$X \sim Beta(1,b)$

$$f(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1} (1-x)^{b-1}$$

$Beta(1,b)$

$$\int_0^x \frac{\Gamma(1+b)}{\Gamma(b)} (1-t)^{b-1} dt$$

$$\frac{\Gamma(1+b)}{\Gamma(b)} \int_0^x (1-t)^{b-1} dt \quad \rightarrow \quad u = 1-t$$

$$\frac{du}{dt} = -1$$

$$dt = -du$$

$$\int_0^x u^{b-1} - du = -\frac{u^b}{b}\Big|_0^x = -\frac{1}{b}\left((1-x)^b - (1-0)^b\right)$$

$$= \frac{1}{b}\left(1-(1-x)^b\right)$$

$$F(x|a,b) = \frac{\Gamma(1+b)}{b\Gamma(b)}\left(1-(1-x)^b\right)$$

$$F(x|a,b) = u$$

$$\frac{\Gamma(1+b)}{b\Gamma(b)}\left(1-(1-x)^b\right) = u \rightarrow 1-(1-x)^b = \frac{u\Gamma(b)b}{\Gamma(b+1)}$$

$$-1+(1-x)^b = \frac{-u\Gamma(b)b}{\Gamma(b+1)} \rightarrow (1-x)^b = \frac{-u\Gamma(b)b}{\Gamma(b+1)} + 1$$

$$b\ln(1-x) = \ln\left(-\frac{u\Gamma(b)b}{\Gamma(b+1)} + 1\right)$$

$$\ln(1-x) = \ln\left(-\frac{u\Gamma(b)b}{\Gamma(b+1)} + 1\right)/b \qquad 1-x = e^{\ln\left(-\frac{u\Gamma(b)b}{\Gamma(b+1)} + 1\right)/b}$$

$$-1+x = -e^{\ln\left(-\frac{u\Gamma(b)b}{\Gamma(b+1)} + 1\right)/b} \qquad x = 1 - e^{\ln\left(-\frac{u\Gamma(b)b}{\Gamma(b+1)} + 1\right)/b}$$

$$1-(1-x)^b = \frac{\Gamma'(b)}{\Gamma'(b+1)}\, b.u \rightarrow \quad (1-x)^b = -\frac{\Gamma'(b)}{\Gamma'(b+1)}\, b.u + 1$$

$$b\ln(1-x) = \ln\left(-\frac{\Gamma'(b)}{\Gamma'(b+1)}\, b.u + 1\right)$$

$$1-x = e^{\ln\left(-\frac{\Gamma'(b)}{\Gamma'(b+1)} b.u+1\right)/b}$$

$$-1+x = -e^{\ln\left(-\frac{\Gamma'(b)}{\Gamma'(b+1)} b.u+1\right)/b}$$

$$x = -e^{\ln\left(-\frac{\Gamma'(b)}{\Gamma'(b+1)} b.u+1\right)/b} + 1$$

Função de distribuição weibull $\quad X \sim Weibull(\alpha, \beta)$

$$F(x) = 1 - \exp\left\{-\left(\frac{x}{\beta}\right)^{\alpha}\right\}$$

$x > 0$

$\alpha \rightarrow$ forma $\quad \alpha > 0.$

$\beta \rightarrow$ escala $\quad \beta > 0.$

$$1-\exp\left\{-\left(\frac{x}{\beta}\right)^{\alpha}\right\} = u$$

$$-\exp\left\{-\left(\frac{x}{\beta}\right)^{\alpha}\right\} = u-1 \rightarrow \quad \exp\left\{-\left(\frac{x}{\beta}\right)^{\alpha}\right\} = 1-u$$

$$-\left(\frac{x}{\beta}\right)^{\alpha} = \ln(1-u) \rightarrow \quad x^{\alpha} = \frac{-\ln(1-u)}{\beta^{\alpha}}$$

