00001	Equal variances assumed	.384	.543	4.000	18
	Equal variances not assumed			4.000	17.780

Independent Samples Test

		t-test for Equality of Means				
					95% Confidence Interval of the Difference	
		Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
VAR00001	Equal variances assumed	.001	8.00000	2.00000	3.79816	12.20184
	Equal variances not assumed	.001	8.00000	2.00000	3.79443	12.20557

THANK YOU