$$\alpha\ln(x) = \ln\left(\frac{-\ln(1-u)}{\beta^{\alpha}}\right) \rightarrow \ln(x) = \frac{1}{\alpha}\ln\left(\frac{-\ln(1-u)}{\beta^{\alpha}}\right)$$

$$x = e^{\frac{1}{\alpha}\ln\left(\frac{-\ln(1-u)}{\beta^{\alpha}}\right)}$$

Função de distribuição Logística

$X \sim Log(\mu, \sigma)$

$$f(x) = \frac{1}{\sigma} \cdot \frac{e^{\frac{(x-\mu)}{\sigma}}}{(1+e^{(x-\mu)/\sigma})^2}$$

$$F(x) = \frac{1}{1+e^{-(x-\mu)/\sigma}}$$

$\mu \in \mathbb{R}$ locação
$\sigma > 0$ Escala

$$u = \frac{1}{1+e^{-(x-\mu)/\sigma}} \rightarrow 1+e^{-(x-\mu)/\sigma} = \frac{1}{u}$$

$$e^{-(x-\mu)/\sigma} = \frac{1}{u} - 1 \rightarrow -(x-\mu)/\sigma = \ln\left(\frac{1}{u}-1\right)$$

$$-x+\mu = \sigma \ln\left(\frac{1}{u}-1\right) \rightarrow -x = \sigma \ln\left(\frac{1}{u}-1\right) - \mu$$

$$x = \mu - \sigma \ln\left(\frac{1}{u}-1\right)$$

Cauchy não-Central $X \sim Cauchy(\theta, \lambda)$

$$f(x|\theta,\lambda) = \frac{1}{\pi\lambda}\left(1+\left(\frac{x-\theta}{\lambda}\right)^2\right)^{-1}$$

$\theta \in \mathbb{R}$.
$\lambda > 0$.

$$F(x|\theta,\lambda) = \frac{1}{\pi\lambda}\int_0^x \frac{1}{1+\left(\frac{t-\theta}{\lambda}\right)^2}\,dt$$

$$\frac{1}{1+\left(\frac{t-\theta}{\lambda}\right)^2} = \frac{1}{1+\frac{(t+\theta)^2}{\lambda^2}} = \frac{1}{\frac{\lambda^2+(t+\theta)^2}{\lambda^2}} = \frac{\lambda^2}{\lambda^2+(t+\theta)^2}$$

$$F(x|\theta,\lambda) = \frac{1}{2} + \frac{1}{\pi}\arctan\left(\frac{x-\theta}{\lambda}\right)$$

$$u = \frac{1}{2} + \frac{1}{\pi}\arctan\left(\frac{x-\theta}{\lambda}\right)$$

$$\left(u-\frac{1}{2}\right)\pi = \arctan\left(\frac{x-\theta}{\lambda}\right)$$

$$\tan\left[\left(u-\frac{1}{2}\right)\pi\right] = \frac{x-\theta}{\lambda}$$

$$x = \lambda\tan\left[\left(u-\frac{1}{2}\right)\pi\right] + \theta$$

$\mathrm{Beta}(a,1)$ $\qquad$ $x \sim \mathrm{Beta}(a,1)$

$$f(x|a,b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1} \qquad \begin{matrix} 0<x<1 \\ a,b>0 \end{matrix}$$

$$f(x|a,b) = \frac{\Gamma(a+1)}{\Gamma(a)} x^{a-1}$$

$$F(x|a,b) = \int_0^x \frac{\Gamma(a+1)}{\Gamma(a)} t^{a-1}\,dt = \frac{\Gamma(a+1)}{\Gamma(a)}\int_0^x t^{a-1}\,dt$$

$$= \frac{\Gamma(a+1)}{\Gamma(a)} \left.\frac{t^a}{a}\right|_0^x = \frac{\Gamma(a+1)}{a\,\Gamma(a)}(x^a - 0^a)$$

$$= \frac{\Gamma(a+1)}{a\,\Gamma(a)} x^a, \quad 0<x<1.$$